Using Sampling Techniques to do Bayesian Inference: Part III

Bayesian model implementation for object localization project from multiple observations

Last week we have implemented a location estimation problem from multiple observations using discretize-space method. In this exercise, we will use a special prior distribution and multiple independent observations to implement the model with the sampling approach.

Below are the mathematical specifications:

Let t be the random variable of the target location.
Let x be the observed location reported from your sensor. You measured the location three times. These observations are independent.
Assume the following:

Prior
We have prior information that the object is likely to occur in one of the two locations. Use $\mu_1 = -3, \mu_2 = 7, \sigma = 2$. Construct the prior distribution by using the sum of two Gaussian distributions:

$$P(t) \propto N(t;\ \mu_1, \sigma^2) + N(t;\ \mu_2, \sigma^2).$$

Likelihood
The three location measures are $x_1 = 3,\ x_2 = 3.4,\ x_3 = 2.6$. Use a Gaussian distribution to model the uncertainty of the sensor with $\sigma_x = 2$. Because the three observations are independent, we can multiple likelihood term for individual observation to compute the likelihood of the data.

$$P(x_1, x_2, x_3|t) = P(x_1|t)P(x_2|t)P(x_3|t);$$

$$P(x_1|t) = N(x_1; t, \sigma_x^2);\ P(x_2|t) = N(x_2; t, \sigma_x^2);\ \ P(x_3|t) = N(x_3; t, \sigma_x^2);$$

How does this work?

We can perform the same inference by using the **sampling method**:

1. Sample N possible locations t, i.e., obtain $t_1, t_2, \ldots, t_N$, based on the prior distribution
2. Compute the likelihood for the observed location x, i.e., $P(x_1, x_2, x_3|t_i)$ for each sampled t_i
3. Assign the a weight w_i to each sampled t_i such that the weight is proportional to the likelihood, i.e., $w_i \propto P(x_1, x_2, x_3|t_i)$. Remember to normalize the weights: i.e., divide each of them by $\sum_{i=1}^{N} w_i$
4. Resample from the N locations you obtained step 1 based on their weights, with replacement. In other words, the probability for a certain location t_i to be resampled should be equal to its normalized weight w_i. This new sample is, in fact, a sample from the posterior distribution.
5. Obtain an estimate from the posterior distribution (to make this comparable with the analytical method above, use mean here).

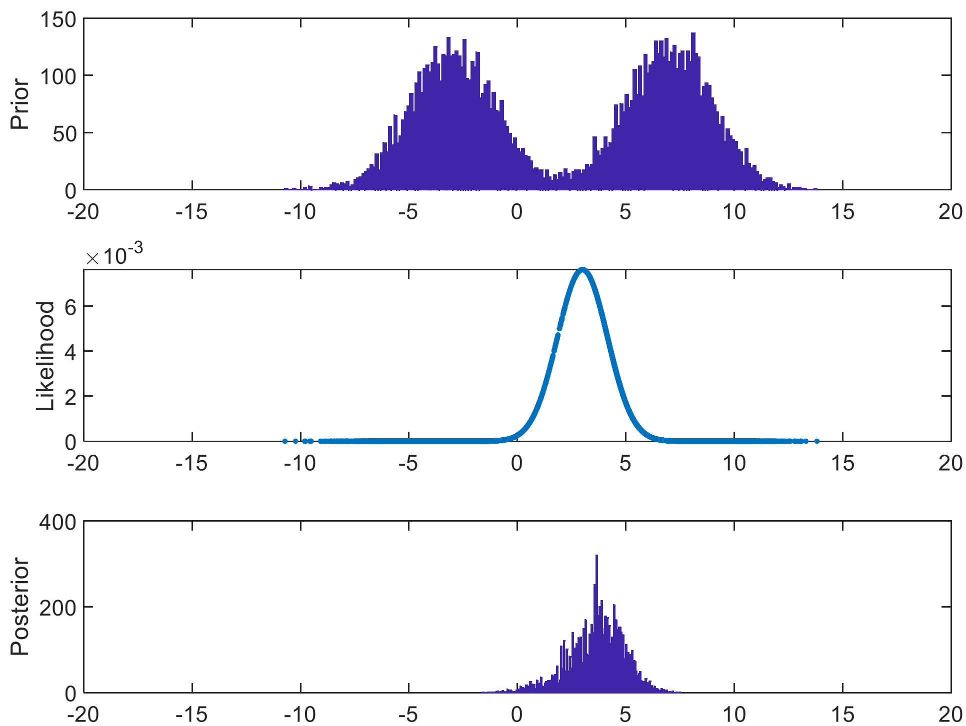
In-class Exercise

(1) Complete **SamplingExercise4.m** to get posterior samples.
(2) Compute the **mean** and **median** estimate for the target location based on the posterior samples.
(3) **Plot** prior histogram, likelihood and posterior histogram.
(4) **Compute the probability** that the target is located within the range between 3 and 5, P(3<X<5)

(5) **Compute model evidence** for the observed data

$$P(\boldsymbol{x}) = \int_{-\infty}^{+\infty} P(\boldsymbol{x}|t)P(t)dt \approx \frac{1}{N}\sum_{j=1}^{N} P(\boldsymbol{x}|t_j)$$

For each sample t_j, you have computed its likelihood $P(\boldsymbol{x}|t_j)$. The model evidence is the average of likelihoods that you computed in the previous step.




```
EstMeanSampling =

    3.6739

EstMedianSampling =

    3.8100

prob(3<X<5) = 0.594600
Model evidence P(data):
   4.7086e-04
```