Statistical Methods for Reliability Data

Second Edition

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Part of *Statistical Methods for Reliability Data*
Second Edition

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Preface to the Second Edition

The first edition of *Statistical Methods for Reliability Data* (SMRD1) was published 23 years ago. We believe SMRD1 successfully met its goal of providing a comprehensive overview of statistical methods for reliability data analysis and test planning for practitioners and statisticians and we have received much positive feedback. Despite, and perhaps because of this, there were compelling reasons for a second edition (SRMD2). We (both the profession and the authors) have learned much since 1998. In our experiences consulting on reliability applications and in presenting on the order of one hundred short courses to both statisticians and reliability engineers in industry and government, we have gained a solid appreciation for what are the most important topics in the field of statistical methods for reliability, suggesting a slight change in focus for SMRD2. In teaching our respective university courses, we have discovered some improved methods for presenting some topics. These experiences helped us to develop our plan for SMRD2. For the SMRD2 project, Bill and Luis have been most fortunate to have Jave Pascual join them as a co-author.

Goals for SMRD2

Our goals for the SMRD2 were to:

- Update and improve or expand on various previously presented statistical methods for reliability data, using statistical and computational methods that have been developed or become readily available since SMRD1 was published.

- Improve the organization of the material to make it possible to cover more and the most important topics in a one-semester course.

- Completely rewrite chapters where there have been important developments or changes in what are considered to be best practices in the analysis of reliability data.

- Provide a more extensive treatment of the use and application of Bayesian methods in reliability data analysis.

- Provide, via technical appendices, additional justification and theory underlying the statistical methods presented in this book.

- Provide a webpage that gives up-to-date information about available software for doing reliability data analysis, supplementary information such as presentation slides, additional data sets, and exercises, as well as other important up-to-date information developed or coming to our attention after publication.

What Has Not Changed

In SMRD2, we have paid special care to retain the appealing features of SMRD1. Specifically, the *special features of the book*, listed in the preface of SMRD1 are still intact.

Important Changes in SMRD2

Important changes in SMRD2 include the following:

- Although SMRD1 has been popular with both engineers and statisticians, in the preparation of SMRD2 there has been a concerted effort to look for ways to improve the presentation and usability of the book. Means for doing this have included additional words of explanation, additional examples using simulation to provide insights, moving some technical details to appendices, and omitting topics that, while of some technical interest, have had little or no value in practical applications.
- We have added a section on the important topic of the distribution of remaining life to the chapter *Models, Censoring, and Likelihood for Failure-Time Data* (Chapter 2).
- We now include a section describing the important Fréchet distribution in the chapter *Some Parametric Distributions Used in Reliability Applications* (Chapter 4).
- SMRD1 Chapter 5 (*Other Parametric Distributions*) has been eliminated, with the important and useful material being moved to either Chapter 4 or the chapter *Special Parametric Models* (Chapter 11).
- The chapter *Parametric Bootstrap and Other Simulation-Based Confidence Interval Methods* (Chapter 9), has been completely rewritten to reflect many new developments since SMRD1 was published, including the use of the fractional-random-weight bootstrap and generalized pivotal quantities.
- The chapter *An Introduction to Bayesian Statistical Methods for Reliability* (Chapter 10) replaces, updates, and expands SMRD1 Chapter 14. The chapter has been completely rewritten with a more modern slant on prior distribution specification and computational methods, with several illustrations of Bayesian applications. Then, in most subsequent chapters, Bayesian methods are integrated into the development and presentation of many statistical analyses where some prior information is available (and ignoring it would be wrong) or where there is other strong motivation to use Bayesian methods.
- SMRD1 Chapter 10 has been completely rewritten to focus on *Planning Life Tests for Estimation* (now Chapter 13) and to improve clarity and usability of the material.
- The material in the chapter *Planning Reliability Demonstration Tests* (Chapter 14) is mostly new, where the SMRD1 material (previously in a small part of SMRD1 Chapter 10) has been completely rewritten and generalized to allow planning demonstration tests for any log-location-scale distribution and tests that allow failures to occur (and still pass the test), providing demonstration tests that have much improved probability of successful demonstration (i.e., power).
- The chapter *Prediction of Failure Times and the Number of Future Field Failures* (Chapter 15) has also been completely rewritten to reflect simpler and more direct methods to obtain prediction intervals in reliability applications. These include the use of *predictive distributions* for both non-Bayesian and Bayesian prediction. Also, we now put more focus on the important applications of predicting the number of field failures and warranty returns.
- Instead of one combined chapter on system reliability and the analysis of competing risks (SMRD1 Chapter 15), these topics are now covered more extensively in two separate chapters: *System Reliability Concepts and Methods* (Chapter 5) and *Analysis of Data with More than One Failure Mode* (Chapter 16), respectively.
- The material on regression analysis of failure-time data and accelerated life testing has been reorganized with many improvements and new examples, now in Chapters 17, 18, and 19.
- We have added a completely new chapter, *Degradation Modeling and Destructive Degradation Data Analysis* (Chapter 20), to describe and illustrate the use of these important statistical methods.

- To save space and improve organization, the two SMRD1 chapters on degradation modeling and analysis (SMRD1 Chapters 13 and 21) have been combined and completely rewritten to form *Repeated-Measures Degradation Modeling and Analysis* (Chapter 21). To make inferences from repeated-measures degradation data, we have replaced the previously used maximum likelihood and bootstrap methods for nonlinear mixed-effects models with the more versatile Bayesian hierarchical models and inference methods.

- Almost all of the figures in SMRD2 have been redrawn, both to improve quality and to introduce color (for the electronic versions of SMRD2). We have, however, continued to design our graphics so that having color is not necessary.

- Numerous new references have been added or updated and the Bibliographic Notes and Related Topics sections at the end of each chapter have been expanded and reorganized to make it easier to find references and additional information for particular topics.

- Many new data sets and examples have been added throughout SMRD2. We also have added many new applications to illustrate the use of the methods that we present. As in SMRD1, all of these applications are based on real data. In some of the data sets, however, we have changed the names of the variables and/or the scale of the data to protect sensitive information. The data in Section 23.3 had to be simulated, but they reflect the interesting statistical aspects of the real applications.

- Many new exercises have been added at the end of the chapters.

- Some tables containing small reliability data sets remain in the chapters and in Appendix D. These data sets and all new data sets, used in examples and exercises, are available as csv files from the SRMD2 webpage.

What Was Dropped

Some material from SMRD1 Chapter 5 (*Other Parametric Distributions*) and Chapter 20 (*Planning Accelerated Life Tests*) has been dropped. As mentioned earlier, the useful material from Chapter 5 has been integrated into either Chapter 4 or Chapter 11. The important ideas behind planning accelerated life tests are illustrated and briefly described in some of the accelerated testing examples in Chapters 18 and 19 with a summary of the key points in Section 19.4.3. To save space for more important material, we have also dropped a few tables of lengthy data sets. These data sets (and many others) are, however, available on SMRD2's webpage.

Overview and Paths through SMRD2

There are many paths that readers and instructors might take through this book. Chapters 1–8 cover single distribution models without any explanatory variables. This is basic material that will be of interest to almost all readers and should be read in sequence. It is possible to do only a light reading of Chapter 4 (e.g., focusing on the Weibull and lognormal distributions) before going on to the important methods in Chapters 6–8. Chapter 5 introduces some important system reliability concepts. The material on the reliability of a series system is used in Chapter 16, but otherwise this chapter is not prerequisite for material in subsequent chapters.

Chapters 9–16 and Chapter 22 describe special but important topics in reliability data analysis. While the material in these chapters depends on earlier chapters, the order of reading is not important.

Chapter 9 explains and illustrates the use of parametric bootstrap and other simulation-based methods for obtaining confidence intervals. These methods are becoming more popular, now that they are easy to use in certain software packages (e.g., JMP). These intervals, in general, have better statistical properties (i.e., coverage probabilities closer to nominal confidence levels) than the traditional confidence intervals methods based on Wald approximations. Although confidence intervals based on Wald approximations are usually sufficient for initial and other exploratory analyses of reliability data, we recommend using

these more trustworthy methods before reporting final results. The simulation-based methods in this chapter are also used in Chapter 15 to obtain prediction intervals.

Chapter 10 provides an introduction to the use of modern Bayesian methods in the analysis of reliability data. These methods are becoming increasingly popular, now that we have the software tools that make implementation possible without the need for complicated computer programming. Bayesian applications are illustrated in numerous examples in subsequent chapters.

Chapter 11 describes and illustrates several important statistical models that arise in applications. These include the limited-failure population model (also known as the defective sub-population model), truncated data, and maximum likelihood estimation for the generalized gamma, Birnbaum–Saunders, and threshold distributions.

Chapter 12 is a new chapter that describes nonparametric and parametric statistical methods for comparing two or more failure-time distributions, extending methods in Chapters 3 and 8, respectively.

Chapter 13 focuses on test planning: evaluating the effects of choosing sample size and test length when the purpose is *estimation*. Chapter 14 covers reliability *demonstration* tests.

Chapter 15 shows how to obtain prediction intervals for failure times and for the number of failures in a future time interval. The latter application is important for warranty and other field-failure prediction applications. We present both non-Bayesian and Bayesian methods.

Chapter 16 describes statistical methods for estimating failure-time distributions for individual failure modes and how to evaluate the effect of eliminating a failure mode.

Chapter 17 introduces models and methods for failure-time regression analysis. Relatedly, Chapters 18 and 19 present physically based acceleration relationships and show how to analyze accelerated life-test data.

The first two sections of Chapter 20 provide background and motivation for degradation analysis and degradation path models that are used for both destructive and repeated-measures degradation data. The remainder of the chapter focuses on statistical models and methods for *destructive* degradation data. Chapter 21 describes statistical models and methods for *repeated-measures* degradation data.

Chapter 22 presents an introduction to statistical methods for recurrent-event data—an important topic with many applications and one that needs to be contrasted with traditional time-to-event (or failure-time) data (e.g., involving time-to-first failure of systems or components and other replaceable units).

Chapter 23 provides case studies that illustrate, in the context of interesting additional applications, the integration of ideas presented throughout the book. This chapter also illustrates how some of the general methods presented in the earlier chapters can be extended and adapted to deal with new applications.

Appendices

We use appendices to present supplementary material and to provide technical details that are not needed to apply the statistical methods in the body of SMRD2.

- Appendix A provides a summary and index of notation and acronyms used in SMRD2.
- Appendix B provides properties of important probability distributions that are used in SMRD2, but that are not described in Chapter 4.
- Appendix C outlines the general maximum likelihood and some other statistical theory on which most of the methods in SMRD2 are based.
- Appendix D gives tables for some of the data sets used in our examples. These and many other data sets that are used in SMRD2 examples, and exercises are also available on SMRD2's webpage.

Use as a Course Textbook or for a Workshop

A two-semester course would be required to cover thoroughly all of the material in the book. For a one-semester course, aimed at engineers and/or statisticians, one could cover Chapters 1–8 and Chapters 17–19, along with selected material from the appendices, and a few other chapters according to the interests,

background, and tastes of the students. We have written a collection of solutions for selected exercises and these will be available from John Wiley for instructors who adopt SMRD2 as a textbook for their class.

SMRD2 could be used as the basis for workshops or short courses aimed at engineers or statisticians working in industry. For an audience with a working knowledge of basic statistical tools, *key sections* from Chapters 1–8 could be covered in one day. If the purpose of the short course is to introduce the basic ideas and illustrate with examples, then some material from Chapters 17–19 could also be covered. For a less experienced audience or a presentation allowing time for computer exercises and discussion, at least two days would be needed to cover this material. Extending the course to three or four days would allow covering selected topics in Chapters 9–16, 20 and 21.

In our SMRD2 writing process, we first developed presentation slides to provide a detailed outline for each chapter. These slides are used in our university courses. We will make these available on our webpage, as they might be useful for others wanting to teach from SMRD2. In our short courses, we use a relatively small subset of these slides focusing on key technical points, applications, and interpretation of results, combined with an extensive set of hands-on computer exercises.

SMRD2 Reliability Data

The data sets used in the SMRD2 examples and exercises have been archived and are publicly available on DataShare, Iowa State University's open data repository, and can be accessed at: https://doi.org/10.25380/iastate.c.5395665.

Computer Software

Most of the numerical examples in this book were done using the `R` system for graphics and data analysis (**?**). `R` functions were developed in parallel with the writing of SMRD2 and these are part of the `R` package RSplida. Although we have not included explicit information about RSplida in the chapters, a version of RSplida and commands that were used for most of the examples in SMRD2 and all of the data sets are available on SMRD2's webpage and at:
https://wqmeeker.stat.iastate.edu/RSplida.zip. We also illustrate, in many examples, the use of `R` as a sophisticated calculator. In another development branch from RSplida, the `R` package SMRD, with lead developer Jason Freels, provides much of the functionality to do the examples in SMRD1 and SMRD2. This package can be found and installed from: https://github.com/Auburngrads/SMRD.

Many commercial statistical software products (e.g., JMP, MINITAB, SAS, and WEIBULL++) do reliability data analysis. New versions of these packages with improved capabilities for doing reliability data analysis, such as those discussed in this book, are released periodically. Therefore, instead of directly discussing current features of popular software packages—which would become rapidly outdated—we provide this information in an Excel spreadsheet accessible from SMRD2's webpage at:
https://www.wiley.com\go\meeker\reliability2e and plan to update this webpage to keep it current.

More on SMRD2's Webpage

The Wiley webpage for SMRD2 can be found at https://www.wiley.com\go\meeker\reliability2e. In addition to the link to the RSplida package and the Excel spreadsheet on current reliability capabilities of popular software, this webpage will provide other resources such as our presentation slides, supplementary tables and figures for reliability data analysis and reliability test planning.

We plan to update this webpage periodically by adding new materials and references, (numerous, we hope) reader comments and experiences, and (few, we hope) corrections.

Happy reading!

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