Tutorial 1 Solutions: Monte Carlo Simulation of the 2D Ising Model

May 27, 2019

1 Monte Carlo algorithm

a) Add the following to the marked section of the code:

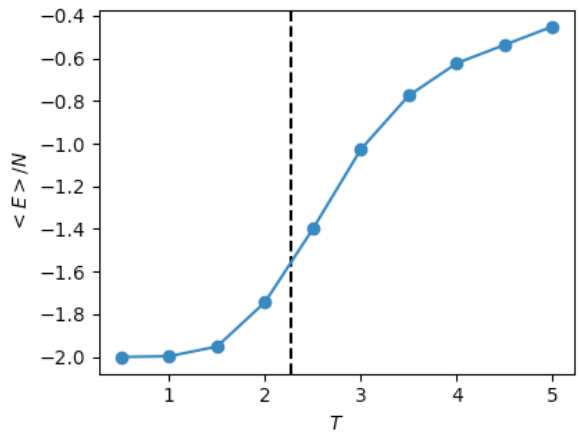
```
#neighbour to the left:
neighbours[i,2]=i-1
if i%L==0:
  neighbours[i,2]=i-1+L


#downwards neighbour:
neighbours[i,3]=i-L
if i <= (L-1):
  neighbours[i,3]=i-L+N_spins
```

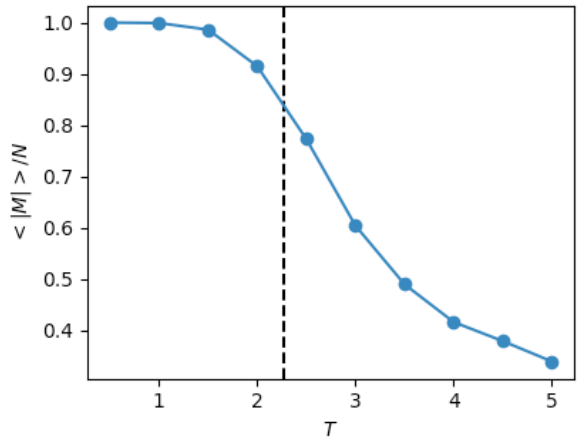
c) Comment out the five lines of code that calculate `deltaE` and replace them with:

```
deltaE = 0
#calculate deltaE of the proposed move from only the nearest neighbours:
for j in range(4):
  deltaE += 2*J*spins[site]*spins[neighbours[site,j]]
```

2 Estimating the critical temperature

a) Using the data from Question 1, the code `plot_ising.py` (without modification) should produce results similar to the following for $\langle E \rangle / N$ and $\langle |M| \rangle / N$:





When $T \to 0$, we expect to find the system in a ground state (with $\sigma_i = 1$ for all i or $\sigma_i = -1$ for all i). In this case, we expect that

$$\left.\frac{\langle E \rangle}{N}\right|_{T \to 0} = \frac{1}{N}\left(-J \sum_{\langle ij \rangle}(1)\right) = \frac{1}{N}(-2JN) = -2J$$

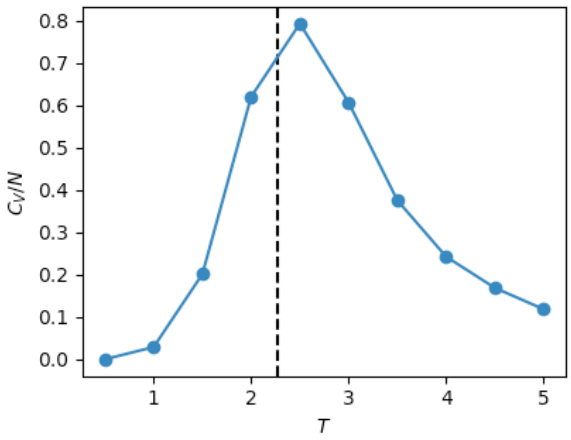
and

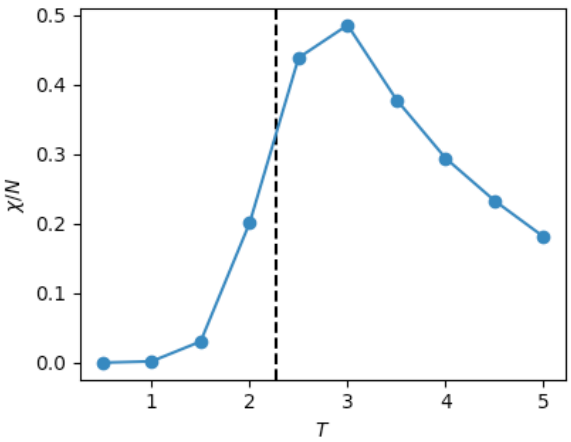
$$\left.\frac{\langle M\rangle}{N}\right|_{T\to 0} = \frac{1}{N}\left(\sum_i (1)\right) = \frac{1}{N}(N) = 1.$$

b) Modify the marked two lines of the code such that:

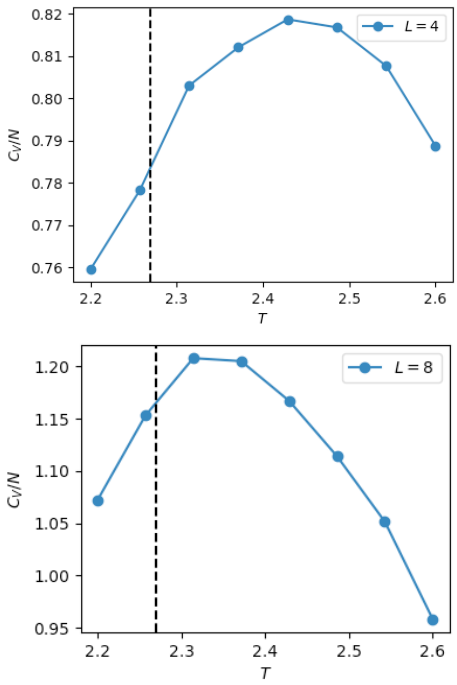
```
specHeat[iT] = (np.mean(E**2) - np.mean(E)**2)/(1.0*T**2)
susc[iT]     = (np.mean(M**2) - np.mean(M)**2)/(1.0*T)
```

Using the data from Question 1e, the code should then produce results similar to the following for C_V/N and χ/N:

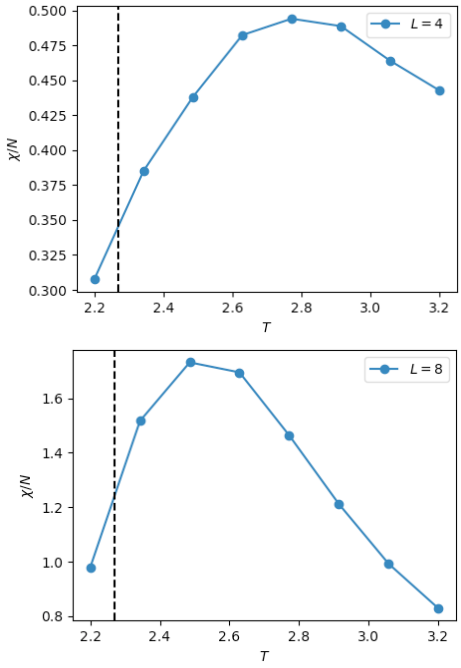




c) Using the parameters `T_list = np.linspace(2.6,2.2,8)`, `n_eqSweeps = 10000` and
`n_measSweeps = 100000`, one can generate data that leads to the following plots for C_V/N:



Similarly, using `T_list = np.linspace(3.2,2.2,8)`, `n_eqSweeps = 10000` and
`n_measSweeps = 100000`, one can generate data that leads to the following plots for χ/N:



In both cases, we see that the peak shifts closer to T_c as L increases.