<u>Modeling Perception: Part II</u>

In previous tutorial of inferring a target position from noisy input an prior knowledge, we encountered one observation. In this exercise, we will include multiple observations and estimate the target location using the mean of posterior distribution.

Suppose you're a surveillance robot. You need to report to your superior if a target object appears within your duty area, which is a one-dimensional space, from -20 to +20. Before carrying out your duty, you have been briefed that target objects are likely to appear at location = 0. Now, as you're on duty, you receive four measures from your sensing device, telling you that it detects an object at locations x = [7, 3, -1, 8]; However, you know that your sensing device is not 100% accurate. The object could actually appear at another position even though your sensor tells you location x. Now, your task is to construct a Bayesian model, with the goal to infer the target object's location θ based on both the intelligence you obtained in the briefing and the signal from your sensor.

Below are the mathematical specifications:

Let θ be the random variable of the true target location. We define the range of possible location from -20 to 20 with a stepsize of 0.1. Let $\boldsymbol{x}$ be the observed locations reported from your sensor. In this example, we have four reported locations, x = [7, 3, -1, 8];
Assume the following:
Prior for θ is a normal distribution, with mean μ_θ and variance σ_θ^2,
i.e., $P(\theta) \sim Normal(\mu_\theta, \sigma_\theta^2)$ $\mu_\theta = 0, \sigma_\theta = 8$

Likelihood of a observed locations x_i follows a normal distribution, with mean θ and variance σ_x^2,
i.e., $P(x_i|\theta) \sim Normal(\theta, \sigma_x^2)$ $\sigma_x = 4$
As we have multiple observed locations, we assume that these observations are independent. The likelihood of observing the four observations in the this study would be

$$P(\boldsymbol{x}|\theta) = \prod_i P(x_i|\theta) = P(x_1|\theta)P(x_2|\theta)P(x_3|\theta)P(x_4|\theta)$$

Recall the steps for the <u>**discretize-space method**</u>:

1. Define the range of all possible locations t for the target to appear (i.e., all possible hypotheses)
2. Compute the prior probability for each hypothesized target location, i.e., $P(\theta)$
3. Compute the likelihood for the observed location x, i.e., $P(x|\theta)$ for each θ in the range defined in step 1.
4. Obtain the posterior distribution by
   a. Multiplying the prior and likelihood probabilities, correspondingly for each location t
   b. Normalizing the products
5. Obtain an estimate from the posterior distribution. We'll use the expected value of the location using its posterior distribution.

$$\mathrm{E}[X] = \sum_{i=1}^{\infty} x_i\, p_i. \qquad \mathrm{E}[X] = \int_{\mathbb{R}} x f(x)\, dx,$$

<u>In-class Exercise</u>

Complete Q1.m with the five steps above.

(1) Report the expected location using the posterior mean.

(2) Plot prior and posterior distribution in one figure, and likelihood in a second figure.

Your program should find mean posterior estimate as 4 using the current settings.

The two figures should look like: