### **Question 6**

$$P(\text{one specific number is green}) = \frac{1}{2}$$
 
$$P(\text{one specific number all green on 5 cards}) = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$$
 
$$P(\text{one specific number not all green on 5 cards}) = 1 - \frac{1}{32} = \frac{31}{32}$$
 
$$P(\text{16 numbers are not all green on 5 cards}) = \left(\frac{31}{32}\right)^{16}$$

P(there are some numbers that are green on 5 cards) = 1 - P(16 numbers are not all green on 5 cards)=  $1 - \left(\frac{31}{32}\right)^{16}$ 

# **Question 7**

$$P(\text{pick one green 5's}) = \frac{1}{2}$$
 
$$P(\text{pick 5 green 5's}) = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$$

# **Question 8**

Since there are M machines and a total of t pulls, the size of the hypothesis set number is Mt. And  $E_{\text{out}} = \mu_m$ ,  $E_{\text{in}} = \frac{c_m}{N_m}$ .

$$\begin{split} \operatorname{Let} P(\operatorname{bad} \operatorname{at} \operatorname{t}) &= P\left(\mu_m > \frac{c_m}{N_m} + \sqrt{\frac{\ln t - \frac{1}{2}\ln \delta}{N_m}}\right). \\ \sum_{t=1}^\infty \delta t^{-2} &\geq \sum_{t=1}^\infty P(\operatorname{bad} \operatorname{at} \operatorname{t}) \geq P(\operatorname{bad} \operatorname{at} \operatorname{t} = 1 \operatorname{or} \operatorname{bad} \operatorname{at} \operatorname{t} = 2 \ldots \operatorname{bad} \operatorname{at} \operatorname{t} = \infty) \\ \operatorname{because} \quad \sum_{t=1}^\infty \delta t^{-2} &= \frac{\pi^2}{6} \delta > \sum_{t=1}^\infty P(\operatorname{bad} \operatorname{at} \operatorname{t}). \end{split}$$

Thus, P(bad at t) will be constrained to a certain value and will not approach infinity.

$$\sum_{t=1}^{\infty} P(\mathrm{bad} \ \mathrm{at} \ \mathrm{t}) \geq P(\mathrm{bad} \ \mathrm{at} \ \mathrm{t} = 1 \ \mathrm{or} \ \ldots \ \mathrm{bad} \ \mathrm{at} \ \infty).$$

We can get:

$$egin{aligned} P\left(\mu_m > rac{c_m}{N_m} + \epsilon
ight) & \leq 2 \cdot ext{hypothesis number} \cdot \exp(-2\epsilon^2 N_m). \ \ & \Rightarrow P\left(\mu_m > rac{c_m}{N_m} + \epsilon
ight) \leq 2 \cdot Mt \cdot \exp(-2\epsilon^2 N_m). \ \ & ext{Let} \quad \delta = 2 \cdot Mt \cdot \exp(-2\epsilon^2 N_m). \end{aligned}$$

With  $1 - \delta$ , good case,  $E_{\text{out}} \leq E_{\text{in}} + \epsilon$ .

$$\Rightarrow \mu_m \leq rac{c_m}{N_m} + \sqrt{rac{\ln t + \ln M - rac{1}{2}\ln \delta}{N_m}} \quad ext{is proved.}$$

## **Question 9**

We can find k different positive numbers of ones in  $\{-1,1\}^k$ , and all zeros can get 0, so  $\{-1,1\}^k$  can shatter the hypothesis set of k+1, so  $d_{vc} \ge k+1$ 

And we can't find any k+2 different numbers offered by  $\{-1,1\}^k$ , so  $d_{vc} < k+2$  with  $d_{vc} \ge k+1$  and  $d_{vc} < k+2$ , we can get  $d_{vc} = k+1$ 

#### **Question 10**

With x in uniform distribution [-1, 1], the pdf is  $\frac{1}{2}$ .

Hence 
$$E_{\mathrm{out}}(h_{s, heta}) = rac{1}{2} \int_{-1}^{1} P\left(y 
eq h_{s, heta}(x)
ight) dx.$$

And with  $E_{\text{out}}(h_{s,\theta}) = u + v \cdot |\theta|$ , we have:

$$egin{aligned} s &= rac{1}{2} - v + v | heta| \ u &= rac{1}{2} + v \cdot | heta| (1-1) \ E_{ ext{out}}(h_{s, heta}) &= rac{1}{2} + s \left(rac{1}{2} - p
ight) \cdot (| heta| - 1). \end{aligned}$$

We can divide  $s, \theta$  into four cases:

$$(s=+1, heta \geq 0), (s=+1, heta < 0), (s=-1, heta \geq 0), (s=-1, heta < 0).$$
 This leads to:  $E_{
m out}(h_{s, heta})_{s=+1, heta \geq 0}, \quad E_{
m out}(h_{s, heta})_{s=+1, heta < 0}, \ E_{
m out}(h_{s, heta})_{s=-1, heta > 0}, \quad E_{
m out}(h_{s, heta})_{s=-1, heta < 0}.$ 

We can then calculate  $E_{\text{out}}(h_{s,\theta})$  by conquering these cases.

Case sign = +1, has  $2 \theta$  cases:

$$\theta > 0, \theta < 0$$

We have 3 cases:  $x \in [-1, 0], x \in (0, \theta], x \in (\theta, 1]$ 

$$x \in [-1, 0], \text{sign}(x) = -1, \ P(y = 1) = p, \ P(y = -1) = 1 - p, \ h_{s,\theta} = -1, \ P(E_{\text{out of this case}}) = p,$$
  
The range of this case is  $-1 \sim 0$ .

$$x \in (0, \theta], \text{sign}(x) = +1, \ P(y = 1) = 1 - p, \ P(y = -1) = p, \ h_{s,\theta} = -1, \ P(E_{\text{out of this case}}) = 1 - p,$$
  
The range of this case is  $0 \sim \theta$ .

$$x \in [\theta, 1], \operatorname{sign}(x) = 1, \ P(y = 1) = 1 - p, \ P(y = -1) = p, \ h_{s,\theta} = 1, \ P(E_{\operatorname{out\ of\ this\ case}}) = p$$
  
The range of this case is  $\theta \sim 1$ .

In sign = +1,  $\theta \geq 0$ :

$$egin{aligned} E_{ ext{out}}(h_{s, heta})_{s=+1, heta>0} &= rac{1}{2} \left[ \int_{-1}^{0} p \, dx + \int_{0}^{ heta} (1-p) \, dx + \int_{ heta}^{1} p \, dx 
ight] \ &= rac{1}{2} \left[ p + (1-p) heta + p (1- heta) 
ight] \ &= rac{1}{2} \left[ p + (1-p) heta + p - p heta 
ight] \ &= p + rac{1}{2} (1-2p) heta \end{aligned}$$

With v, u in case  $s = +1, \theta > 0$ :

$$egin{split} E_{ ext{out}}(h_{s, heta}) &= rac{1}{2} + s\left(rac{1}{2} - p
ight)(| heta| - 1) \ &= rac{1}{2} + rac{ heta}{2} - rac{1}{2} + p heta + p \end{split}$$

$$=p+rac{1}{2}(1-2p) heta,\quad E_{\mathrm{out}}(h_{s, heta})_{s=+1, heta>0} ext{ is proved.}$$

In  $\theta < 0$  case:

We have 3 cases:  $x \in [-1, \theta], x \in (\theta, 0], x \in (0, 1]$ 

$$x \in [-1, \theta], \text{sign}(x) = -1, \ P(y = 1) = p, \ P(y = -1) = 1 - p, \ h_{s,\theta} = -1, \ P(E_{\text{out of this case}}) = p$$
  
The range of this case is  $-1 \sim \theta$ .

$$x \in (\theta, 0], \text{sign}(x) = -1, \ P(y = 1) = p, \ P(y = -1) = 1 - p, \ h_{s,\theta} = 1, \ P(E_{\text{out of this case}}) = 1 - p,$$
 The range of this case is  $\theta \sim 0$ .

$$x \in (0,1], ext{sign}(x) = +1, \; P(y=1) = 1-p, \; P(y=-1) = p, \; h_{s,\theta} = 1, \; P(E_{ ext{out of this case}}) = p, \; ext{The range of this case is } 0 \sim 1.$$

In sign = +1,  $\theta < 0$ :

$$egin{align} E_{ ext{out}}(h_{s, heta})_{s=+1, heta<0} &= rac{1}{2} \left[ \int_{-1}^{ heta} p \, dx + \int_{ heta}^{0} (1-p) \, dx + \int_{0}^{1} p \, dx 
ight] \ &= rac{1}{2} \left[ p( heta+1) + (1-p) heta + p 
ight] \ &= rac{1}{2} \left[ p( heta+1) - (1-p) heta + p 
ight] \ &= rac{1}{2} \left[ 2p - (1-2p) heta 
ight] = p - rac{1}{2} (1-2p) heta \end{split}$$

With v, u, in case  $+1, \theta < 0$ :

$$egin{align} E_{ ext{out}}(h_{s, heta}) &= rac{1}{2} + s\left(rac{1}{2} - p
ight)(| heta| - 1) \ &= rac{1}{2} - \left(rac{1}{2} - p
ight)( heta + 1) \ &= rac{1}{2} - rac{ heta}{2} - rac{1}{2} + p heta + p \ &= p - rac{1}{2}(1 - 2p) heta \quad E_{ ext{out}}(h_{s, heta})_{s=+1, heta < 0} ext{ is proved.} \end{split}$$

Case sign = -1, has  $2 \theta$  cases:

$$\theta \geq 0, \theta < 0$$

In 
$$\theta > 0$$

We have 3 cases:  $x \in [-1, 0), x \in [0, \theta), x \in [\theta, 1]$ 

$$x \in [-1,0], \mathrm{sign}(x) = -1, \ P(y=1) = p, \ P(y=-1) = 1-p, \ h_{s,\theta} = 1, \ P(E_{\mathrm{out\ of\ this\ case}}) = 1-p,$$
 The range of this case is  $-1 \sim 0$ .

$$x \in [0, \theta), \mathrm{sign}(x) = 1, \; P(y = 1) = 1 - p, \; P(y = -1) = p, \; h_{s,\theta} = 1, \; P(E_{\mathrm{out \; of \; this \; case}}) = p,$$
 The range of this case is  $0 \sim \theta$ .

$$x \in [\theta, 1], \operatorname{sign}(x) = 1, \ P(y = 1) = 1 - p, \ P(y = -1) = p, \ h_{s,\theta} = -1, \ P(E_{\operatorname{out of this case}}) = p$$
  
The range of this case is  $\theta \sim 1$ .

In sign = 
$$-1$$
,  $\theta \ge 0$ :

$$egin{aligned} E_{ ext{out}}(h_{s, heta})_{s=-1, heta\geq 0} &= rac{1}{2} \left[ \int_{-1}^{0} (1-p) \, dx + \int_{0}^{ heta} p \, dx + \int_{ heta}^{1} (1-p) \, dx 
ight] \ &= rac{1}{2} \left[ (1-p) \cdot (0+1) + p \cdot heta + (1-p) \cdot (1- heta) 
ight] \ &= rac{1}{2} \left[ 1-p + p heta + (1-p)(1- heta) 
ight] \ &= rac{1}{2} \left[ 1-p + p heta + (2p-1)(1- heta) 
ight] \ &= 1-p + rac{1}{2} (2p-1) heta \end{aligned}$$

With v, u, in case  $s = -1, \theta \ge 0$ :

$$egin{split} E_{ ext{out}}(h_{s, heta}) &= rac{1}{2} - \left(rac{1}{2} - p
ight)(| heta| - 1) \ &= rac{1}{2} - \left(rac{1}{2} - p
ight)( heta - 1) \end{split}$$

$$=1-p+rac{1}{2}(2p-1) heta \quad E_{\mathrm{out}}(h_{s, heta})_{s=-1, heta\geq 0} ext{ is proved.}$$

In 
$$\theta < 0$$
:

We have 3 cases:  $x \in [-1, \theta], x \in (\theta, 0], x \in (0, 1]$ 

$$x \in [-1,\theta], \text{sign}(x) = -1, \ P(y=1) = p, \ P(y=-1) = 1-p, \ h_{s,\theta} = 1, \ P(E_{\text{out of this case}}) = 1-p$$

The range of this case is  $-1 \sim \theta$ .

$$x \in (\theta, 0], \operatorname{sign}(x) = -1, \ P(y = 1) = p, \ P(y = -1) = 1 - p, \ h_{s,\theta} = -1, \ P(E_{\operatorname{out of this case}}) = p$$
  
The range of this case is  $\theta \sim 0$ .

$$x \in (0,1], \operatorname{sign}(x) = 1, \ P(y=1) = 1-p, \ P(y=-1) = p, \ h_{s,\theta} = -1, \ P(E_{\operatorname{out\ of\ this\ case}}) = 1-p$$
  
The range of this case is  $0 \sim 1$ .

In sign = 
$$-1$$
,  $\theta < 0$ 

$$egin{split} E_{ ext{out}}(h_{s, heta}) &= rac{1}{2} \left[ \int_{-1}^{ heta} p(1-p) \, dx + \int_{ heta}^{0} p \, dx + \int_{0}^{1} (1-p) \, dx 
ight] \ &= rac{1}{2} \left[ heta(1-p) + p + (1-p) 
ight] \ &= rac{1}{2} \left[ heta - p heta + 1 - p + 1 - p 
ight] = 1 - p - p heta + rac{ heta}{2} \ &= 1 - p + rac{1}{2} (1 - 2p) heta \end{split}$$

with 
$$v, u$$
, in case  $s = -1, \theta < 0$ 

$$egin{align} E_{ ext{out}}(h_{s, heta}) &= rac{1}{2} - \left(rac{1}{2} - p
ight)(1 - | heta|) \ &= rac{1}{2} + \left(rac{1}{2} - p
ight)(| heta| + 1) = rac{1}{2} + rac{ heta}{2} - p heta + 1 - p \ &= 1 - p + rac{1}{2}(1 - 2p) heta, \; E_{ ext{out}}(h_{s, heta})_{\{s = -1, heta < 0\}} \; ext{is proved.} \end{split}$$

Hence,  $E_{\text{out}}(h_{s,\theta})_{s=\pm 1,\theta\geq 0}, E_{\text{out}}(h_{s,\theta})_{s=\pm 1,\theta< 0}$  are proved.

$$E_{ ext{out}}(h_{s, heta}) = u + v \cdot | heta|, ext{ where } u = rac{1}{2} - v ext{ is proved.}$$