Modeling Perception

In this tutorial, we're going to build a very simple model to do a very simple perceptual task: to locate a target object in a one-dimensional world, given some "prior" information.

Suppose you're a surveillance robot. You need to report to your superior if a target object appears within your duty area, which is a one-dimensional space, from -20 to +20.

Before carrying out your duty, you have been briefed that target objects are likely to appear at two locations, $\mu_1 = -3$ and $\mu_2 = 7$.

Now, as you're on duty, you receive a piece of information from your sensing device, telling you that it detects an object at location $x = 3$. However, you know that your sensing device is not 100% accurate. The object could actually appear at another position even though your sensor tells you location x.

Now, your task is to construct a Bayesian model, with the goal to infer the target object's location θ based on both the intelligence you obtained in the briefing and the signal from your sensor.

Below are the model specifications:

Range of possible location θ
From -20 to 20. Define stepsize as a variable and set it to 0.1.

Prior
Use $\mu_1 = -3, \mu_2 = 7, \sigma = 2$. Construct the prior distribution by using the sum of two Gaussian distributions:

$$P(\theta) \propto N(\theta;\ \mu_1, \sigma^2) + N(\theta;\ \mu_2, \sigma^2).$$

In MATLAB, simply add up two normpdf( )'s over the possible range of possible location θ. Make sure you normalize the prior after summing the two normpdf's.

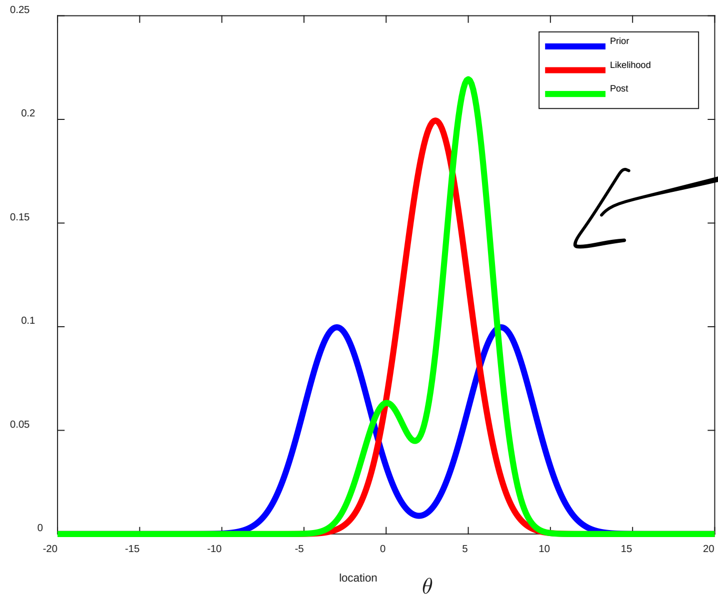
Likelihood
Use $x = 3, \sigma_x = 2$. Again, use a Gaussian distribution to model the uncertainty of the sensor, i.e.,

$$P(x|\theta) = N(x; \theta, \sigma_x^2)$$

Posterior
Multiply the prior and likelihood, and normalize the posterior distribution. Obtain the maximum a posterior (MAP) estimate of location θ (i.e., the θ value that gives you the maximum posterior probability).

[In-class Exercise]
Submit the q1.m that plots likelihood, prior and posterior distribution as below, and compute the MAP estimate of the location.
Your program should find MAP estimate as 5 using the current settings.

The three plots should look like:



green = red * blue

[**Optional**: extra questions for thinking about and testing your model]

1. Suppose the two locations given in the briefing differ in terms of uncertainty. How would you model that in your Bayesian model? Which parameter(s) would you change to represent the difference in uncertainty? And how?

   change σ ; bigger spread = more uncertainty

2. Which parameter(s) in the model would you change if you've become more certain about your sensor's signal because, for example, you've just upgraded the parts in it? How would you change parameter(s)?

   Make σ_x SMALLER

3. Suppose you trust the prior information on location 1 three times more than that on location 2. What would you do when you construct your prior?
   [Hint: You may want to put a "heavier weight" on the Gaussian that you trust more].

   $3 \cdot G_1 + G_2$

4. With the prior distribution fixed, try a range of observed locations x. Obtain the posterior probability of the MAP estimate for each x. Before doing the programming, can you guess at which observed location(s) you will obtain the highest posterior probability for the MAP? Why?

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