

Reasoning model with noisy-or likelihood function

Input: Below is the contingency data that report the number of participants in each condition. Your implementation should use this as the input data to your learning model,

	C = 1, take the drug (experimental group)	C = 0, did not take the drug (control group)
E = 1, with headache	12	0
E = 0, no headache	4	16

In this exercise, we will use the **noisy-or likelihood** function and the **uniform prior** of causal strengths to implement the causal inference with the sampling approach. We will compute the posterior distribution of causal strengths, and then estimate the median value of strength that the drug causes a headache.

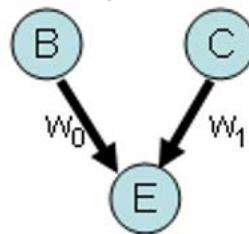
Input: $D, a = 12; b = 4; c = 0, d = 16$

Learning causation from contingencies

	C present (c^+)	C absent (c^-)
E present (e^+)	a	c
E absent (e^-)	b	d

	C = 1, take the drug (experimental group)	C = 0, did not take the drug (control group)
E = 1, with headache	12	0
E = 0, no headache	4	16

e.g., “Does this medicine cause headache?”



Goal: Use the sampling approach to implement the Bayesian inference.

$$P(w_0, w_1 | D, Graph1) = \frac{P(D | w_0, w_1, Graph1) P(w_0, w_1)}{P(D)}$$

In-class exercise [CausalEx.m]

- ✓ (1) Generate samples of causal strengths, w_0 and w_1 , from uniform distribution within the range 0 to 1. [Hint: use MATLAB function **unifrnd()**].
- (2) Use noisy-or function to compute likelihood for each of the four contingency conditions. The lecture slide below provides the functions.

Likelihood function II

- Noisy-OR function for generative case

$$P(e^+ | B, C; w_0, w_1) = 1 - (1 - w_0 B)(1 - w_1 C)$$

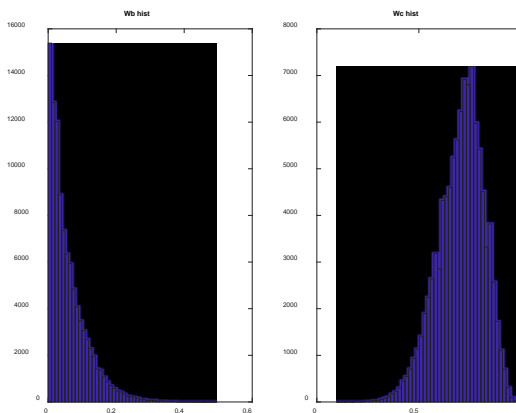
$P(e B, C; w_0, w_1)$	C present (c^+) B = 1, C = 1	C absent (c^-) B = 1, C = 0
E present (e^+)	$1 - (1 - w_0)(1 - w_1)$	w_0
E absent (e^-)	$(1 - w_0)(1 - w_1)$	$1 - w_0$

The likelihood of D, $P(D|w_0, w_1, Graph1)$, can be computed by combining the contingency data and calculated likelihoods for each condition in step 2.

$$\begin{aligned}
 P(D|w_0, w_1, Graph1) \\
 &= P(e^+ | B = 1, C = 1)^a * P(e^- | B = 1, C = 1)^b * P(e^+ | B = 1, C = 0)^c \\
 &\quad * P(e^- | B = 1, C = 0)^d
 \end{aligned}$$

- Normalize the likelihood to compute the sampling weights.
- Use `randsample()` function to draw the samples that approximate the posterior distributions $P(w_0, w_1 | D, Graph1)$.
- plot the histograms of posterior samples for w_0 and w_1 .
- Compute the median value of w_1 .

Results



```

%% calculate the median
estimate of the causal strength

ans =

    0.7166

```