Chapter 18

Analyzing Accelerated Life Test Data

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Chapter 18
Accelerated Life Test Analysis

Topics discussed in this chapter are:

- Motivation and applications of accelerated reliability testing.
- Use rate, temperature, and voltage acceleration methods.
- Graphical and maximum likelihood methods of presenting and analyzing accelerated life test (ALT) data.
- Bayesian analysis on ALT data.
- Diagnostics and model checking.

Chapter 18

Analyzing Accelerated Life Test Data

Segment 1

Introduction to Accelerated Testing and a Simple Example of Temperature Acceleration

Accelerated Tests Increasingly Important

Today's manufacturers need to develop newer, higher technology products in record time while improving productivity, reliability, and quality.

Important issues:

- Rapid product development.
- Rapidly changing technologies.
- More complicated products with more components.
- Higher customer expectations for better reliability.

Need for Accelerated Tests

Need timely information on high-reliability products.

- Modern products designed to last for years or decades.
- Accelerated Tests used for timely assessment of reliability of product components and materials.
- Tests at high levels of use rate, temperature, voltage, pressure, humidity, etc.
- Estimate life at **use conditions**.

  Note: Estimation/prediction from Accelerated Tests involves **extrapolation.**

Applications of Accelerated Tests

Applications of Accelerated Tests include:

- Evaluating the effect of stress on life.
- Assessing component reliability.
- Demonstrating component reliability.
- Comparing two or more competing products.
- Evaluation of a proposed material change.
- Establishing safe warranty times.

Different Methods of Acceleration

Three fundamentally different methods of accelerating a reliability test:

- Increase the use-rate of the product (e.g., test a toaster 200 times/day). Higher use rates reduce test time. This is useful if life adequately modeled by cycles of operation. It is reasonable if cycling simulates actual use and if test units return to steady state after each cycle.
- Use elevated temperature or humidity to increase rate of failure-causing chemical/physical process.
- Increase stress (e.g., voltage or pressure) to make degrading units fail more quickly.

  Use a **physical/chemical** (preferable) or **empirical** model relating degradation or lifetime to **use** conditions.

Example: Temperature-Accelerated Life Test on Device-A

- Device-A units were tested at 10, 40, 60, and 80°C for 5000 hours.

- Even though no failures were expected at the use conditions of 10°C, useful information from degradation measurements was obtained.

- There was heavy censoring at the lower levels of temperature.

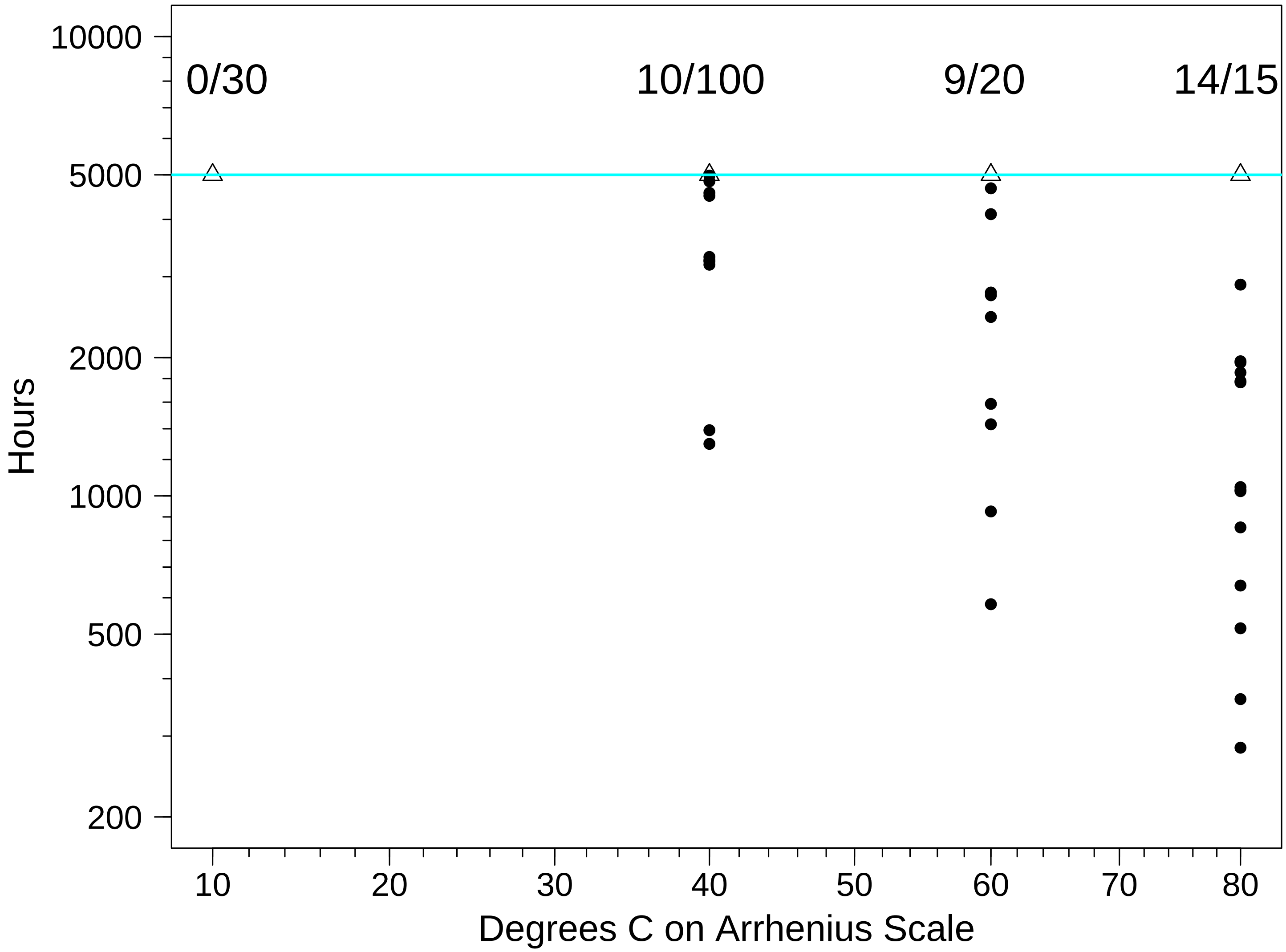
- The purpose of the test was to determine if the device would meet its reliability needs at 10,000 and 30,000 hours at an operating temperature of 10°C.

- Data were originally analyzed in Hooper and Amster (1998).

Hours Versus Temperature Data from a Temperature-Accelerated Life Test on Device-A

		Number	Temperature	In Subexperiment	
Hours	Status	of Devices	°C	Units	Failures
5000	Censored	30	10	30	0/30
1298	Failed	1	40	100	10/100
1390	Failed	1	40		
⋮	⋮	⋮	⋮		
5000	Censored	90	40		
581	Failed		60	20	9/20
925	Failed		60		
1432	Failed		60		
⋮	⋮	⋮	⋮		
5000	Censored	11	60		
283	Failed	1	80	15	14/15
361	Failed	1	80		
515	Failed	1	80		
638	Failed	1	80		
⋮	⋮	⋮	⋮		
5000	Censored	1	80		

Device-A Hours Versus Temperature
Log-Arrhenius Scale

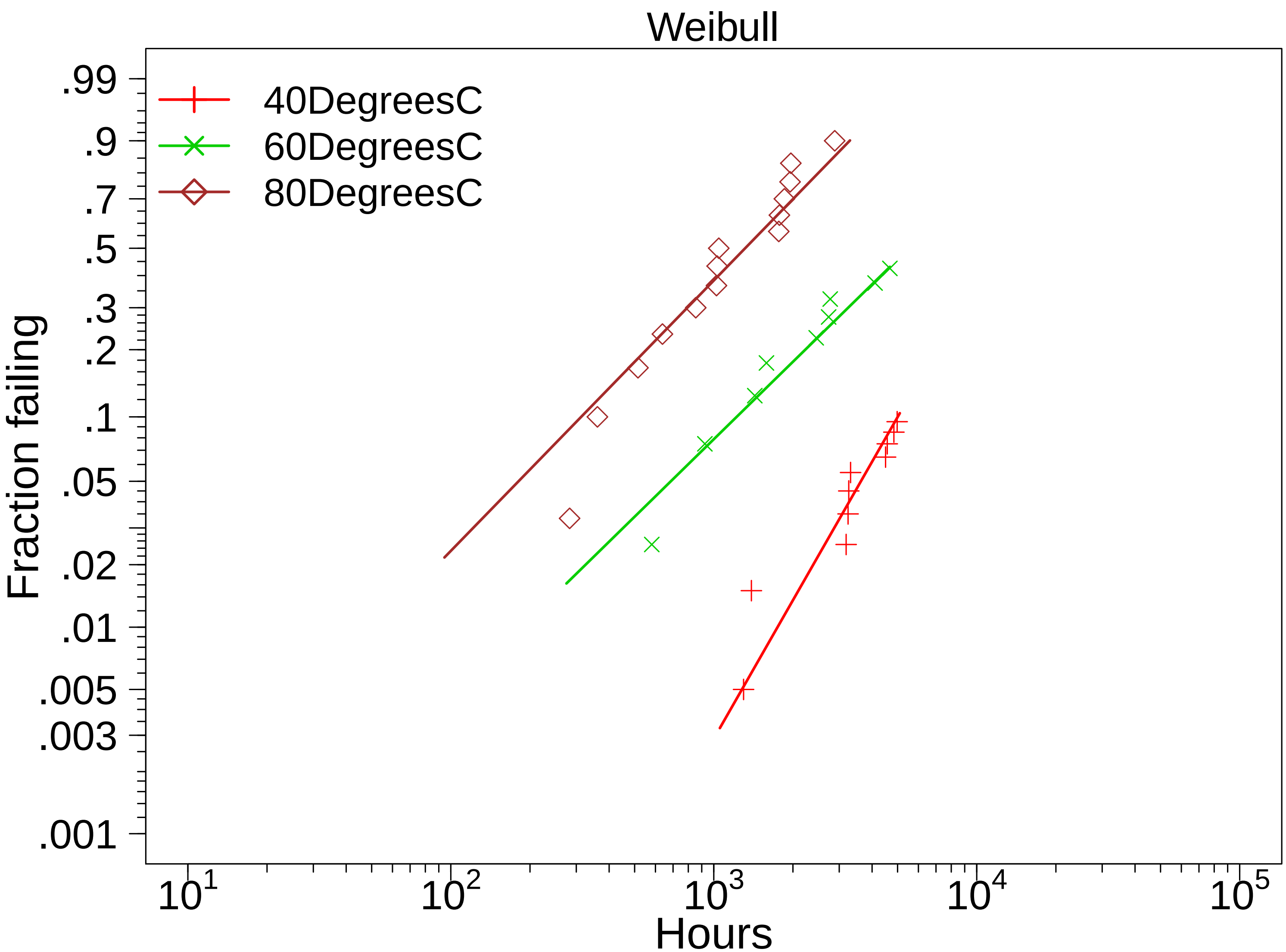


Strategy for Analyzing ALT Data

- Examine a scatterplot of lifetime versus stress, using different symbols for censored observations.
- Use a multiple probability plot to study the data from the individual subexperiments.
- Use a multiple probability plot to study the data from the individual subexperiments with a equal shape (σ or β) parameter.
- Fit an overall model involving a life/stress relationship. Display results on a scatterplot and a multiple probability plot.
- Perform residual analysis and other diagnostic checks.
- Perform a sensitivity analysis.
- Assess the reasonableness of using the ALT data to make the desired inferences.

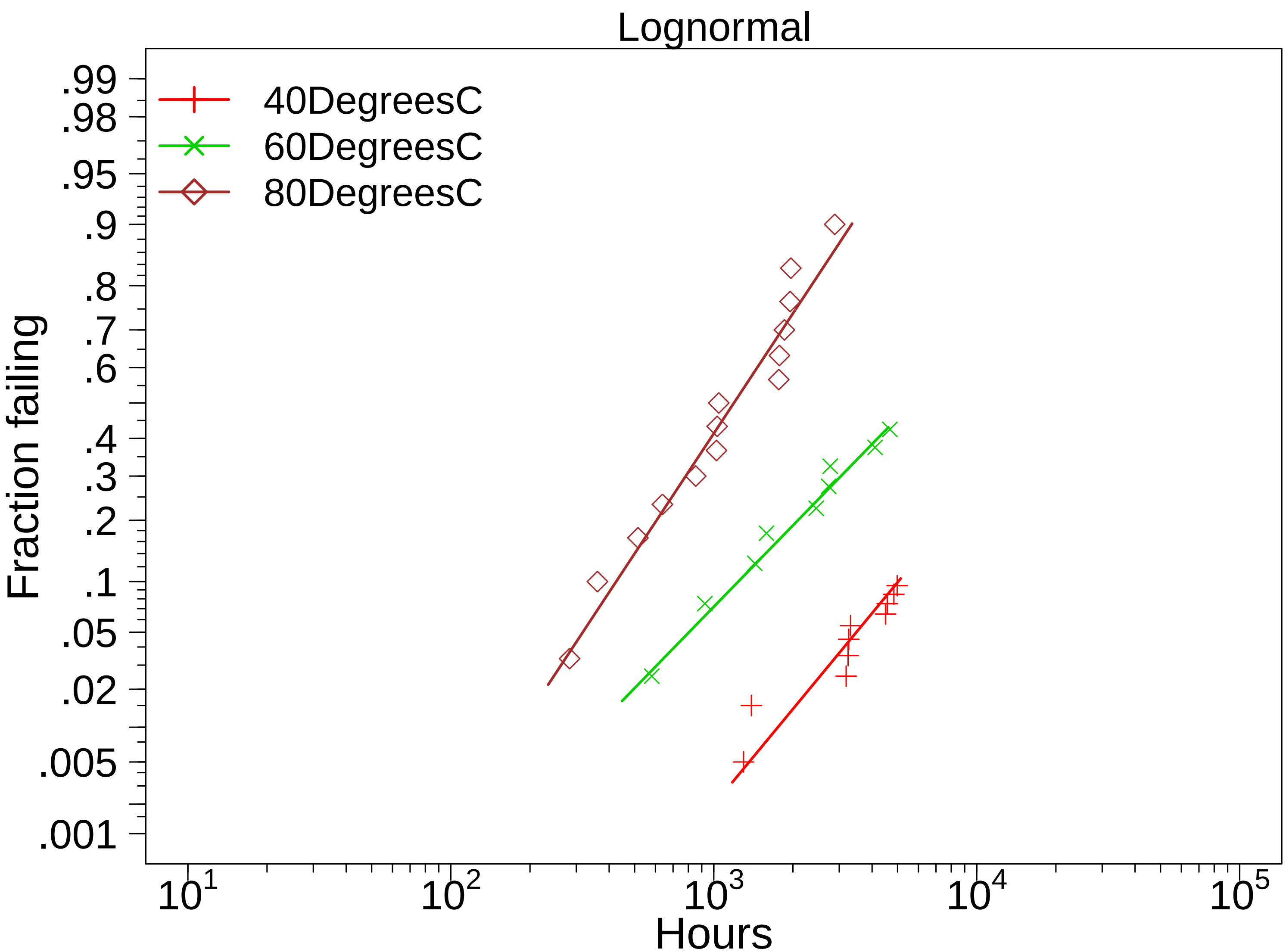
Weibull Multiple Probability Plot Device-A ALT
Weibull ML Fits Each Level of Temperature
Different Shape Parameters

$$\widehat{\Pr}[T(\mathbf{Temp}_i) \le t] = \Phi_{\mathbf{sev}}\left[\frac{\log(t) - \widehat{\mu}_i}{\widehat{\sigma}_i}\right], \qquad i = 40, 60, 80$$



Lognormal Multiple Probability Plot Device-A ALT Lognormal ML Fits Each Level of Temperature Different Shape Parameters

$$\widehat{\Pr}[T(\mathbf{Temp}_i) \le t] = \Phi_{\mathbf{norm}}\left[\frac{\log(t) - \widehat{\mu}_i}{\widehat{\sigma}_i}\right], \quad i = 40, 60, 80$$

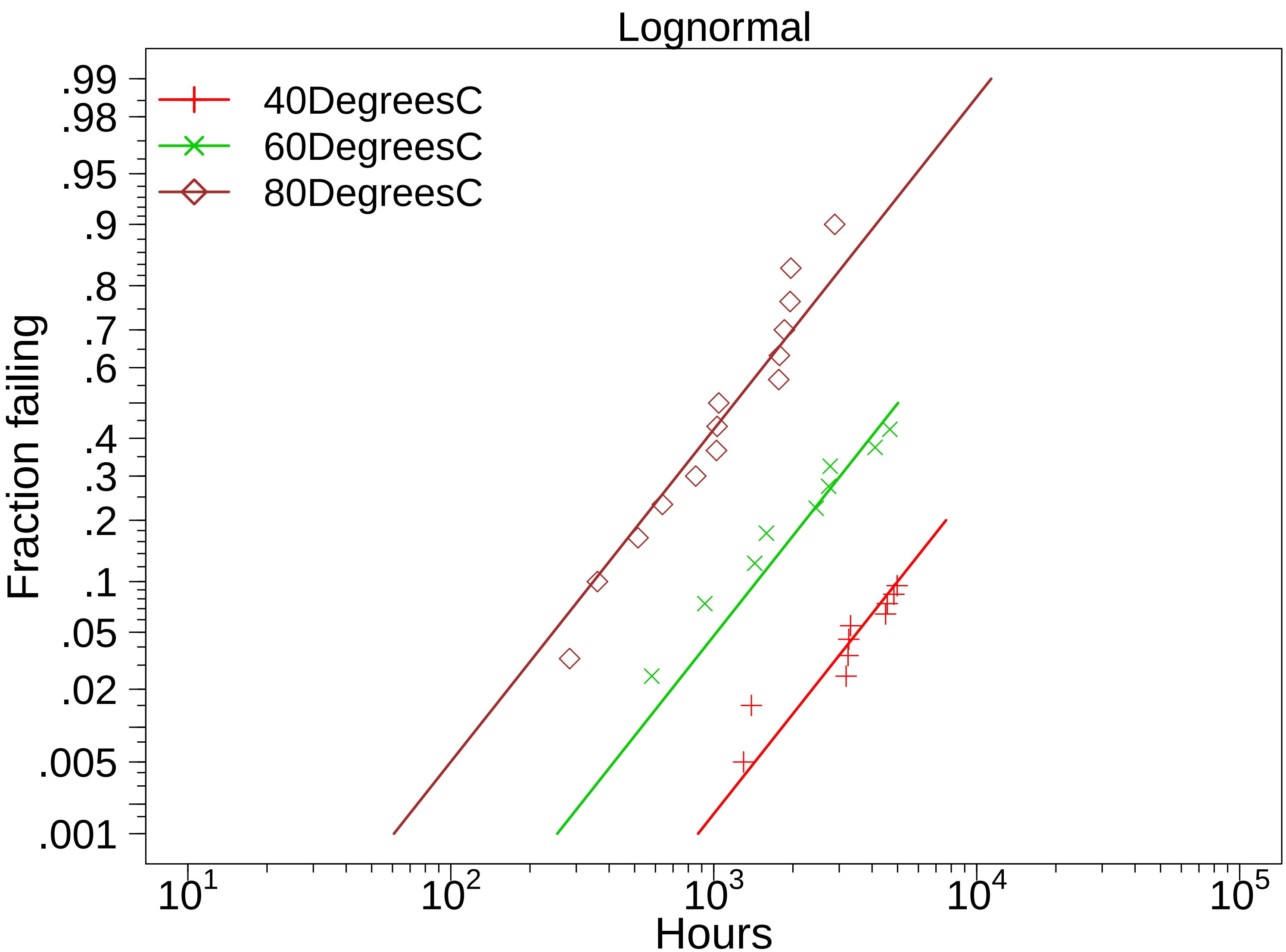


Device-A ALT Lognormal ML Estimation Results at Individual Temperatures

	Parameter	ML Estimate	Standard Error	95% Approximate Confidence Interval Lower	Upper
40°C	μ	9.81	0.42	8.9	10.6
	σ	1.0	0.27	0.59	1.72
60°C	μ	8.64	0.35	8.0	9.3
	σ	1.19	0.32	0.70	2.0
80°C	μ	7.08	0.21	6.7	7.5
	σ	0.80	0.16	0.55	1.17

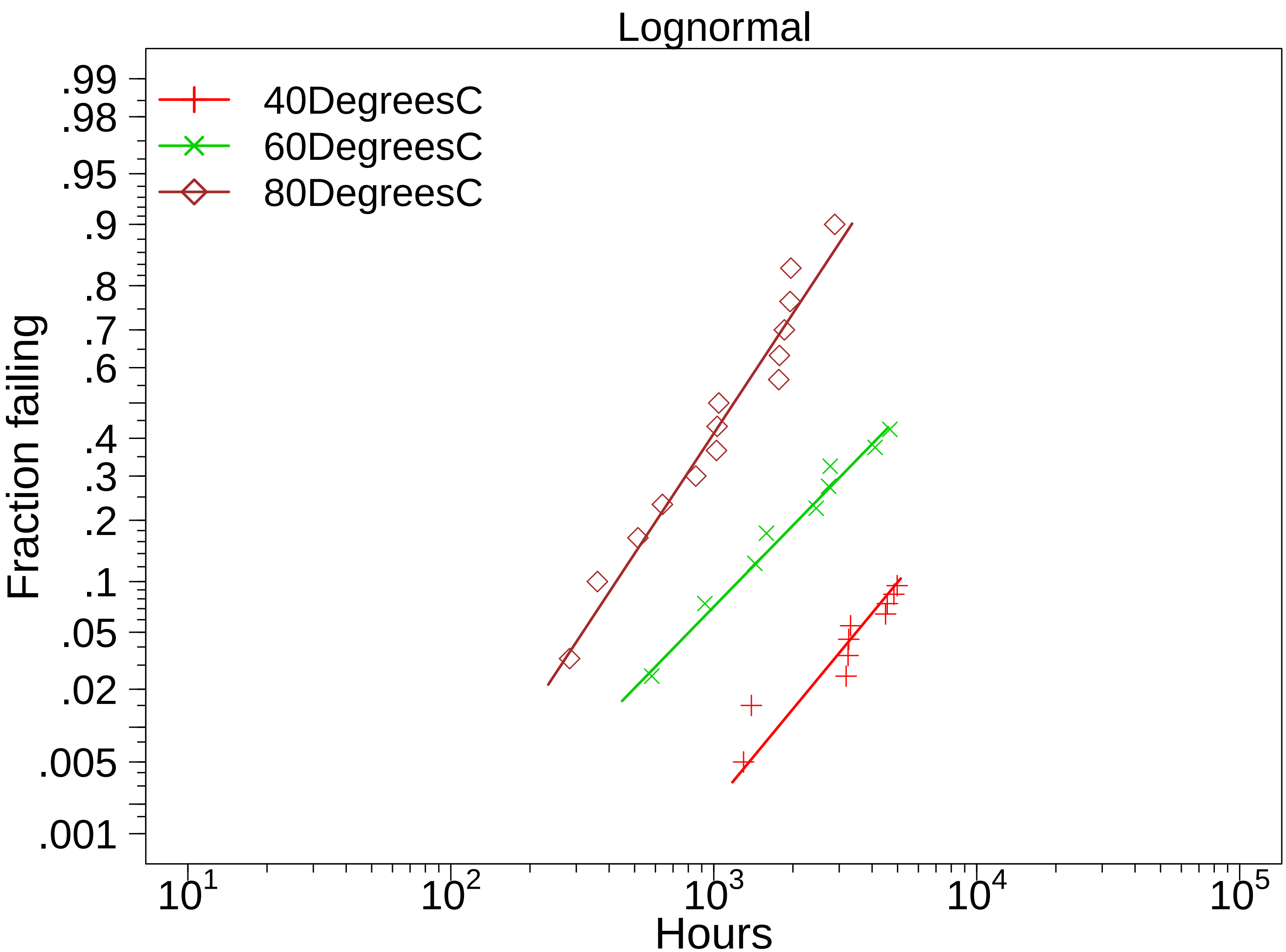
Lognormal Multiple Probability Plot Device-A ALT
Lognormal ML Fits Each Level of Temperature
Equal Shape Parameter

$$\widehat{\Pr}[T(\mathbf{Temp}_i) \leq t] = \Phi_{\mathbf{norm}}\left[\frac{\log(t) - \widehat{\mu}_i}{\widehat{\sigma}}\right], \qquad i = 40, 60, 80$$



Lognormal Multiple Probability Plot Device-A ALT
Lognormal ML Fits Each Level of Temperature
Different Shape Parameters

$$\widehat{\Pr}[T(\mathbf{Temp}_i) \le t] = \Phi_{\mathbf{norm}}\left[\frac{\log(t) - \widehat{\mu}_i}{\widehat{\sigma}_i}\right], \quad i = 40, 60, 80$$



Device-A ALT Lognormal ML Estimation Results at Individual Temperatures with Equal Shape σ

	Parameter	ML Estimate	Standard Error	95% Approximate Confidence Interval Lower	95% Approximate Confidence Interval Upper
40°C	μ	9.75	0.25	9.3	10.2
60°C	μ	8.52	0.25	8.0	9.0
80°C	μ	7.09	0.25	6.6	7.6
	σ	0.97	0.13	0.71	1.22

The Arrhenius-Lognormal Regression Model

The Arrhenius-lognormal regression model is

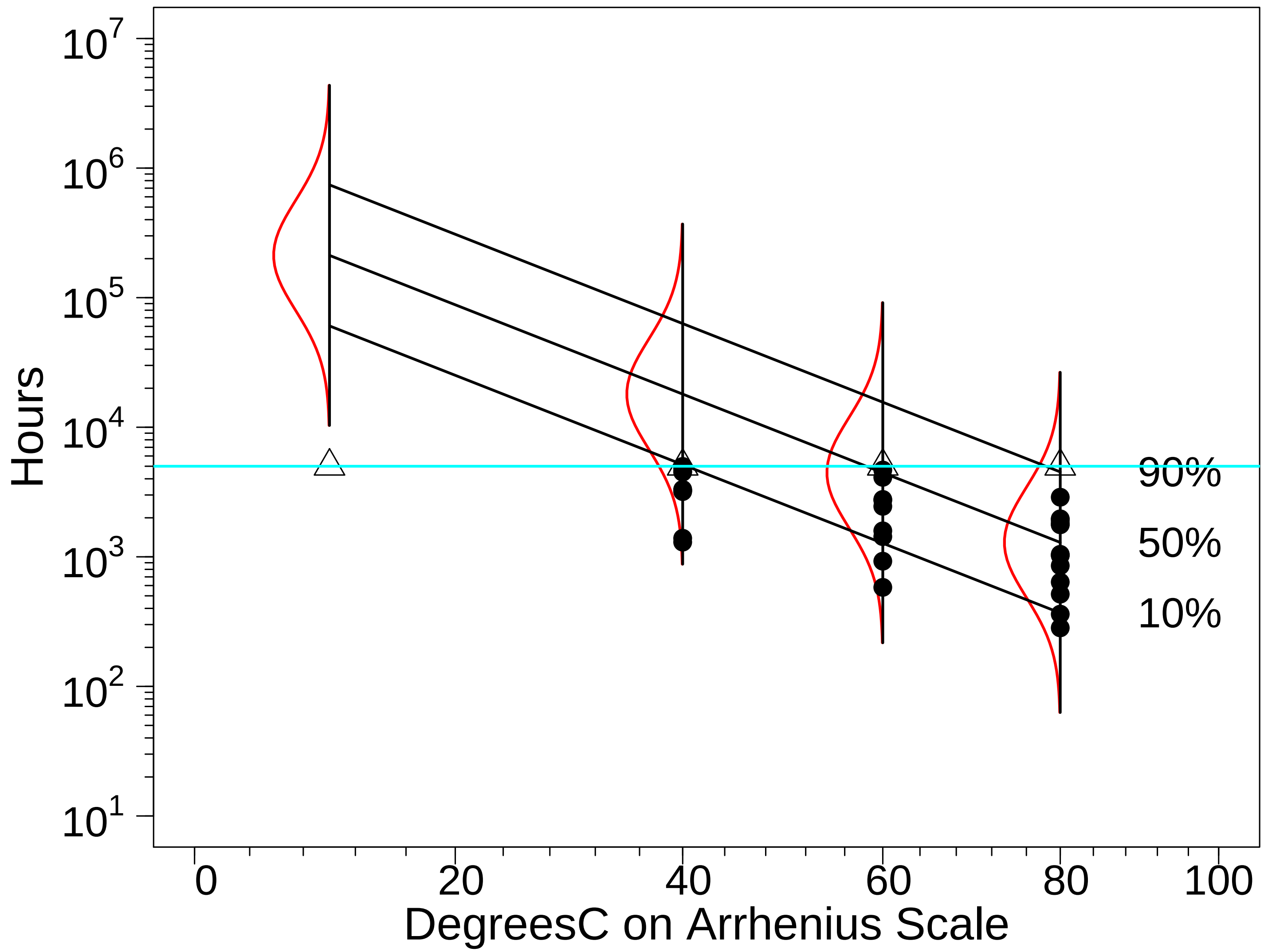
$$\Pr[T(\text{Temp}) \leq t] = \Phi_{\text{norm}}\left[\frac{\log(t) - \mu(x)}{\sigma}\right]$$

where

- $\mu(x) = \beta_0 + \beta_1 x$,
- $x = \frac{11604.52}{\text{Temp}\,^\circ\text{C} + 273.15}$,
- $\beta_1 = E_a$ is the effective activation energy in electron volts (eV), and
- σ is assumed to be constant.

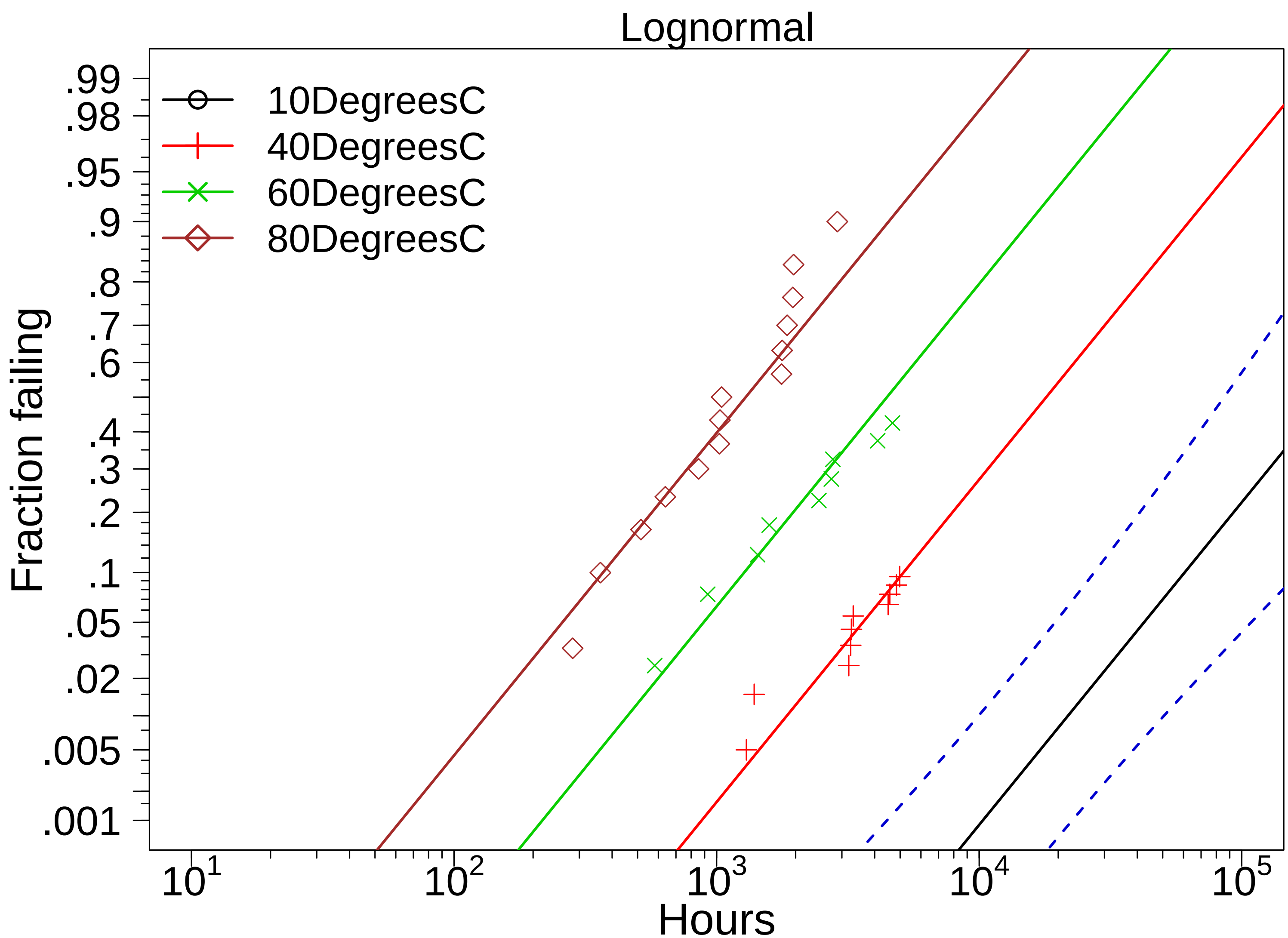
Scatterplot Showing the Arrhenius-Lognormal Regression Model Fit to the Device-A ALT Data

$$\log[\widehat{t}_p(x)] = \widehat{\mu}(x) + \Phi^{-1}_{\mathbf{norm}}(p)\widehat{\sigma}, \qquad \widehat{\mu}(x) = \widehat{\beta}_0 + \widehat{\beta}_1 x$$



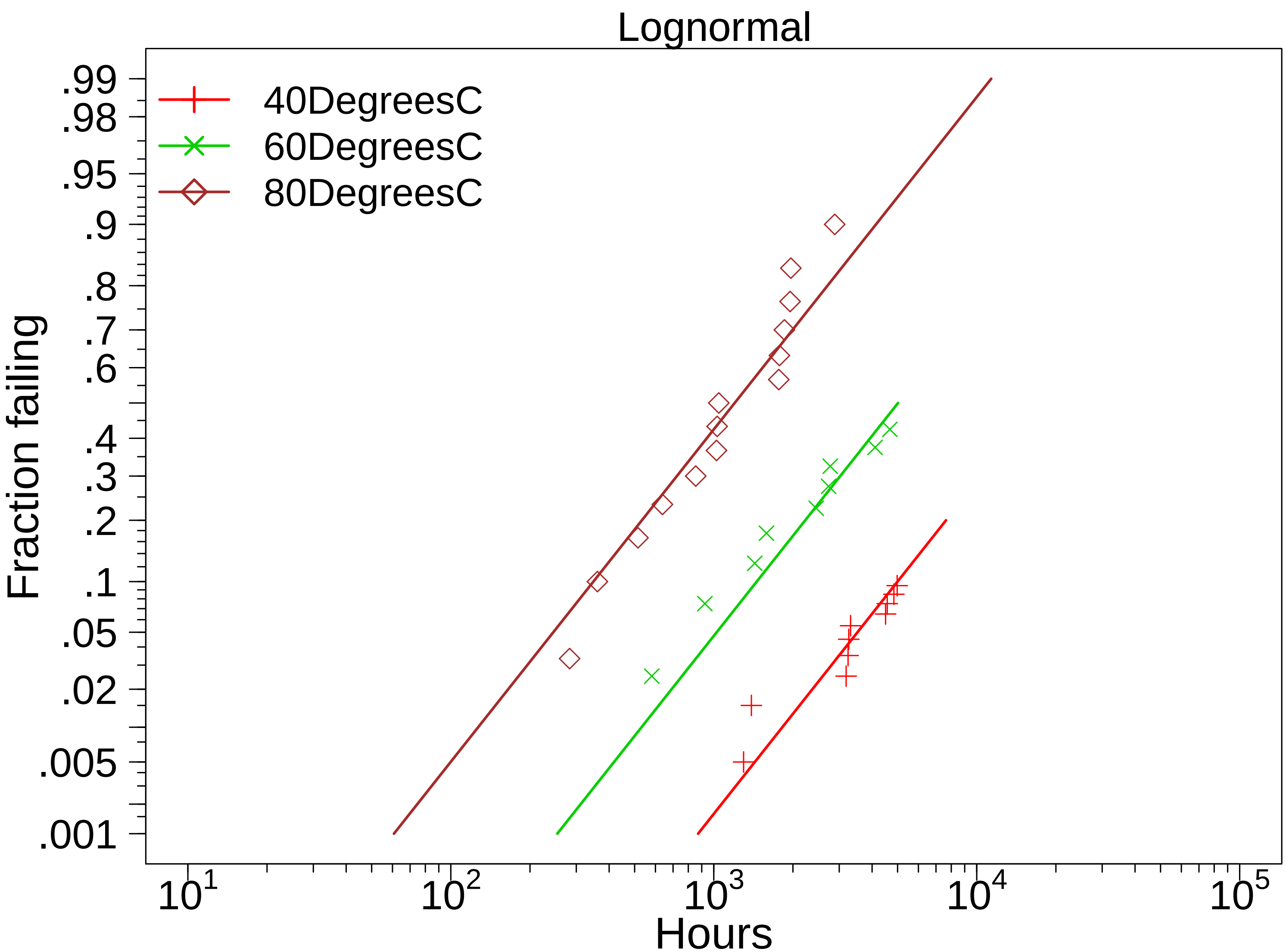
Lognormal Multiple Probability Plot of the Arrhenius-Lognormal Regression Model ML Fit to the Device-A ALT Data

$$\widehat{\Pr}[T(\textbf{Temp}) \leq t] = \Phi_{\textbf{norm}}\left[\frac{\log(t)-\widehat{\mu}(x)}{\widehat{\sigma}}\right], \qquad \widehat{\mu}(x) = \widehat{\beta}_0 + \widehat{\beta}_1 x$$



Lognormal Multiple Probability Plot Giving Individual Lognormal ML Fits with Equal Shape Parameter to Each Level of Temperature for Device-A ALT Data

$$\widehat{\Pr}[T(\textbf{Temp}_i) \le t] = \Phi_{\textbf{norm}}\left[\frac{\log(t) - \widehat{\mu}_i}{\widehat{\sigma}}\right], \qquad i = 40, 60, 80$$



ML Estimation Results for the Device-A ALT Data and the Arrhenius-Lognormal Regression Model

Parameter	ML Estimate	Standard Error	95% Approximate Confidence Intervals Lower	Upper
β_0	-13.5	2.9	-19.1	-7.8
β_1	0.63	0.08	0.47	0.79
σ	0.98	0.13	0.75	1.28

Chapter 18

Analyzing Accelerated Life Test Data

Segment 2

Diagnostics and Model Checking
Confidence Intervals for Quantiles
and Failure Probabilities

Checking Model Assumptions

It is important to check model assumptions by using likelihood-ratio (LR) tests and residual analysis.

- Define standardized residuals as
$$\exp\left\{\frac{\log[t(x_i)] - (\widehat{\beta}_0 + \widehat{\beta}_1 x_i)}{\widehat{\sigma}}\right\}$$
where $t(x_i)$ is a failure or censoring time at x_i.
- Residuals corresponding to censored observations are called **censored** standardized residuals.
- Make a probability plot of the residuals.
- Plot residuals versus the fitted values given by $\exp\left(\widehat{\beta}_0 + \widehat{\beta}_1 x_i\right)$.
Note: For the Device-A data, these plots do not indicate departures from the model assumptions.

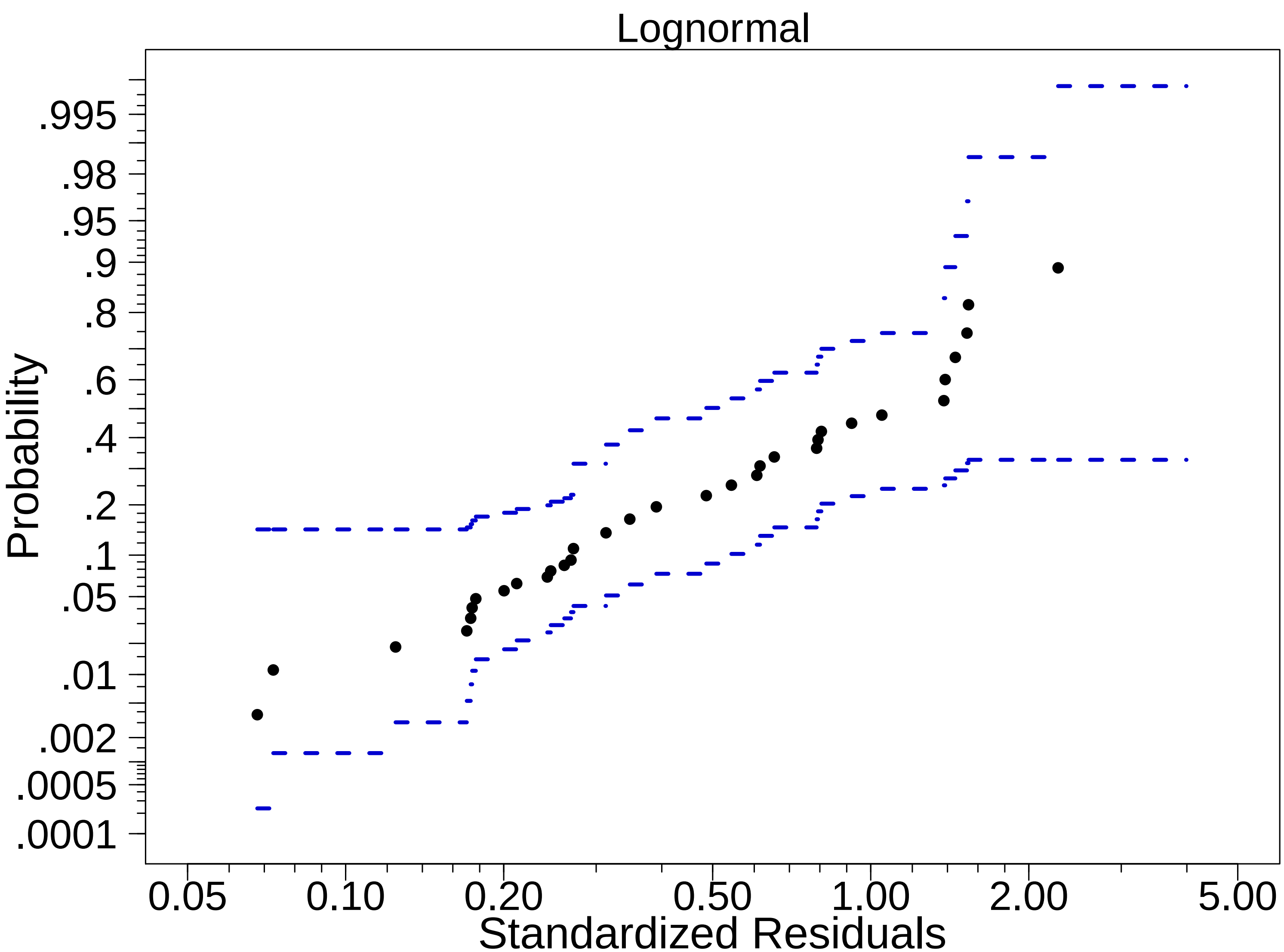
Device-A Lognormal Model-Fitting Summary

Model	-2LogLike	AIC	# Param
SepDists	641.5	653.5	6
EqualSig	643.0	653.0	4/5
RegrModel	643.4	649.4	3
Pooled	724.1	728.1	2

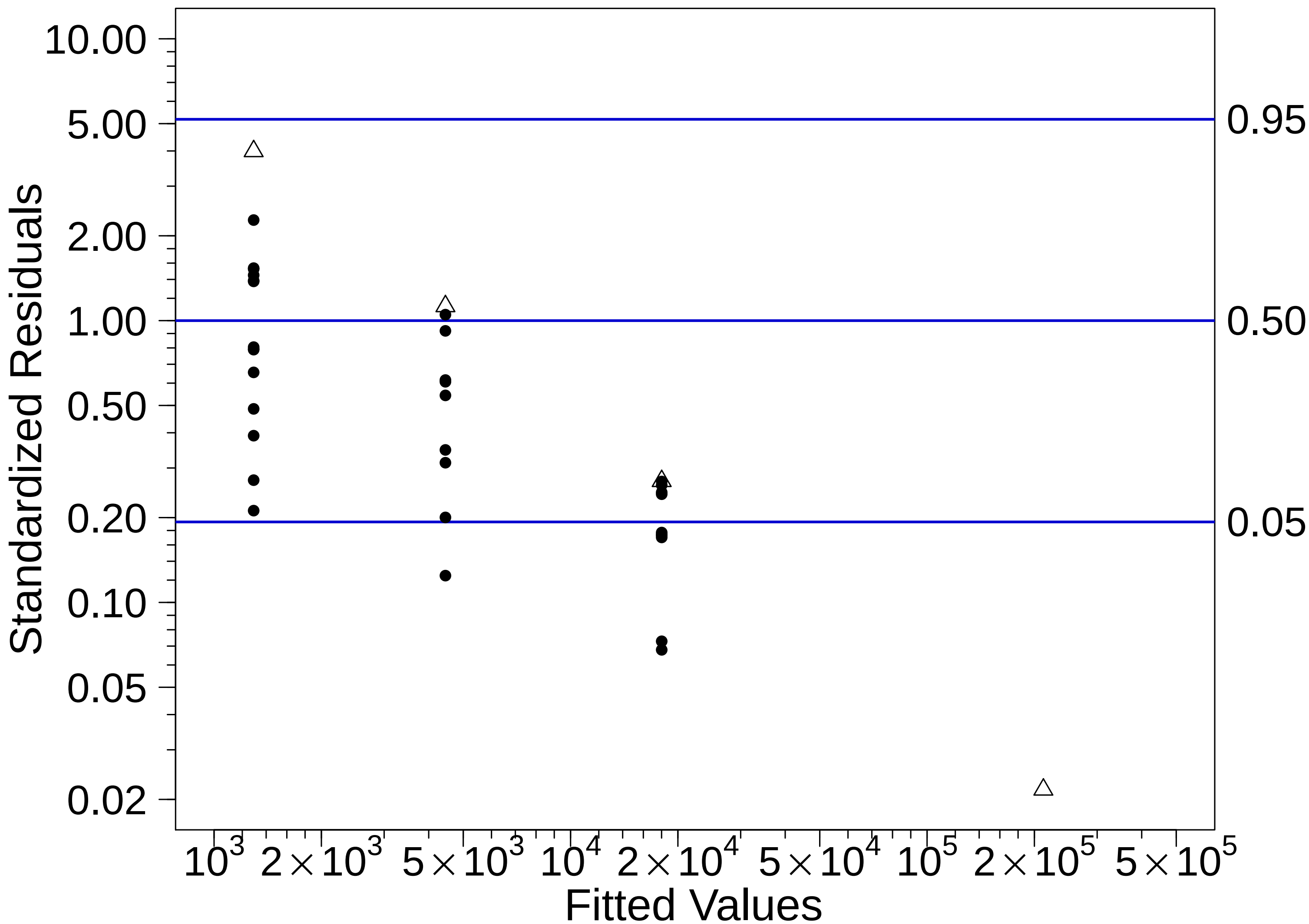
Device-A Lognormal LR Tests

Comparison	LR Statistic	dof	p-value
SepDists vs EqualSig	1.5032	2	0.22
EqualSig vs RegrModel	0.3873	2	0.82
RegrModel vs Pooled	80.7141	1	< 0.001

Probability Plot of the Standardized Residuals from the Arrhenius-Lognormal Regression Model fit to the Device-A ALT Data



Plot of Standardized Residuals Versus Fitted Values for the Arrhenius-Lognormal Regression Model Fit to the Device-A ALT Data



Device-A Model Residual Analysis

- The lognormal probability plot of the standardized residuals suggests that the lognormal distribution fits well.

- Similar distributions of the residuals around the median line would suggest no evidence against the Arrhenius model fit.

- Censored observations are included in the residual-versus-fitted plot, but one must keep in mind that their locations indicate **lower bounds** for the actual residuals if the exact lifetimes had actually been observed.

- For the Device-A ALT data, heavy censoring particularly at lower temperatures (longer lifetimes) **limits the interpretability** of the plot in assessing the model fit.

- Generally with heavy censoring, the multiple probability plots provide better information about model lack of fit.

ML Estimation for the Device-A Lognormal Distribution 0.01 Quantile at 10°C

$$\begin{aligned}\widehat{\mu}(x) &= \widehat{\beta}_0 + \widehat{\beta}_1 x \\ &= -13.4686 + 0.6279 \times 11604.52/(10 + 273.15) \\ &= 12.2641\end{aligned}$$

$$\widehat{t}_{0.01} = \exp(\widehat{\mu}(x) + \widehat{\sigma} \times \Phi^{-1}_{\text{norm}}(0.01))$$

$$\widehat{t}_{0.01} = \exp(12.2641 + 0.97782 \times -2.3263) = 21{,}793$$

$$\widehat{\Sigma}_{\widehat{\mu},\widehat{\sigma}} = \begin{bmatrix} \widehat{\text{Var}}(\widehat{\mu}) & \widehat{\text{Cov}}(\widehat{\mu},\widehat{\sigma}) \\ \widehat{\text{Cov}}(\widehat{\mu},\widehat{\sigma}) & \widehat{\text{Var}}(\widehat{\sigma}) \end{bmatrix} = \begin{bmatrix} 0.28677 & 0.04782 \\ 0.04782 & 0.01760 \end{bmatrix}.$$

$$\begin{aligned}\text{se}_{\log(\widehat{t}_{0.01})} &= \left[\widehat{\text{Var}}(\widehat{\mu}) + 2\Phi^{-1}_{\text{norm}}(0.01)\widehat{\text{Cov}}(\widehat{\mu},\widehat{\sigma}) + [\Phi^{-1}_{\text{norm}}(0.01)]^2\widehat{\text{Var}}(\widehat{\sigma})\right]^{1/2} \\ &= \left[0.28677 + 2 \times (-2.3263) \times 0.04782 + (-2.3263)^2 \times 0.01760\right]^{1/2} \\ &= 0.39941.\end{aligned}$$

$$[\underset{\sim}{t}_{0.01},\ \widetilde{t}_{0.01}] = \exp[\log(\widehat{t}_{0.01}) \mp z_{0.975}\text{se}_{\log(\widehat{t}_{0.01})}] = [9{,}962,\ 47{,}676]$$

ML Estimation for the Device-A Lognormal Distribution $F(30000)$ at 10°C

$$\begin{aligned}\widehat{\mu}(x) &= \widehat{\beta}_0 + \widehat{\beta}_1 x \\ &= -13.4686 + 0.6279 \times 11604.52/(10 + 273.15) = 12.2641\end{aligned}$$

$$\begin{aligned}\widehat{z}_e &= [\log(t_e) - \widehat{\mu}]/\widehat{\sigma} = [\log(30000) - 12.2641]/0.97782 \\ &= -2.000\end{aligned}$$

$$\widehat{F}(30000) = \Phi_{\text{norm}}(\widehat{z}_e) = \Phi_{\text{norm}}(-2.000) = 0.02281$$

$$\begin{aligned}\text{se}_{\widehat{z}_e} &= \frac{1}{\widehat{\sigma}}\left[\widehat{\text{Var}}(\widehat{\mu}) + 2\widehat{z}_e\widehat{\text{Cov}}(\widehat{\mu}, \widehat{\sigma}) + \widehat{z}_e^2\widehat{\text{Var}}(\widehat{\sigma})\right]^{1/2} \\ &= \frac{1}{0.97782}\left[0.28677 + 2 \times (-2.000) \times 0.04782 + (-2.000)^2 \times 0.01760\right]^{1/2} \\ &= 0.42.\end{aligned}$$

$$[\underset{\sim}{F}(30000),\ \widetilde{F}(30000)] = \Phi\text{norm}(\widehat{z}_e \mp z_{0.975}\text{se}_{\widehat{z}_e}) = [0.00238,\ 0.120]$$

Chapter 18

Analyzing Accelerated Life Test Data

Segment 3

Practical Suggestions and Application of Bayesian Methods to Accelerated Testing

Some Practical Suggestions

- Build on previous experience with similar products and materials.
- Use pilot experiments; evaluate the effect of stress on degradation and life.
- Seek physical understanding of failure mechanisms.
- Use results from physical failure mode analysis.
- Seek physical justification for the life/stress relationships.
- Design tests to limit the amount extrapolation needed for desired inferences and to have at least a few failures at three or four levels of the accelerating variable.
- Study the pitfalls of accelerated tests (Chapter 19 and references)
- See Nelson (2004).

Bayesian Analysis of Accelerated Life Test Data

- Engineers conducting accelerated tests often have prior information about one or more of the model parameters.

- In temperature-accelerated life test, there is often information, based on previous experience with the same device and failure mechanism, on the effective activation energy.

- Can use weakly informative prior distributions for parameters for which there is no informative prior information.

- Reparameterization: Generally the intercept parameter β_0 has no practical interpretation. It needs to be replaced by an alternative parameter for which it is possible to elicit a prior distribution.

Bayesian Analysis of the Device-A Data

- For purposes of **eliciting marginal prior information** about parameters and to have a **better-behaved likelihood and posterior distribution**, reparameterize by replacing β_0 with the 0.01 quantile of the failure time distribution at 40°C (near the center of the data). More generally,

$$\log[t_p(\text{Degrees C})] = \beta_0 + \beta_1 \frac{11604.52}{\text{Degrees C} + 273.15} + \Phi^{-1}_{\text{norm}}(p)\sigma$$

- Based on previous experience with similar devices, engineers believe that the effective activation energy Ea$= \beta_1$ is, with high probability, between 0.5 and 0.8. The informative prior distribution for β_1 was chosen to be lognormal with probability 0.99 between these limits.

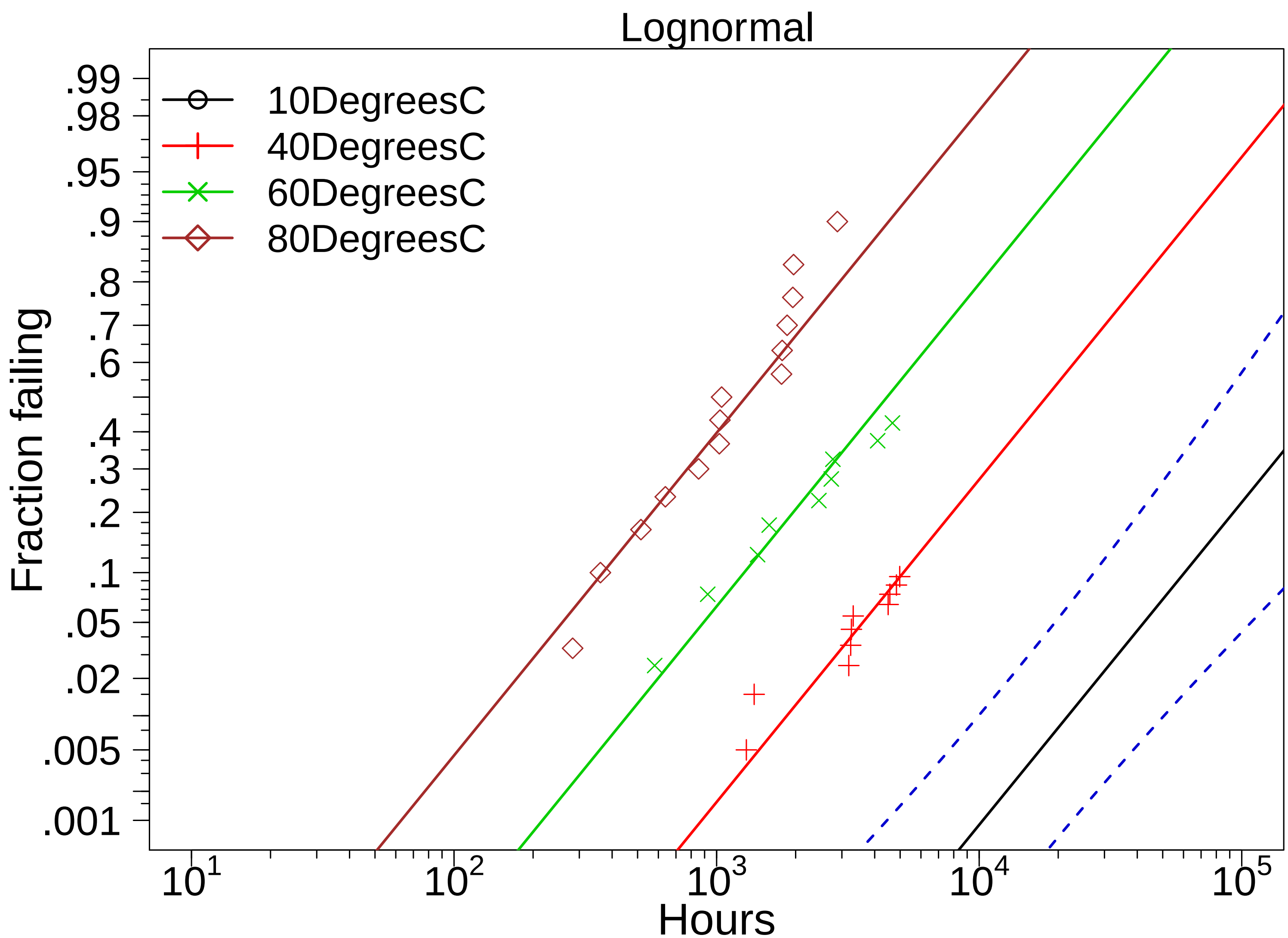
- The weakly informative prior distribution is $< \mathrm{LNORM} >(100, 32000)$ for $t_{0.01}(40)$ and $< \mathrm{LNORM} >(0.05, 3.0)$ for σ.

Summary
Device-A ALT Prior Distributions

	Prior Distributions	
Parameter	Weakly informative	Informative
$t_{0.01}(40)$	<LNORM>$(100, 32000)$	<LNORM>$(100,\ 32000)$
β_1	<**LNORM**>$(0.10, 3.0)$	<**LNORM**>$(0.50, 0.80)$
σ	<LNORM>$(0.05, 3.0)$	<LNORM>$(0.05, 3.0)$

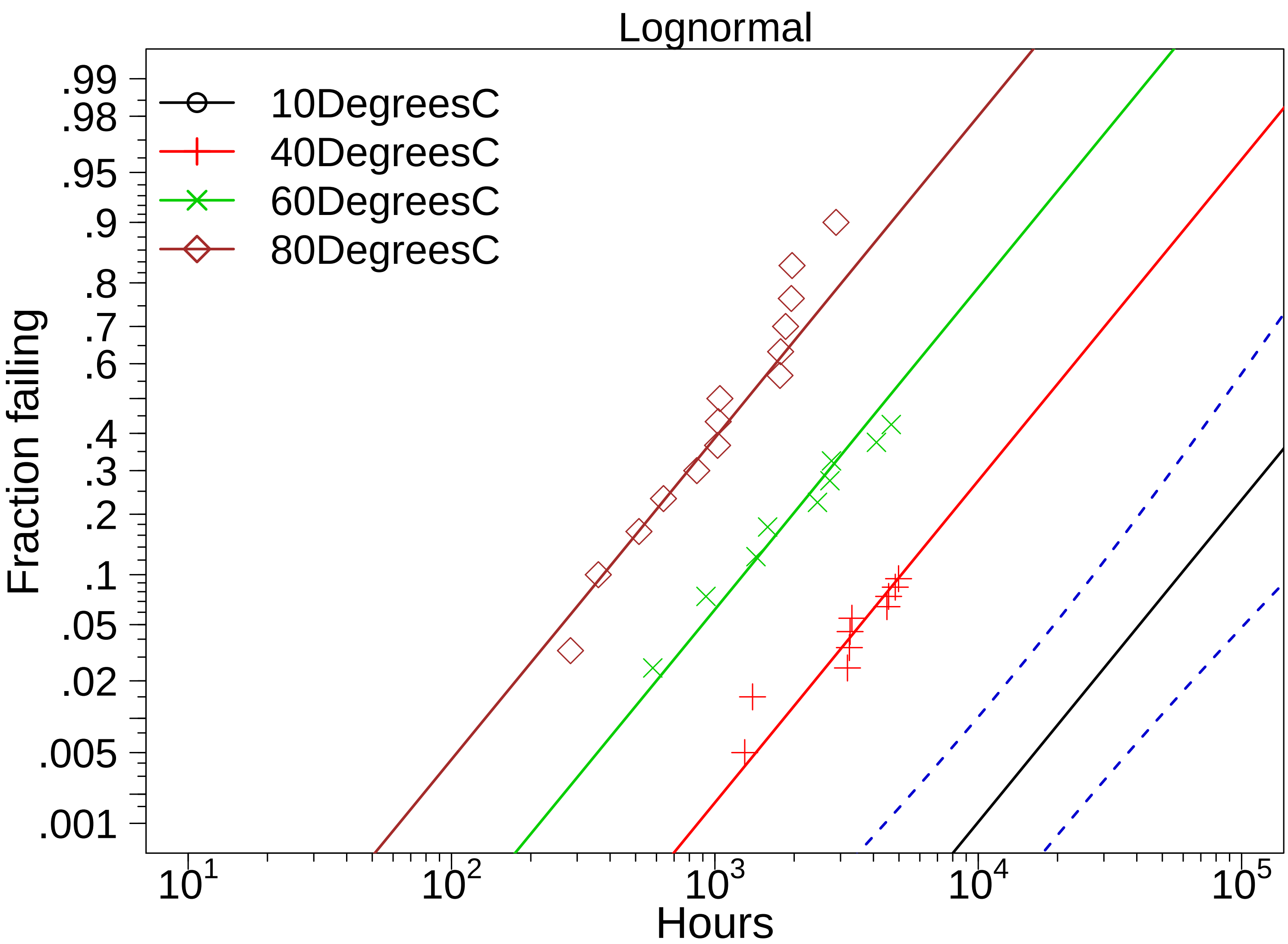
Lognormal Multiple Probability Plot of the Arrhenius-Lognormal Regression Model ML Fit to the Device-A ALT Data

$$\widehat{\Pr}[T(\textbf{Temp}) \leq t] = \Phi_{\textbf{norm}}\left[\frac{\log(t) - \widehat{\mu}(x)}{\widehat{\sigma}}\right], \qquad \widehat{\mu}(x) = \widehat{\beta}_0 + \widehat{\beta}_1 x$$



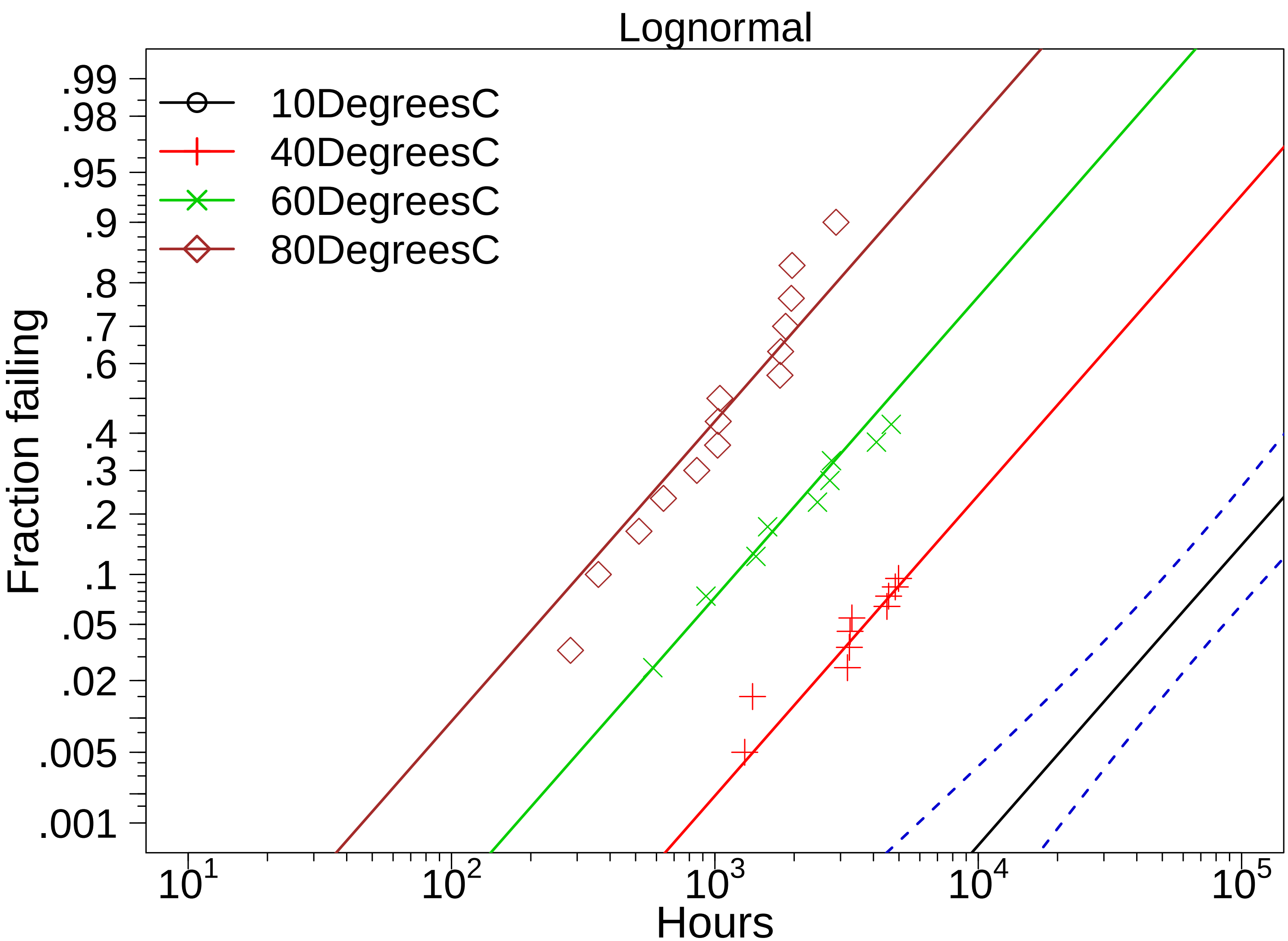
Lognormal Multiple Probability Plot of the Arrhenius-Lognormal Regression Model Fit to the Device-A ALT Data

Weakly Informative Prior Distribution



Lognormal Multiple Probability Plot of the Arrhenius-Lognormal Regression Model Fit to the Device-A ALT Data

Informative Prior Distribution



Chapter 18

Analyzing Accelerated Life Test Data

Segment 4

Voltage Acceleration and the Inverse-Power Relationship An Example of Accelerated Test Model Breakdown

Voltage and Voltage Stress Acceleration
Inverse-Power Relationship

Depending on the failure mode, voltage (or voltage stress) can be increased to:

- Increase the stress level (e.g., `volt` = voltage stress), relative to declining **dielectric strength**).

- Increase the strength of electric fields which can
  - Accelerate certain failure-causing chemical reactions.
  - Increase micro-currents through the dielectric, resulting in generation of heat that will cause failures that would not be seen at usual operation voltages.

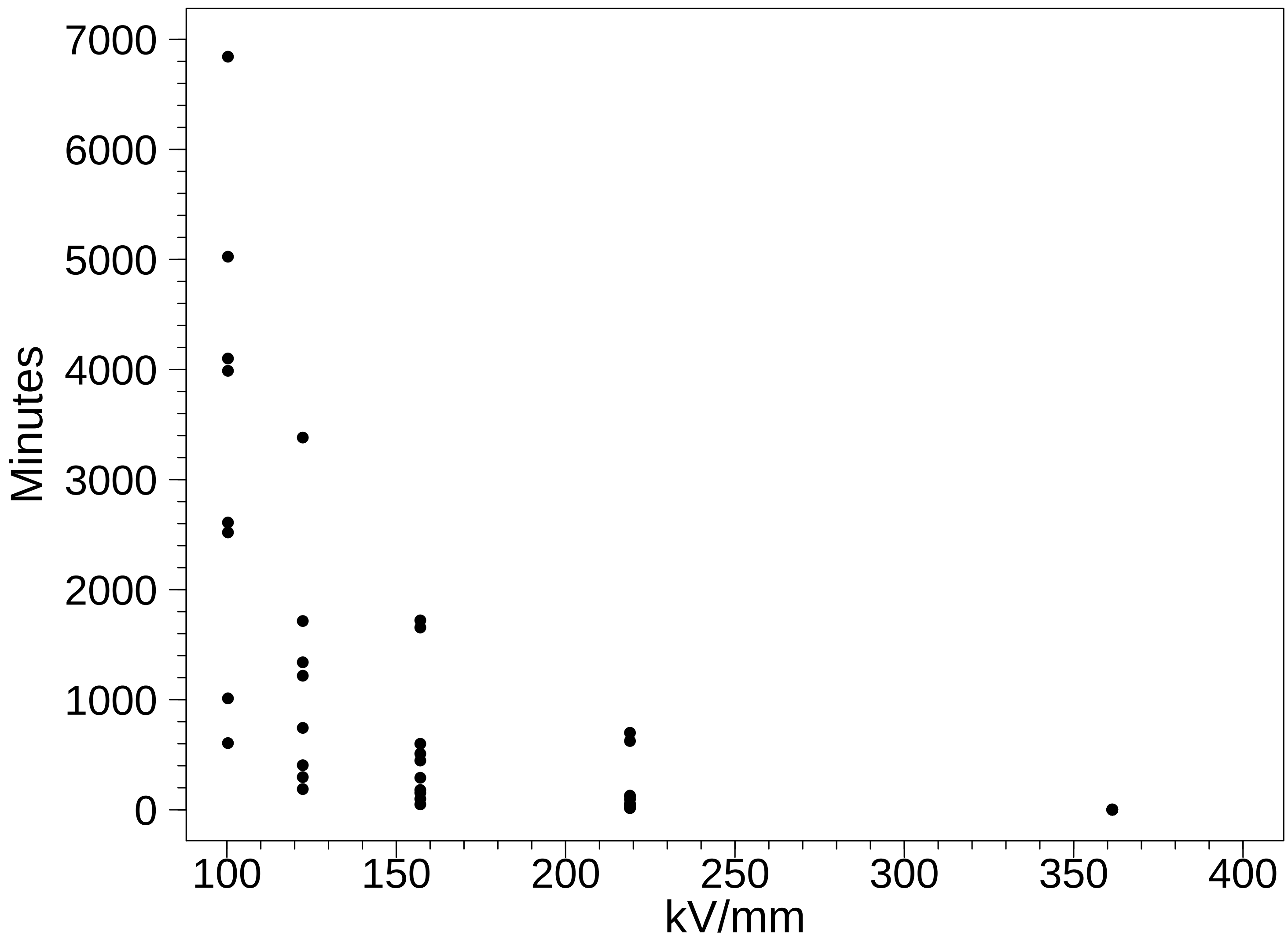
Accelerated Life Test of a Mylar-Polyurethane Laminated Direct Current High Voltage Insulating Structure

- Time to dielectric breakdown of units tested at 100.3, 122.4, 157.1, 219.0, and 361.4 kV/mm.

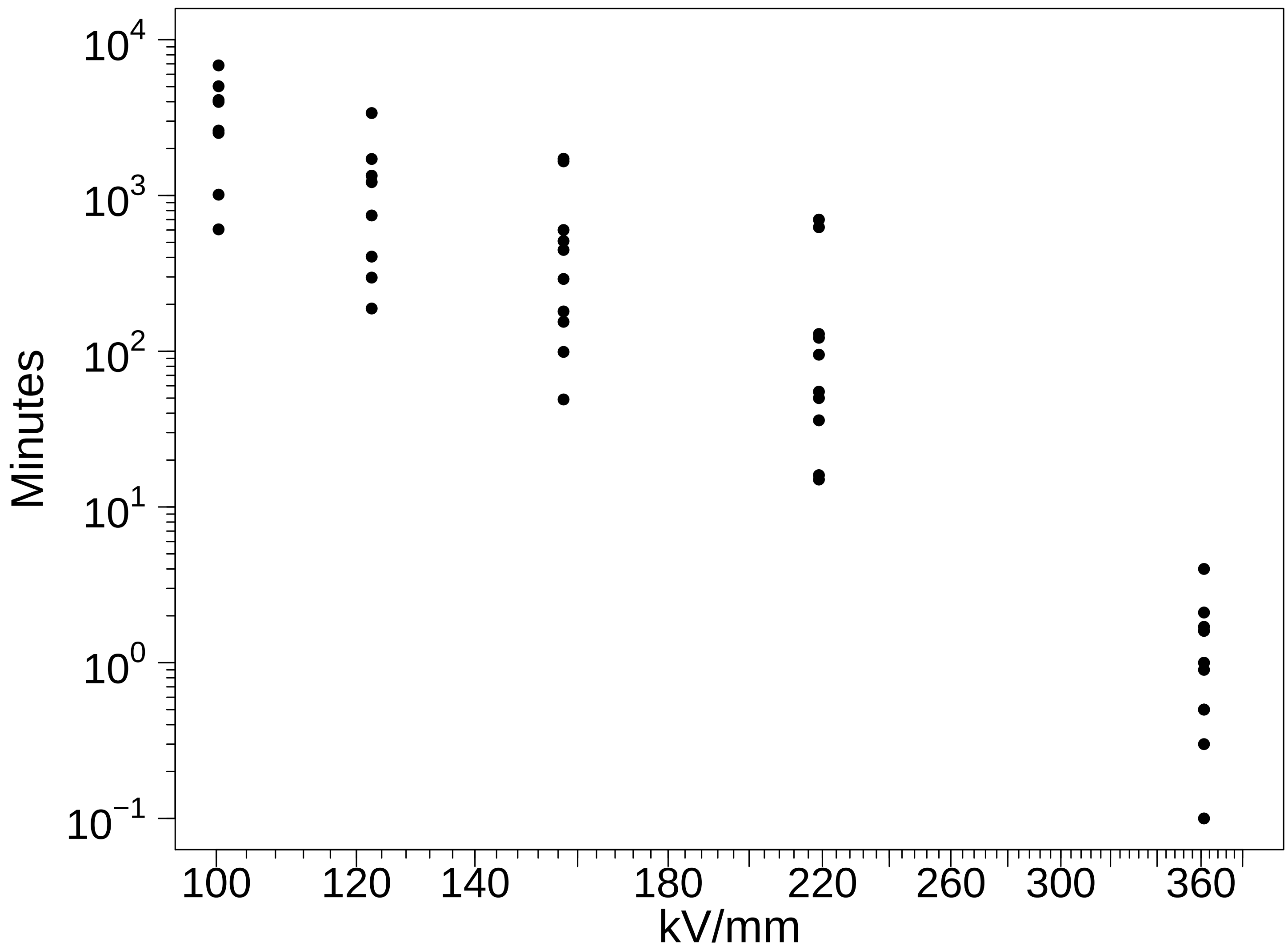
- Needed to evaluate the reliability of the insulating structure and to estimate the life distribution at a multiple of the system design voltages (50 kV/mm).

- Data from Kalkanis and Rosso (1989).

Breakdown Times in Minutes of a Mylar-Polyurethane Insulating Structure Linear-Linear



Breakdown Times in Minutes of a Mylar-Polyurethane Insulating Structure Log-Log



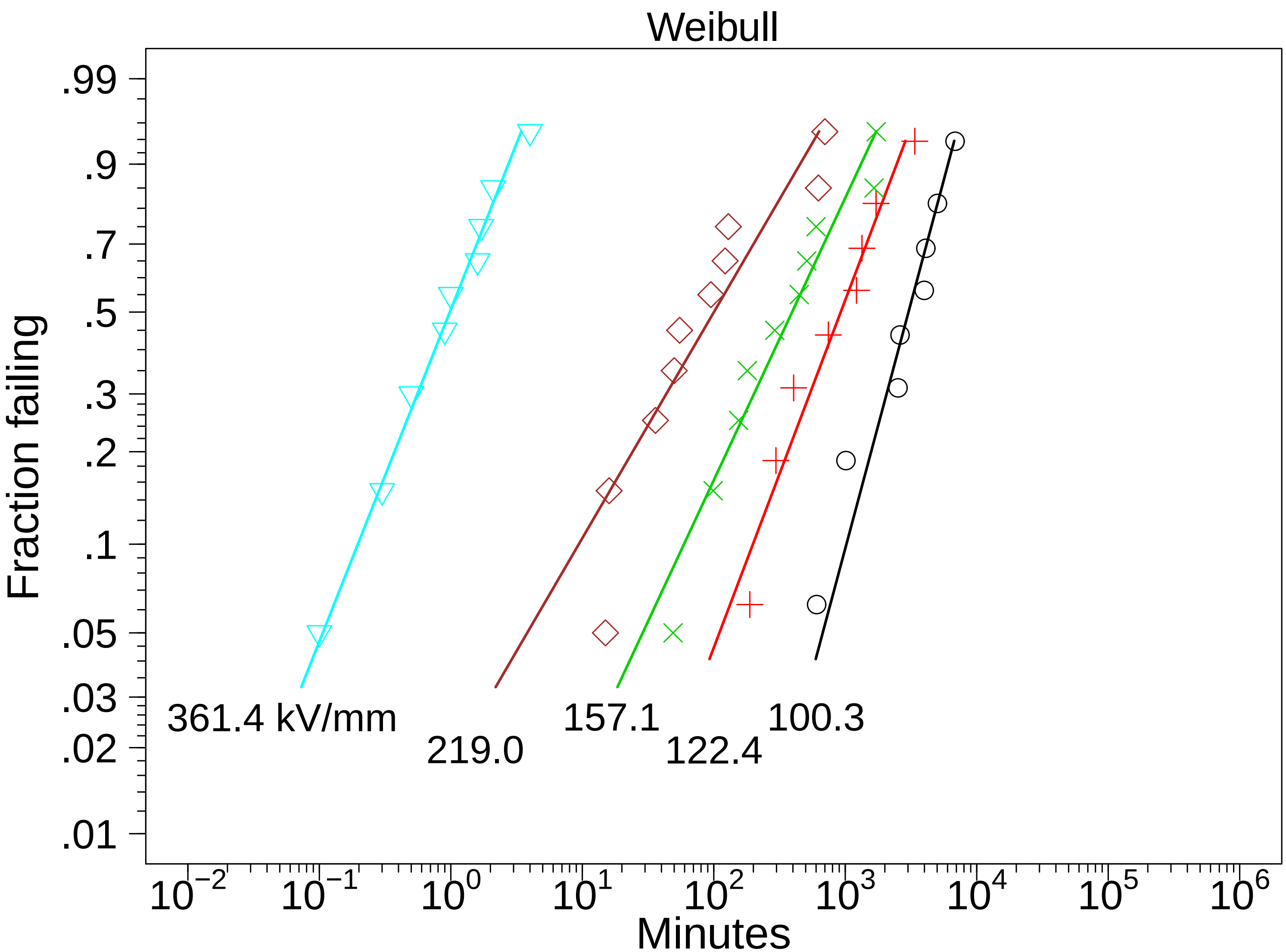
Features of the
Mylar-Polyurethane Insulating Structure Data

- Except for the highest level of voltage stress at 361.4 kV/mm, the relationship between log life and log voltage stress appears to be approximately linear.

- The failure mechanism at 361.4 kV/mm is probably different.

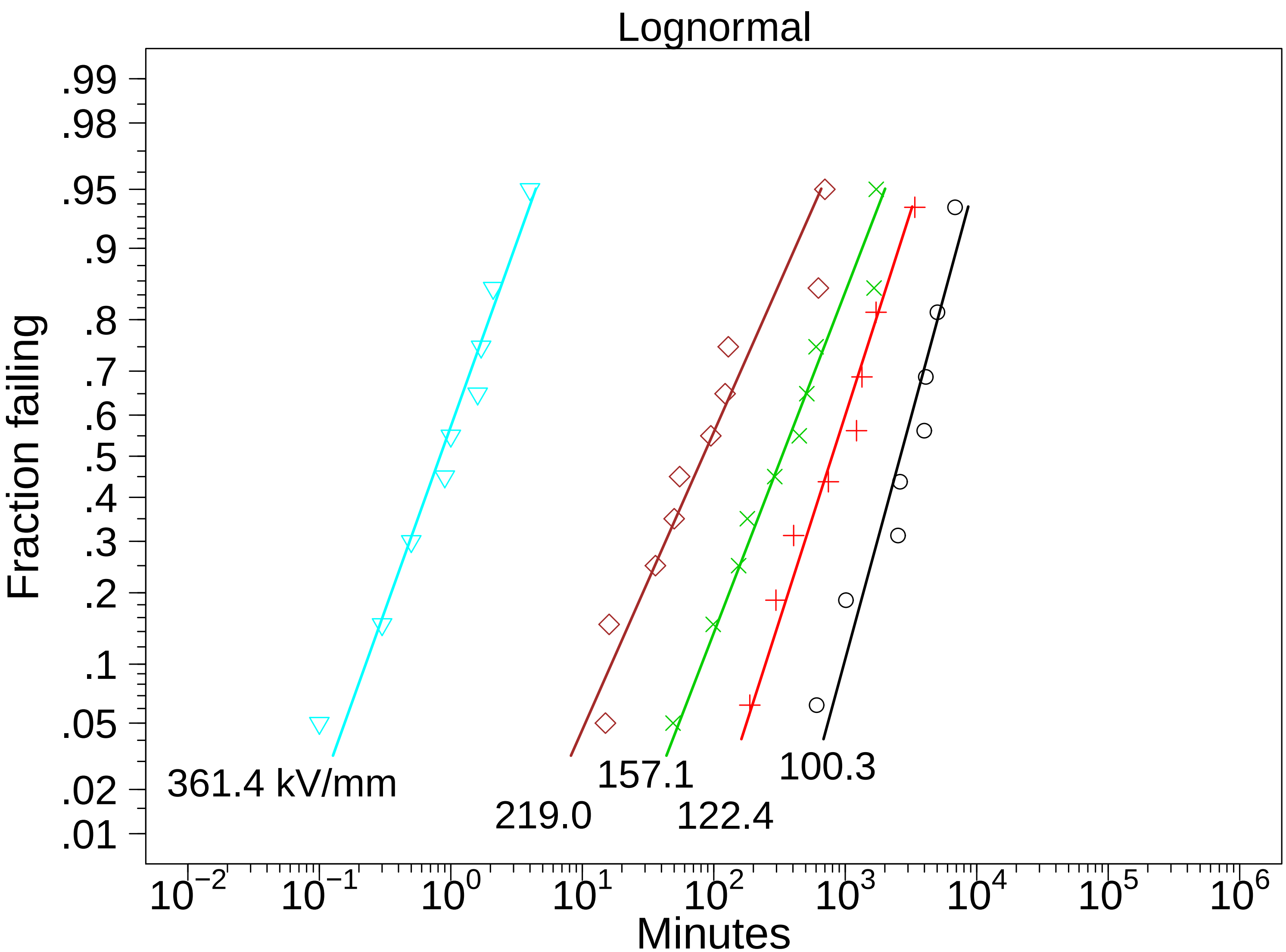
Weibull Multiple Probability Plot of the Mylar-Polyurethane ALT Individual Stress Levels ML Estimates with Different Shape Parameters

$$\widehat{\Pr}[T(\texttt{volt}_i) \leq t] = \Phi_{\textbf{sev}}\left[\frac{\log(t) - \widehat{\mu}_i}{\widehat{\sigma}_i}\right], \qquad i = 100.3, \ldots, 361.4$$



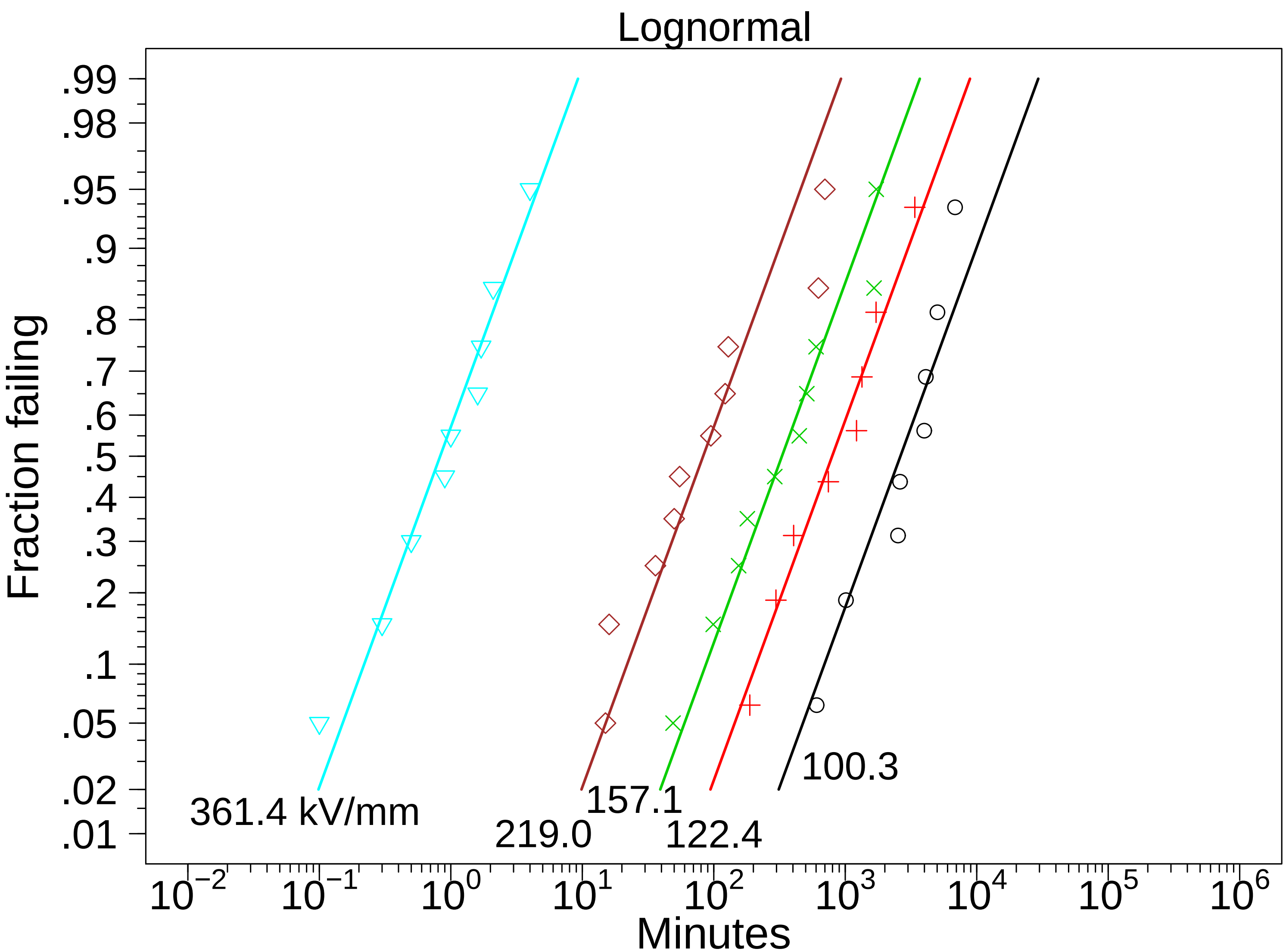
Lognormal Multiple Probability Plot of the Mylar-Polyurethane ALT Individual Stress Levels ML Estimates with Different Shape Parameters

$$\widehat{\Pr}[T(\texttt{volt}_i) \le t] = \Phi_{\textbf{norm}}\left[\frac{\log(t)-\widehat{\mu}_i}{\widehat{\sigma}_i}\right], \qquad i = 100.3, \ldots, 361.4$$



Lognormal Probability Plot of the Mylar-Polyurethane ALT Individual Stress Levels ML Estimates with Equal Shape Parameter

$$\widehat{\Pr}[T(\mathtt{volt}_i) \le t] = \Phi_{\mathbf{norm}}\left[\frac{\log(t)-\widehat{\mu}_i}{\widehat{\sigma}}\right], \qquad i = 100.3, \ldots, 361.4$$



Lognormal Inverse-Power Relationship

- The lognormal inverse-power relationship is

$$F(t) = \Pr[T(\texttt{volt}) \le t] = \Phi_{\text{norm}}\left[\frac{\log(t) - \mu(x)}{\sigma}\right]$$

  where

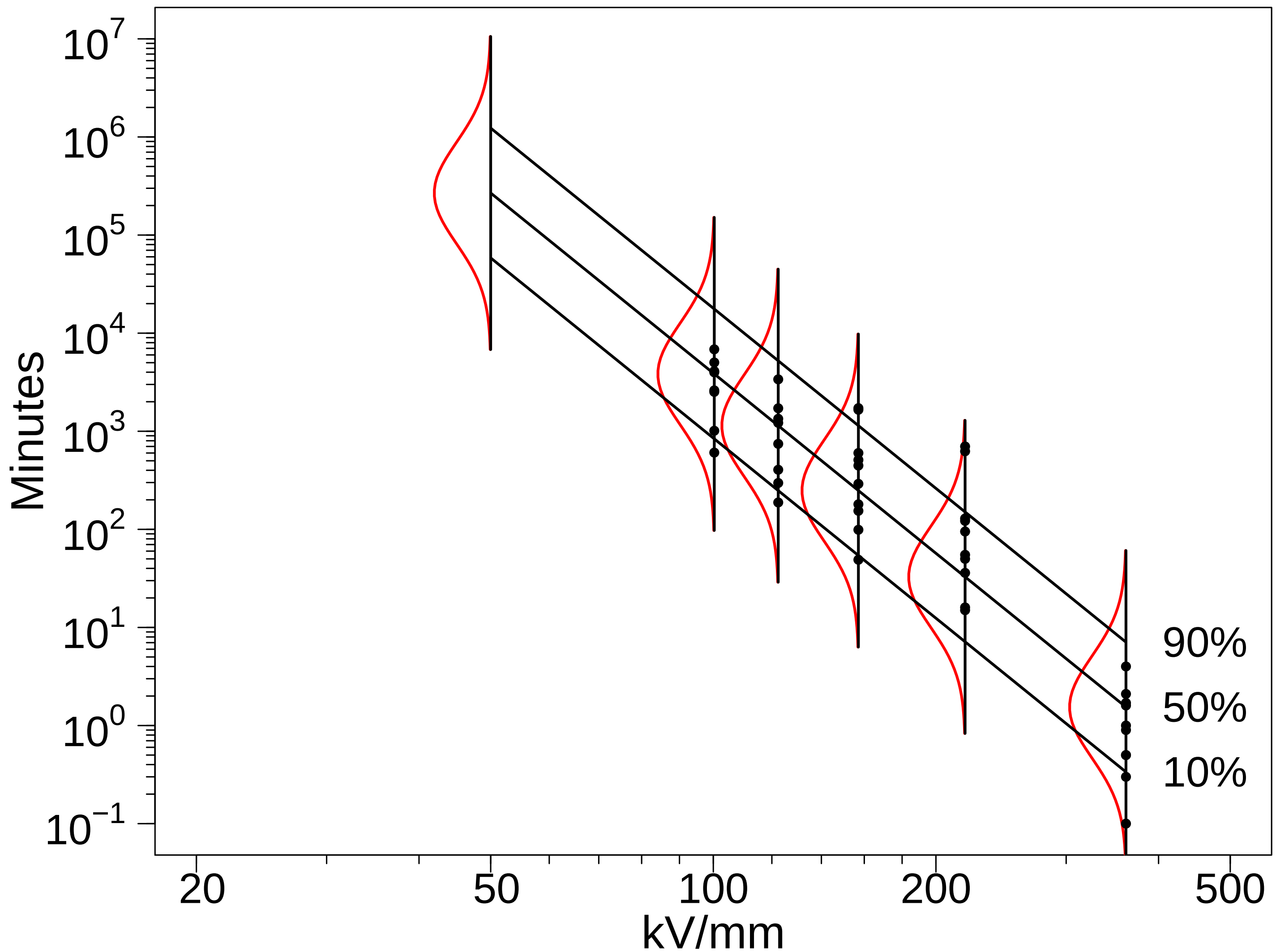
- $\mu(x) = \beta_0 + \beta_1 x$,

- $x = \log(\text{Voltage Stress})$, and

- σ is assumed to be constant.

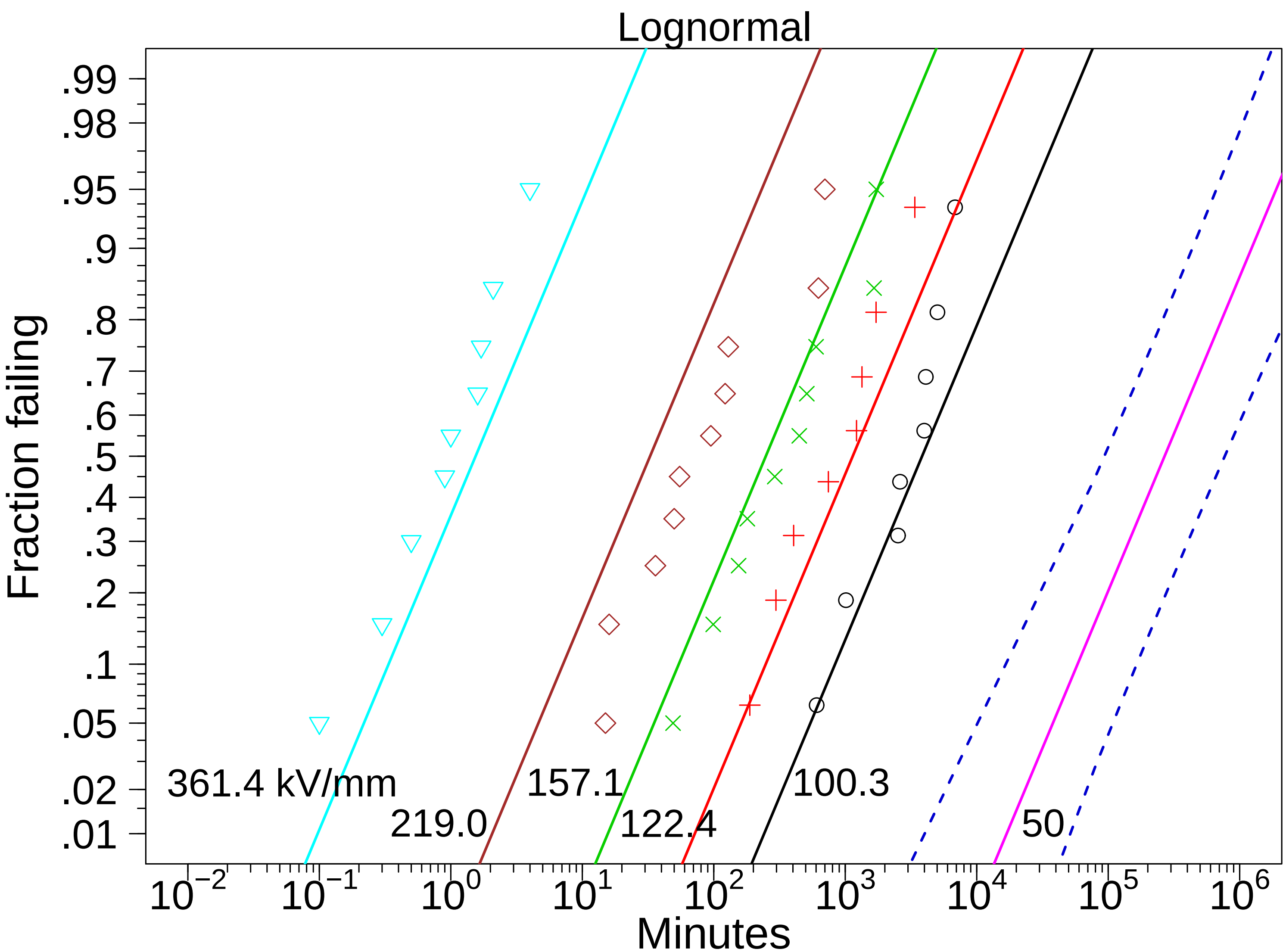
Plot of Inverse-Power Lognormal Model Fitted to the Mylar-Polyurethane Data Including the 361.4 kV/mm Data

$$\log[\widehat{t_p}(x)] = \widehat{\mu}(x) + \Phi_{\textbf{norm}}^{-1}(p)\widehat{\sigma}, \qquad \widehat{\mu}(x) = \widehat{\beta}_0 + \widehat{\beta}_1 x$$



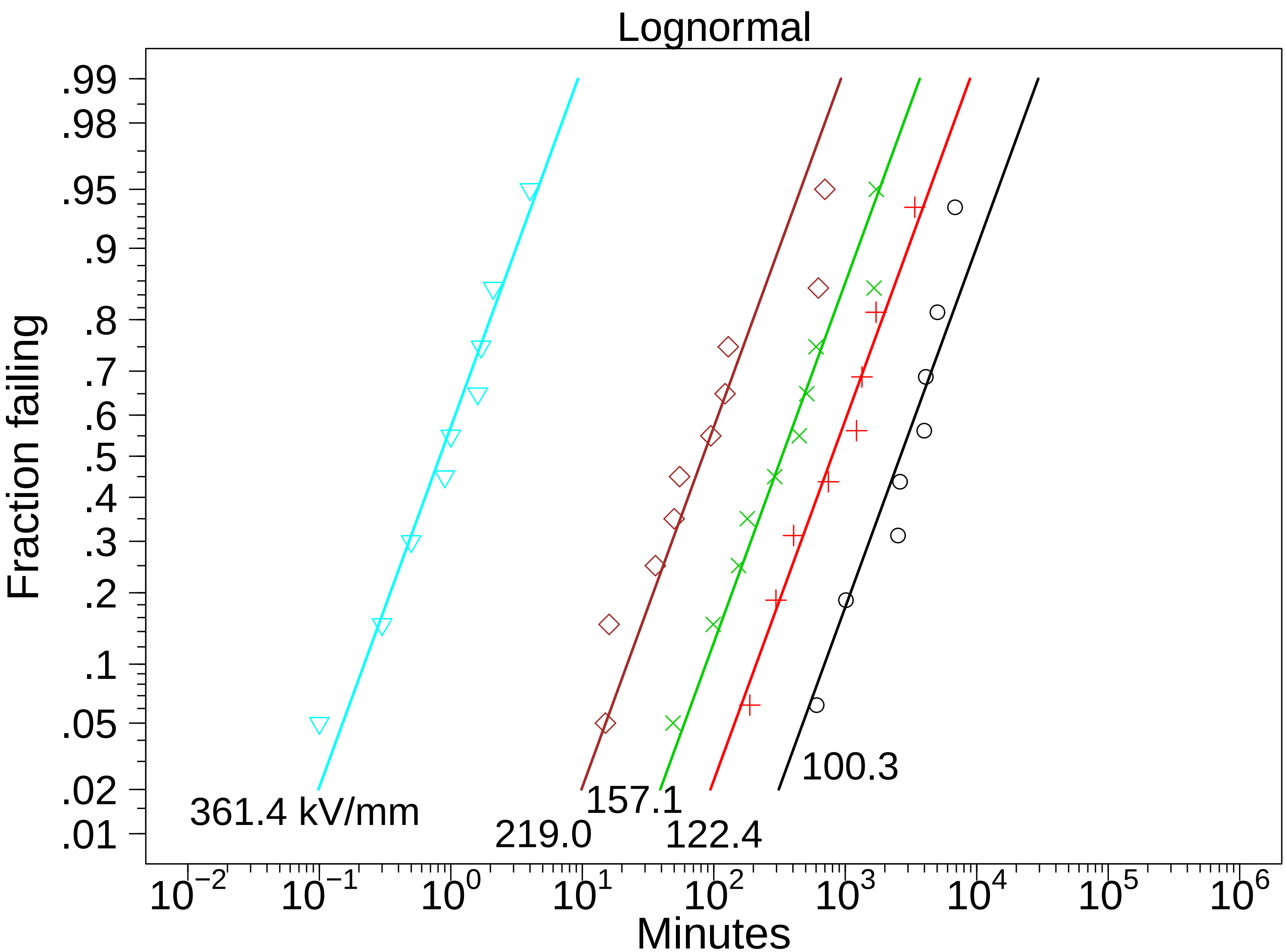
Lognormal Probability Plot of the Inverse-Power Lognormal Model Fitted to the Mylar-Polyurethane Data Including 361.4 kV/mm

$$\widehat{\Pr}[T(\texttt{volt}_i) \le t] = \Phi_{\textbf{norm}}\left[\frac{\log(t)-\widehat{\mu}_i}{\widehat{\sigma}}\right], \qquad i = 100.3, \ldots, 361.4$$



Lognormal Probability Plot of the Mylar-Polyurethane ALT Individual Stress Levels ML Estimates with Equal Shape Parameter

$$\widehat{\Pr}[T(\texttt{volt}_i) \le t] = \Phi_{\textbf{norm}}\left[\frac{\log(t)-\widehat{\mu}_i}{\widehat{\sigma}}\right], \qquad i = 100.3, \ldots, 361.4$$



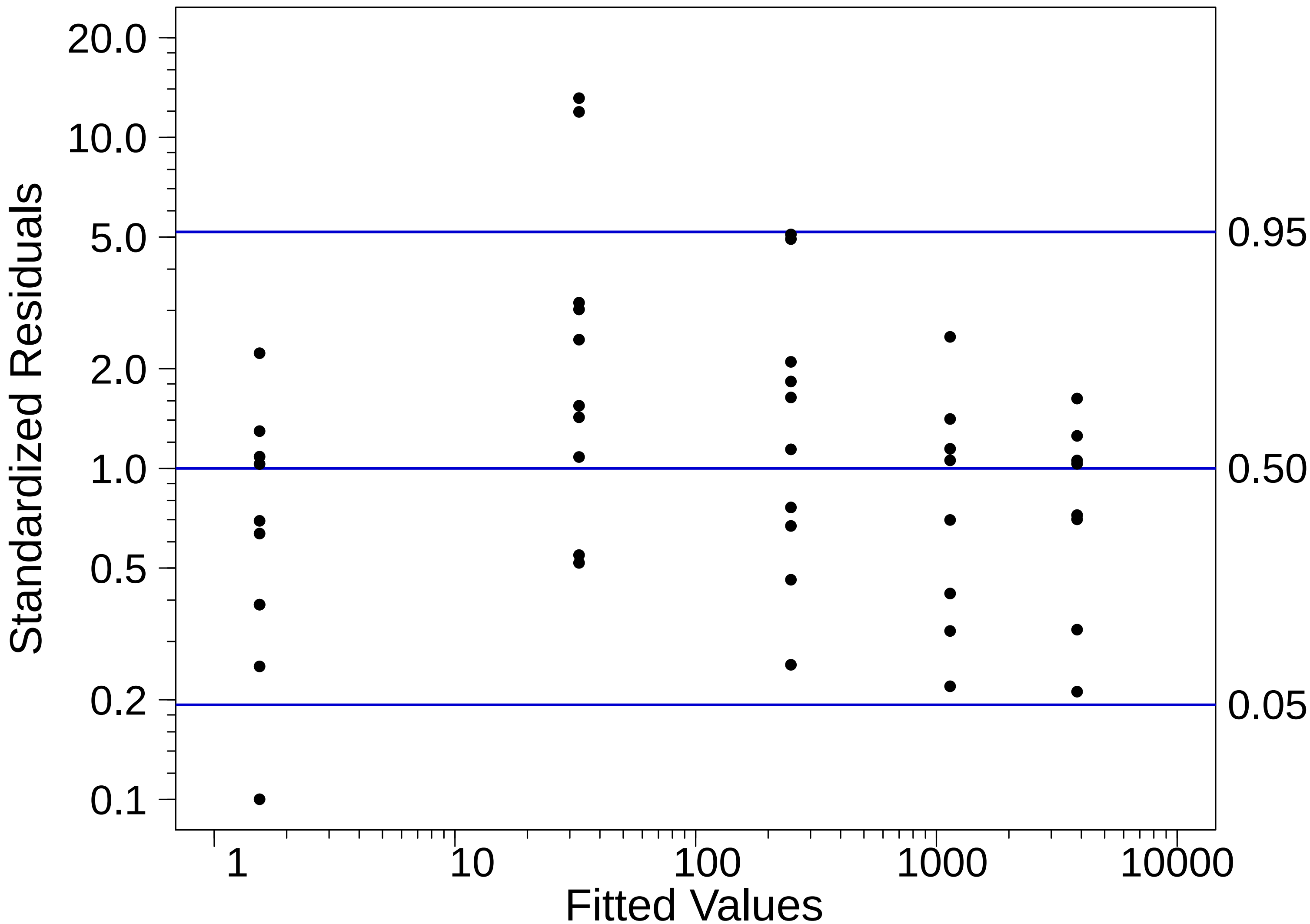
Mylar-Polyurethane ALT Data
Lognormal Model-Fitting Summary
Including the 361.4 kV/mm Data

Model	-2LogLike	AIC	# Param
SepDists	565.1	585.1	10
EqualSig	567.4	579.4	6
RegrModel	579.9	585.9	3
Pooled	665.0	669.0	2

Lognormal LR Tests
Including the 361.4 kV/mm Data

Comparison	LR Statistic	dof	p-value
SepDists vs EqualSig	2.295	4	0.68
EqualSig vs RegrModel	12.515	3	0.005
RegrModel vs Pooled	85.055	1	< 0.001

Lognormal Plot of the Standardized Residuals versus $\exp[\widehat{\mu}(x)]$ for the Inverse-Power Lognormal Model Fitted to the Mylar-Polyurethane Data with the 361.4 kV/mm Data



Mylar-Polyurethane Model Diagnostics

- In contrast to that for the Device-A ALT fit, the plot of standardized residuals provides useful information regarding the fit of the model of the data because all data are exact observations (no censoring).

- The plot indicates an uneven distribution of residuals about the median line.

- Note that the left-most column of residuals are those for 361.4 kV/mm. This plot and the plot of breakdown times against voltage suggest a lack of fit of the model at this voltage level.

- Units at 361.4 kV/mm failed prematurely due to a different failure mode (probably caused by excess heat in the structure).

Chapter 18

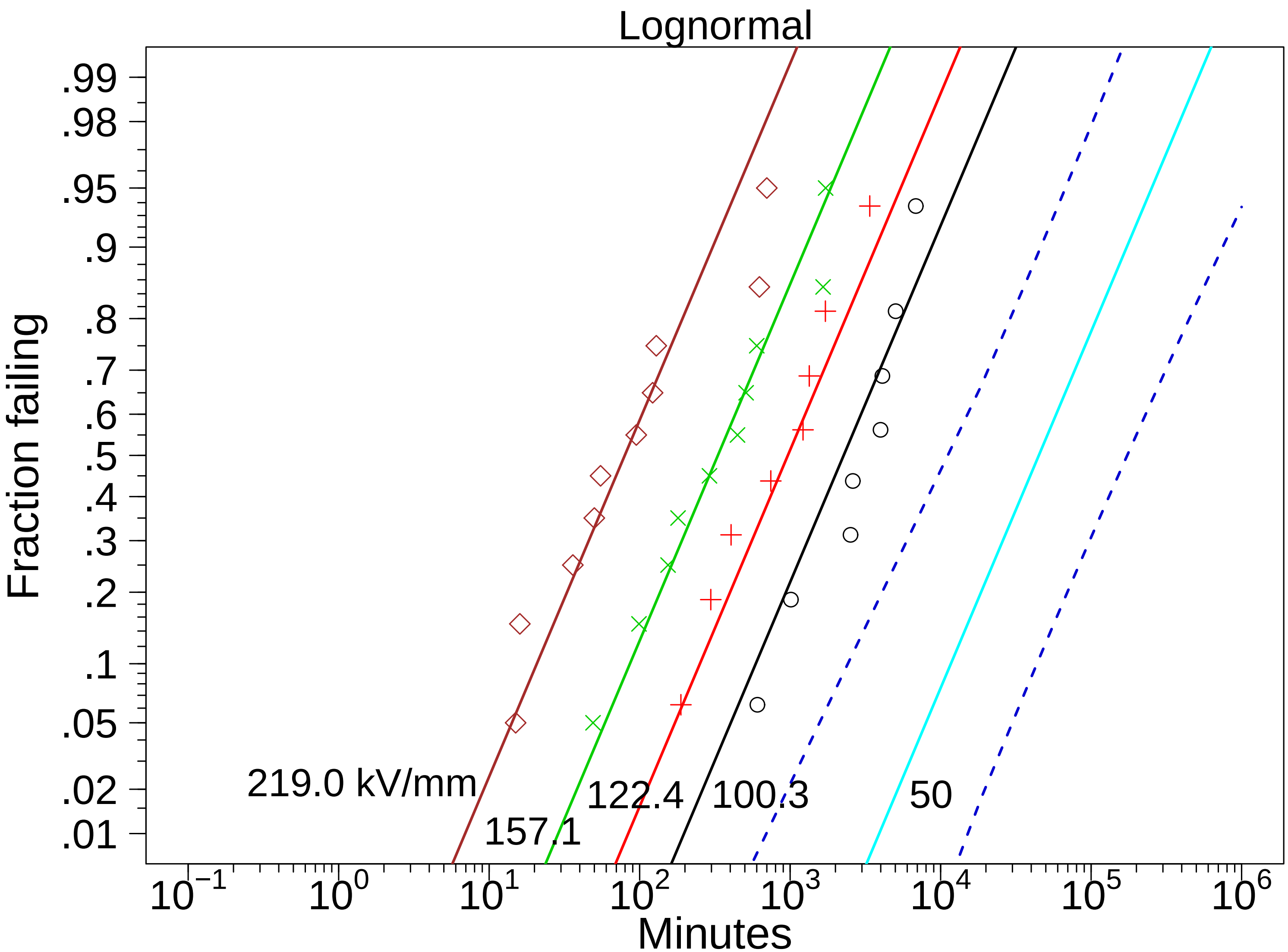
Analyzing Accelerated Life Test Data

Segment 5

Fixing the
Mylar-Polyurethane Insulating Structure Analysis

Lognormal Probability Plot of the Inverse-Power Lognormal Model Fitted to the Mylar-Polyurethane Data Excluding the 361.4 kV/mm Data

$$\widehat{\Pr}[T(\texttt{volt}) \le t] = \Phi_{\textbf{norm}}\left[\frac{\log(t) - \widehat{\mu}(x)}{\widehat{\sigma}}\right], \qquad \widehat{\mu}(x) = \widehat{\beta}_0 + \widehat{\beta}_1 x$$



Mylar-Polyurethane ALT Data
Lognormal Model-Fitting Summary
Excluding the 361.4 kV/mm Data

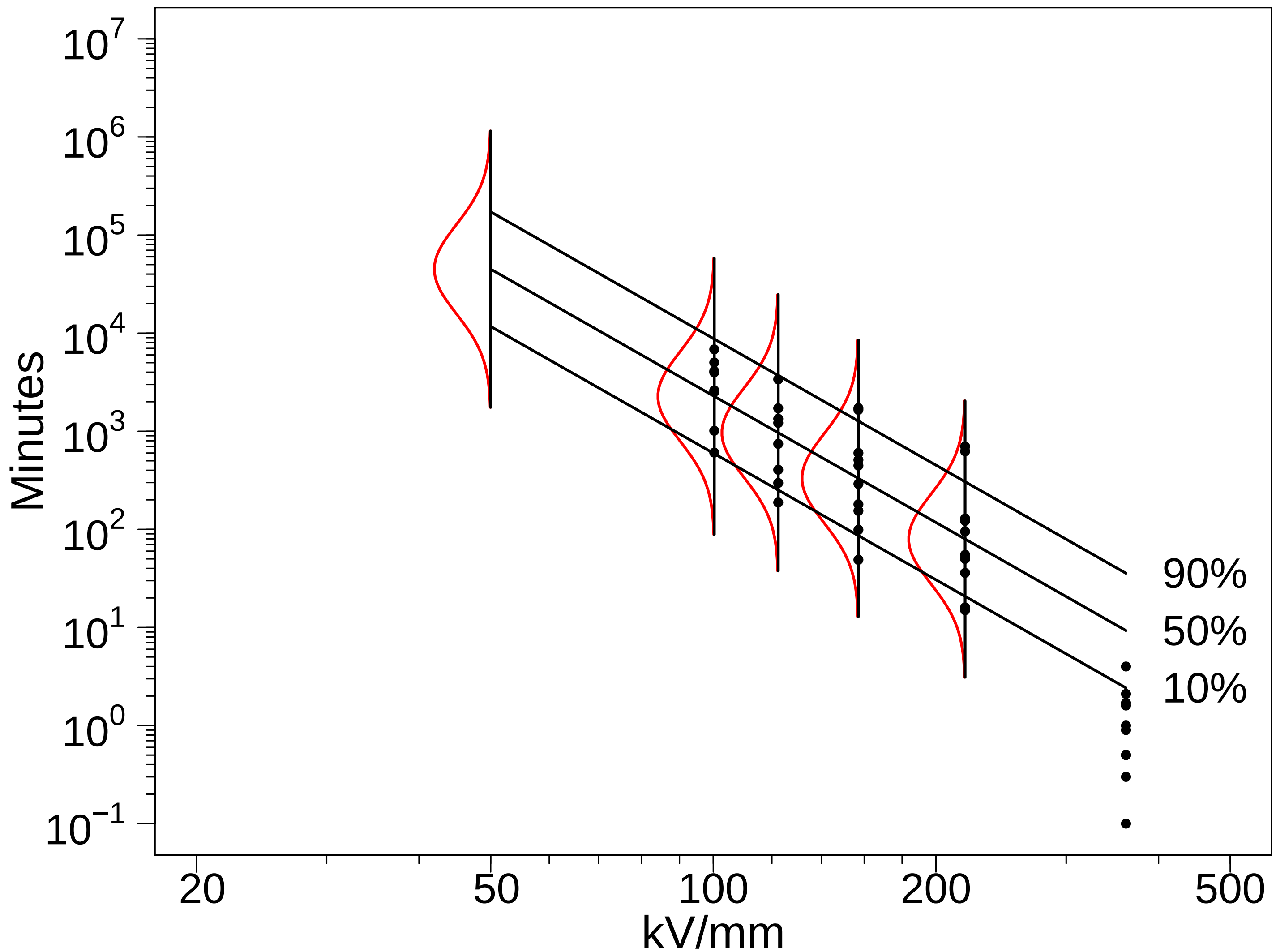
Model	-2LogLike	AIC	# Param
SepDists	540.1	556.1	8
EqualSig	542.4	552.4	5
RegrModel	542.8	548.8	3
Pooled	574.7	578.7	2

Lognormal LR Tests
Excluding the 361.4 kV/mm Data

Comparison	LR Statistic	dof	p-value
SepDists vs EqualSig	2.29	3	0.52
EqualSig vs RegrModel	0.45	2	0.80
RegrModel vs Pooled	31.87	1	< 0.001

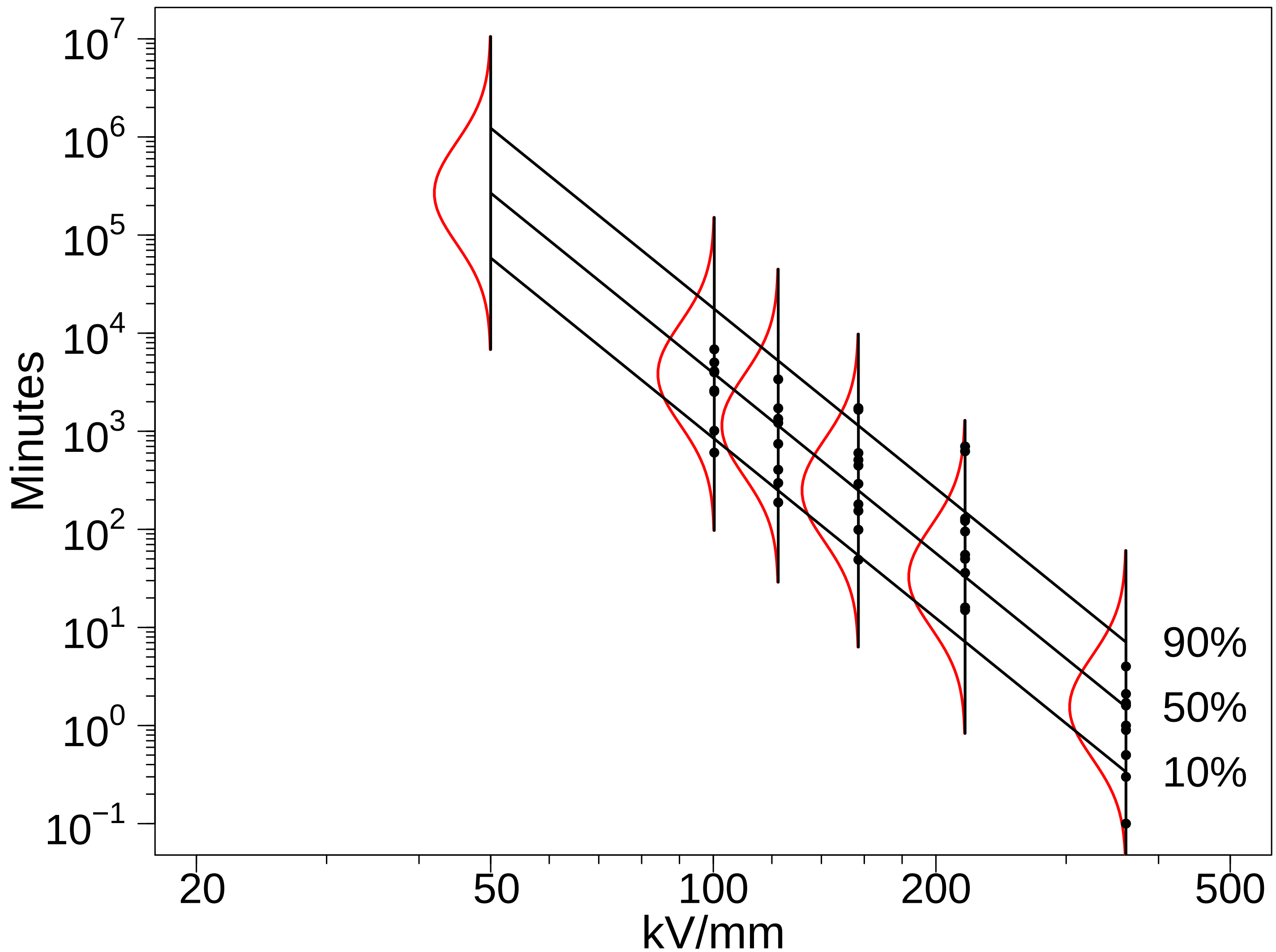
Plot of Inverse-Power Lognormal Model Fitted to the Mylar-Polyurethane Data Excluding the 361.4 kV/mm Data

$$\log[\widehat{t}_p(x)] = \widehat{\mu}(x) + \Phi^{-1}_{\mathbf{norm}}(p)\widehat{\sigma}, \qquad \widehat{\mu}(x) = \widehat{\beta}_0 + \widehat{\beta}_1 x$$

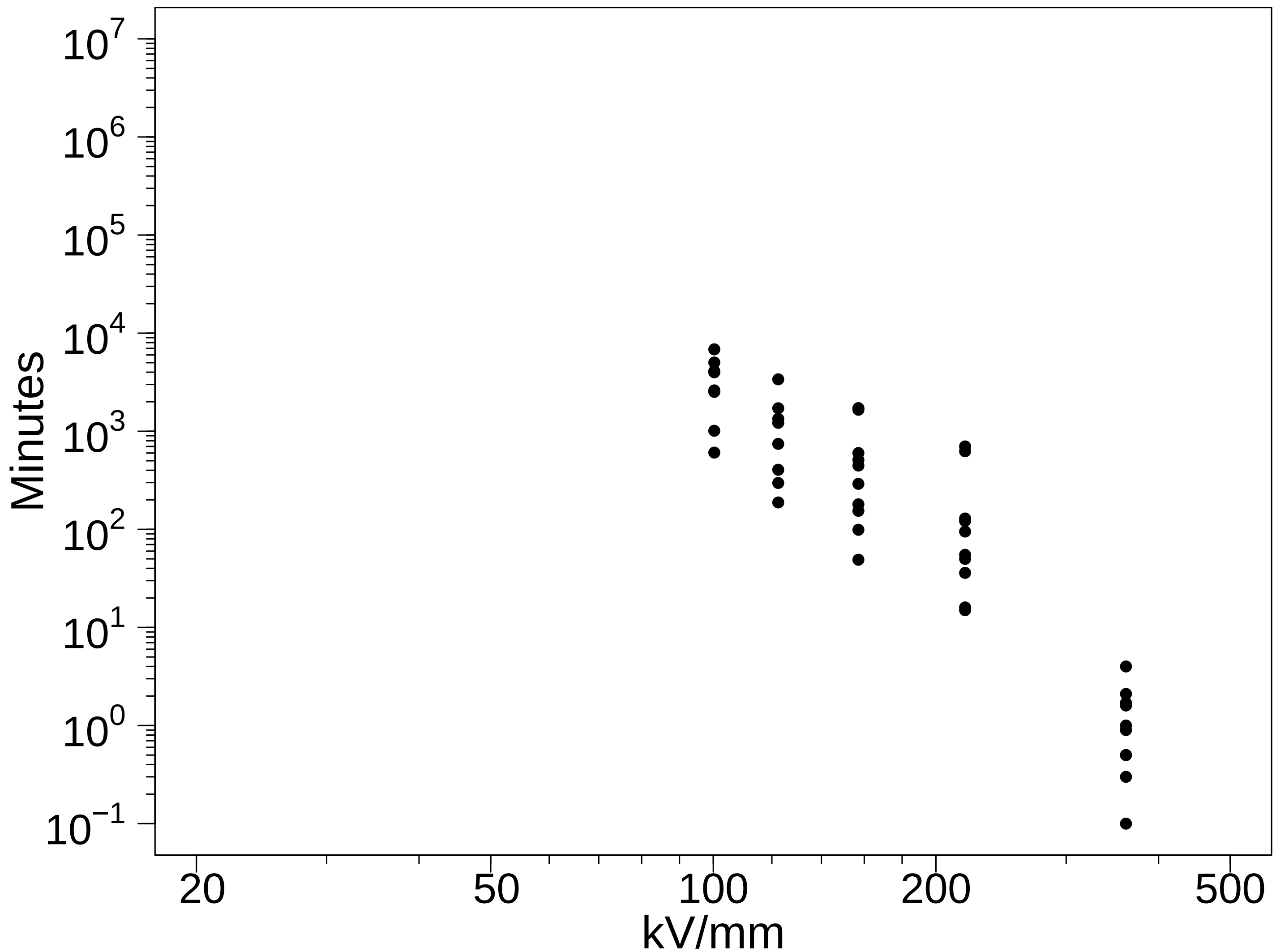


Plot of Inverse-Power Lognormal Model Fitted to the Mylar-Polyurethane Data Including the 361.4 kV/mm Data

$$\log[\widehat{t}_p(x)] = \widehat{\mu}(x) + \Phi^{-1}_{\textbf{norm}}(p)\widehat{\sigma}, \qquad \widehat{\mu}(x) = \widehat{\beta}_0 + \widehat{\beta}_1 x$$



Mylar-Polyurethane Data

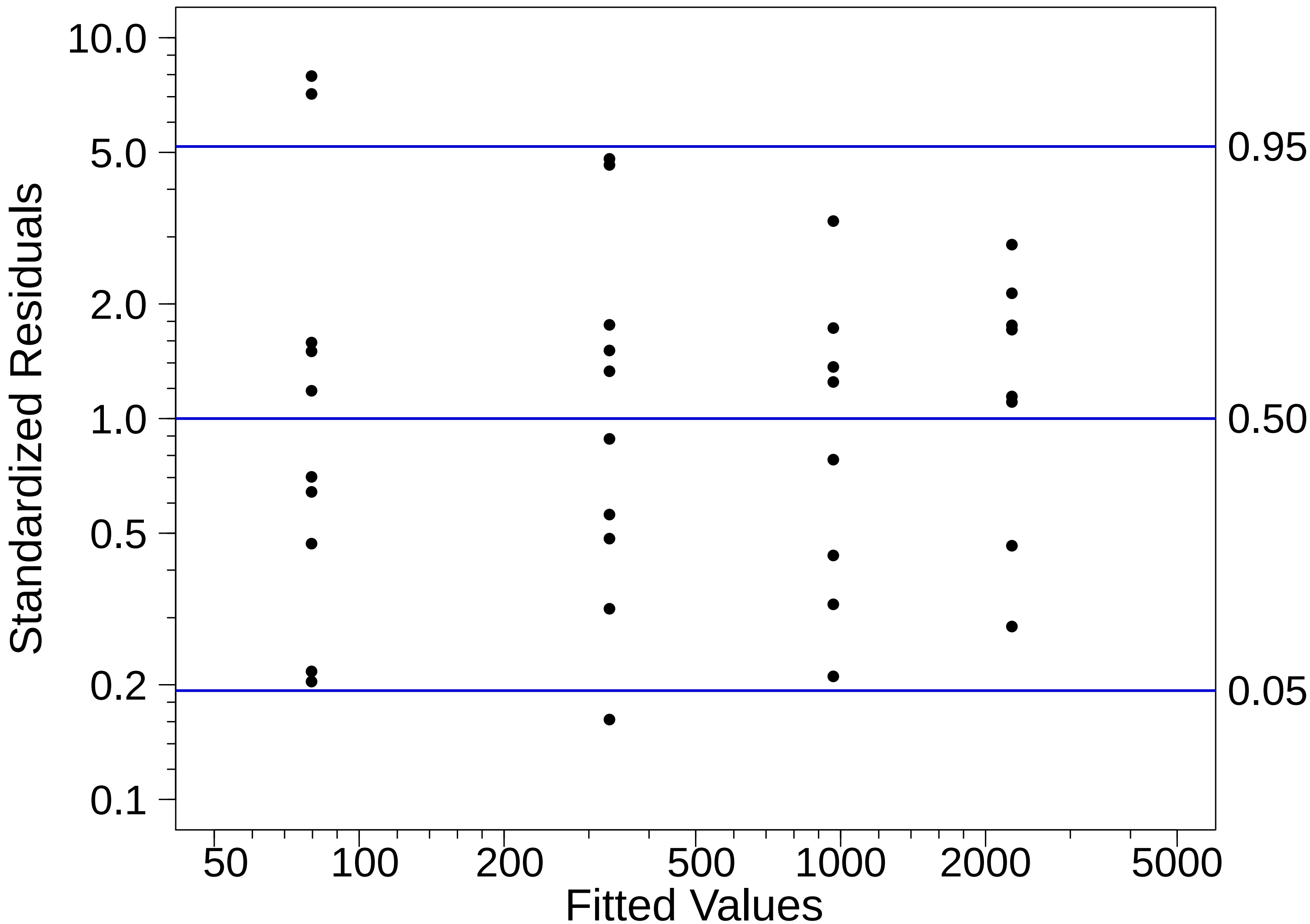


Inverse-Power Lognormal Model ML Estimation Results for the Mylar-Polyurethane ALT Data Excluding the 361.4 kV/mm Data

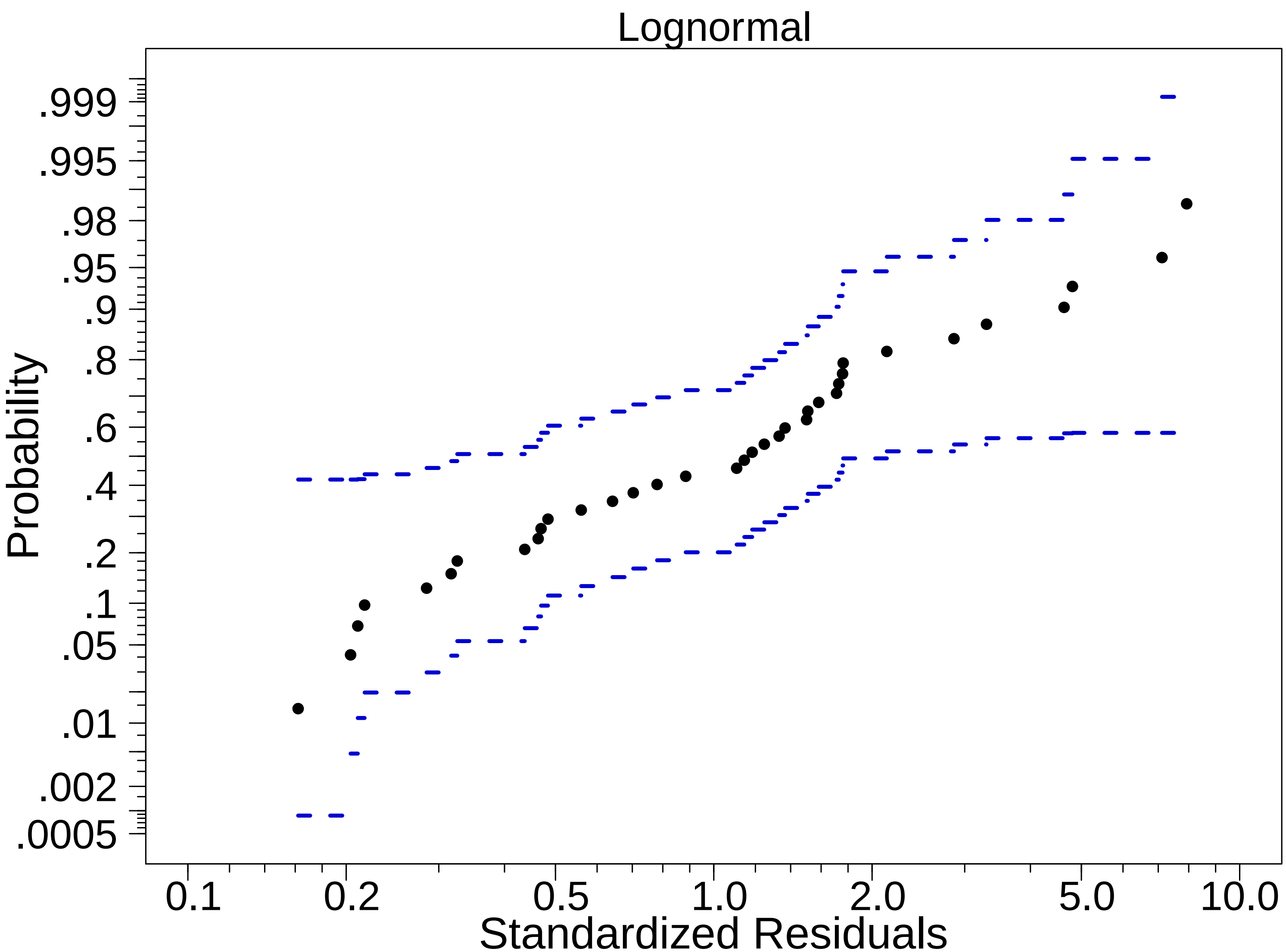
Parameter	ML Estimate	Standard Error	95% Approximate Confidence Intervals Lower	Upper
β_0	27.5	3.0	21.6	33.4
β_1	−4.29	0.60	−5.46	−3.11
σ	1.05	0.12	0.83	1.32

The confidence intervals are based on the Wald method.

Lognormal Plot of the Standardized Residuals versus $\exp(\widehat{\mu})$ for the Inverse-Power Lognormal Model Fitted to the Mylar-Polyurethane Data Excluding the 361.4 kV/mm Data



Lognormal Probability Plot of the Standardized Residuals for the Inverse-Power Lognormal Model Fitted to the Mylar-Polyurethane Data Excluding the 361.4 kV/mm Data



Chapter 18

Analyzing Accelerated Life Test Data

Segment 6

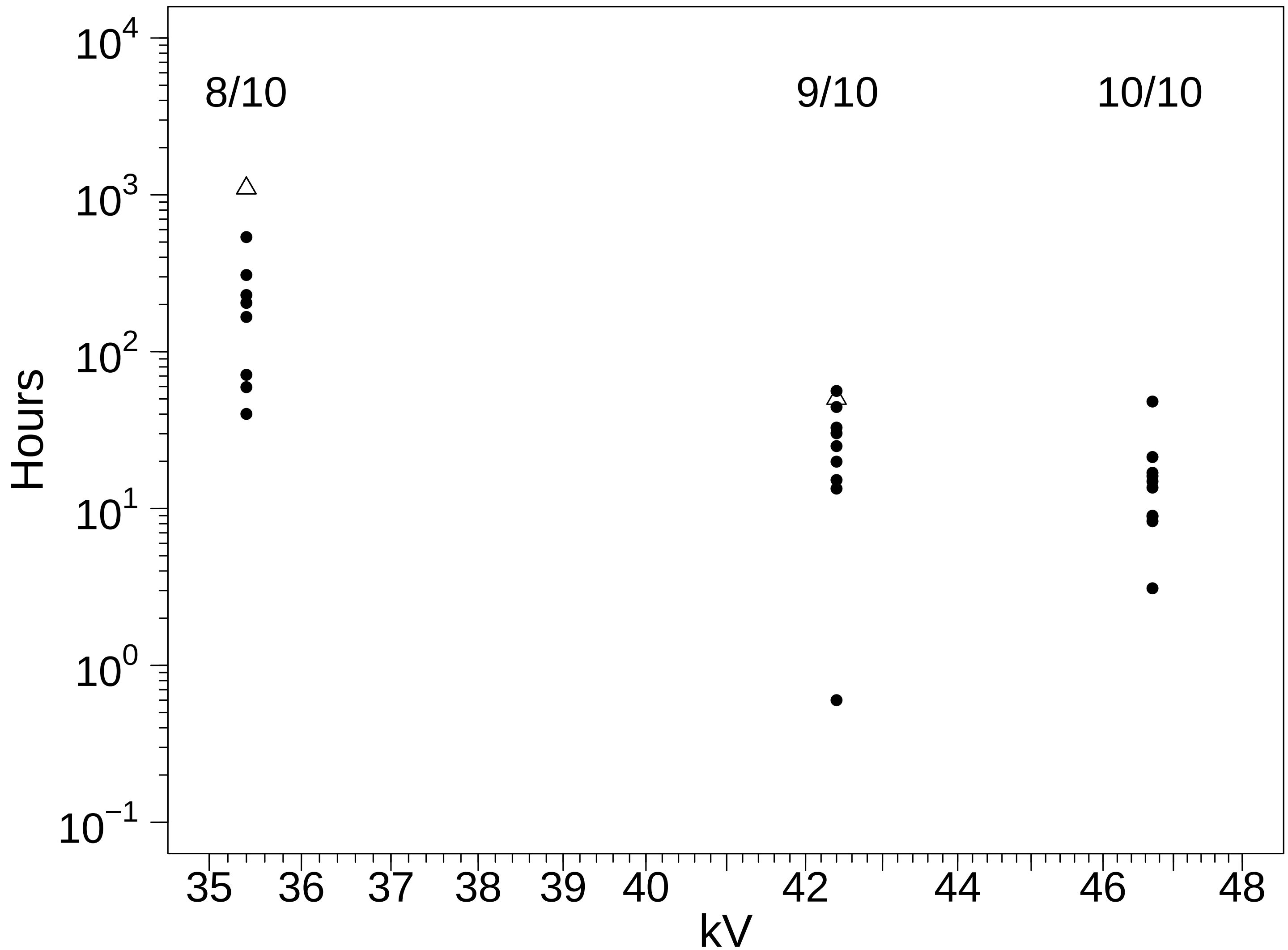
An Inverse-Power Weibull Model Example
An Accelerated Life Test of

Transformer Insulation Turn-to-Turn Failures

Accelerated Life Test of Transformer Insulation Turn-to-Turn Failures

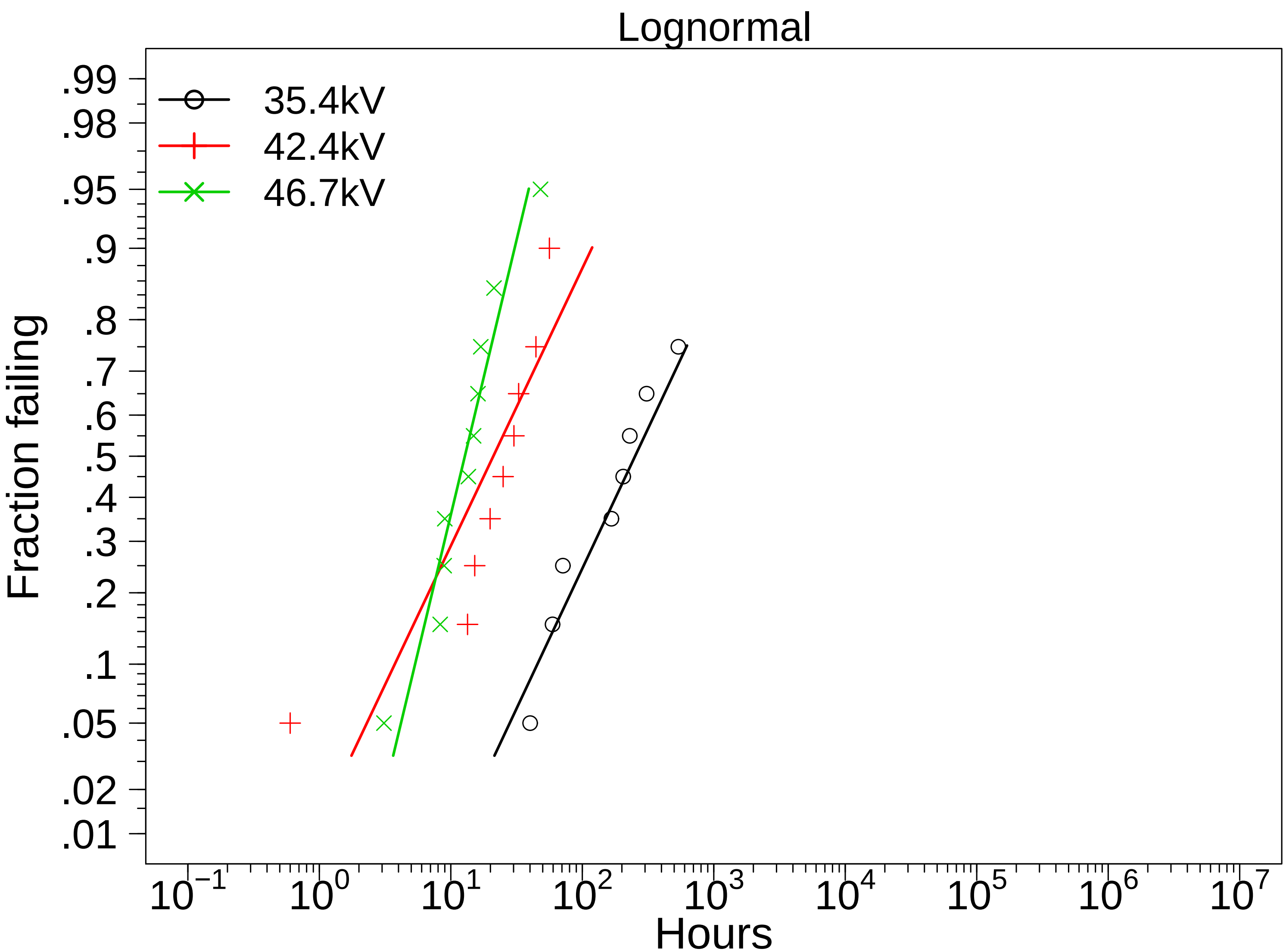
- Time to a turn-to-turn failure of transformer primary insulation.
- Units tested at 35.4, 42.4, and 46.7 kV.
- Needed to estimate the B01 life (the 0.01 quantile) at 15.8kV (110% of the design voltage 14.4 kV).
- Engineers believed (based on previous experience) that the inverse-power Weibull model can usefully describe the data.
- Data from Nelson (2004).

Turn-to-Turn Transformer Insulation Failure Times



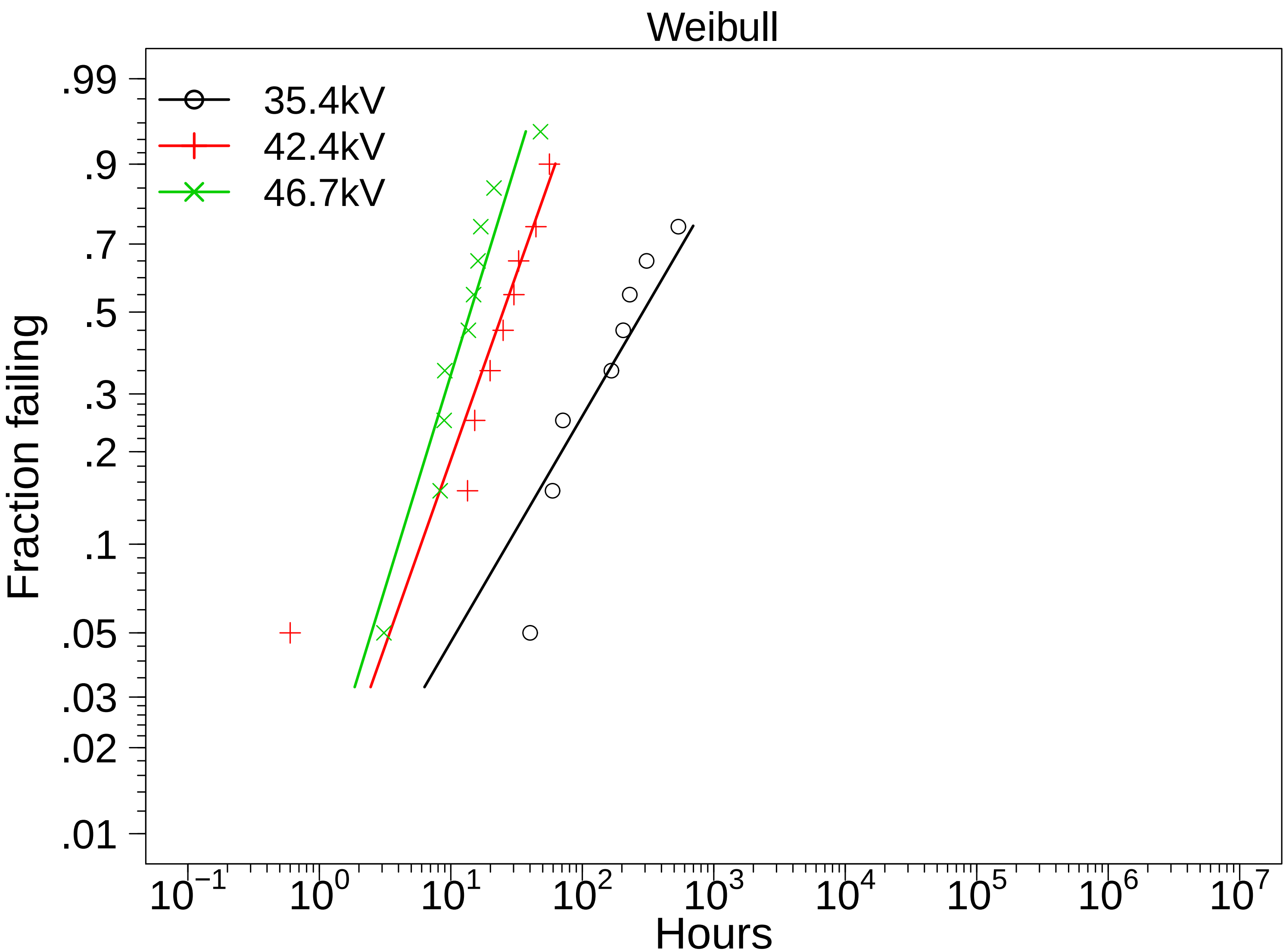
Lognormal Multiple Probability Plot of the Transformer Insulation ALT Individual Stress Levels ML Estimates with Different Shape Parameters

$$\widehat{\Pr}[T(\texttt{volt}_i) \le t] = \Phi_{\textbf{sev}}\left[\frac{\log(t) - \widehat{\mu}_i}{\widehat{\sigma}_i}\right], \qquad i = 35.4, 42.4, 46.7 \textbf{ kV}$$



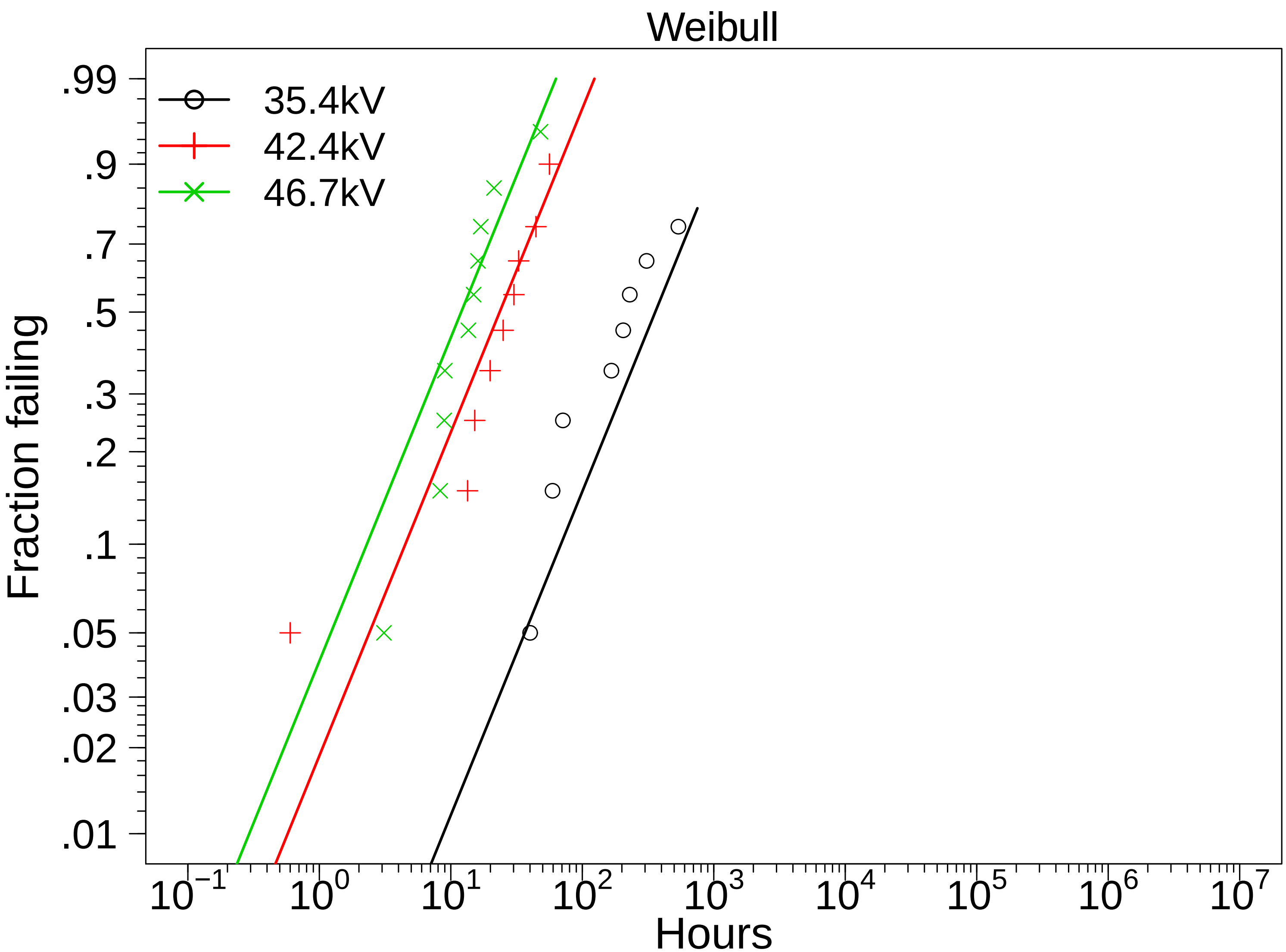
Weibull Multiple Probability Plot of the Transformer Insulation ALT Individual Stress Levels ML Estimates with Different Shape Parameters

$$\widehat{\Pr}[T(\texttt{volt}_i) \le t] = \Phi_{\textbf{sev}}\left[\frac{\log(t)-\widehat{\mu}_i}{\widehat{\sigma}_i}\right], \qquad i = 35.4, 42.4, 46.7 \ \textbf{kV}$$



Weibull Probability Plot of the Transformer Insulation ALT Individual Stress Levels ML Estimates with Equal Shape Parameter

$$\widehat{\Pr}[T(\texttt{volt}_i) \leq t] = \Phi_{\textbf{sev}}\left[\frac{\log(t) - \widehat{\mu}_i}{\widehat{\sigma}}\right], \qquad i = 35.4, 42.4, 46.7 \textbf{ kV}$$



Weibull Inverse-Power Relationship

The Weibull inverse-power relationship is

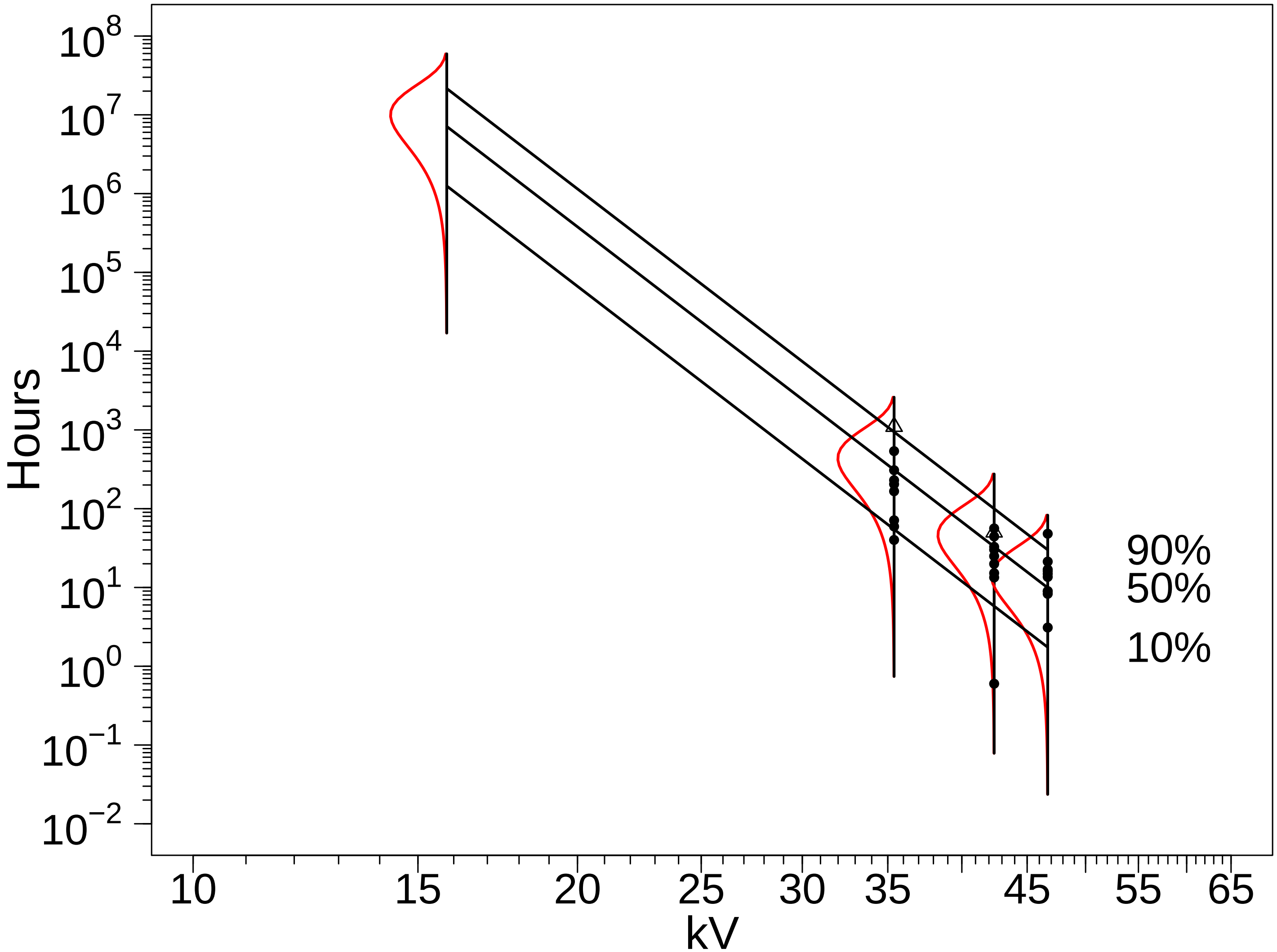
$$\Pr[T \le t; \texttt{volt}] = \Phi_{\text{sev}}\left[\frac{\log(t) - \mu}{\sigma}\right]$$

where

- $\mu = \beta_0 + \beta_1 x$,
- $x = \log(\texttt{volt})$, where $\texttt{volt} =$ voltage (or voltage stress), and
- σ is assumed to be constant.

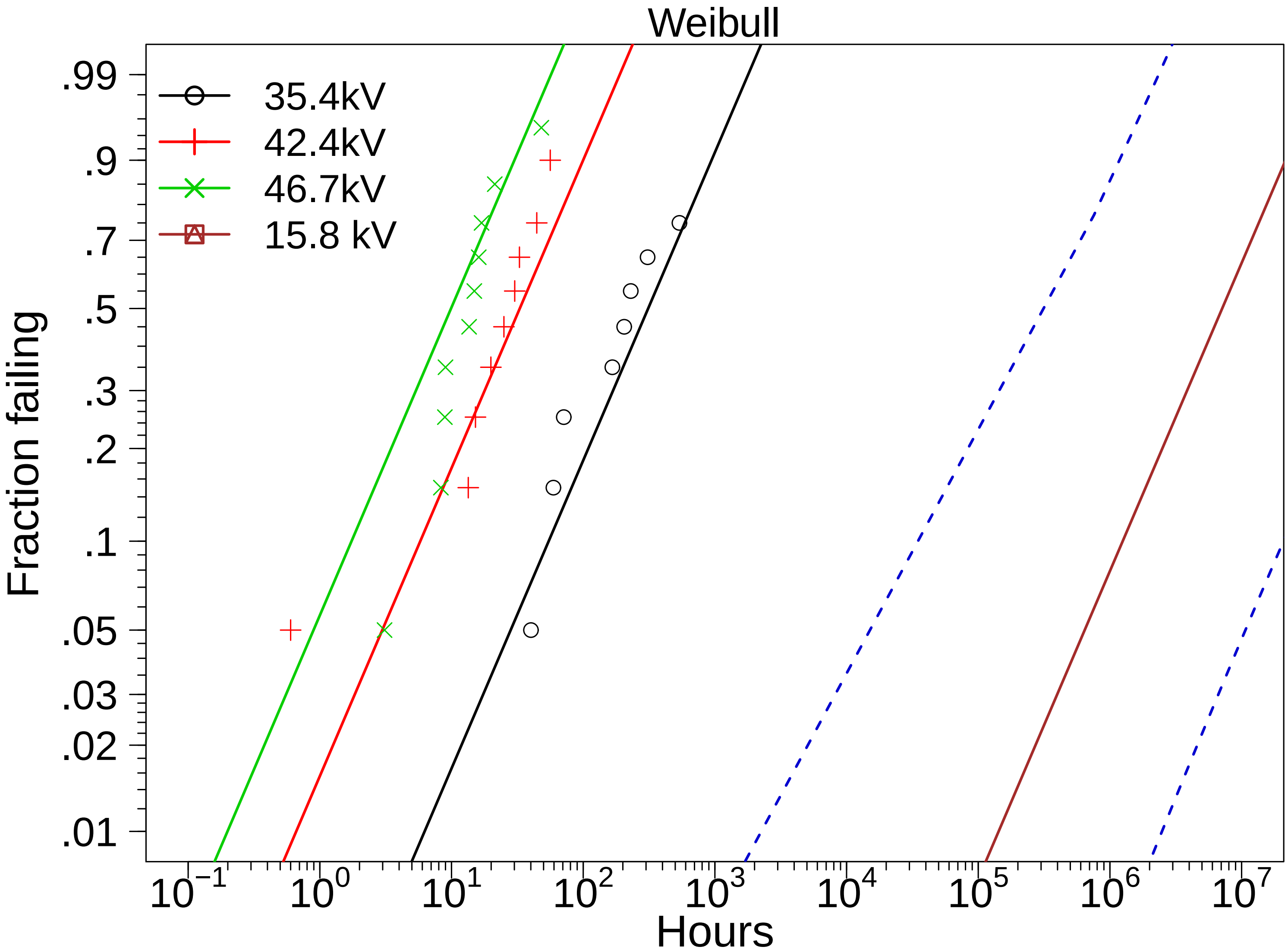
Plot of Inverse-Power Weibull Model Fitted to the Transformer Insulation Data

$$\log[\widehat{t}_p(x)] = \widehat{\mu}(x) + \Phi^{-1}_{\textbf{sev}}(p)\widehat{\sigma}, \qquad \widehat{\mu}(x) = \widehat{\beta}_0 + \widehat{\beta}_1 x$$



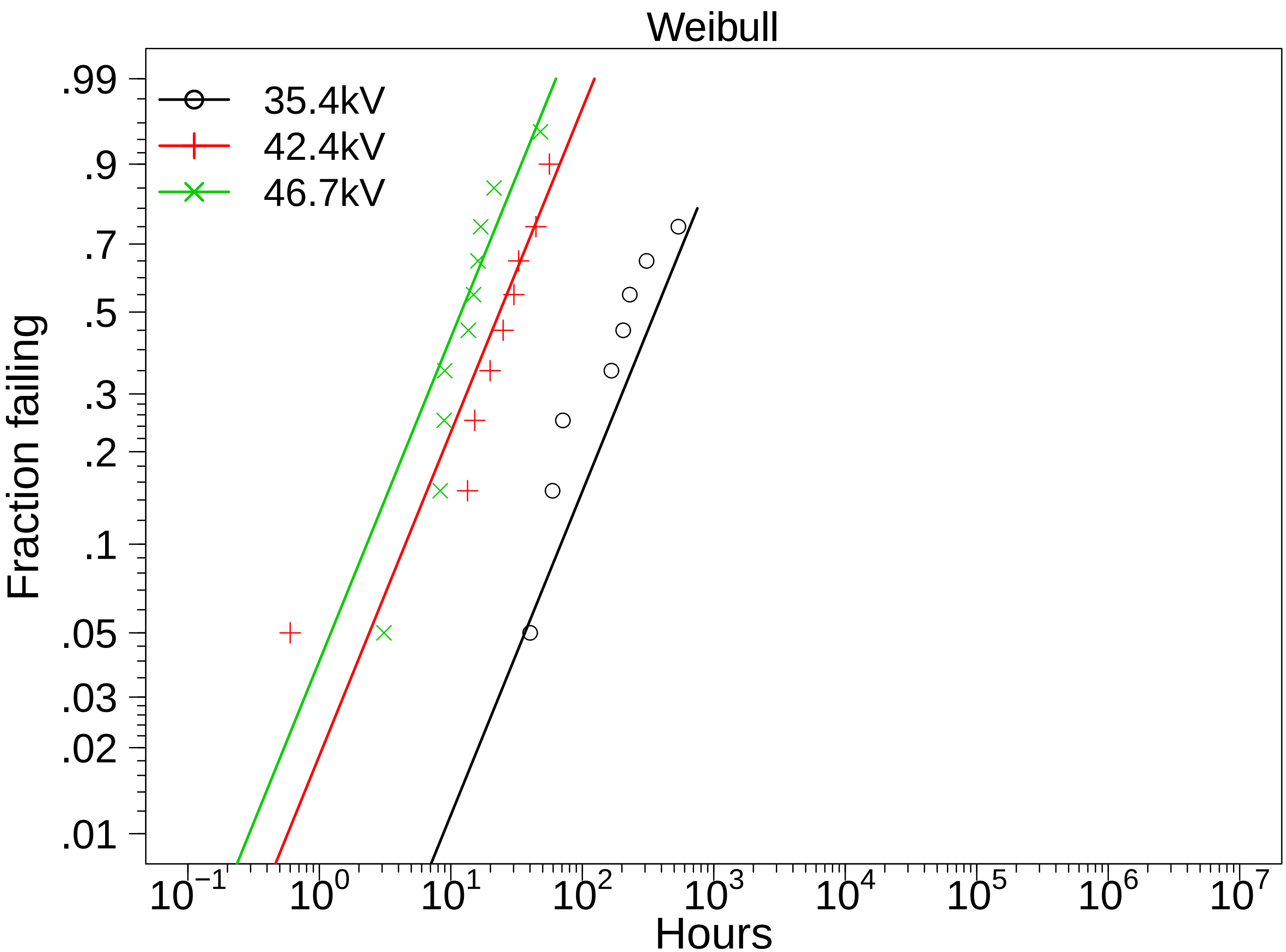
Weibull Probability Plot of the Inverse-Power Weibull Model Fitted to the Transformer Insulation Data

$$\widehat{\Pr}[T(\texttt{volt}_i) \le t] = \Phi_{\textbf{sev}}\left[\frac{\log(t) - \widehat{\mu}_i}{\widehat{\sigma}}\right], \qquad i = 35.4, 42.4, 46.7 \textbf{ kV}$$



Weibull Probability Plot of the Transformer Insulation ALT Individual Stress Levels ML Estimates with Equal Shape Parameter

$$\widehat{\Pr}[T(\texttt{volt}_i) \leq t] = \Phi_{\textbf{sev}}\left[\frac{\log(t)-\widehat{\mu}_i}{\widehat{\sigma}}\right], \qquad i = 35.4, 42.4, 46.7 \textbf{ kV}$$



Transformer Insulation ALT Data
Weibull Model-Fitting Summary

Model	-2LogLike	AIC	# Param
SepDists	266.5	278.5	6
EqualSig	269.9	277.9	4
RegrModel	271.6	277.6	3
Pooled	304.3	308.3	2

Transformer Insulation ALT Data
Weibull LR Tests

Comparison	LR Statistic	dof	p-value
SepDists vs EqualSig	3.366	2	0.19
EqualSig vs RegrModel	1.756	1	0.19
RegrModel vs Pooled	32.686	1	< 0.001

References

Hooper, J. H. and S. J. Amster (1998). Analysis and presentation of reliability data. In H. M. Wadsworth (Editor), *Handbook of Statistical Methods for Engineers and Scientists* (Second Edition). McGraw-Hill. []

Kalkanis, G. and E. Rosso (1989). The inverse power law model for the lifetime of a mylar-polyurethane laminated DC HV insulating structure. *Nuclear Instruments and Methods in Physics Research A281*, 489–496. []

Meeker, W. Q., L. A. Escobar, and F. G. Pascual (2021). *Statistical Methods for Reliability Data* (Second Edition). Wiley. [1]

Nelson, W. B. (2004). *Accelerated Testing: Statistical Models, Test Plans, and Data Analyses* (Paperback Edition). Wiley. [32, 65]