CS455 Wk4 HW1

Prove by mathematical induction: The sum of consecutive cubes

$$1^3 + 2^3 + 3^3 + ... + n^3 = (1 + 2 + 3 + ... +n)^2$$

$$1^3 + 2^3 + 3^3 + ... + n^3 = \frac{n^2 * (n + 1)^2}{4}$$

- Answer=>
- Step 1. Claim=> For any n >= 0, let P(n) be the property that

  1^3 + 2^3 + 3^3 + 4^3 + …. + n^3 =(n^2 *(n+1)^2)/4

  We want to show that P(n) is true for all n >= 0

- Step 2. Base case=> As a base case, consider when n = 1.

  We will show that P(1) is true

  Left-hand side = 1^3+ … +1^3= 1 =(1^2 *(1+1)^2)/4= right-hand side

  Step 3. Induction Hypothesis=>

  Suppose P(k) were true for some fixed k >= 1

  1^3 + 2^3 + 3^3 + … + (k-1)^3 + k^3 = (k^2 * (k+1)^2)/4

  Step 4. Prove=>Now we prove that P(k+1) is true (given that P(k)is true) is as follows

Left-hand side = $1^3 + 2^3 + 3^3 + \ldots + k^3 + (k+1)^3$

$= (1^3 + 2^3 + 3^3 + \ldots + k^3) + (k+1)^3$

$= (k^2 * (k+1)^2)/4 + (k+1)^3$ by the induction hypothesis P(k)

$= (k^2 (k+1)(k+1))/4 + ((k+1)(k+1)(k+1))$

$= (k^2(k+1)(k+1))/4 + 4((k+1)(k+1)(k+1))/4$

$= (k^2 (k+1)(k+1) + 4(k+1)(k+1)(k+1)/4$

$= (k+1)(k+1)(k^2+4(k+1))/4$

$= ((k+1)^2 * (k^2 +4k+4)/4$

$=(k+1)^2 * (k^2+2k+2k+4)/4$

$= (k+1)^2 * k(k+2)+2(k+2)/4$

$=(k+1)^2*(k+2)(k+2)/4$

$=(k+1)^2 * (k+2)^2/4$

= right hand side

Therefore we have shown that if P(k)is true ,then P(k+1) is also true, for any k>=1

Step 5. The steps above have shown that for any k >= 0, if P(k) is true , then P(k+1) is also true. Combined with the base case, which shows that P(0) is true, we have shown that for all n >= 0, P(n)is true, as desired.