Question 6

$$\text{P(one specific number is green)} = \frac{1}{2}$$

$$\text{P(one specific number all green on 5 cards)} = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$$

$$\text{P(one specific number not all green on 5 cards)} = 1 - \frac{1}{32} = \frac{31}{32}$$

$$\text{P(16 numbers are not all green on 5 cards)} = \left(\frac{31}{32}\right)^{16}$$

$$\text{P(there are some numbers that are green on 5 cards)} = 1 - \text{P(16 numbers are not all green on 5 cards)}$$
$$= 1 - \left(\frac{31}{32}\right)^{16}$$

Question 7

$$\text{P(pick one green 5's)} = \frac{1}{2}$$

$$\text{P(pick 5 green 5's)} = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$$

Question 8

Since there are M machines and a total of t pulls, the size of the hypothesis set number is Mt.And $E_{\text{out}} = \mu_m$, $E_{\text{in}} = \frac{c_m}{N_m}$.

$$\text{Let } P(\text{bad at t}) = P\left(\mu_m > \frac{c_m}{N_m} + \sqrt{\frac{\ln t - \frac{1}{2}\ln\delta}{N_m}}\right).$$

$$\sum_{t=1}^{\infty} \delta t^{-2} \geq \sum_{t=1}^{\infty} P(\text{bad at t}) \geq P(\text{bad at t} = 1 \text{ or bad at t} = 2 \ldots \text{bad at t} = \infty)$$

$$\text{because} \quad \sum_{t=1}^{\infty} \delta t^{-2} = \frac{\pi^2}{6}\delta > \sum_{t=1}^{\infty} P(\text{bad at t}).$$

Thus, $P(\text{bad at t})$ will be constrained to a certain value and will not approach infinity.

$$\sum_{t=1}^{\infty} P(\text{bad at t}) \geq P(\text{bad at t} = 1 \text{ or } \ldots \text{ bad at } \infty).$$

We can get:

$$P\left(\mu_m > \frac{c_m}{N_m} + \epsilon\right) \leq 2 \cdot \text{hypothesis number} \cdot \exp(-2\epsilon^2 N_m).$$

$$\Rightarrow P\left(\mu_m > \frac{c_m}{N_m} + \epsilon\right) \leq 2 \cdot Mt \cdot \exp(-2\epsilon^2 N_m).$$

$$\text{Let} \quad \delta = 2 \cdot Mt \cdot \exp(-2\epsilon^2 N_m).$$

$$\frac{\delta}{2Mt} = \exp(-2\epsilon^2 N_m).$$

$$\Rightarrow 2\ln M + 2\ln t - \ln\delta = 2\epsilon^2 N_m.$$

$$\Rightarrow \ln t + \ln M - \frac{1}{2}\ln\delta = \epsilon^2.$$

$$\Rightarrow \sqrt{\frac{\ln t + \ln M - \frac{1}{2}\ln\delta}{N_m}} = \epsilon.$$

$$\text{With } 1-\delta, \text{ good case, } E_{\text{out}} \le E_{\text{in}} + \epsilon.$$

$$\Rightarrow \mu_m \le \frac{c_m}{N_m} + \sqrt{\frac{\ln t + \ln M - \frac{1}{2}\ln\delta}{N_m}} \quad \text{is proved.}$$

Question 9

We can find k different positive numbers of ones in $\{-1,1\}^k$,and all zeros can get 0, so $\{-1,1\}^k$ can shatter the hypothesis set of k+1, so $d_{vc} \ge k+1$
And we can't find any k+2 different numbers offered by $\{-1,1\}^k$, so $d_{vc} < k+2$
with $d_{vc} \ge k+1$ and $d_{vc} < k+2$, we can get $d_{vc} = k+1$

Question 10

$$\text{With } x \text{ in uniform distribution } [-1,1], \text{ the pdf is } \frac{1}{2}.$$

$$\text{Hence } E_{\text{out}}(h_{s,\theta}) = \frac{1}{2}\int_{-1}^{1} P\left(y \ne h_{s,\theta}(x)\right) dx.$$

$$\text{And with } E_{\text{out}}(h_{s,\theta}) = u + v \cdot |\theta|, \text{ we have:}$$

$$s = \frac{1}{2} - v + v|\theta|$$

$$u = \frac{1}{2} + v \cdot |\theta|(1-1)$$

$$E_{\text{out}}(h_{s,\theta}) = \frac{1}{2} + s\left(\frac{1}{2} - p\right) \cdot (|\theta| - 1).$$

$$\text{We can divide } s, \theta \text{ into four cases:}$$

$$(s=+1, \theta \ge 0), (s=+1, \theta < 0), (s=-1, \theta \ge 0), (s=-1, \theta < 0).$$

$$\text{This leads to:}$$

$$\begin{array}{ll} E_{\text{out}}(h_{s,\theta})_{s=+1,\theta\ge 0}, & E_{\text{out}}(h_{s,\theta})_{s=+1,\theta<0}, \\ E_{\text{out}}(h_{s,\theta})_{s=-1,\theta\ge 0}, & E_{\text{out}}(h_{s,\theta})_{s=-1,\theta<0}. \end{array}$$

$$\text{We can then calculate } E_{\text{out}}(h_{s,\theta}) \text{ by conquering these cases.}$$

$$\textbf{Case } \text{sign} = +1, \text{ has } 2\ \theta \text{ cases:}$$

$$\theta \ge 0, \theta < 0$$

In $\theta \geq 0$:

We have 3 cases: $x \in [-1, 0],\ x \in (0, \theta], x \in (\theta, 1]$

$x \in [-1, 0], \text{sign}(x) = -1,\ P(y=1) = p,\ P(y=-1) = 1-p,\ h_{s,\theta} = -1,\ P(E_{\text{out of this case}}) = p,$
The range of this case is $-1 \sim 0$.

$x \in (0, \theta], \text{sign}(x) = +1,\ P(y=1) = 1-p,\ P(y=-1) = p,\ h_{s,\theta} = -1,\ P(E_{\text{out of this case}}) = 1-p,$
The range of this case is $0 \sim \theta$.

$x \in [\theta, 1], \text{sign}(x) = 1,\ P(y=1) = 1-p,\ P(y=-1) = p,\ h_{s,\theta} = 1,\ P(E_{\text{out of this case}}) = p$
The range of this case is $\theta \sim 1$.

In sign $= +1, \theta \geq 0$:

$$E_{\text{out}}(h_{s,\theta})_{s=+1,\theta>0} = \frac{1}{2}\left[\int_{-1}^{0} p\,dx + \int_{0}^{\theta} (1-p)\,dx + \int_{\theta}^{1} p\,dx\right]$$

$$= \frac{1}{2}\left[p + (1-p)\theta + p(1-\theta)\right]$$

$$= \frac{1}{2}\left[p + (1-p)\theta + p - p\theta\right]$$

$$= p + \frac{1}{2}(1-2p)\theta$$

With v, u in case $\text{s} = +1, \theta > 0$:

$$E_{\text{out}}(h_{s,\theta}) = \frac{1}{2} + s\left(\frac{1}{2} - p\right)(|\theta| - 1)$$

$$= \frac{1}{2} + \frac{\theta}{2} - \frac{1}{2} + p\theta + p$$

$$= p + \frac{1}{2}(1-2p)\theta, \quad E_{\text{out}}(h_{s,\theta})_{s=+1,\theta>0} \text{ is proved.}$$

In $\theta < 0$ case:

We have 3 cases: $x \in [-1, \theta],\ x \in (\theta, 0], x \in (0, 1]$

$x \in [-1, \theta], \text{sign}(x) = -1,\ P(y=1) = p,\ P(y=-1) = 1-p,\ h_{s,\theta} = -1,\ P(E_{\text{out of this case}}) = p$
The range of this case is $-1 \sim \theta$.

$x \in (\theta, 0], \text{sign}(x) = -1,\ P(y=1) = p,\ P(y=-1) = 1-p,\ h_{s,\theta} = 1,\ P(E_{\text{out of this case}}) = 1-p,$
The range of this case is $\theta \sim 0$.

$x \in (0, 1], \text{sign}(x) = +1,\ P(y=1) = 1-p,\ P(y=-1) = p,\ h_{s,\theta} = 1,\ P(E_{\text{out of this case}}) = p,$
The range of this case is $0 \sim 1$.

In sign $= +1,\ \theta < 0$:

$$E_{\text{out}}(h_{s,\theta})_{s=+1,\theta<0} = \frac{1}{2}\left[\int_{-1}^{\theta} p\,dx + \int_{\theta}^{0} (1-p)\,dx + \int_{0}^{1} p\,dx\right]$$

$$= \frac{1}{2}\left[p(\theta+1) + (1-p)\theta + p\right]$$

$$= \frac{1}{2}\left[p(\theta+1) - (1-p)\theta + p\right]$$

$$= \frac{1}{2}\left[2p - (1-2p)\theta\right] = p - \frac{1}{2}(1-2p)\theta$$

$$\text{With } v, u, \text{ in case } +1, \theta < 0:$$

$$E_{\text{out}}(h_{s,\theta}) = \frac{1}{2} + s\left(\frac{1}{2} - p\right)(|\theta| - 1)$$

$$= \frac{1}{2} - \left(\frac{1}{2} - p\right)(\theta + 1)$$

$$= \frac{1}{2} - \frac{\theta}{2} - \frac{1}{2} + p\theta + p$$

$$= p - \frac{1}{2}(1 - 2p)\theta \quad E_{\text{out}}(h_{s,\theta})_{s=+1,\theta<0} \text{ is proved.}$$

$$\textbf{Case } \text{sign} = -1, \text{ has } 2\ \theta \text{ cases:}$$

$$\theta \geq 0, \theta < 0$$

$$\text{In } \theta \geq 0$$

$$\text{We have 3 cases: } x \in [-1, 0),\ x \in [0, \theta), x \in [\theta, 1]$$

$$x \in [-1, 0], \text{sign}(x) = -1,\ P(y = 1) = p,\ P(y = -1) = 1 - p,\ h_{s,\theta} = 1,\ P(E_{\text{out of this case}}) = 1 - p,$$
$$\text{The range of this case is } -1 \sim 0.$$

$$x \in [0, \theta), \text{sign}(x) = 1,\ P(y = 1) = 1 - p,\ P(y = -1) = p,\ h_{s,\theta} = 1,\ P(E_{\text{out of this case}}) = p,$$
$$\text{The range of this case is } 0 \sim \theta.$$

$$x \in [\theta, 1], \text{sign}(x) = 1,\ P(y = 1) = 1 - p,\ P(y = -1) = p,\ h_{s,\theta} = -1,\ P(E_{\text{out of this case}}) = p$$
$$\text{The range of this case is } \theta \sim 1.$$

$$\textbf{In } \text{sign} = -1,\ \theta \geq 0\textbf{:}$$

$$E_{\text{out}}(h_{s,\theta})_{s=-1,\theta\geq 0} = \frac{1}{2}\left[\int_{-1}^{0}(1 - p)\,dx + \int_{0}^{\theta} p\,dx + \int_{\theta}^{1}(1 - p)\,dx\right]$$

$$= \frac{1}{2}\left[(1 - p)\cdot(0 + 1) + p\cdot\theta + (1 - p)\cdot(1 - \theta)\right]$$

$$= \frac{1}{2}\left[1 - p + p\theta + (1 - p)(1 - \theta)\right]$$

$$= \frac{1}{2}\left[1 - p + p\theta + (2p - 1)(1 - \theta)\right]$$

$$= 1 - p + \frac{1}{2}(2p - 1)\theta$$

$$\text{With } v, u, \text{ in case } s = -1, \theta \geq 0:$$

$$E_{\text{out}}(h_{s,\theta}) = \frac{1}{2} - \left(\frac{1}{2} - p\right)(|\theta| - 1)$$

$$= \frac{1}{2} - \left(\frac{1}{2} - p\right)(\theta - 1)$$

$$= 1 - p + \frac{1}{2}(2p - 1)\theta \quad E_{\text{out}}(h_{s,\theta})_{s=-1,\theta\geq 0} \text{ is proved.}$$

$$\textbf{In } \theta < 0\textbf{:}$$

$$\text{We have 3 cases: } x \in [-1, \theta],\ x \in (\theta, 0], x \in (0, 1]$$

$$x \in [-1, \theta], \text{sign}(x) = -1,\ P(y = 1) = p,\ P(y = -1) = 1 - p,\ h_{s,\theta} = 1,\ P(E_{\text{out of this case}}) = 1 - p$$

The range of this case is $-1 \sim \theta$.

$$x \in (\theta, 0], \text{sign}(x) = -1,\ P(y=1) = p,\ P(y=-1) = 1-p,\ h_{s,\theta} = -1,\ P(E_{\text{out of this case}}) = p$$

The range of this case is $\theta \sim 0$.

$$x \in (0, 1], \text{sign}(x) = 1,\ P(y=1) = 1-p,\ P(y=-1) = p,\ h_{s,\theta} = -1,\ P(E_{\text{out of this case}}) = 1-p$$

The range of this case is $0 \sim 1$.

In sign $= -1,\ \theta < 0$

$$E_{\text{out}}(h_{s,\theta}) = \frac{1}{2}\left[\int_{-1}^{\theta} p(1-p)\,dx + \int_{\theta}^{0} p\,dx + \int_{0}^{1} (1-p)\,dx\right]$$

$$= \frac{1}{2}\left[\theta(1-p) + p + (1-p)\right]$$

$$= \frac{1}{2}\left[\theta - p\theta + 1 - p + 1 - p\right] = 1 - p - p\theta + \frac{\theta}{2}$$

$$= 1 - p + \frac{1}{2}(1-2p)\theta$$

with $v, u,$ in case $s = -1, \theta < 0$

$$E_{\text{out}}(h_{s,\theta}) = \frac{1}{2} - \left(\frac{1}{2} - p\right)(1 - |\theta|)$$

$$= \frac{1}{2} + \left(\frac{1}{2} - p\right)(|\theta| + 1) = \frac{1}{2} + \frac{\theta}{2} - p\theta + 1 - p$$

$$= 1 - p + \frac{1}{2}(1-2p)\theta,\ E_{\text{out}}(h_{s,\theta})_{\{s=-1,\theta<0\}} \text{ is proved.}$$

Hence, $E_{\text{out}}(h_{s,\theta})_{s=\pm1,\theta\geq0}, E_{\text{out}}(h_{s,\theta})_{s=\pm1,\theta<0}$ are proved.

$$E_{\text{out}}(h_{s,\theta}) = u + v \cdot |\theta|,\ \text{where } u = \frac{1}{2} - v \text{ is proved.}$$