6.034 Exam 3 Cheat Sheet

Neural Nets

Useful Information

1) Neural Nets are numerical classifiers with binary (`0/1`) output
2) The neuron is a primitive circuit element
3) Forward propagation computes the overall output of a neural net

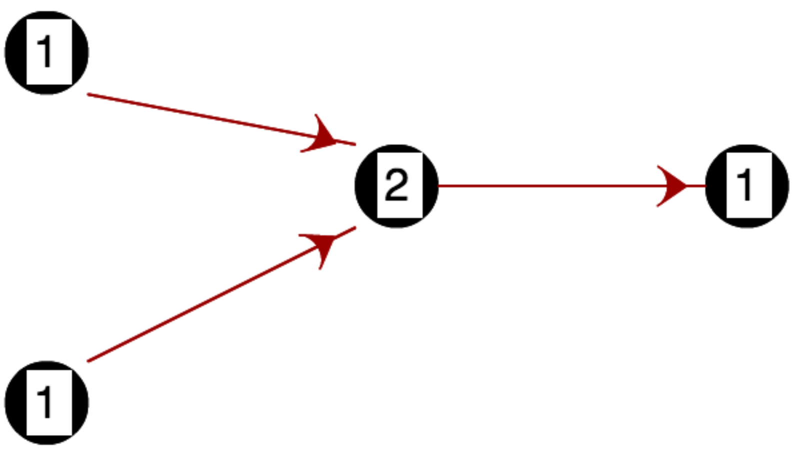
(Input Layer) -> (Logic Function Layers) -> Output (`0/1`)

A single neuron can draw one line and shade above or below it

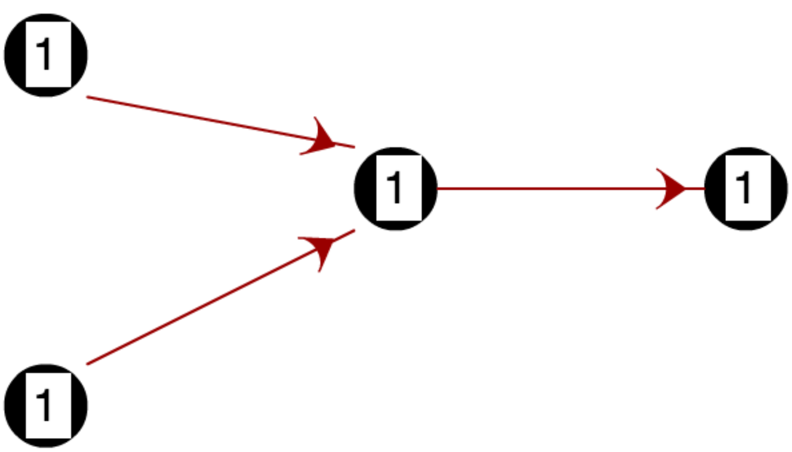
Primitive Logic Functions Computable by a Single Neuron

Note: used in the **logic** layer

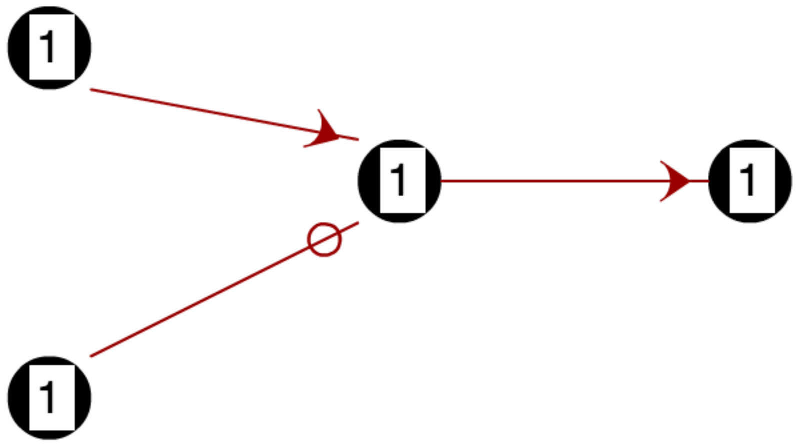
- `AND(x,y)`



- `OR(x,y)`

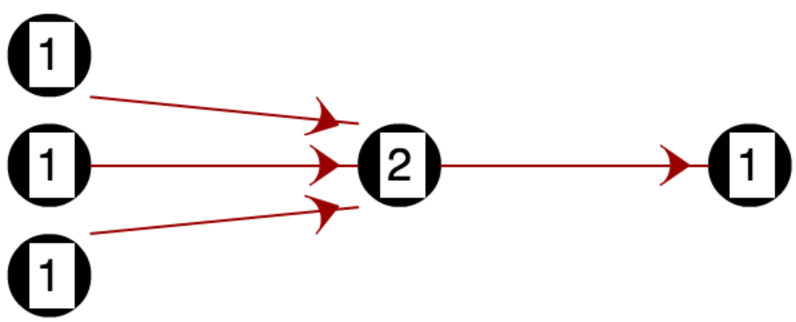


- `NOT(x, _)`



* note, the circle on the line means that the weight is `-1`)

- "`MAJORITY(x1, x2, x3, x4, ...)`" (3 input example)



- note, doubling the weight of the bottom input (`x3` for instance) makes this gate act like `OR(AND(x1, x2), x3)`

Helper Functions

$Stairstep\,T\,(x) =$

$$\begin{cases} 1 & \text{if } x \geq T \\ 0 & \text{if } x < T \end{cases}$$

$Sigmoid\,S,\ M\,(x) =$

$$\frac{1}{1+e^{-S(x-m)}}$$

$Performance\ =\ Accuracy(out*, out)\ =\ \frac{1}{2}(out * - out)^2$

- * means **desired** output

Quick Formulas For Backward Propagation

$$W'_{A\to B} = W_{A\to B} + \Delta W_{A\to B}$$
$$\Delta W_{A\to B} = r \cdot out_A \cdot \delta_B$$

$$\delta_B = \begin{cases} out_B(1-out_B)(out* - out) & \text{if neuron B is in final (output) layer} \\ out_B(1-out_B)\sum_{outgoingC_i} W_{B\to C_i}\delta_{C_i} & \text{if neuron B is not in final (output) layer} \end{cases}$$

Backwards Propagation Steps

1. Computing output of each neuron using forward Propagation and $Stairstep_T$ function
2. Compute δ_B for final layer
3. Compute δ_B for earlier layers
4. Compute updates for weights
5. Update all weights

Miscellaneous Notes

- You can never classify all points correctly if you have a + data point and a - data point (contradictory) right on top of each other

Overfitting - too strict with regards to the data it's trying to model

Underfitting - too simple with regards to the data it's trying to model

Support Vector Machines

Useful Information

- like Neural Nets, classifies numerical data into two classes: + and -
- draws the decision boundary line that separates the training data with the widest possible margin

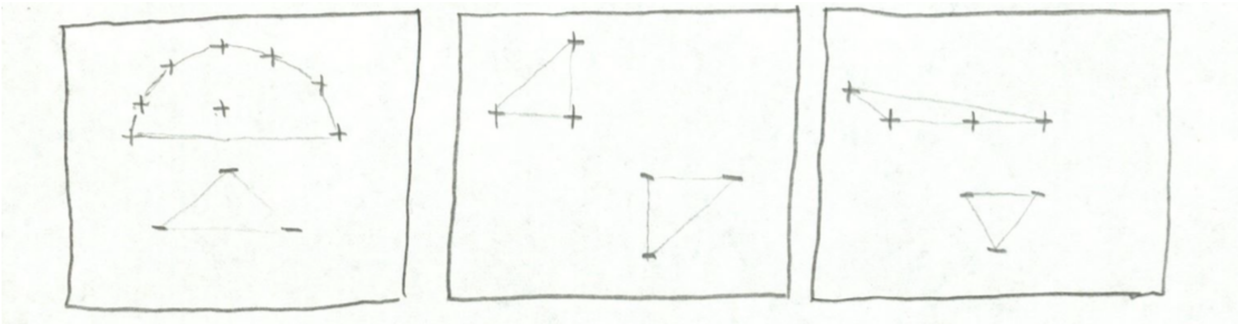
Boundaries

- 1-D - just a point
- 2-D - some sort of line or curve
- 3-D - some sort of plane

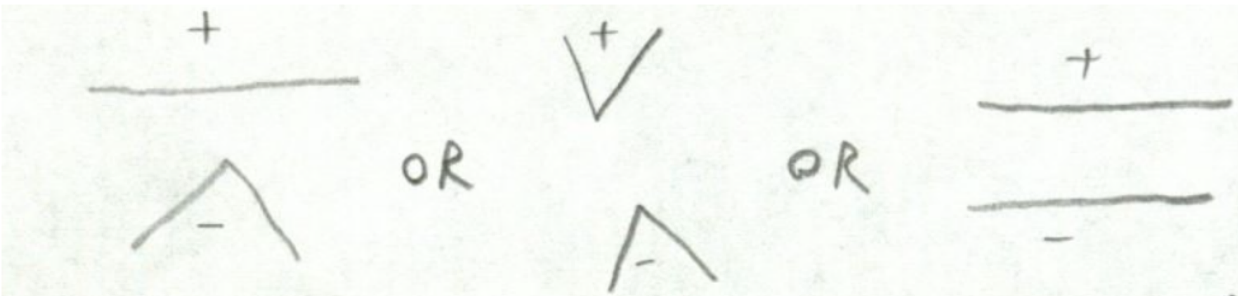
How to Draw SVM Boundaries (2D)

1. Draw the *convex hulls* for the + and - training points.

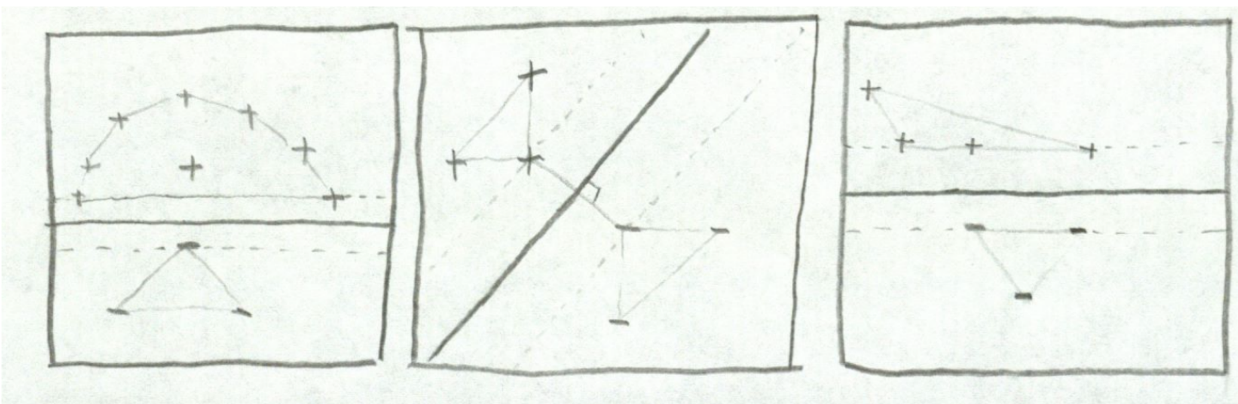
- a convex hull is the shape you get when you wrap a rubber band around the points and let it contract



2. Look at the regions where the convex hulls are closest.
3 Cases:

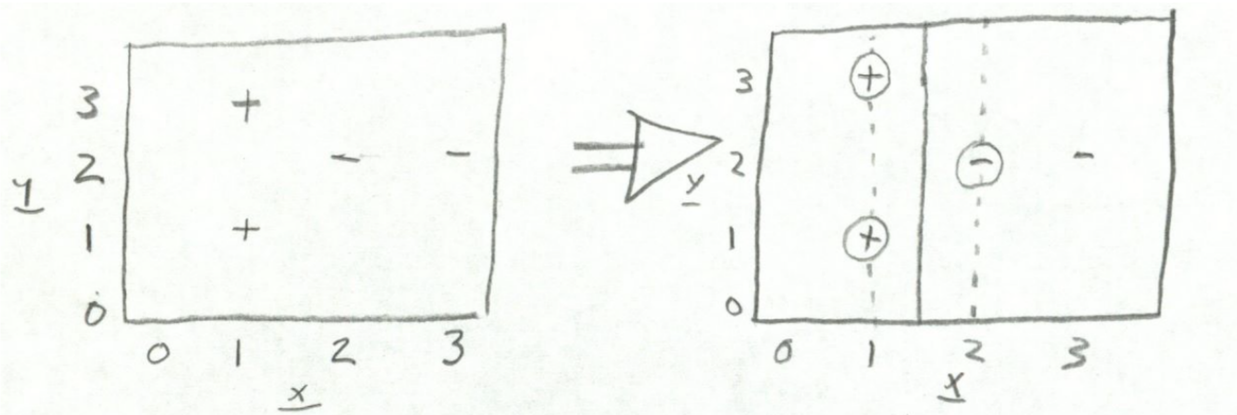


3. The corresponding boundaries look like this:



How to find the equation of the boundary line

1. Find the boundary line:



2. Write equation the for the line:

In the above example:

$$x = 1.5$$

3. Re-write the line equation to the form: $\vec{w} \cdot \vec{x} + b \Rightarrow \vec{w} \cdot \begin{pmatrix} x \\ y \end{pmatrix} + b = 0$

In the above example:

$$x = 1.5 \Rightarrow 0y + 1x - 1.5 = 0 \Rightarrow \underbrace{\begin{pmatrix} 1 & 0 \end{pmatrix}}_{\vec{w}} \cdot \underbrace{\begin{pmatrix} x \\ y \end{pmatrix}}_{\vec{x}} + \underbrace{(-1.5)}_{b} = 0$$

4. So, our decision boundary needs to follow a few conventions:

- `positiveness(x)` = output of $\vec{w} \cdot \vec{x} + b$
- `positiveness` for `+` support vectors: $+1$
- `positiveness` for `-` support vectors: -1
- `positiveness` for `+` training points: ≥ 1
- `positiveness` for `-` training points: ≤ -1

So we need to scale $\vec{w} \cdot \vec{x} + b$ so that it actually follows these conventions.

We can do this by taking one of the support vectors and scaling the equation so that `positiveness` outputs the correct thing ($+1$ or -1 appropriately).

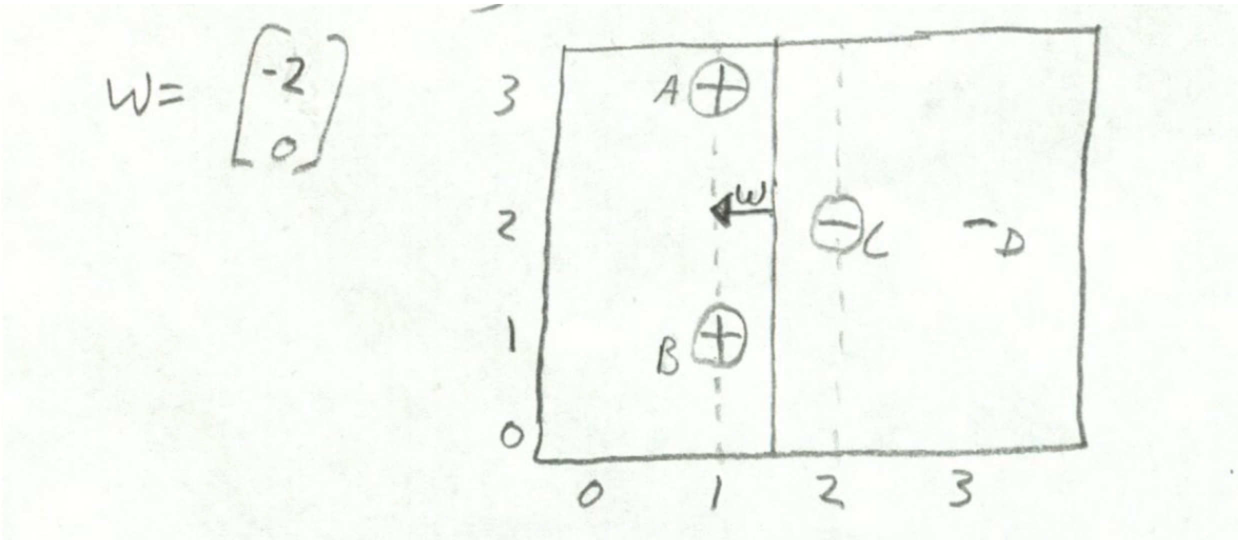
Using the `-` support vector $(2, 2)$:

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 2 \end{pmatrix} + (-1.5) \Rightarrow 2 - 1.5 = .5$$

`postiveness(2,2)` should be -1, so we need to multiply the whole equation by -2.

$$\underbrace{\begin{pmatrix}-2 & 0\end{pmatrix}}_{\vec{w}} \cdot \underbrace{\begin{pmatrix}x \\ y\end{pmatrix}}_{\vec{x}} + \underbrace{(3)}_{b} = 0$$

Note that $\vec{w}$ should be $\perp$ to the decision boundary, and also pointing towards the positive points.



How to find the α values for the training points

- $\alpha = 0$ for non-support vectors
- $\sum_{\vec{p} \in \text{"+" points}} \alpha_p = \sum_{\vec{p} \in \text{"-" points}} \alpha_p$
- solve $\vec{w} = \sum_{\vec{p} \in \text{"+" points}} \alpha_p \vec{p} - \sum_{\vec{p} \in \text{"-" points}} \alpha_p \vec{p}$

In this case (from the above example):

- $\alpha_d = 0$, since it isn't a support vector
- $\underbrace{\alpha_a + \alpha_b}_{\text{"+" points}} = \underbrace{\alpha_c}_{\text{"-"points}}$
- $\underbrace{\begin{pmatrix}-2 \\ 0\end{pmatrix}}_{\vec{w}} = \alpha_a \begin{pmatrix}1 \\ 3\end{pmatrix} + \alpha_b \begin{pmatrix}1 \\ 1\end{pmatrix} - \alpha_c \begin{pmatrix}2 \\ 2\end{pmatrix}$
- solving these gives you $\alpha_c = 2$ and $\alpha_a = \alpha_b = 1$