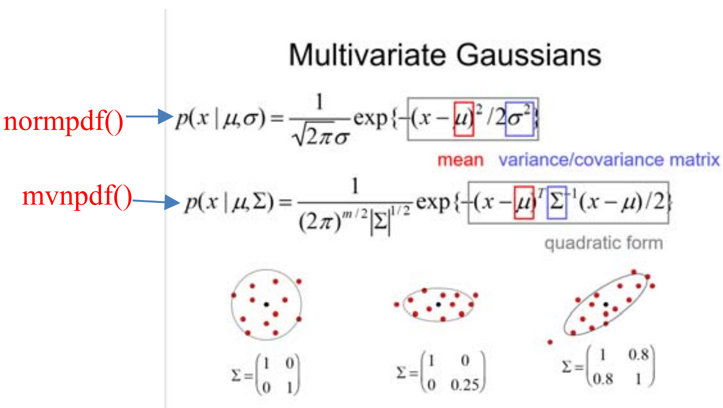
Bayesian Inference in Two Dimensions

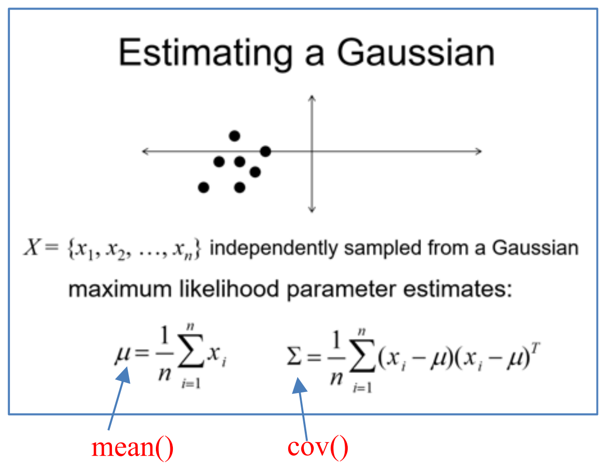
So far, all the problems we've looked at are one-dimensional problems. In other words, the hypothesis world contains hypotheses that can be quantified into one dimension (e.g., P(Head) in coin flipping is a real number between 0 and 1; location of the target object in the robot-sensor example is a real number within the robot's "1D" duty range; RT of a word in the categorization model).

However, in many real-life problems, the possible hypotheses usually span across multiple dimensions. This tutorial will show you how to apply the Bayesian inference techniques you learned for one-dimensional problems on a two-dimensional problem.

A Categorization Problem in 2D

Suppose you are given 50 exemplars from one of the two categories. Each exemplar is represented using two feature values. Load the input file **inputdata.mat**, and the exemplars are saved in two variables "cat1data" and "cat2data". Your job is to first learn the two categories using parametric method, and then use the learned representations to categorize three new cases with feature values of [1, -2]; [2, 0]; [3, 2.5] respectively. The two lecture slides and corresponding matlab functions are useful for the implementation





Parametric method

1. Define the feature space, we make region spans across -10 to 10 in both feature dimensions.

2. We assume that the two categories can be represented using 2-dimensional multivariate Gaussian distributions center at the prototypical feature values. The parametric method estimates the mean and the covariance matrix to form the prototype representation of categories. You can compute sample mean $\boldsymbol{\mu}$ and sample covariance $\boldsymbol{\Sigma}$ using the exemplars for each category. [Hint: use Matlab build-in function mean() and cov()].

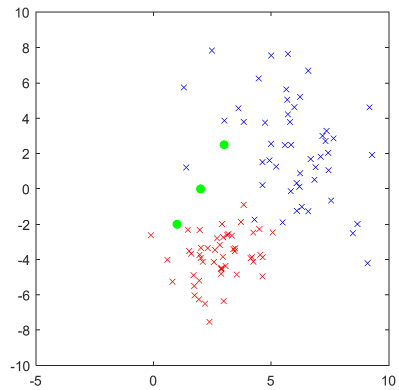
3. Compute and plot the two category distributions, P(X | category 1) and P(X | category2). [Hint: use Matlab build-in function mvnpdf() for multivariate Gaussian distribution]

$$P(\boldsymbol{X}|\boldsymbol{category}) \sim \mathcal{N}(\boldsymbol{X};\ \boldsymbol{\mu}, \boldsymbol{\Sigma})$$

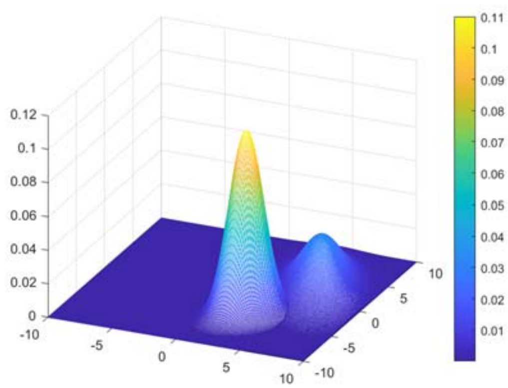
4. For each of the three testing cases, use the distributions calculated in step 3 to compute the probability of the corresponding case belonging to category 1.

Results:

Input: Exemplars in category learning and three testing cases



Category learning result: P(X | category 1) and P(X | category 2)



Categorization task result: Posterior probability for the three testing cases

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Post. prob of belonging to Category 1

postcat =

    0.9977    0.5155    0.0001
```