6.034 Exam 3 Cheat Sheet

Neural Nets

Useful Information

1) Neural Nets are numerical classifiers with binary (`0/1`) output
2) The neuron is a primitive circuit element
3) Forward propagation computes the overall output of a neural net

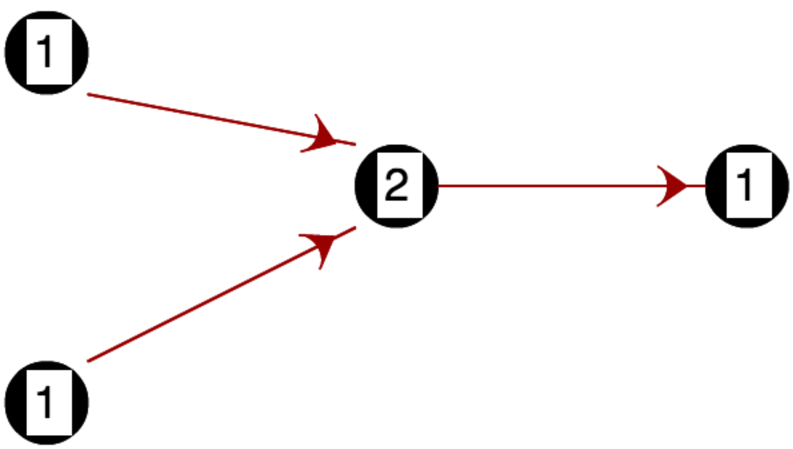
(Input Layer) -> (Logic Function Layers) -> Output (`0/1`)

A single neuron can draw one line and shade above or below it

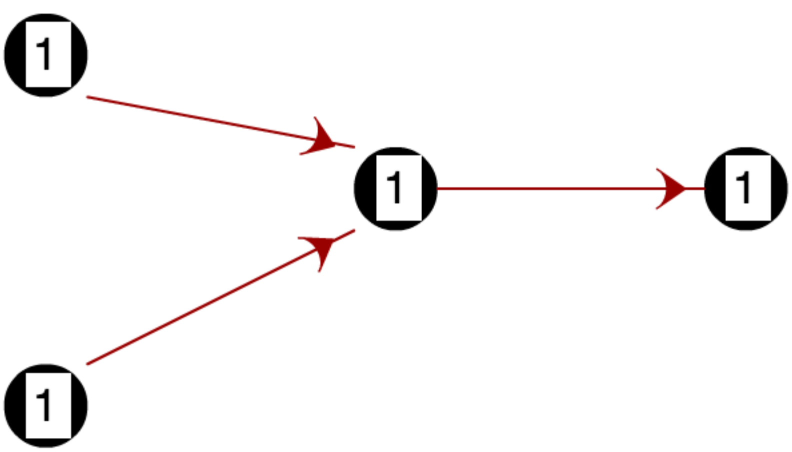
Primitive Logic Functions

Computable by a Single Neuron Note: used in the **logic** layer

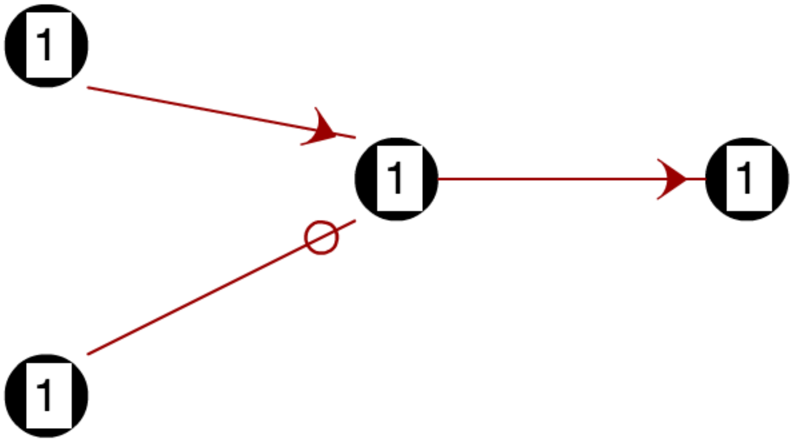
- `AND(x,y)`



- `OR(x,y)`

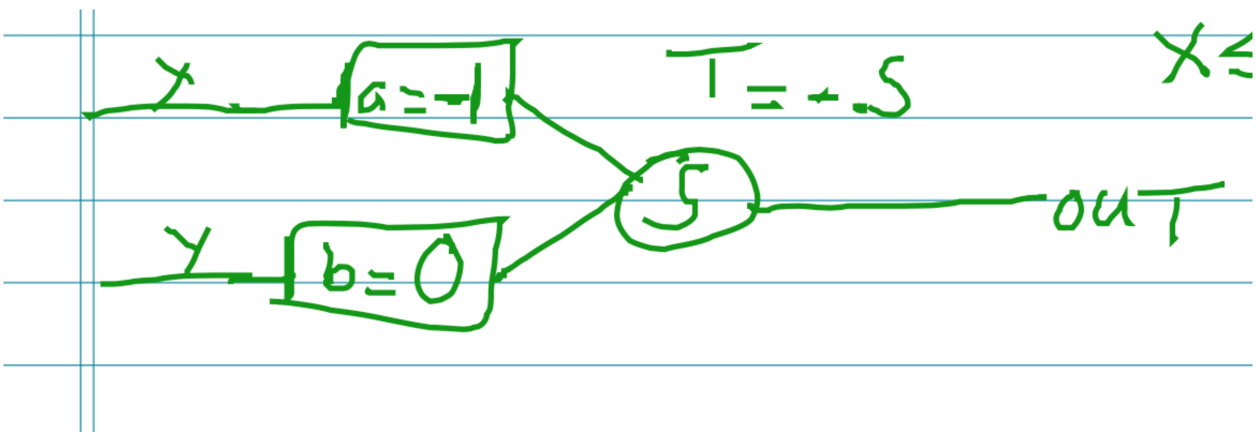


- `AND(x, NOT(y))`

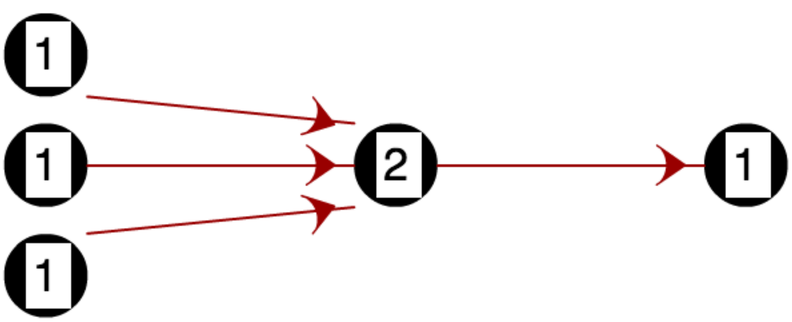


* note, the circle on the line means that the weight is `-1`)

- `NOT(x)`



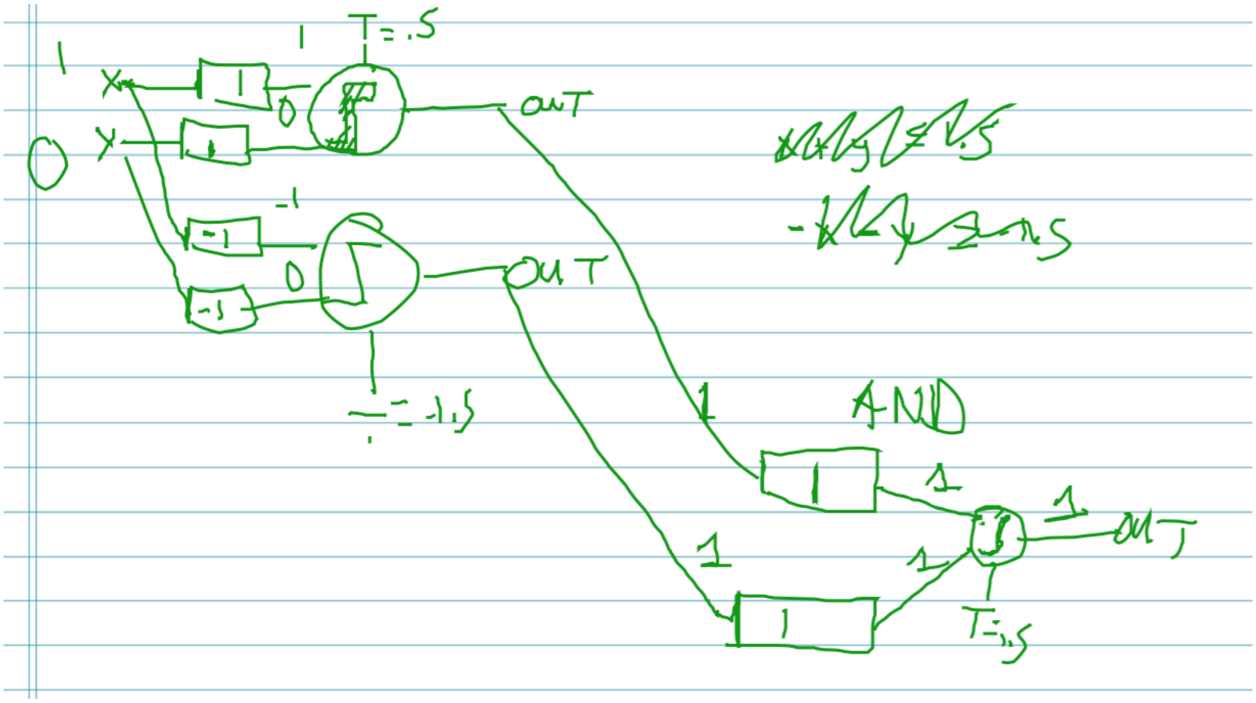
- “`MAJORITY(x1, x2, x3, x4, ...)`” (3 input example)



- note, doubling the weight of the bottom input (`x3` for instance) makes this gate act like `OR(AND(x1, x2), x3)`

Need More than One Neuron

- `XOR(x, y)`(3 neurons to compute)



The T at the end is $+1.5$

Helper Functions

$$\text{Stairstep}_T(x) = \begin{cases} 1 & \text{if } x \geq T \\ 0 & \text{if } x < T \end{cases}$$

$$\text{Sigmoid}_{S,M}(x) = \frac{1}{1 + e^{-S(x-m)}}$$

$$\text{Performance} = \text{Accuracy}(out*, out) = \frac{1}{2}(out * - out)^2$$

- * means **desired** output

Quick Formulas For Backward Propagation

$$W'_{A \to B} = W_{A \to B} + \Delta W_{A \to B}$$
$$\Delta W_{A \to B} = r \cdot out_A \cdot \delta_B$$

$$\delta_B = \begin{cases} out_B(1 - out_B)(out * - out) & \text{if neuron B is in final (output) layer} \\ out_B(1 - out_B) \sum_{outgoingC_i} W_{B \to C_i} \delta_{C_i} & \text{if neuron B is not in final (output) layer} \end{cases}$$

Backwards Propagation Steps

1. Computing output of each neuron using forward Propagation and $Stairstep_T$ function
2. Compute δ_B for final layer
3. Compute δ_B for earlier layers
4. Compute updates for weights
5. Update all weights

Miscellaneous Notes

- You can never classify all points correctly if you have a + data point and a − data point (contradictory) right on top of each other

Overfitting - too strict with regards to the data it's trying to model

Underfitting - too simple with regards to the data it's trying to model

Support Vector Machines

Useful Information

- like Neural Nets, classifies numerical data into two classes: + and −
- draws the decision boundary line that separates the training data with the widest possible margin

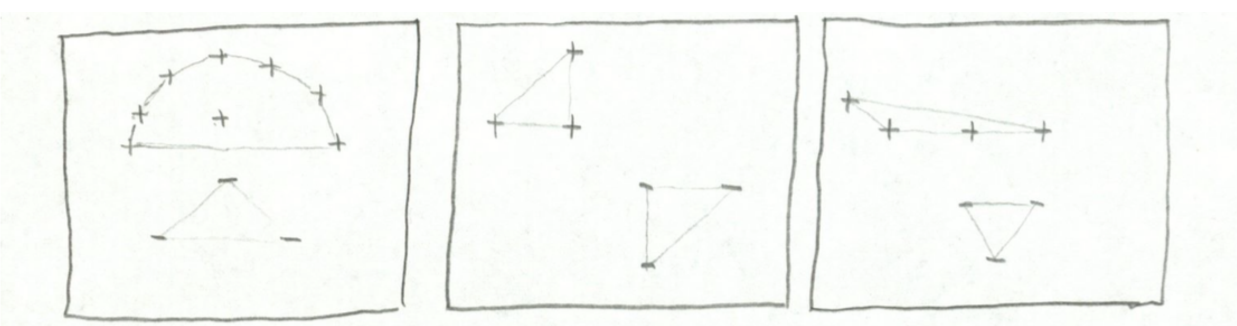
Boundaries

- 1-D - just a point
- 2-D - some sort of line or curve
- 3-D - some sort of plane

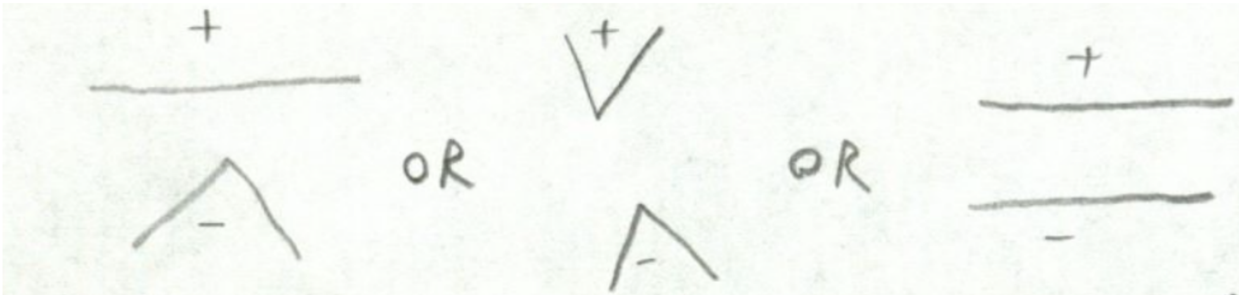
How to Draw SVM Boundaries (2D)

1. Draw the *convex hulls* for the + and − training points.

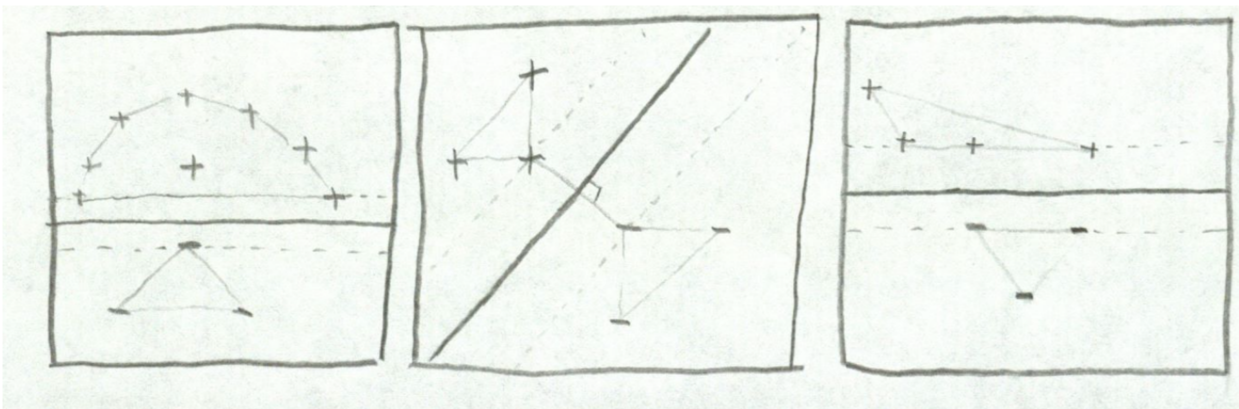
- a convex hull is the shape you get when you wrap a rubber band around the points and let it contract



2. Look at the regions where the convex hulls are closest.
3 Cases:

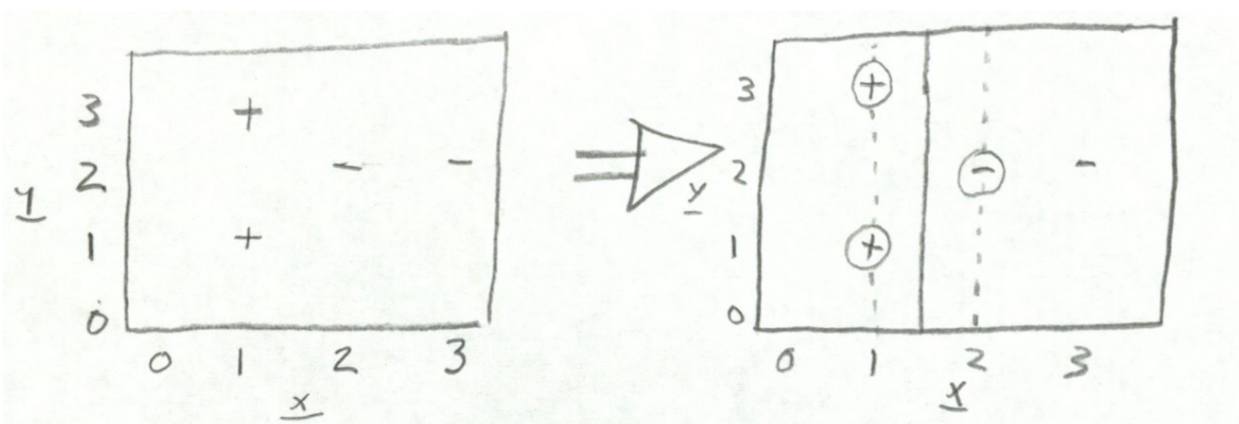


3. The corresponding boundaries look like this:



How to find the equation of the boundary line

1. Find the boundary line:



2. Write equation the for the line:

In the above example:

$$x = 1.5$$

3. Re-write the line equation to the form: $\vec{w} \cdot \vec{x} + b \Rightarrow \vec{w} \cdot \begin{pmatrix} x \\ y \end{pmatrix} + b = 0$

In the above example:

$$x = 1.5 \Rightarrow 0y + 1x - 1.5 = 0 \Rightarrow \underbrace{\begin{pmatrix}1 & 0\end{pmatrix}}_{\vec{w}} \cdot \underbrace{\begin{pmatrix}x \\ y\end{pmatrix}}_{\vec{x}} + \underbrace{(-1.5)}_{b} = 0$$

4. So, our decision boundary needs to follow a few conventions:

- `positiveness(x)` = output of $\vec{w} \cdot \vec{x} + b$
- `positiveness` for `+` support vectors: $+1$
- `positiveness` for `-` support vectors: -1
- `positiveness` for `+` training points: ≥ 1
- `positiveness` for `-` training points: ≤ -1

So we need to scale $\vec{w} \cdot \vec{x} + b$ so that it actually follows these conventions.

We can do this by taking one of the support vectors and scaling the equation so that `positiveness` outputs the correct thing ($+1$ or -1 appropriately).

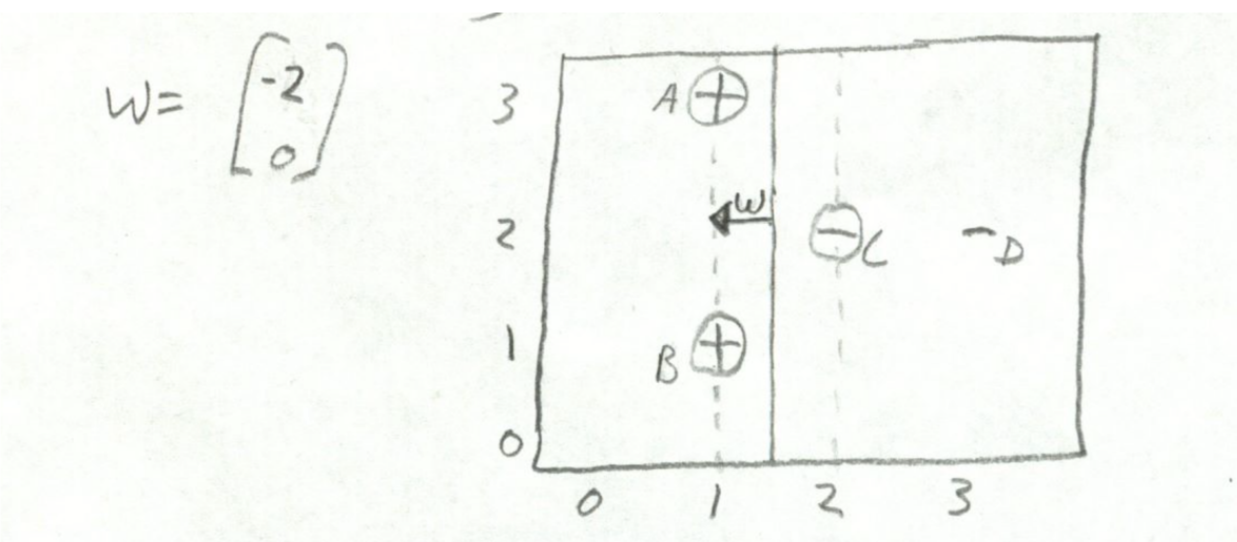
Using the `-` support vector $(2, 2)$:

$$\begin{pmatrix}1 & 0\end{pmatrix} \cdot \begin{pmatrix}2 \\ 2\end{pmatrix} + (-1.5) \Rightarrow 2 - 1.5 = .5$$

`postiveness(2,2)` should be -1, so we need to multiply the whole equation by -2.

$$\underbrace{\begin{pmatrix}-2 & 0\end{pmatrix}}_{\vec{w}} \cdot \underbrace{\begin{pmatrix}x \\ y\end{pmatrix}}_{\vec{x}} + \underbrace{(3)}_{b} = 0$$

Note that $\vec{w}$ should be $\perp$ to the decision boundary, and also pointing towards the positive points.



How to find the α values for the training points

The α of a training point refers to how important it is when determining the boundary line.

- $\alpha = 0$ for non-support vectors
- $\sum_{\vec{p} \in \text{"+" points}} \alpha_p = \sum_{\vec{p} \in \text{"-" points}} \alpha_p$
- solve $\vec{w} = \sum_{\vec{p} \in \text{"+" points}} \alpha_p \vec{p} - \sum_{\vec{p} \in \text{"-" points}} \alpha_p \vec{p}$

In this case (from the above example):

- $\alpha_d = 0$, since it isn't a support vector
- $\underbrace{\alpha_a + \alpha_b}_{\text{"+" points}} = \underbrace{\alpha_c}_{\text{"-"points}}$
- $\underbrace{\begin{pmatrix} -2 \\ 0 \end{pmatrix}}_{\vec{w}} = \alpha_a \begin{pmatrix} 1 \\ 3 \end{pmatrix} + \alpha_b \begin{pmatrix} 1 \\ 1 \end{pmatrix} - \alpha_c \begin{pmatrix} 2 \\ 2 \end{pmatrix}$
- solving these gives you $\alpha_c = 2$ and $\alpha_a = \alpha_b = 1$

Note - the larger the margin width ($\frac{2}{||\vec{w}||}$), the smaller the α values ###Kernels

Sometimes our training data isn't linearly separable. So, try transforming the data (e.g, polar coords).

Kernel Trick - replace the "$\cdot$" in $\vec{w} \cdot \vec{x} + b$ with another function. - this function is called a **Kernel function** - this effectively transforms the space

A **Kernel function** is a similarity measure.

$$
\begin{aligned}
\text{class}(\vec{x}) &= \text{SIGN}[\vec{w} \cdot \vec{x} + b] && (1) \\
&= \text{SIGN}[(\sum_i y_i \alpha_i x_i) \cdot \vec{x} + b] && (2) \\
&= \text{SIGN}[(\sum_i y_i \alpha_i K(\vec{x_i}, \vec{x}) + b] && (3)
\end{aligned}
$$

So basically we're taking this test point $\vec{x}$ and comparing it with all of our support vectors, and then taking a weighted sum of that.

Common Kernel Functions

1. Linear Kernel (e.g.) dot product

$$K(\vec{u}, \vec{v}) = \vec{u} \cdot \vec{v}$$

2. Quadratic Kernel Function

Draws conic sections: - circle - ellipse - hyperbola - parabola - normal line

Ex:

$$K(\vec{u}, \vec{v}) = (\vec{u} \cdot \vec{v} + 1)^2$$

3. Radial (Gaussian) Basis Kernel Function

Basically draws circles around each points. Has a parameter to adjust "how tight" the circles and grouping is.

Can perfectly classify any set of training data, as long as you don't have contradictory test data.

- can sometimes overfit

$$e^{\frac{|\vec{u}-\vec{v}|^2}{\sigma}}$$