Sampling III: Computing the Need Probability in the Human Memory Model

Last lecture we went through Anderson and Milson's (1989) rational model for human memory. The core idea of their model is that memory depends on the need probability: how probable an item is needed in the future. The full version of the model is too complicated to be discussed in only one week. Therefore, we focus on one of the key components in computing the need probability: the history factor $P(A|H_A)$. For the purpose of this tutorial, we simply take $P(A|H_A)$ as the need probability, which describes how probable an item A is needed if it has a retrieval history of H_A.

$P(A|H_A)$ is defined as the product between the expected value of desirability λ and a decay function:

$$P(A|H_A) = E(\lambda)Decay(d,t)$$

Using the Bayes' Rule and going through some mathematical derivation, $E(\lambda)$ is given by

$$E(\lambda) = \frac{v+n}{b+M(t)}$$

, and the decay function is given by

$$Decay(d,t) = e^{-dt}$$

, where
$v/b =$ average use rate, which means that items, on average, are used v times within b time units,
n and t characterize H_A, which refers to an item having been retrieved n times in the past t time units,
$d =$ decay rate, which governs how fast desirability drops over time.

$M(t) = \left(1 - e^{-dt}\right)/d$ is a function based on the decay rate d and time t.

Open **HumanMemBasic.m**. Below is the default parameter setting:

v	b	n	d
2	100	1	0.4

a. Define the parameters values as above. Assume t = 1. (Thought question: Interpret what this parameter setting means).

b. Implement the equations for $M(t)$, $E(\lambda)$, and $Decay(d,t)$.

c. Compute the need probability $P(A|H_A)$ when t=1.

d. Construct a vector T with 100 elements, from 1 to 100. Write a FOR loop to loop through each element in T. For each value T(i) in T, compute the need probability.
[Hint: You don't need to modify the equations you implemented in b. If t is the variable you used in your equations, simply assign t = T(i) within the FOR loop. You may want to store each need probability as an element in a vector.]

e. Plot the need probability over the time range 1:100.

In Class Exercise

Sampling the decay rate d

If you've successfully implemented the program above, you may play around with the model by changing the decay rate d (e.g., .01, .05, .1, .5, 1, etc.). You may notice that different values of d can give quite curves that are of different shapes. Since different items may have different decay rates, the question is which value of d we should choose.

Ideally, we should consider all possible values of d, i.e, from 0 to $+\infty$, and use the sum operation (marginalization) to compute

$$P(A|H_A) = \int_0^{+\infty} P(A, d|H_A)\, d(d) = \int_0^{+\infty} P(A|H_A, d)P(d)\, d(d)$$

However, this is challenging for our implementation in MATLAB to consider an infinite range of numbers. Here, we can make use of the sampling technique. We can start by assuming that d is a random variable following a certain distribution $P(d)$. Then, we can rewrite the integral as follows:

$$P(A|H_A) = \int_0^{+\infty} P(A|H_A, d)P(d)\, d(d)$$

In this example, we assume d follows an exponential distribution, with the mean being α. Therefore,

$$d \sim \frac{1}{\alpha} Exponential\left(-\frac{d}{\alpha}\right)$$

Let's assume $\alpha = .4$. We can then approximate the above integral by the sampling method. If we can generate N samples of d_i, for $i = 1, \ldots, N$, we can approximate the integral as follows:

$$P(A|H_A) \approx \frac{1}{N}\sum_{i=1}^{N} P(A|H_A, d_i)$$

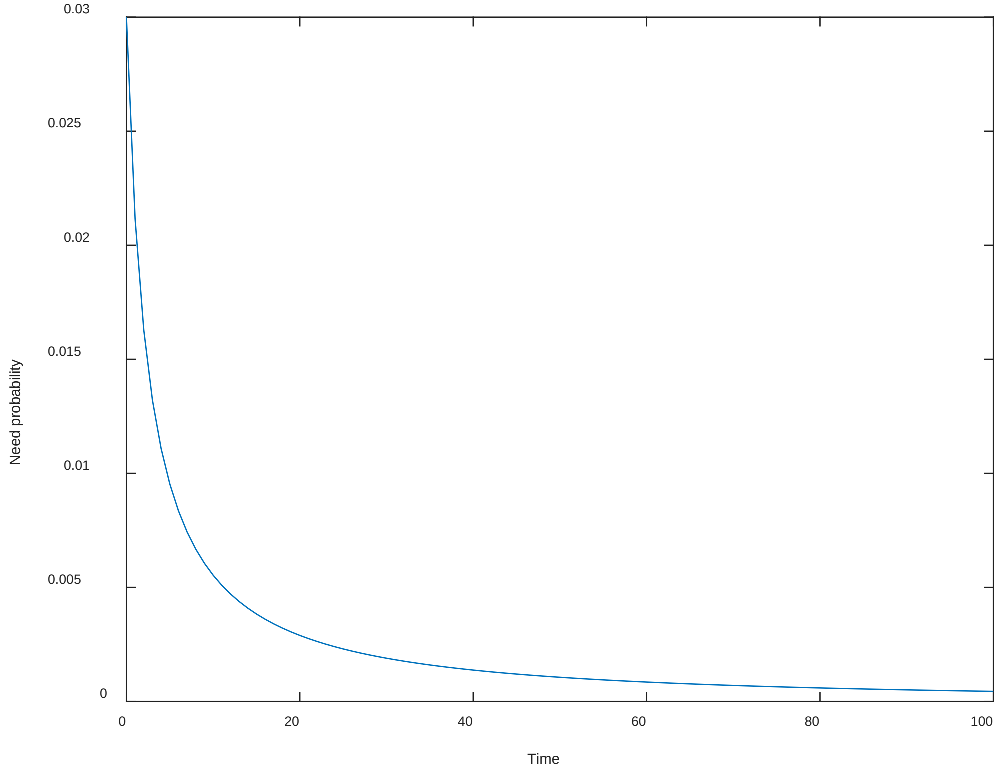
You may try to modify your previous program following the steps below:

a. Define two new parameters: $\alpha = .4$ and $N = 1000$.

b. For each value of t, sample N values of d using the MATLAB built-in function `exprnd`(α, 1 , N), which gives you a $1 \times N$ vector of numbers sampled from an exponential distribution, with mean $= \alpha$.

c. For each sampled value, compute the need probability.
[Hint: You may use your previous codes. What you need to do is just to make sure all the operations (e.g., multiplication, division) involving the sample vector d are element-wise.]

d. Sum up all the computed need probabilities and divide the sum by N.


0.03
0.025
0.02
0.015
0.01
0.005
0
Need probability
0
20
40
60
80
100
Time