Chapter 17

Failure-Time Regression Analysis

W. Q. Meeker, L. A. Escobar, and F. G. Pascual
Iowa State University, Louisiana State University, and Washington State University.

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Chapter 17
Failure-Time Regression Analysis

Topics discussed in this chapter are:

- Applications of failure-time regression.
- Graphical methods for displaying censored regression data.
- Simple regression models to relate life to explanatory variables.
- The use of likelihood methods for censored regression data.
- The importance of model diagnostics.
- Extensions to nonstandard multiple regression models.

Chapter 17

Failure-Time Regression Analysis

Segment 1

Introduction to Failure-Time Regression

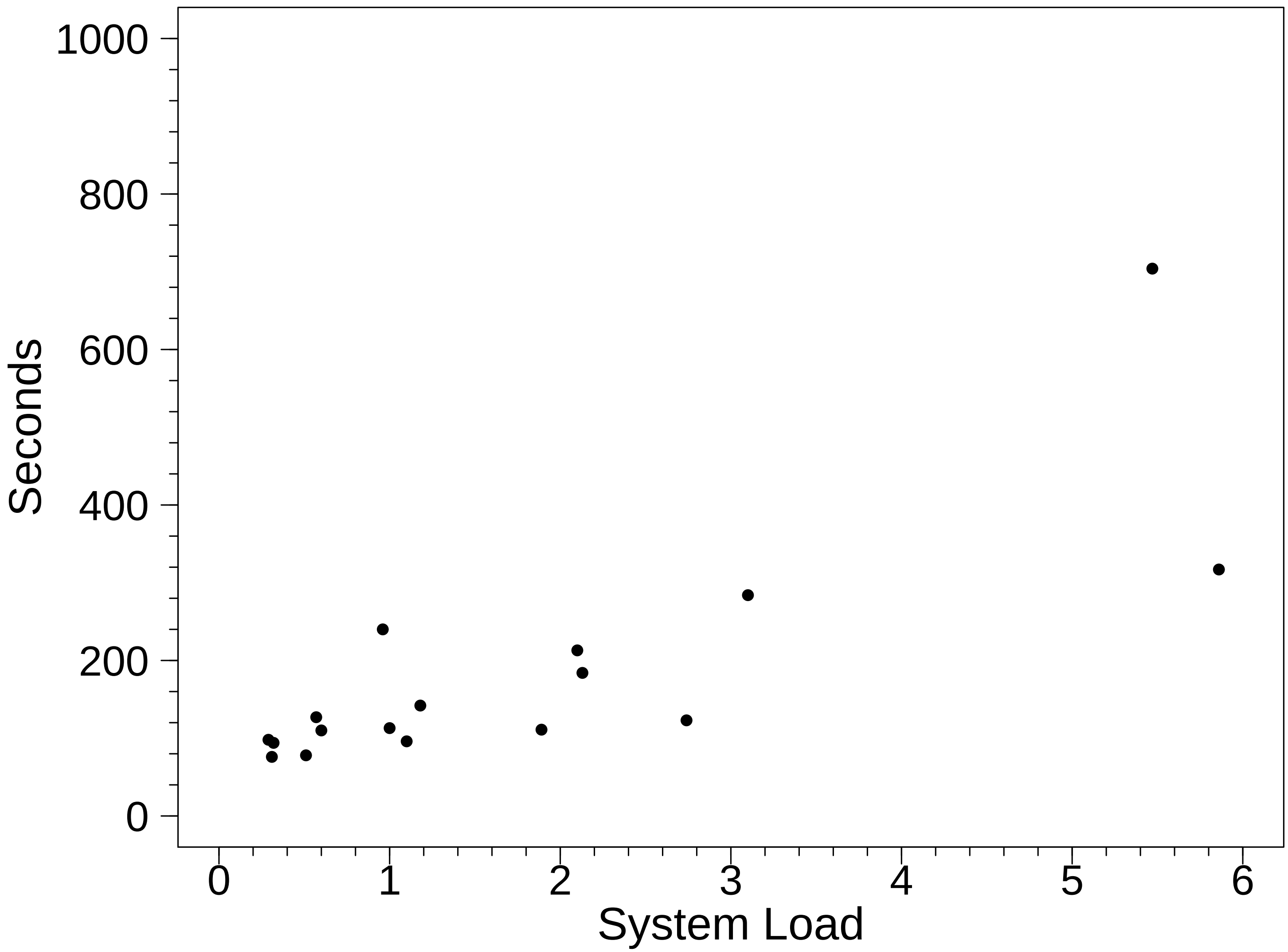
Computer Program Execution Time Versus System Load

- Time to complete a computationally-intensive task.

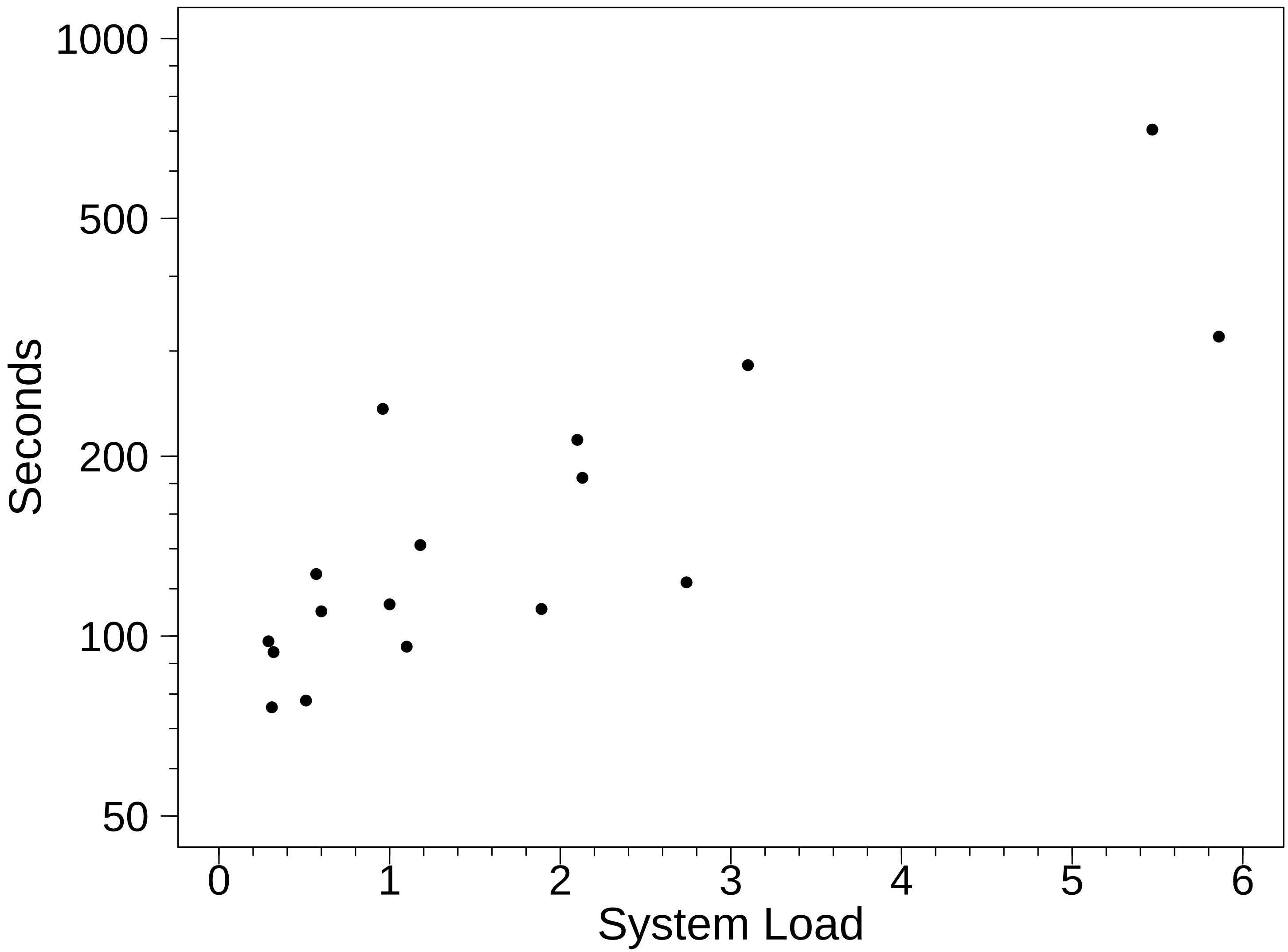
- Information from the Unix `uptime` command.

- Predictions needed for scheduling subsequent steps in a multi-step computational process.

Scatter Plot of Computer Program Execution Time Versus System Load Linear-Linear



Scatter Plot of Computer Program Execution Time Versus System Load Log-Linear



Explanatory Variables for Failure Times

Useful explanatory variables explain/predict why some units fail quickly and some units survive for a long time.

- Continuous variables like stress, temperature, voltage, and pressure.

- Discrete variables like the number of hardening treatments or the number of simultaneous users of a system.

- Categorical variables like manufacturer, design, operator, and location.

Regression model relates failure time distribution to explanatory variables $\boldsymbol{x} = (x_1, \ldots, x_k)$:

$$\Pr(T \leq t) = F(t) = F(t; \boldsymbol{x}).$$

Failure-Time Regression Analysis

- Material in this chapter is an **extension** of statistical regression analysis with normal distributed data and

$$\text{mean} = \beta_0 + \beta_1 x_1 + \cdots + \beta_k x_k$$

  where the x_i are explanatory variables.

- The ideas presented here are more general:
  - ▶ Data not necessarily from a normal distribution.
  - ▶ Data may be censored.
  - ▶ Nonstandard regression models that relate life to explanatory variables.

- Presentation motivated by practical problems in reliability analysis.

Lognormal Distribution Simple Regression Model with Constant Shape Parameter σ

- The lognormal simple regression model is

$$\Pr(T \le t) = F(t; \mu, \sigma) = F(t; \beta_0, \beta_1, \sigma) = \Phi_{\text{norm}}\left[\frac{\log(t) - \mu}{\sigma}\right]$$

where $\mu = \mu(x) = \beta_0 + \beta_1 x$ and σ does not depend on x.

- The failure-time log quantile function

$$\log[t_p(x)] = \mu(x) + \Phi^{-1}_{\text{norm}}(p)\,\sigma$$

is linear in x.

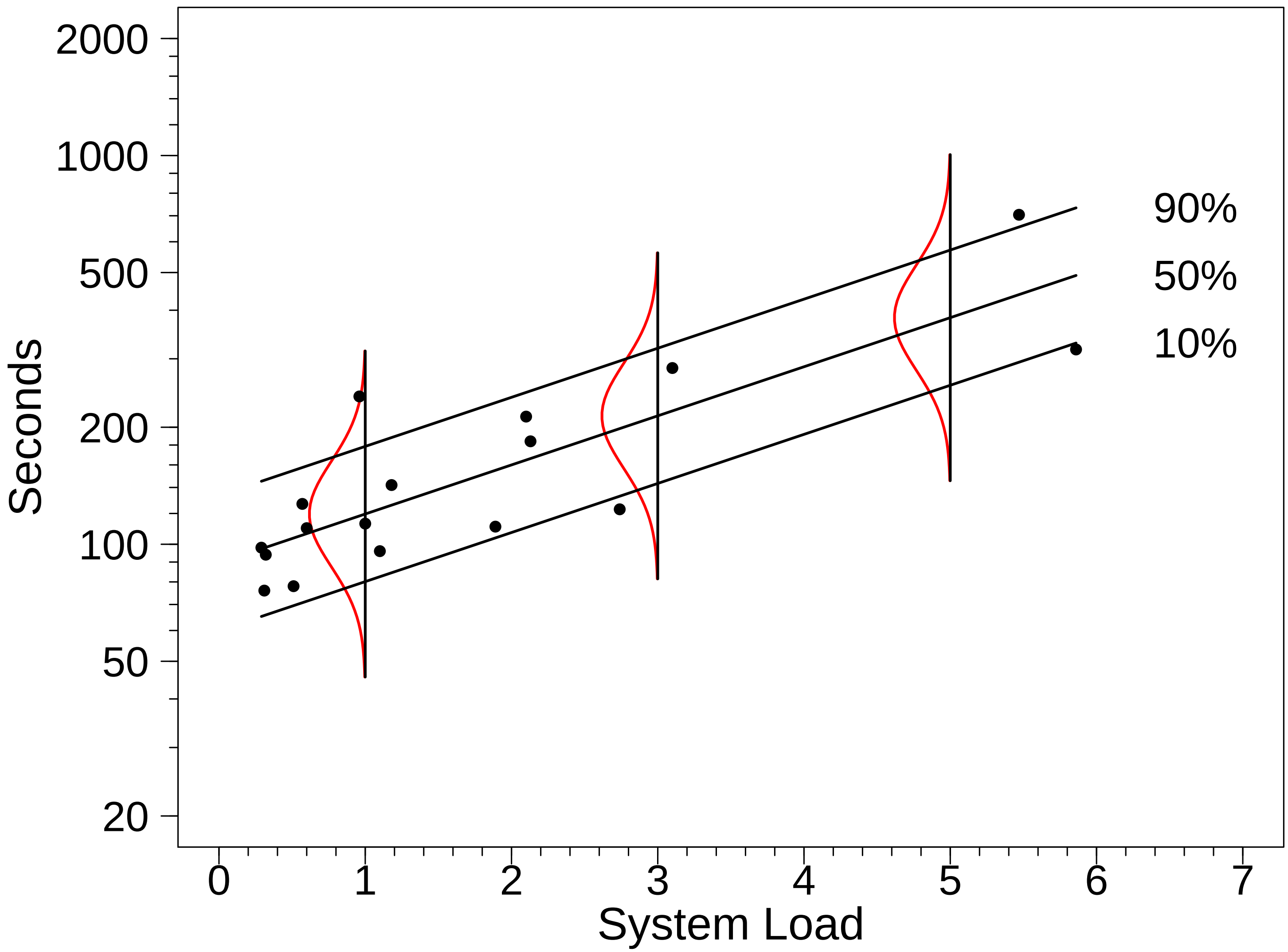
Notice that

$$AF = \frac{t_p(x)}{t_p(0)} = \exp(\beta_1 x)$$

does not depend on p, implying that changes in x only scale time.

Computer Program Execution Time Versus System Load Log-Linear Lognormal Regression Model

$$\log[\widehat{t}_p(x)] = \widehat{\mu}(x) + \Phi^{-1}_{\textbf{norm}}(p)\widehat{\sigma}$$



Likelihood for Lognormal Distribution Simple Regression Model with Right-Censored Data

The likelihood for n independent observations has the form

$$
\begin{aligned}
L(\beta_0, \beta_1, \sigma) &= \prod_{i=1}^{n} L_i(\beta_0, \beta_1, \sigma; \text{data}_i) \\
&= \prod_{i=1}^{n} \left\{ \frac{1}{\sigma t_i} \phi_{\text{norm}} \left[\frac{\log(t_i) - \mu_i}{\sigma} \right] \right\}^{\delta_i} \left\{ 1 - \Phi_{\text{norm}} \left[\frac{\log(t_i) - \mu_i}{\sigma} \right] \right\}^{1-\delta_i}
\end{aligned}
$$

where $\text{data}_i = (x_i, t_i, \delta_i)$, $\mu_i = \beta_0 + \beta_1 x_i$,

$$
\delta_i = \begin{cases} 1 & \text{exact observation} \\ 0 & \text{right-censored observation} \end{cases},
$$

$\phi_{\text{norm}}(z)$ is the standardized normal pdf, and $\Phi_{\text{norm}}(z)$ is the corresponding normal cdf.

The parameters are $\boldsymbol{\theta} = (\beta_0, \beta_1, \sigma)$.

Estimated Parameter Variance-Covariance Matrix

Local (observed information) estimate

$$\hat{\Sigma}_{\widehat{\boldsymbol{\theta}}} = \left[\begin{array}{lll} \widehat{\mathrm{Var}}(\widehat{\beta}_0) & \widehat{\mathrm{Cov}}(\widehat{\beta}_0, \widehat{\beta}_1) & \widehat{\mathrm{Cov}}(\widehat{\beta}_0, \widehat{\sigma}) \\ \widehat{\mathrm{Cov}}(\widehat{\beta}_1, \widehat{\beta}_0) & \widehat{\mathrm{Var}}(\widehat{\beta}_1) & \widehat{\mathrm{Cov}}(\widehat{\beta}_1, \widehat{\sigma}) \\ \widehat{\mathrm{Cov}}(\widehat{\sigma}, \widehat{\beta}_0) & \widehat{\mathrm{Cov}}(\widehat{\sigma}, \widehat{\beta}_1) & \widehat{\mathrm{Var}}(\widehat{\sigma}) \end{array} \right]$$

$$= \left[\begin{array}{ccc} -\frac{\partial^2 \mathcal{L}(\beta_0,\beta_1,\sigma)}{\partial \beta_0^2} & -\frac{\partial^2 \mathcal{L}(\beta_0,\beta_1,\sigma)}{\partial \beta_0 \partial \beta_1} & -\frac{\partial^2 \mathcal{L}(\beta_0,\beta_1,\sigma)}{\partial \beta_0 \partial \sigma} \\ -\frac{\partial^2 \mathcal{L}(\beta_0,\beta_1,\sigma)}{\partial \beta_1 \partial \beta_0} & -\frac{\partial^2 \mathcal{L}(\beta_0,\beta_1,\sigma)}{\partial \beta_1^2} & -\frac{\partial^2 \mathcal{L}(\beta_0,\beta_1,\sigma)}{\partial \beta_1 \partial \sigma} \\ -\frac{\partial^2 \mathcal{L}(\beta_0,\beta_1,\sigma)}{\partial \sigma \partial \beta_0} & -\frac{\partial^2 \mathcal{L}(\beta_0,\beta_1,\sigma)}{\partial \sigma \partial \beta_1} & -\frac{\partial^2 \mathcal{L}(\beta_0,\beta_1,\sigma)}{\partial \sigma^2} \end{array} \right]^{-1}$$

Partial derivatives are evaluated at $\widehat{\beta}_0, \widehat{\beta}_1, \widehat{\sigma}$.

Standard Errors and Confidence Intervals for Parameters

- Lognormal ML estimates for the computer time experiment were $\widehat{\boldsymbol{\theta}} = (\widehat{\beta}_0, \widehat{\beta}_1, \widehat{\sigma}) = (4.49, 0.290, 0.312)$ and an estimate of the variance-covariance matrix for $\widehat{\boldsymbol{\theta}}$ is

$$\widehat{\Sigma}_{\widehat{\boldsymbol{\theta}}} = \begin{bmatrix} 0.012 & -0.0037 & 0 \\ -0.0037 & 0.0021 & 0 \\ 0 & 0 & 0.0029 \end{bmatrix}.$$

- Wald confidence interval for the computer execution time regression slope is

$$[\underset{\sim}{\beta_1},\ \widetilde{\beta_1}] = \widehat{\beta}_1 \pm z_{(0.975)}\text{se}_{\widehat{\beta}_1} = 0.290 \pm 1.96(0.046) = [0.20,\ 0.38]$$

where $\text{se}_{\widehat{\beta}_1} = \sqrt{0.0021} = 0.046$.

Standard Errors and Confidence Intervals for Quantities at Specific Explanatory Variable Conditions

- Unknown values of μ and σ at each level of x.

- $\widehat{\mu} = \widehat{\beta}_0 + \widehat{\beta}_1 x$, σ does not depend on x, and

$$\widehat{\Sigma}_{\widehat{\mu},\widehat{\sigma}} = \begin{bmatrix} \widehat{\text{Var}}(\widehat{\mu}) & \widehat{\text{Cov}}(\widehat{\mu},\widehat{\sigma}) \\ \widehat{\text{Cov}}(\widehat{\mu},\widehat{\sigma}) & \widehat{\text{Var}}(\widehat{\sigma}) \end{bmatrix}$$

  is obtained from $\widehat{\text{Var}}(\widehat{\mu}) = \widehat{\text{Var}}(\widehat{\beta}_0) + 2x\widehat{\text{Cov}}(\widehat{\beta}_1,\widehat{\beta}_0) + x^2\widehat{\text{Var}}(\widehat{\beta}_1)$ and $\widehat{\text{Cov}}(\widehat{\mu},\widehat{\sigma}) = \widehat{\text{Cov}}(\widehat{\beta}_0,\widehat{\sigma}) + x\widehat{\text{Cov}}(\widehat{\beta}_1,\widehat{\sigma})$.

- Use the above results with the methods from Chapter 8 to compute Wald confidence intervals for $F(t)$, $h(t)$, and t_p.

- Could also use likelihood or simulation-based confidence intervals.

Chapter 17

Failure-Time Regression Analysis

Segment 2

Nonconstant Variance in Failure-Time Regression

Nickel-Base Super-Alloy Fatigue Data
26 Observations in Total, 4 Censored

Originally described and analyzed by Nelson (1984) and Nelson (2004).

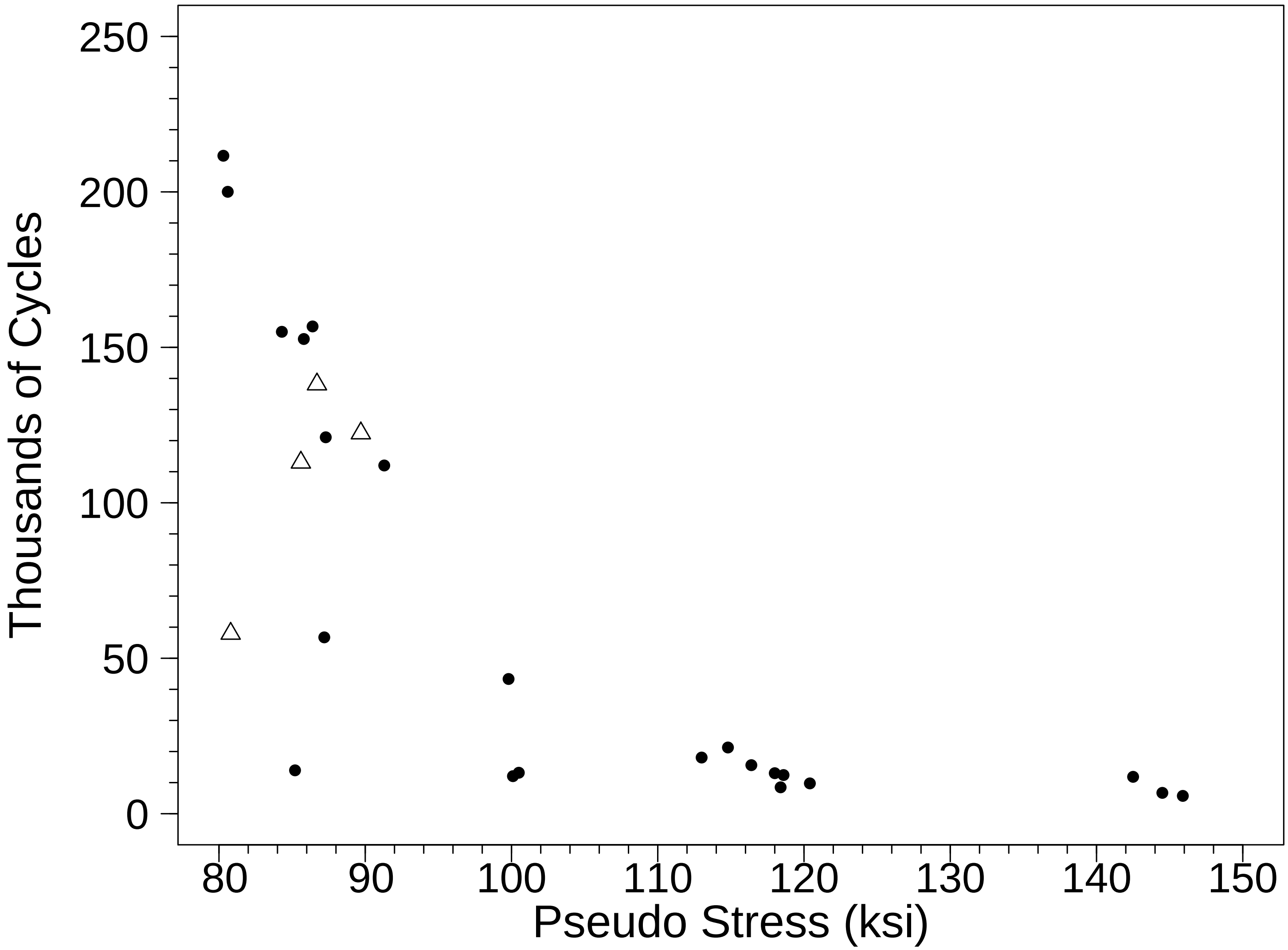
- Thousands of cycles to failure as a function of **pseudo-stress** in ksi.

- Pseudo-stress is Young's modulus multiplied to strain.

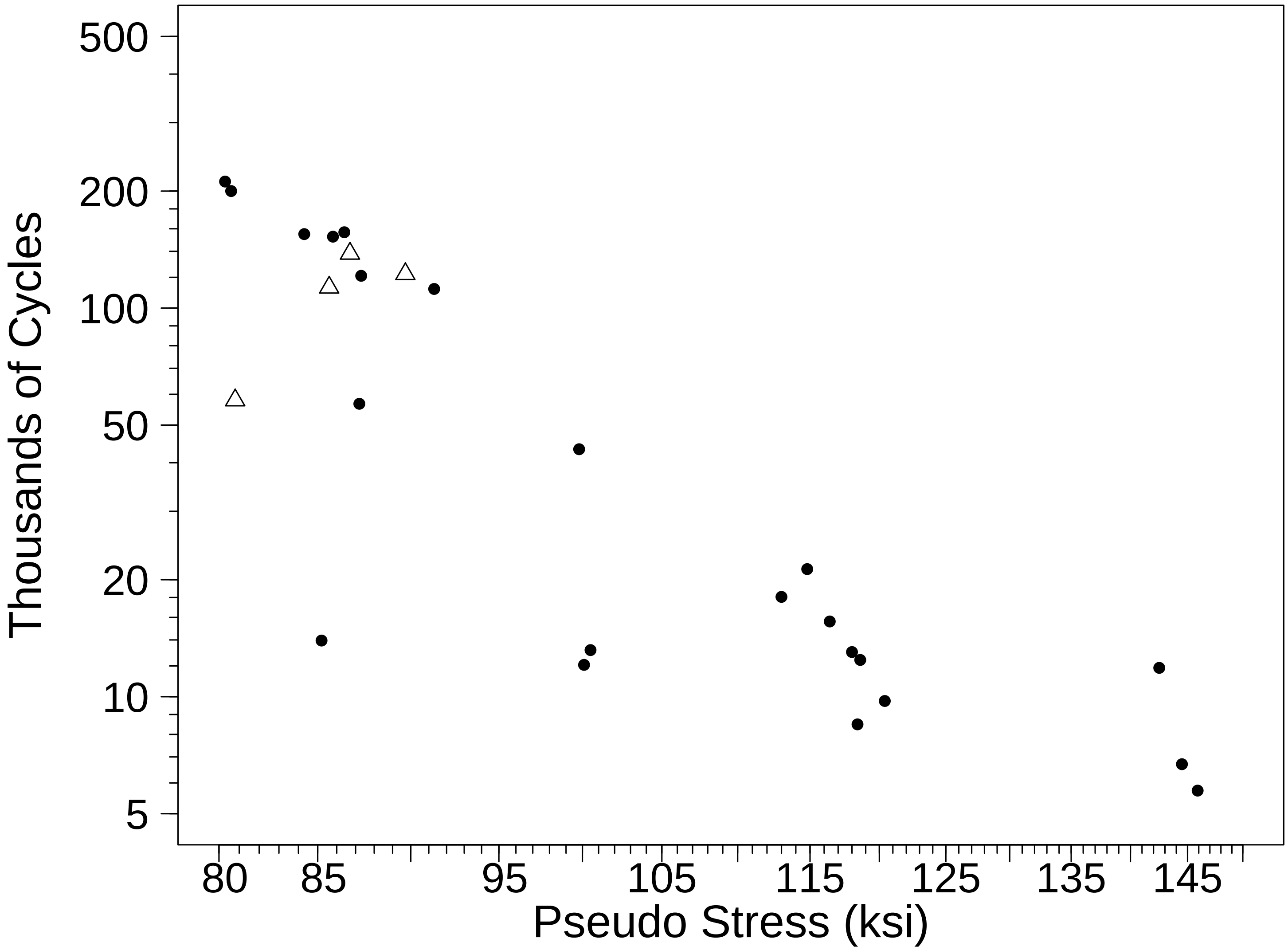
- 26 units tested; 4 units did not fail.

Objective: Find a regression model to describe the relationship between fatigue life and pseudo-stress (i.e., find an S/N curve).

Nickel-Base Super-Alloy Fatigue Data
Linear-Linear



Nickel-Base Super-Alloy Fatigue Data
Log-Log



Weibull Distribution Quadratic Regression Model with Constant Shape Parameter $\beta = 1/\sigma$

This is a lifetime model with the following characteristics:

- The Weibull quadratic regression model is

$$\Pr[T \le t] = \Phi_{\text{sev}}\left[\frac{\log(t) - \mu}{\sigma}\right]$$

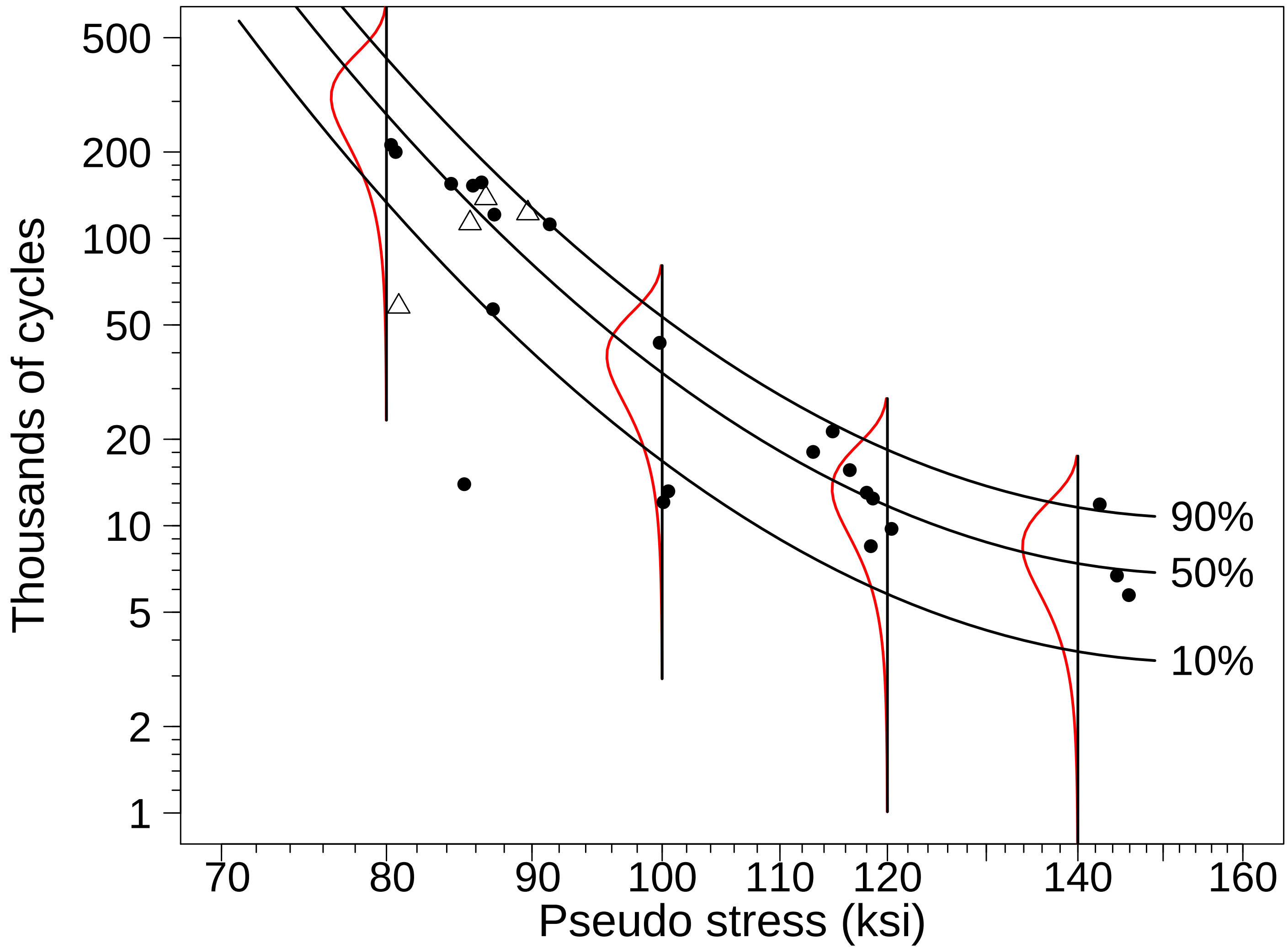
  where $\mu = \mu(x) = \beta_0 + \beta_1 x + \beta_2 x^2$ and σ does not depend on x.

- $x = \log(\text{Pseudo-stress})$.

Log-Quadratic Weibull Regression Model with Constant ($\beta = 1/\sigma$) Fit to the Super-Alloy Data

$$\log[\widehat{t}_p(x)] = \widehat{\mu}(x) + \Phi^{-1}_{\textbf{sev}}(p)\widehat{\sigma},\ x = \log(\textbf{pseudo-stress})$$

$$\widehat{\mu} = \widehat{\beta}_0 + \widehat{\beta}_1 x + \widehat{\beta}_2 x^2$$



Likelihood for Weibull Distribution Quadratic Regression Model with Right-Censored Data

The likelihood for n independent observations is

$$
\begin{aligned}
L(\beta_0, \beta_1, \beta_2, \sigma) &= \prod_{i=1}^{n} L_i(\beta_0, \beta_1, \beta_2, \sigma; \text{data}_i) \\
&= \prod_{i=1}^{n} \left\{ \frac{1}{\sigma t_i} \phi_{\text{sev}} \left[\frac{\log(t_i) - \mu_i}{\sigma} \right] \right\}^{\delta_i} \left\{ 1 - \Phi_{\text{sev}} \left[\frac{\log(t_i) - \mu_i}{\sigma} \right] \right\}^{1-\delta_i}.
\end{aligned}
$$

where $\mu_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2$,

$$
\delta_i = \begin{cases} 1 & \text{exact observation} \\ 0 & \text{right-censored observation} \end{cases}
$$

The parameters are $\boldsymbol{\theta} = (\beta_0, \beta_1, \beta_2, \sigma)$.

Weibull Distribution Quadratic Regression Model with Nonconstant $\beta = 1/\sigma$

- The Weibull quadratic regression model is

$$\Pr[T \le t] = \Phi_{\text{sev}}\{[\log(t) - \mu]/\sigma\},$$

where $\mu = \mu(x) = \beta_0^{[\mu]} + \beta_1^{[\mu]} x + \beta_2^{[\mu]} x^2$ and $\log(\sigma) = \log[\sigma(x)] = \beta_0^{[\sigma]} + \beta_1^{[\sigma]} x$.

- The failure-time log quantile function is

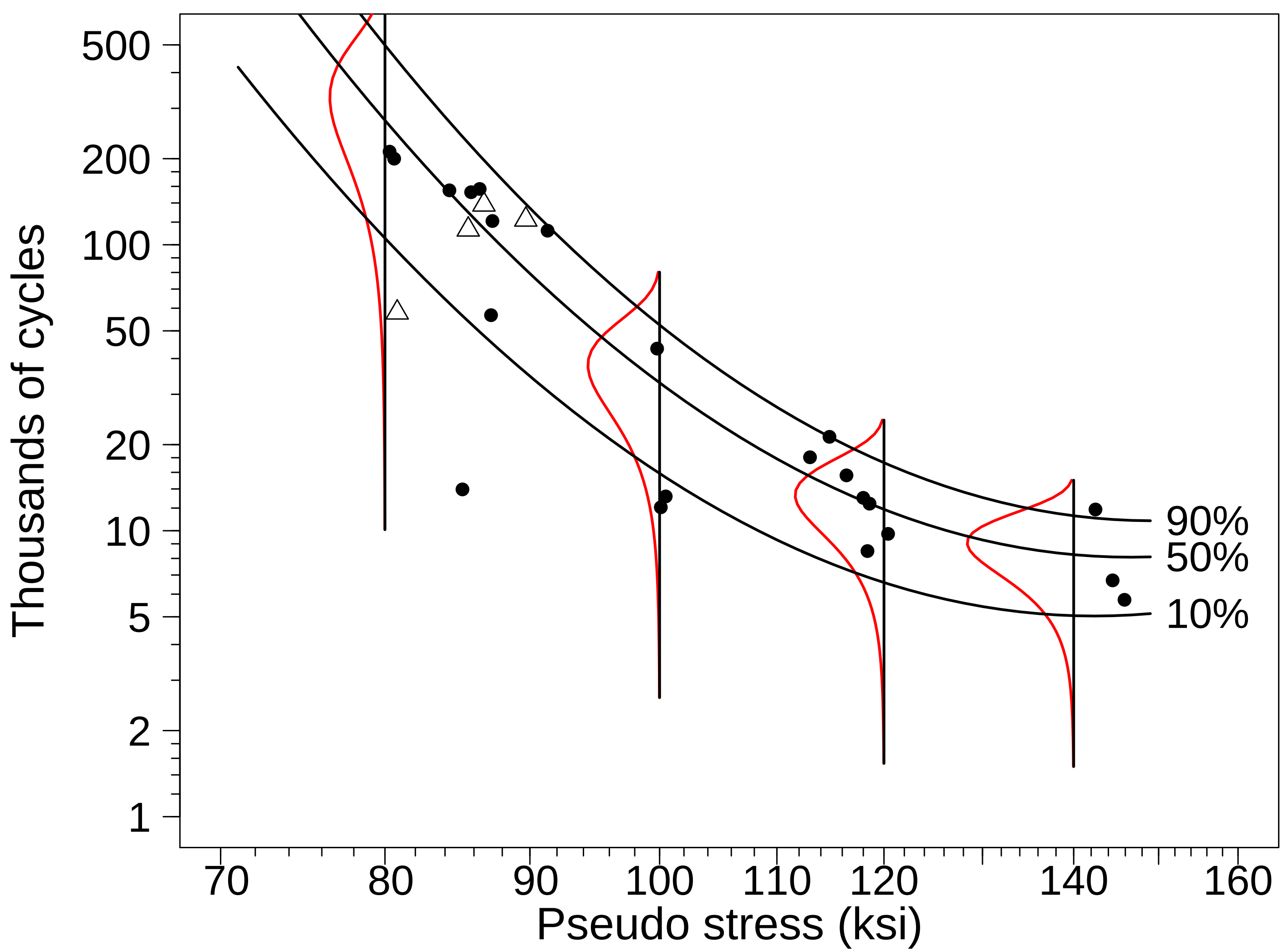
$$\log[t_p(x)] = \mu(x) + \Phi_{\text{sev}}^{-1}(p)\,\sigma(x)$$

which is **not** quadratic in x.

Log-Quadratic Weibull Regression Model with Nonconstant $\beta = 1/\sigma$ Fit to the Super-Alloy Data

$$\log[\widehat{t}_p(x)] = \widehat{\mu}(x) + \Phi^{-1}_{\textbf{sev}}(p)\widehat{\sigma}(x), \; x = \log(\textbf{pseudo-stress})$$

$$\widehat{\mu} = \widehat{\beta}_0 + \widehat{\beta}_1 x + \widehat{\beta}_2 x^2, \; \log(\widehat{\sigma}) = \widehat{\beta}^{[\sigma]}_0 + \widehat{\beta}^{[\sigma]}_1 x$$



Likelihood for Weibull Distribution Quadratic Regression Model with Nonconstant $\beta = 1/\sigma$ and Right-Censored Data

The likelihood for n independent observations has the form

$$
\begin{aligned}
&L(\beta_0^{[\mu]}, \beta_1^{[\mu]}, \beta_2^{[\mu]}, \beta_0^{[\sigma]}, \beta_1^{[\sigma]}) \\
&= \prod_{i=1}^{n} L_i(\beta_0^{[\mu]}, \beta_1^{[\mu]}, \beta_2^{[\mu]}, \beta_0^{[\sigma]}, \beta_1^{[\sigma]}; \text{data}_i) \\
&= \prod_{i=1}^{n} \left\{ \frac{1}{\sigma_i t_i} \phi_{\text{sev}} \left[\frac{\log(t_i) - \mu_i}{\sigma_i} \right] \right\}^{\delta_i} \left\{ 1 - \Phi_{\text{sev}} \left[\frac{\log(t_i) - \mu_i}{\sigma_i} \right] \right\}^{1-\delta_i}.
\end{aligned}
$$

where $\mu_i = \beta_0^{[\mu]} + \beta_1^{[\mu]} x_i + \beta_2^{[\mu]} x_i^2$ and $\sigma_i = \exp\left(\beta_0^{[\sigma]} + \beta_1^{[\sigma]} x_i\right)$.

Parameters are $\boldsymbol{\theta} = (\beta_0^{[\mu]}, \beta_1^{[\mu]}, \beta_2^{[\mu]}, \beta_0^{[\sigma]}, \beta_1^{[\sigma]})$.

Chapter 17

Failure-Time Regression Analysis

Segment 3

Empirical Models and Extrapolation and Checking Model Assumptions

Extrapolation and Empirical Models

- Empirical models can be useful, providing a smooth curve to describe a population or a process.
- When using an empirical model, it is dangerous to extrapolate outside of the range of one's data.
- There are different kinds of extrapolation
  - ▶ To the upper tail of a distribution.
  - ▶ To the lower tail of a distribution.
  - ▶ In an explanatory variable like stress or temperature.
- Need to get the right curve to extrapolate: look toward physical or other process theory.

Checking Model Assumptions

- Graphical checks using generalizations of usual diagnostics (including residual analysis)
  - Residuals versus fitted values.
  - Probability plot of residuals.
  - Residuals versus other potential explanatory variables.
  - Fitted values versus actual response.

- Most analytical tests can be suitably generalized, at least approximately, for censored data (especially using likelihood ratio tests).

Definition of Standardized Residuals

- For location-scale distributions like the normal, logistic, largest extreme value, and smallest extreme value,

$$\widehat{\epsilon}_i = \frac{y_i - \widehat{y}_i}{\widehat{\sigma}}$$

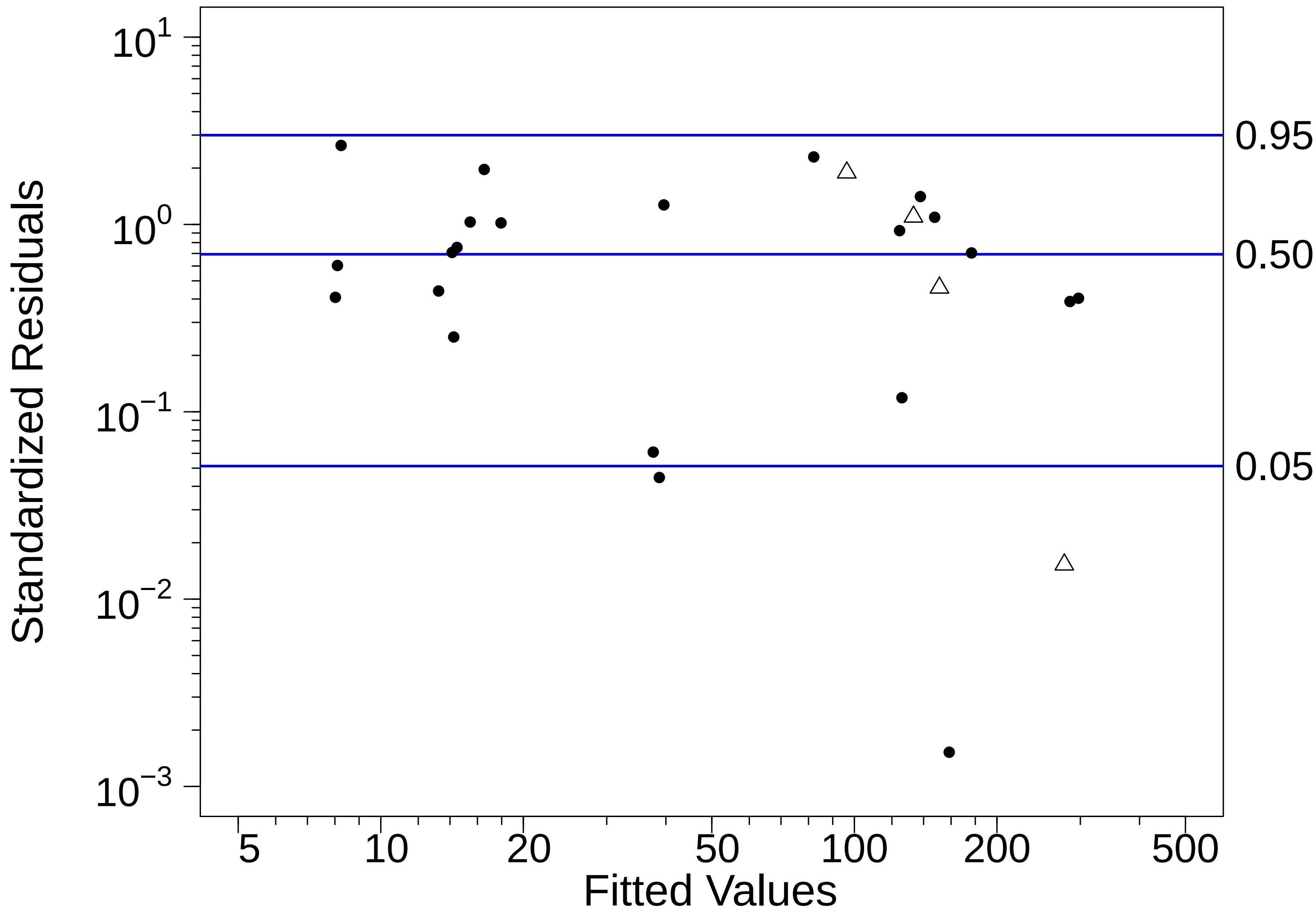
where $\widehat{y}_i$ is an appropriately defined fitted value (e.g., $\widehat{y}_i = \widehat{\mu}_i$).

- With models for positive random variables like Weibull, lognormal, and loglogistic, standardized residuals are defined as

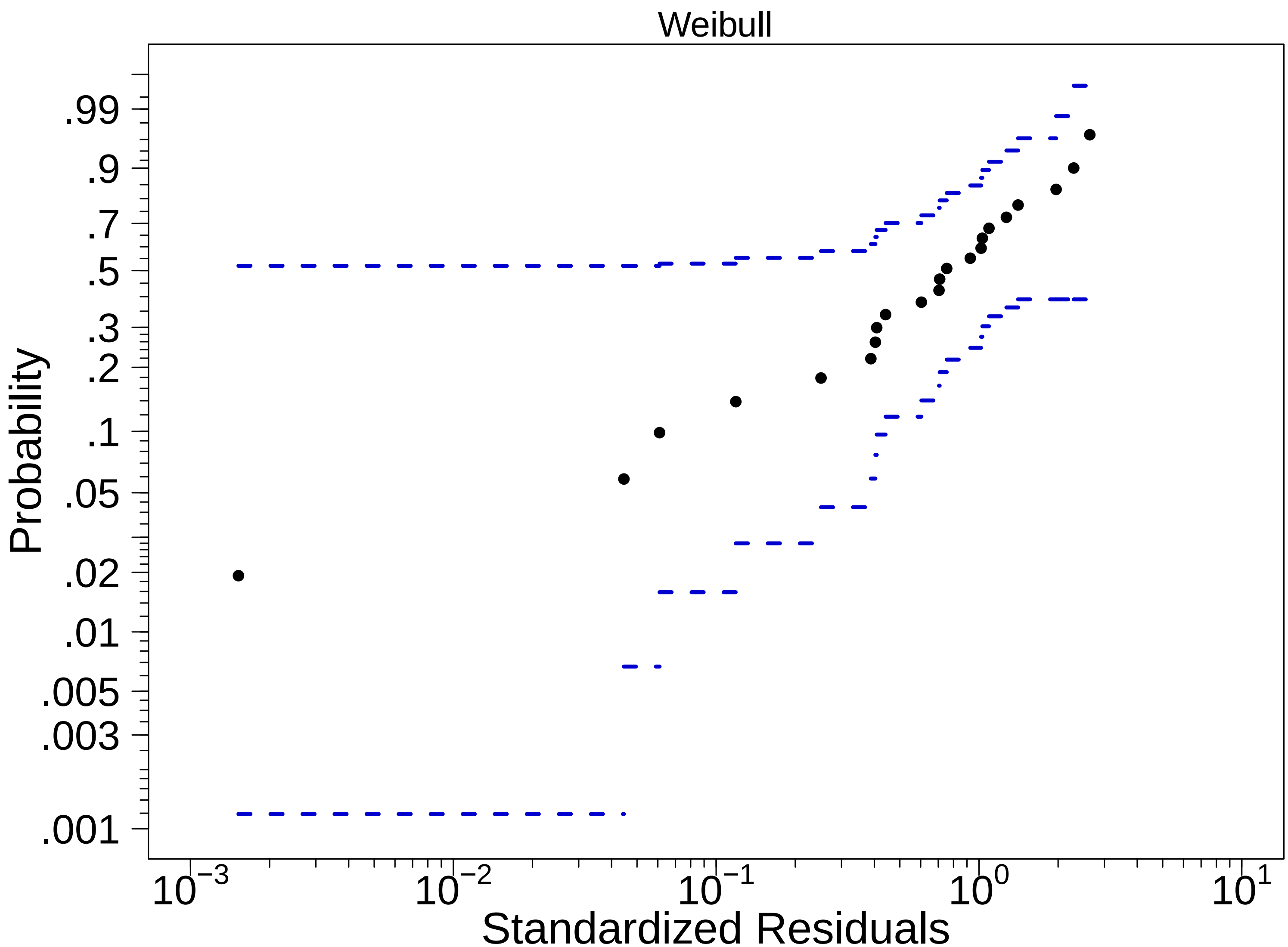
$$\exp(\widehat{\epsilon}_i) = \exp\left[\frac{\log(t_i) - \log(\widehat{t}_i)}{\widehat{\sigma}}\right] = \left(\frac{t_i}{\widehat{t}_i}\right)^{1/\widehat{\sigma}}$$

where $\widehat{t}_i = \exp(\widehat{\mu}_i)$ and when t_i is a censored observation, the corresponding residual is also censored.

Plot of Standardized Residuals Versus Fitted Values for the Log-Quadratic Weibull Regression Model Fit to the Super Alloy Data on Log-Log Axes



Probability Plot of the Standardized Residuals from the Log-Quadratic Weibull Regression Model Fit to the Super Alloy Data



Empirical Regression Models and Sensitivity Analysis Objectives and Strategy

- Describe a class of regression models that can be used to describe the relationship between failure time and explanatory variables. Use data and previous experience to choose a base-line model. Fit the following models to check assumptions:
  - Separate distribution at each condition.
  - Separate distribution at each condition with σ fixed.
  - Regression relationship between explanatory variables and distributions at individual conditions.
- Fit the chosen empirical regression models and use diagnostics (e.g., residual analysis) to check their fits.
- Assess uncertainty
  - Confidence intervals quantify statistical uncertainty.
  - Perturb and otherwise change the model and reanalyze (sensitivity analysis) to assess model uncertainty.

Chapter 17

Failure-Time Regression Analysis

Segment 4

Transformations of a Positive Explanatory Variable

Transformations of a Positive Explanatory Variable

- In choosing an empirical model, it is often necessary to transform the explanatory variable in order to achieve a better fit to data.

- For example, curvature in a scatter plot of y versus x may suggest that a model quadratic in x will provide a better fit than one that is linear in x. In this case, the response t_i might be modeled as a function of $x_i^* = x_i^2$.

- A formal way of choosing an appropriate transformation is to consider one from the Box-Cox family of transformations.

- A sensitivity analysis should be performed to assess the effect of different transformations on the analysis.

Examples of Monotone Increasing Power Transformations of a Positive Explanatory Variable

λ	Transformation
-2	$x_i^* = -1/x_i^2$
-1	$x_i^* = -1/x_i$
-0.5	$x_i^* = -1/\sqrt{x_i}$
-0.333	$x_i^* = -1/x_i^{1/3}$
0	$x_i^* \stackrel{\text{def}}{=} \log(x_i)$
0.333	$x_i^* = x_i^{1/3}$
0.5	$x_i^* = \sqrt{x_i}$
1	$x_i^* = x_i$
2	$x_i^* = x_i^2$

Box–Cox Transformation

- The Box–Cox family of power transformations of a positive explanatory variable is

$$x_i^* = \begin{cases} \dfrac{x_i^\lambda - 1}{\lambda} & \lambda \neq 0 \\ \log(x_i) & \lambda = 0 \end{cases}$$

  where x_i is the original, untransformed explanatory variable for observation i and λ is the power transformation parameter.

- The Box–Cox transformation has the following important properties:

  - The transformed value x_i^* is an increasing function of x_i.

  - For fixed x_i, x_i^* is a continuous function of λ through 0.

Estimation of an S-N Curve for a Laminate Panel Data

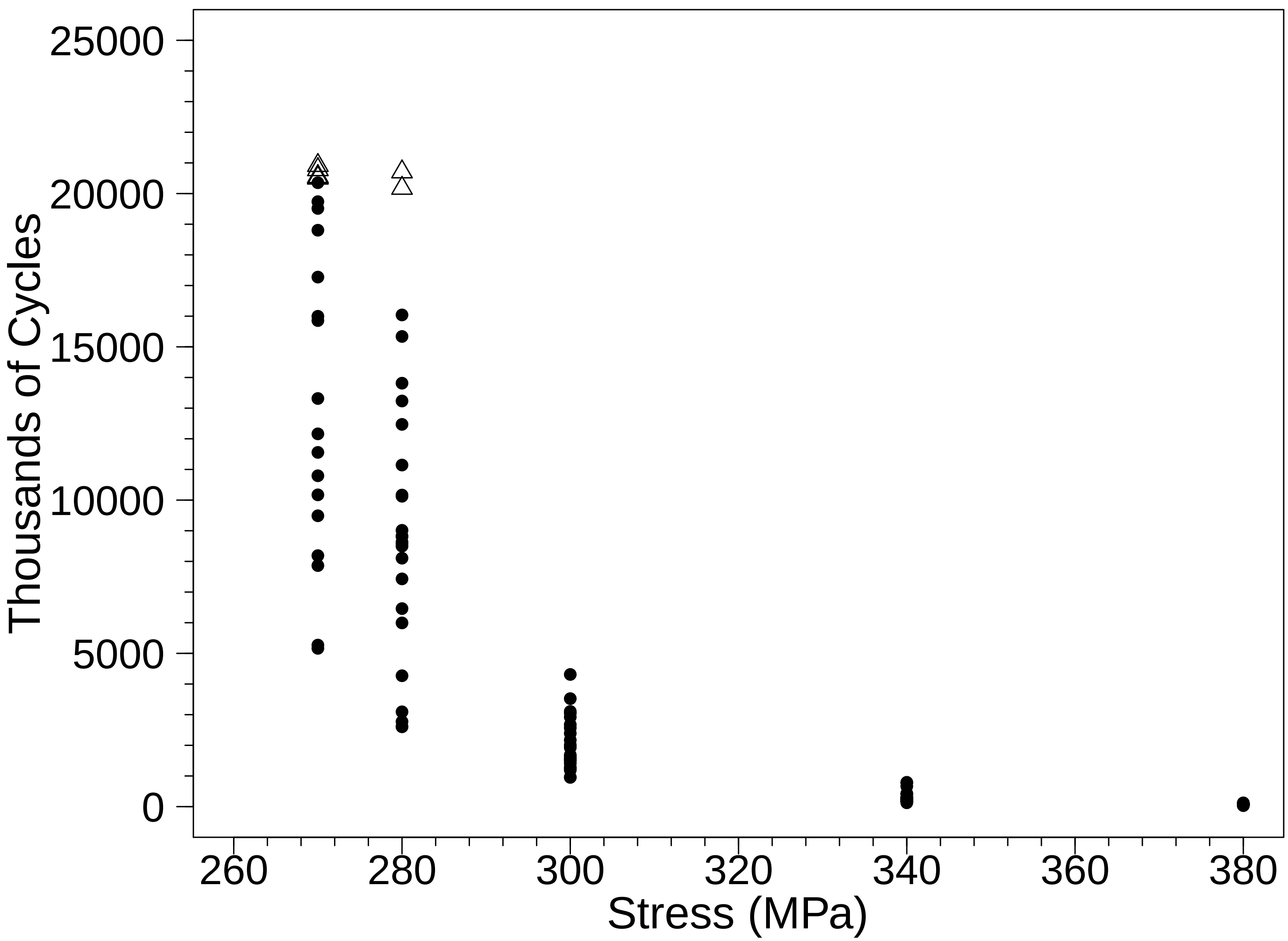
- 125 circular-holed notched specimens of a carbon eight-harness-satin/epoxy laminate panel were subjected to a cyclic four-point out-of-plane bending.

- Units tested at 270, 280, 300, 340, and 380 MPa.

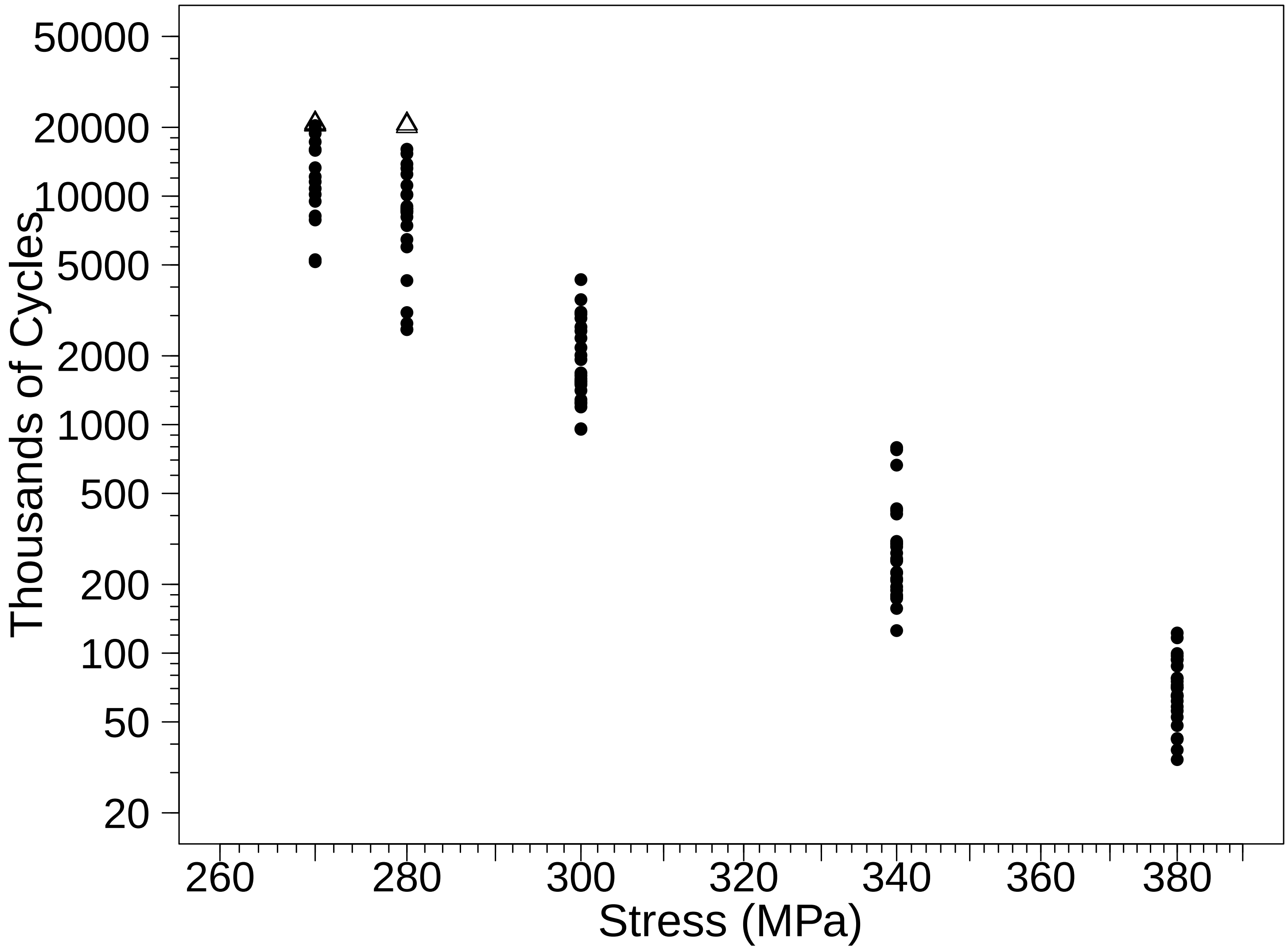
- Some “runouts” at 270 and 280 MPa (8 and 2, respectively).

- Data are from Shimokawa and Hamaguchi (1987).

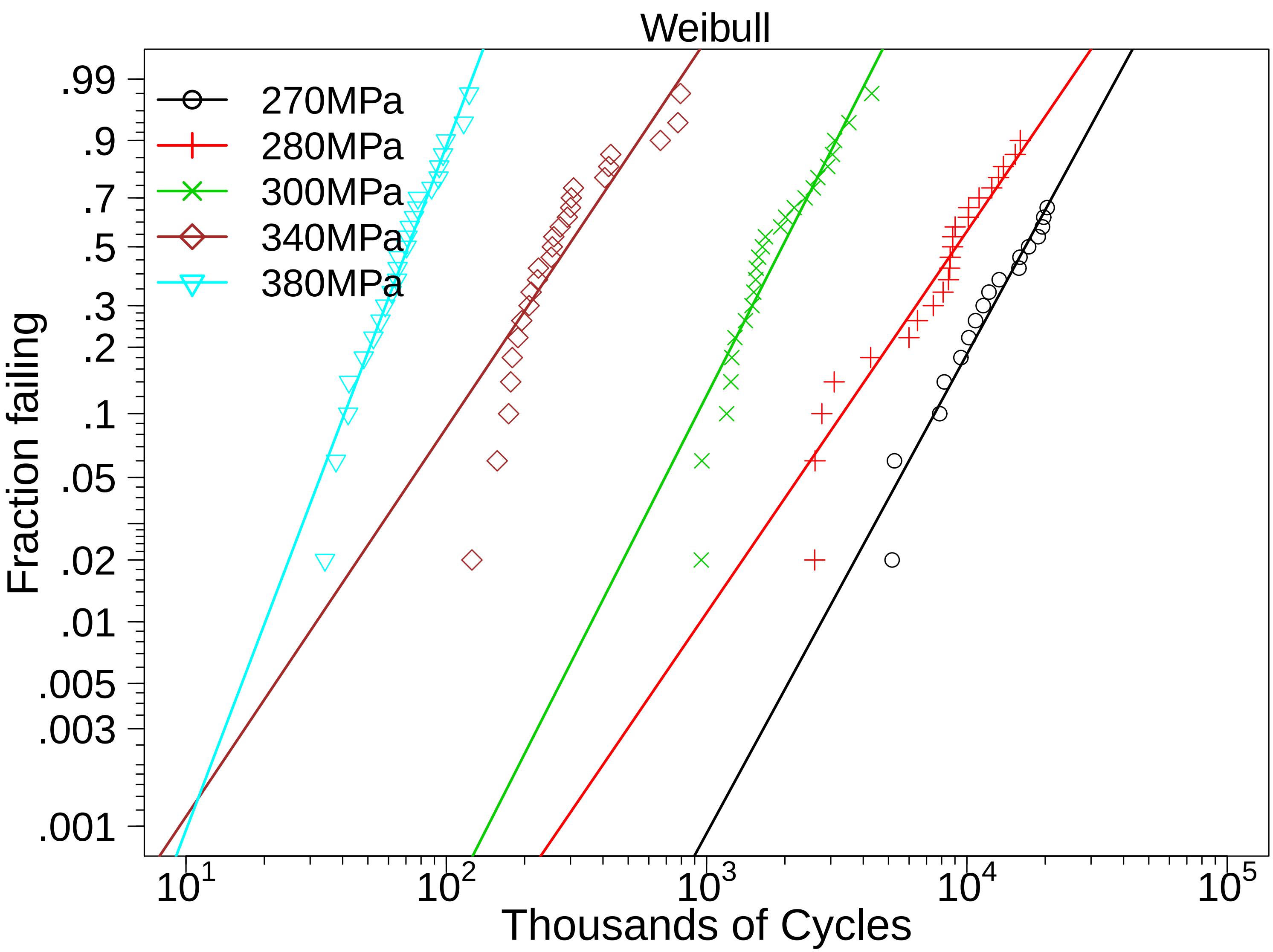
Laminate Panel Data Scatter Plot
Linear-Linear Axes



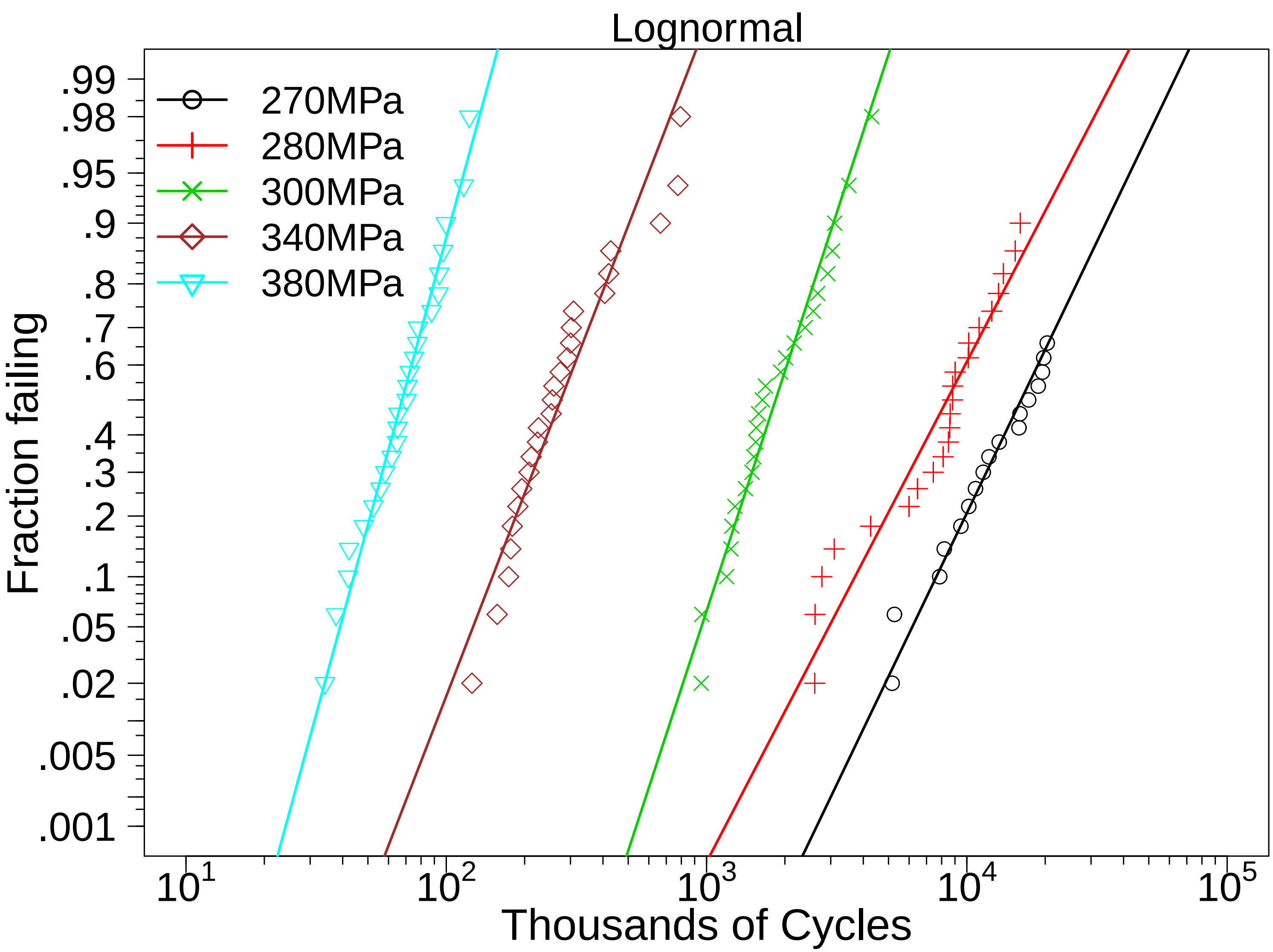
Laminate Panel Data Scatter Plot
Log–Log Axes



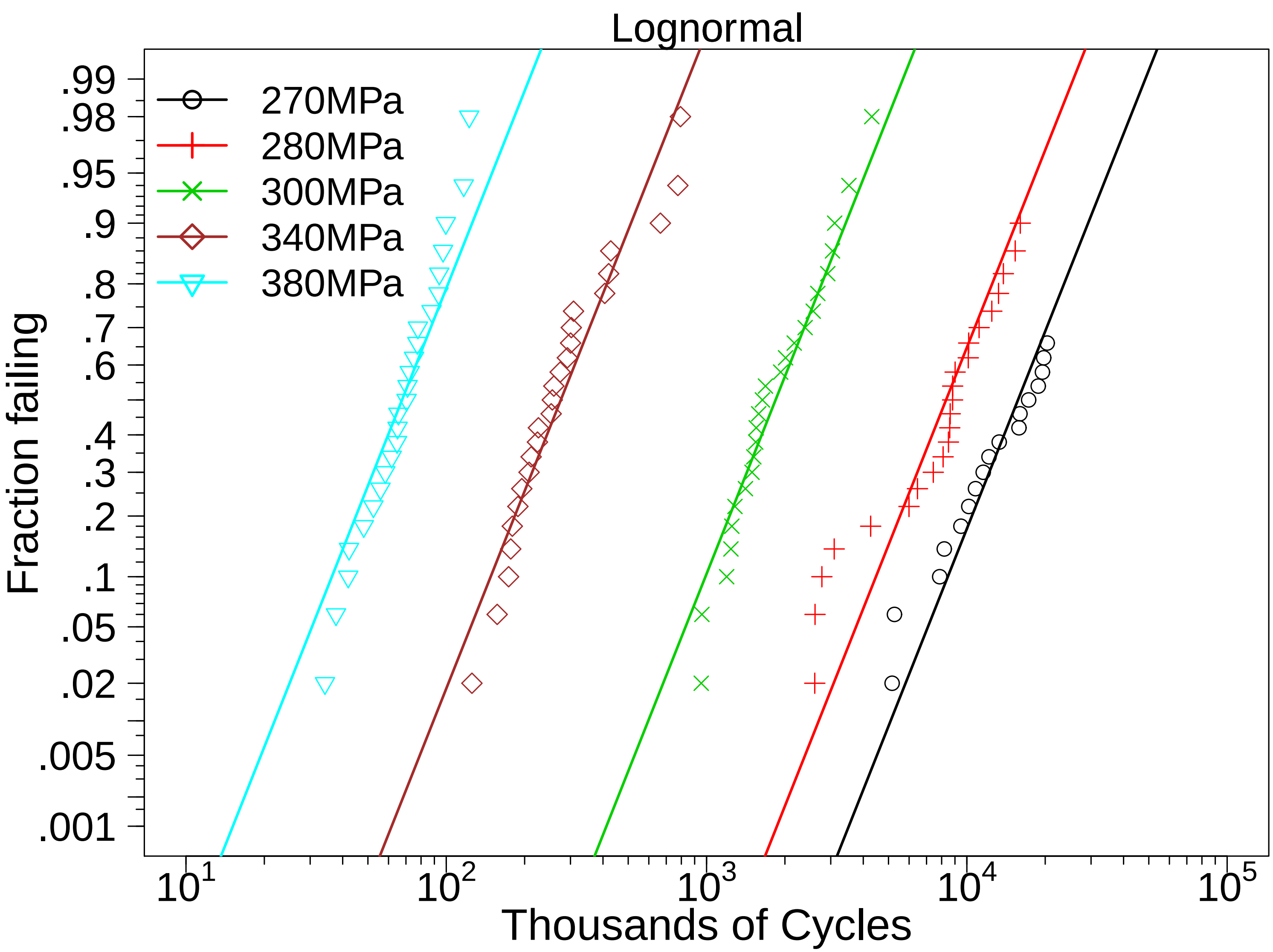
Laminate Panel Data
Multiple Weibull Probability Plot
Different Shape Parameters



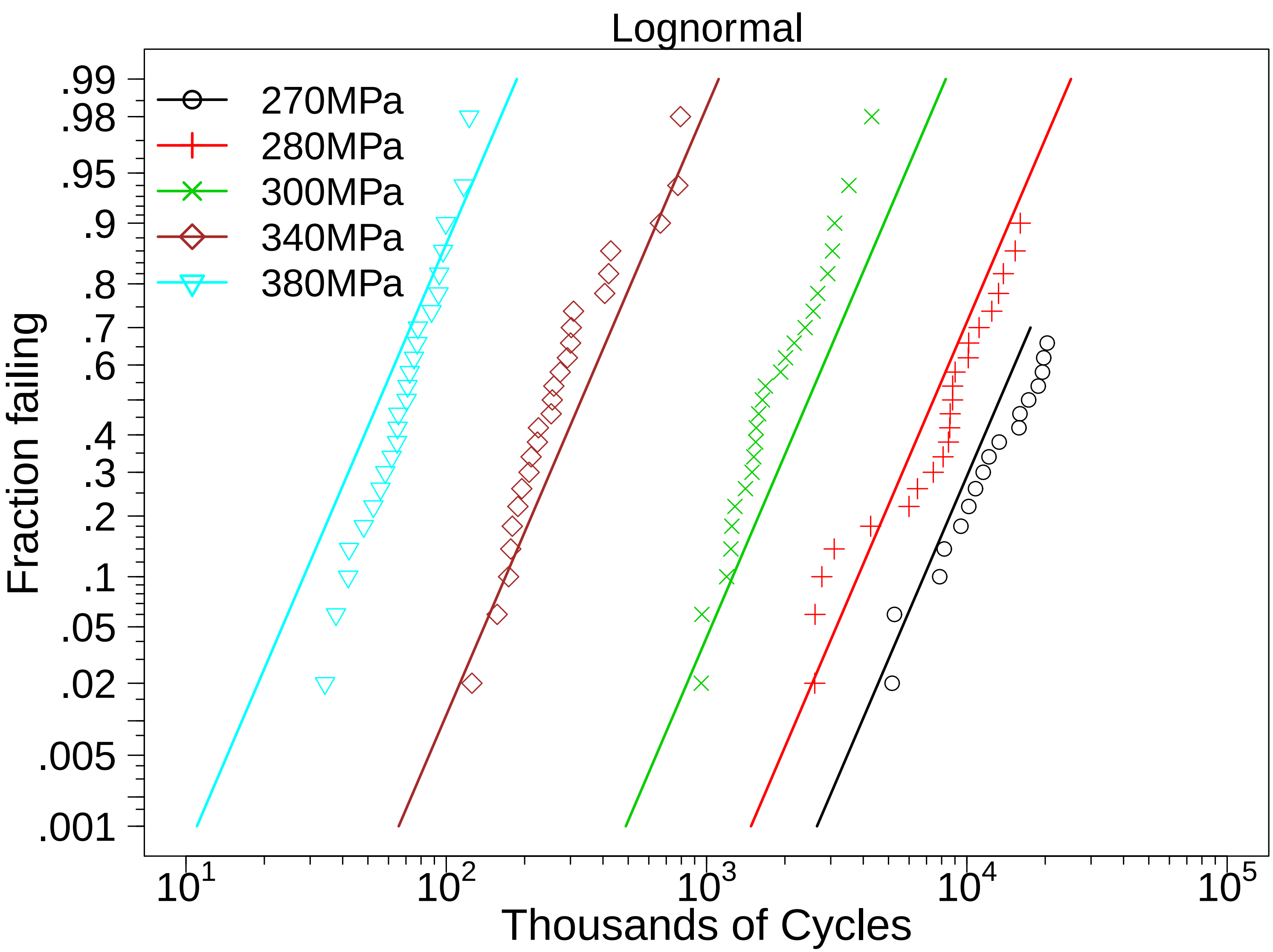
Laminate Panel Data
Multiple Lognormal Probability Plot
Different Shape Parameters



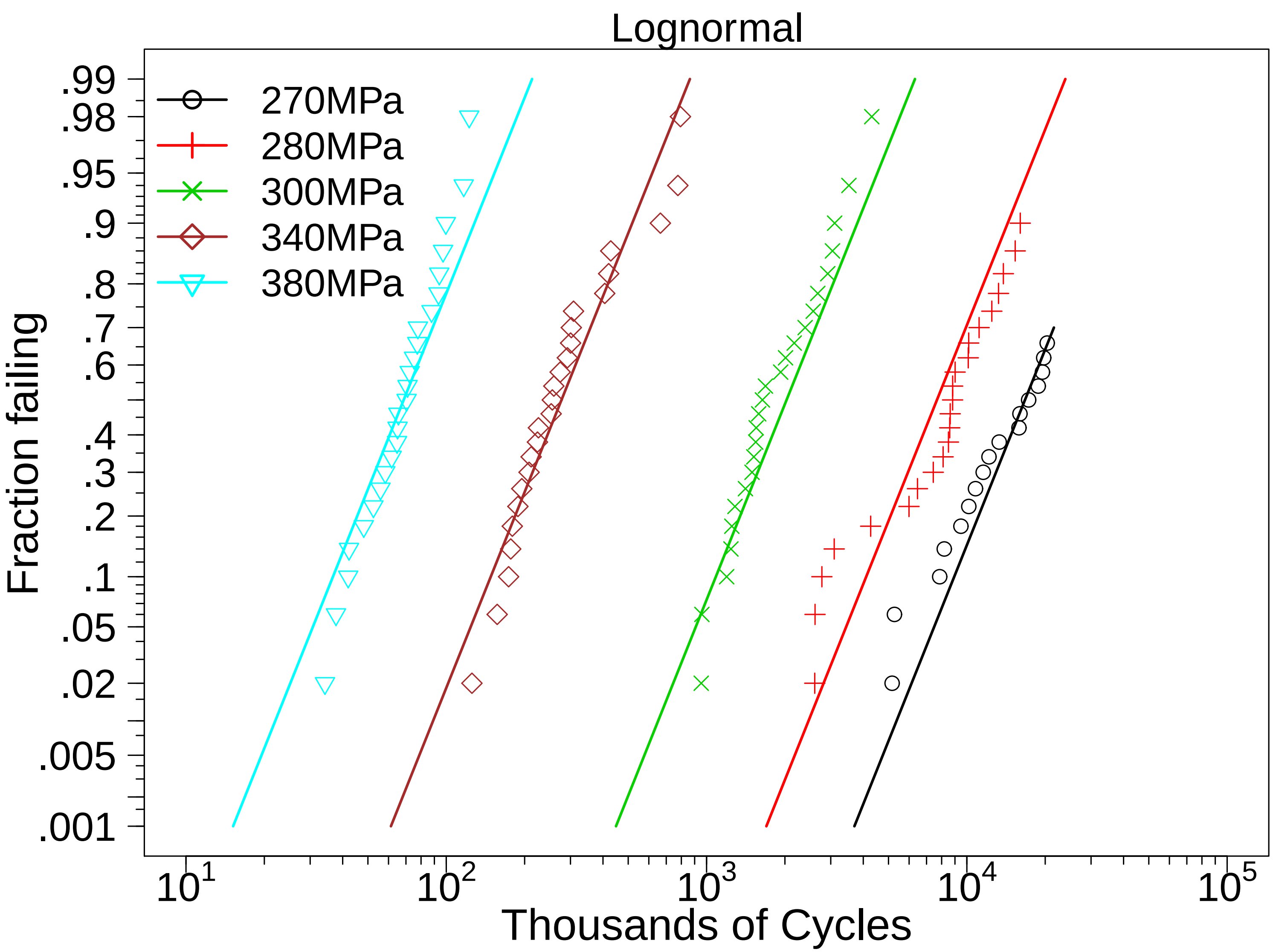
Laminate Panel Data
Multiple Lognormal Probability Plot
Equal Shape Parameter



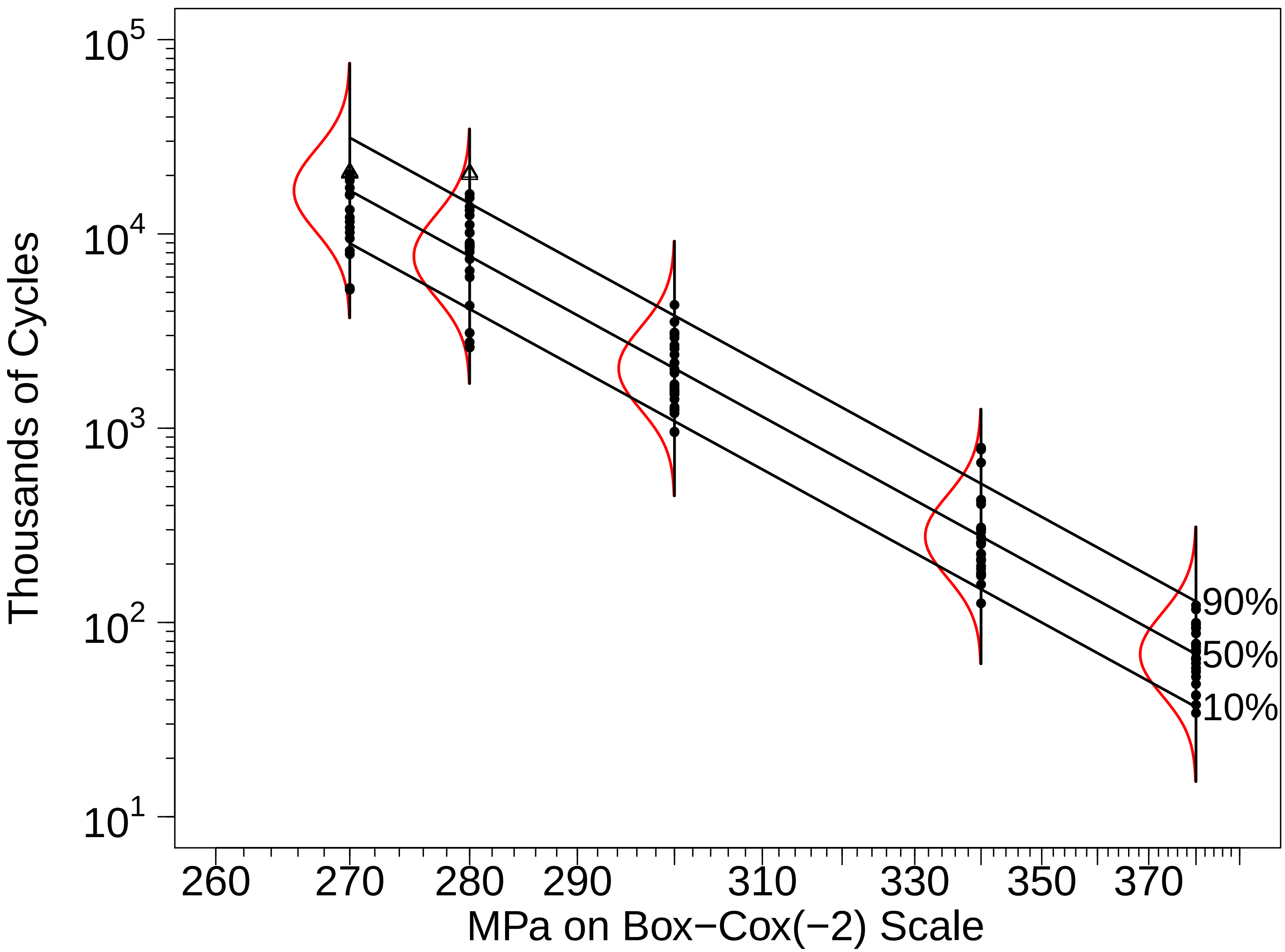
Laminate Panel Data
Multiple Lognormal Probability Plot
Inverse Power Rule Model



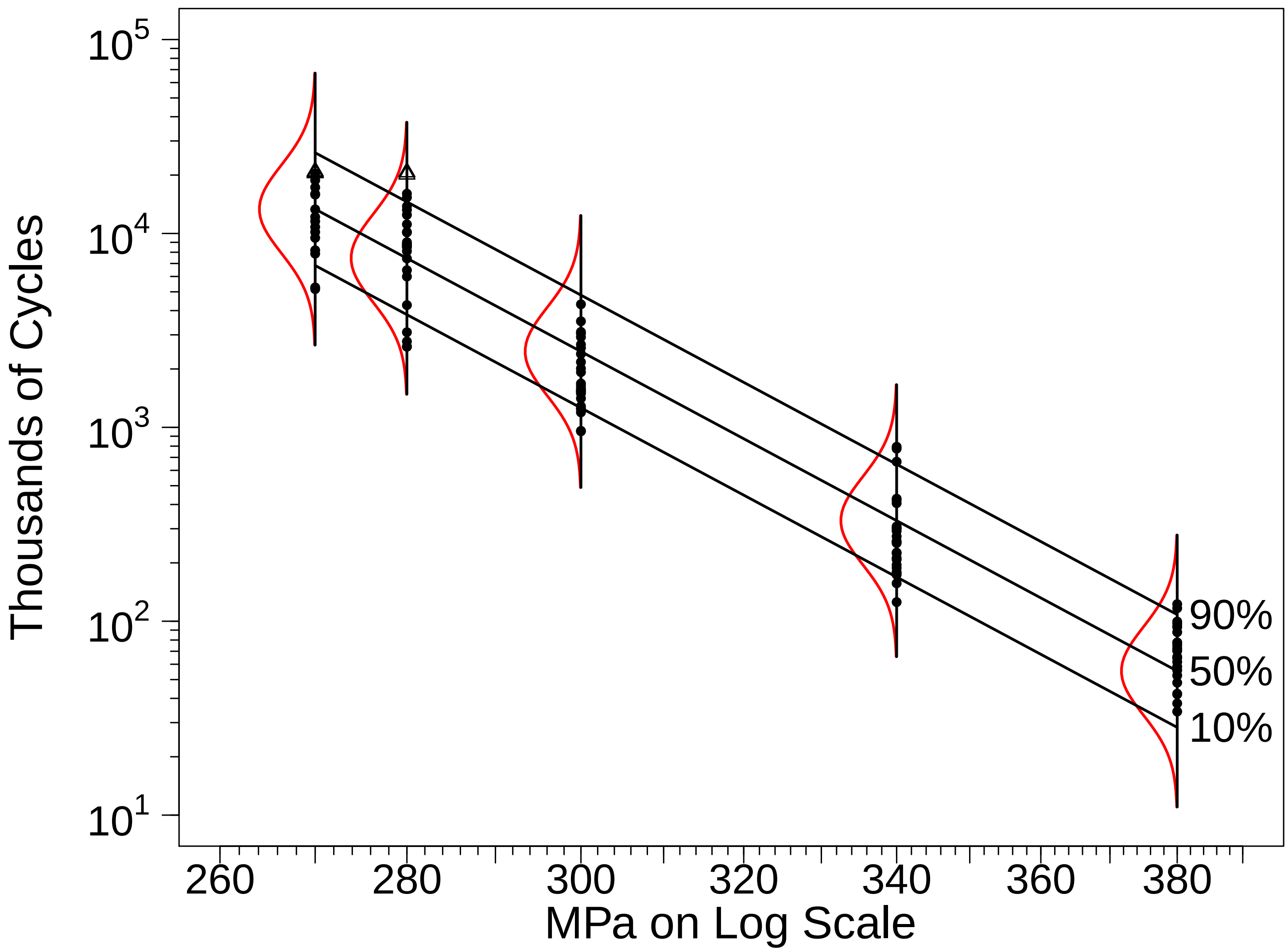
Laminate Panel Data
Multiple Lognormal Probability Plot
Box-Cox Power Law Model $\lambda = -2$



Laminate Panel Data
Model Plot
Box-Cox Power Law Model $\lambda = -2$



Laminate Panel Data
Model Plot
Log Transformation



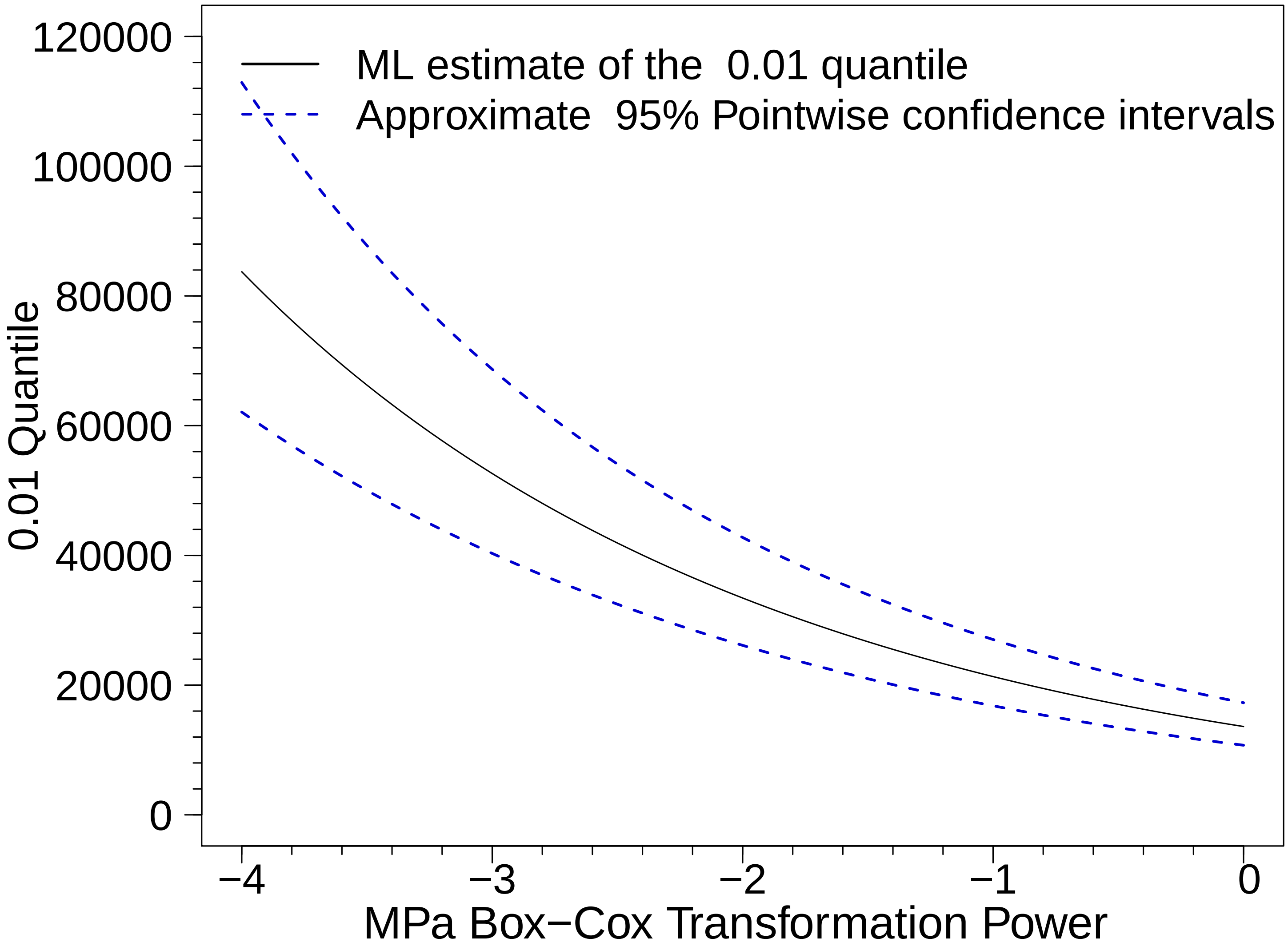
Laminate Panel Data
Lognormal Model-Fitting Summary
Box-Cox Regression Model with Power -2

Model	-2LogLike	AIC	# Param
SepDists	1765	1785	10
EqualSig	1777	1789	6
RegrModel	1779	1785	3
Pooled	2130	2134	2

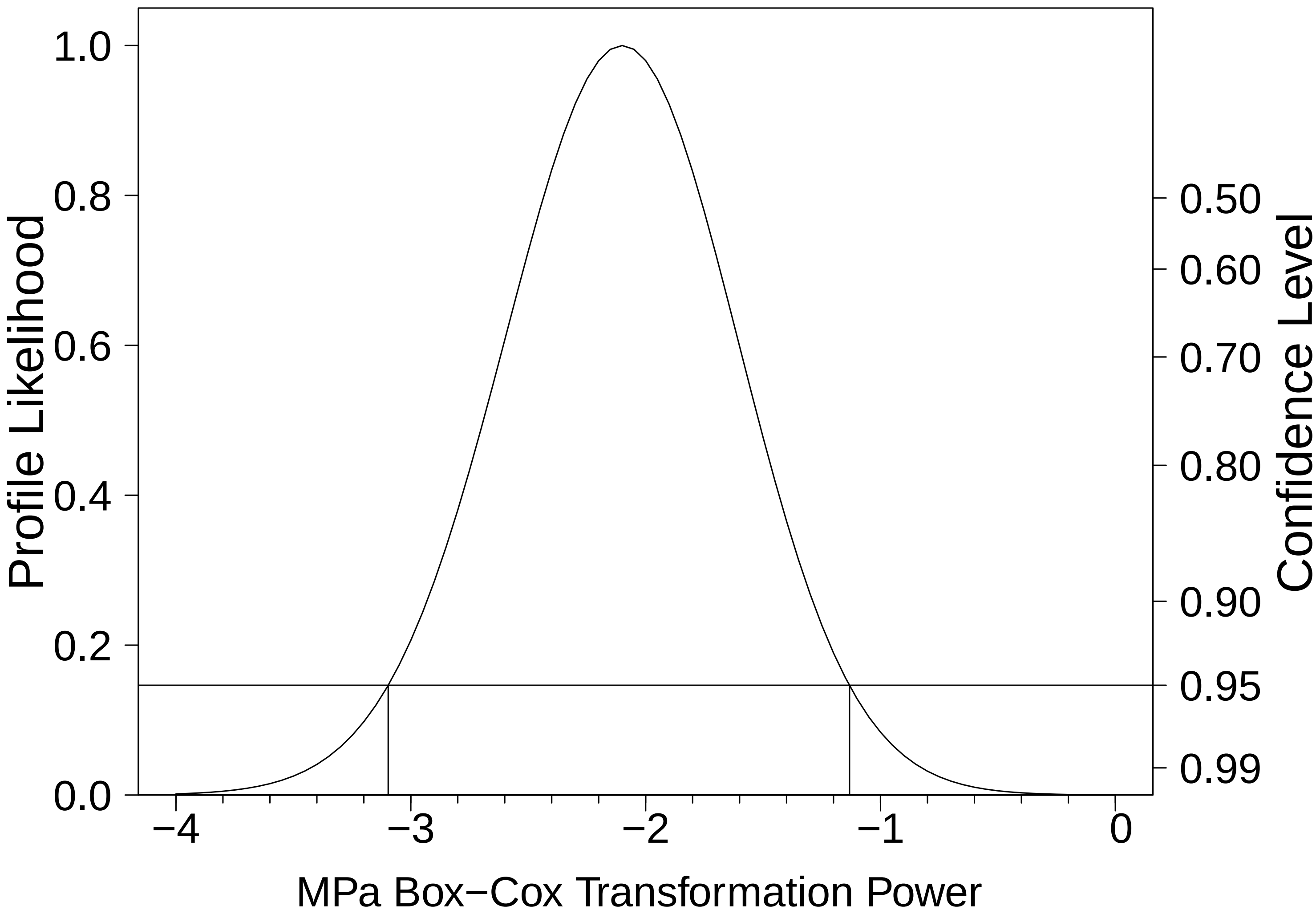
Laminate Panel Data Lognormal Likelihood Ratio Tests
Box-Cox Regression Model with Power -2

Comparison	LR Statistic	dof	p-value
SepDists vs EqualSig	12.31	4	0.015
EqualSig vs RegrModel	1.54	3	0.67
RegrModel vs Pooled	351.31	1	< 0.001

Laminate Panel Box-Cox Sensitivity Analysis at Stress Level 250 MPa



Laminate Panel Box-Cox Sensitivity Analysis
Profile Relative Likelihood



Chapter 17

Failure-Time Regression Analysis

Segment 5

Failure-Time Regression Analysis with Two Explanatory Variables

Two or More Explanatory Variables: Glass Capacitor Failure Data

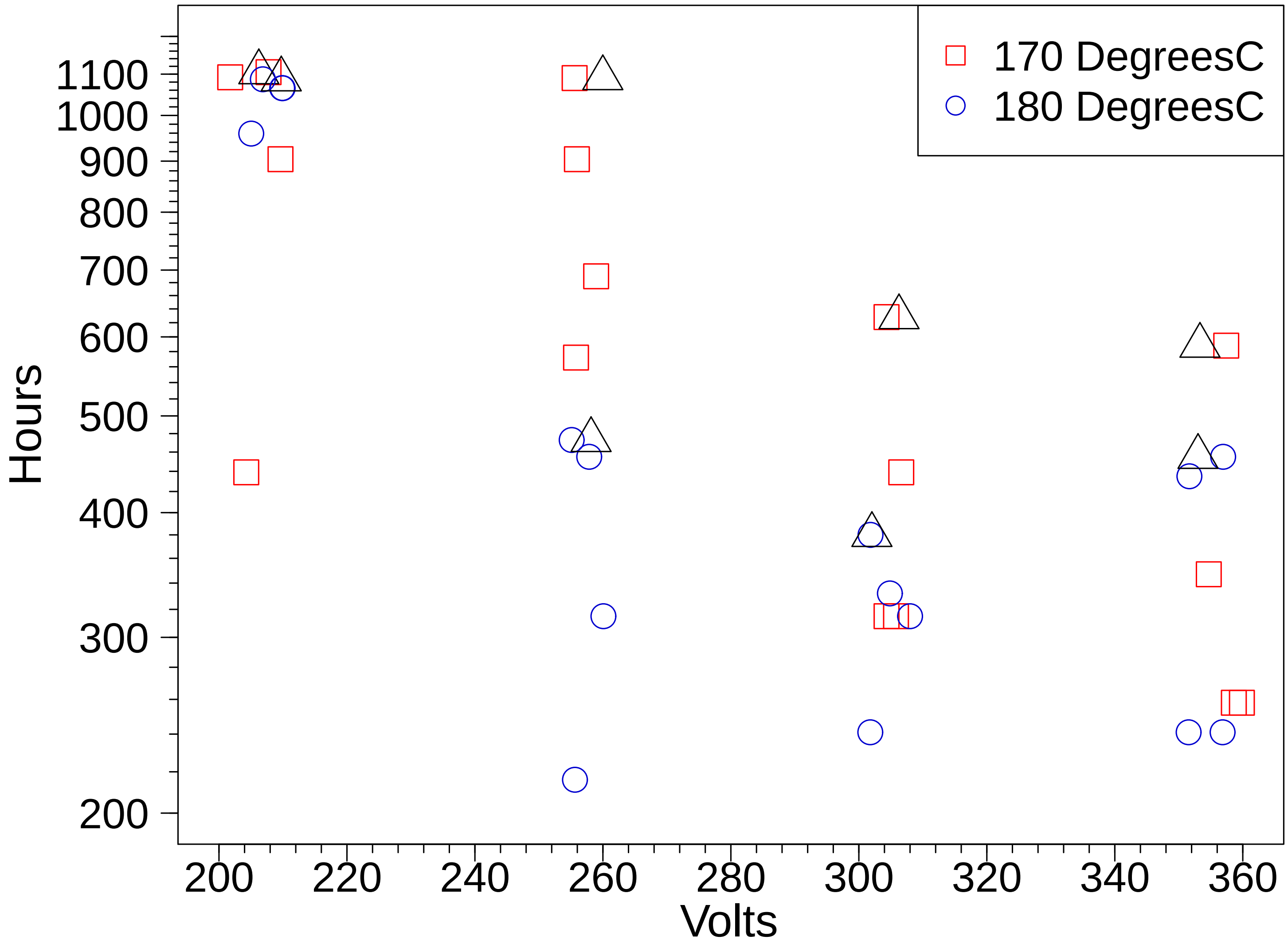
- Experiment designed to determine the effect of voltage and temperature on capacitor life.

- 2×4 factorial, 8 units at each combination.

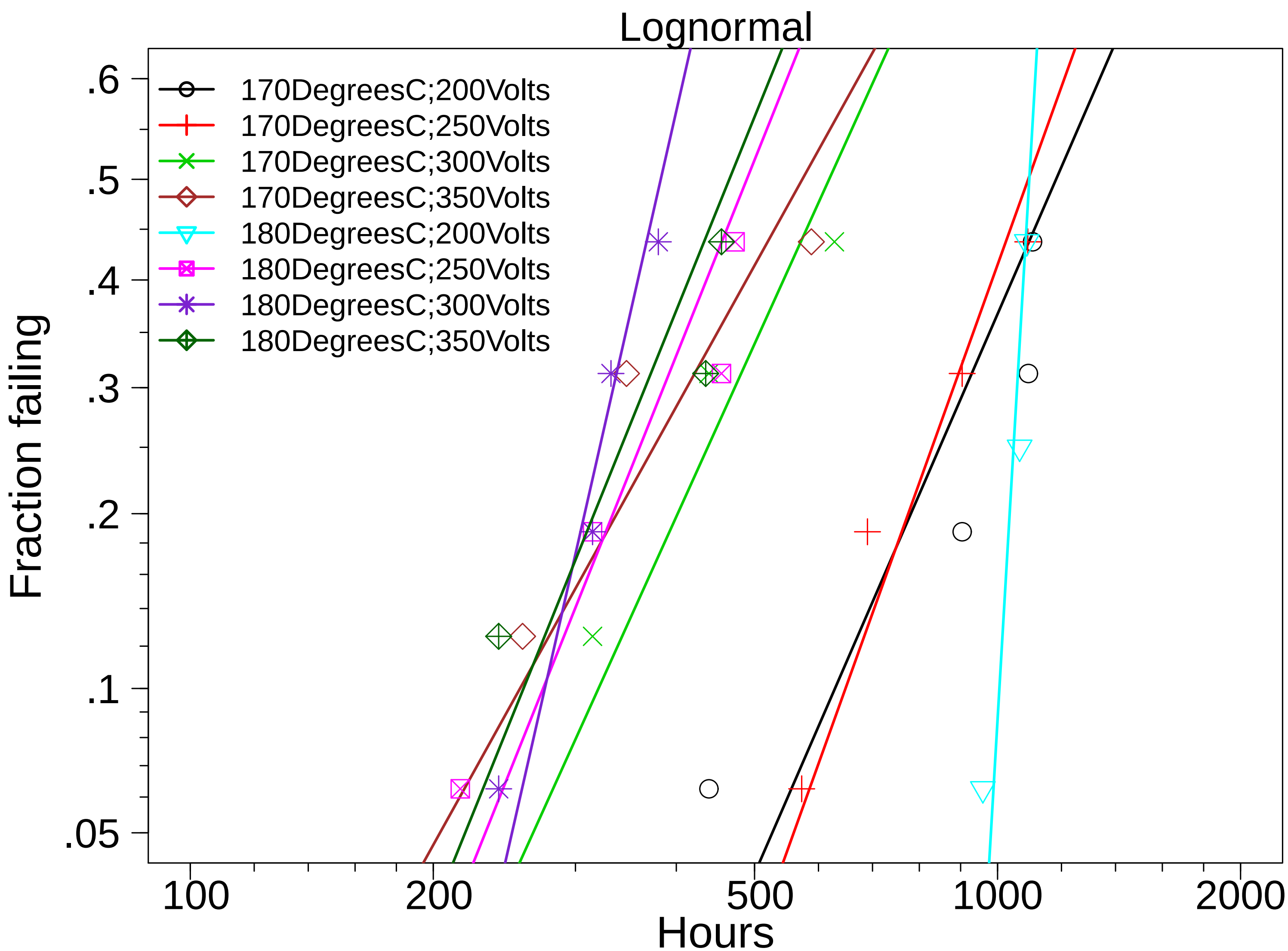
- Test at each combination run until 4 of 8 units failed (Type 2 censoring).

- Original data from Zelen (1959).

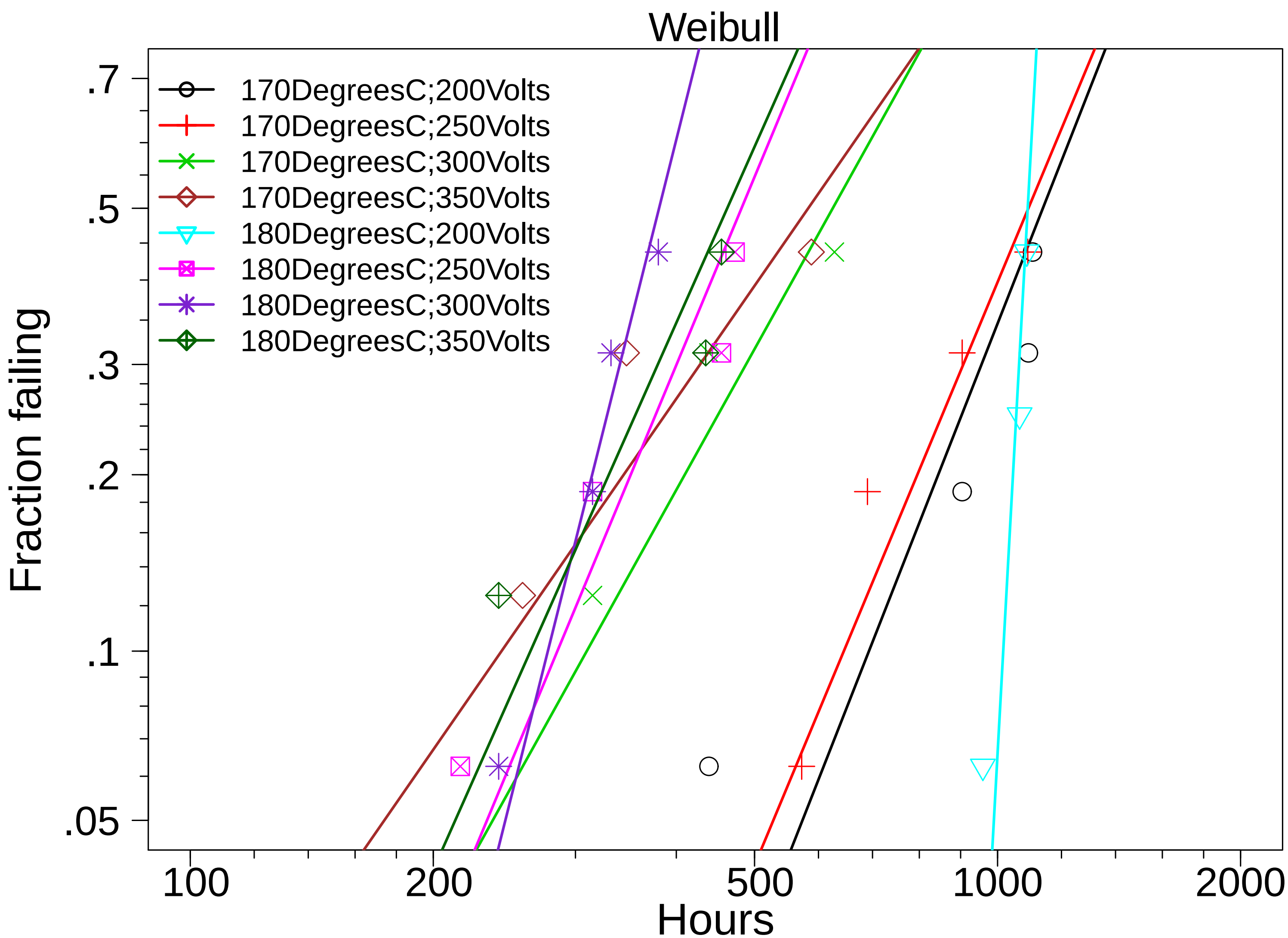
Scatter Plot the Effect of Voltage and Temperature on Glass Capacitor Life



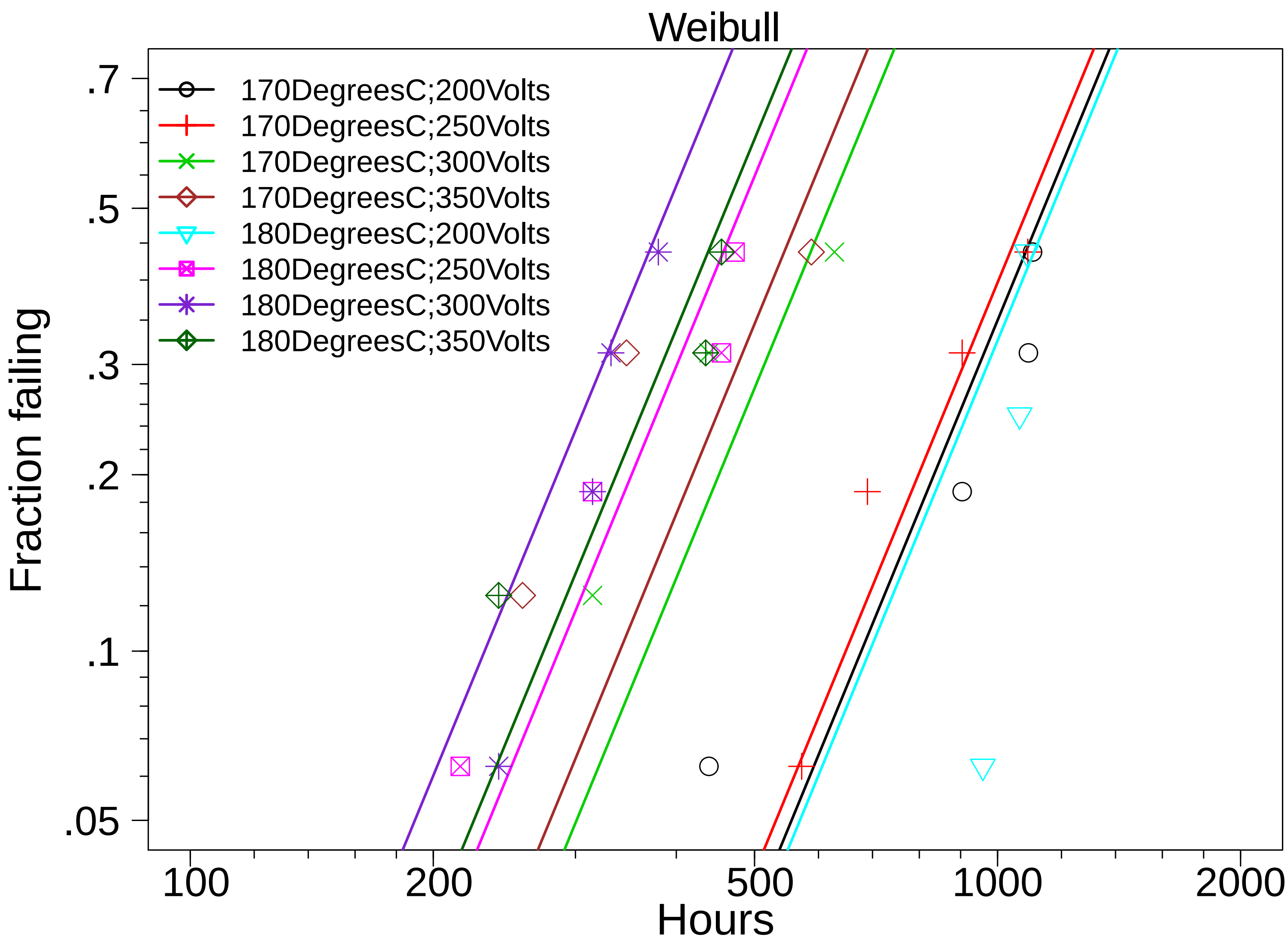
Lognormal Probability Plot
Glass Capacitor Life Test Results
Different Shape Parameters



Weibull Probability Plot
Glass Capacitor Life Test Results
Different Shape Parameters



Weibull Probability Plot
Glass Capacitor Life Test Results
Equal Shape Parameter



Glass Capacitor Life Test
Two-Variable Regression Models

- The additive model is

$$\log[t_p(\boldsymbol{x})] = y_p(\boldsymbol{x}) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \Phi^{-1}(p)\sigma,$$

  where $x_1 =$ Temperature and $x_2 =$ Voltage.

- The interaction model is

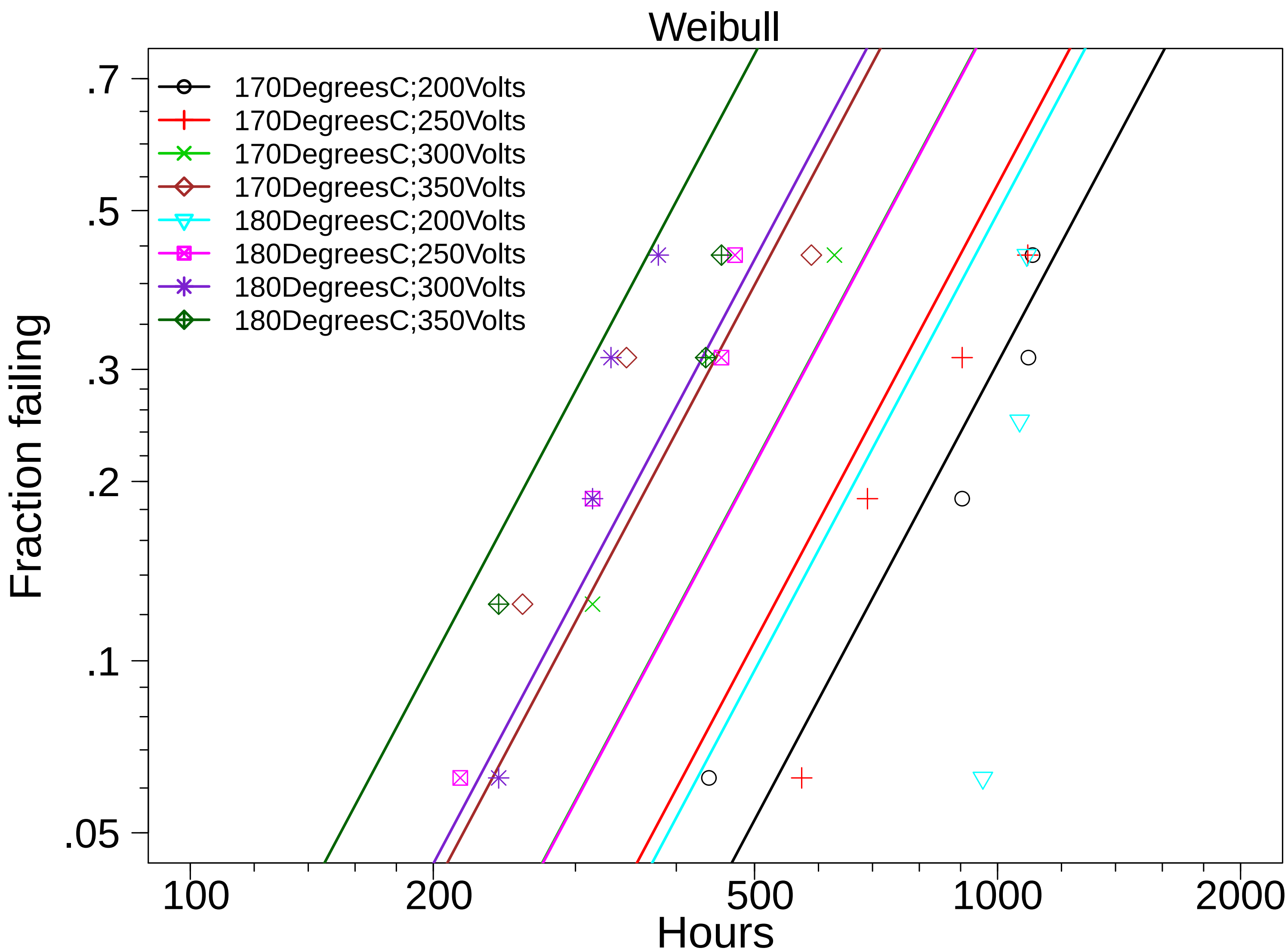
$$\log[t_p(\boldsymbol{x})] = y_p(\boldsymbol{x}) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \Phi^{-1}(p)\sigma.$$

- Comparing the two models gives

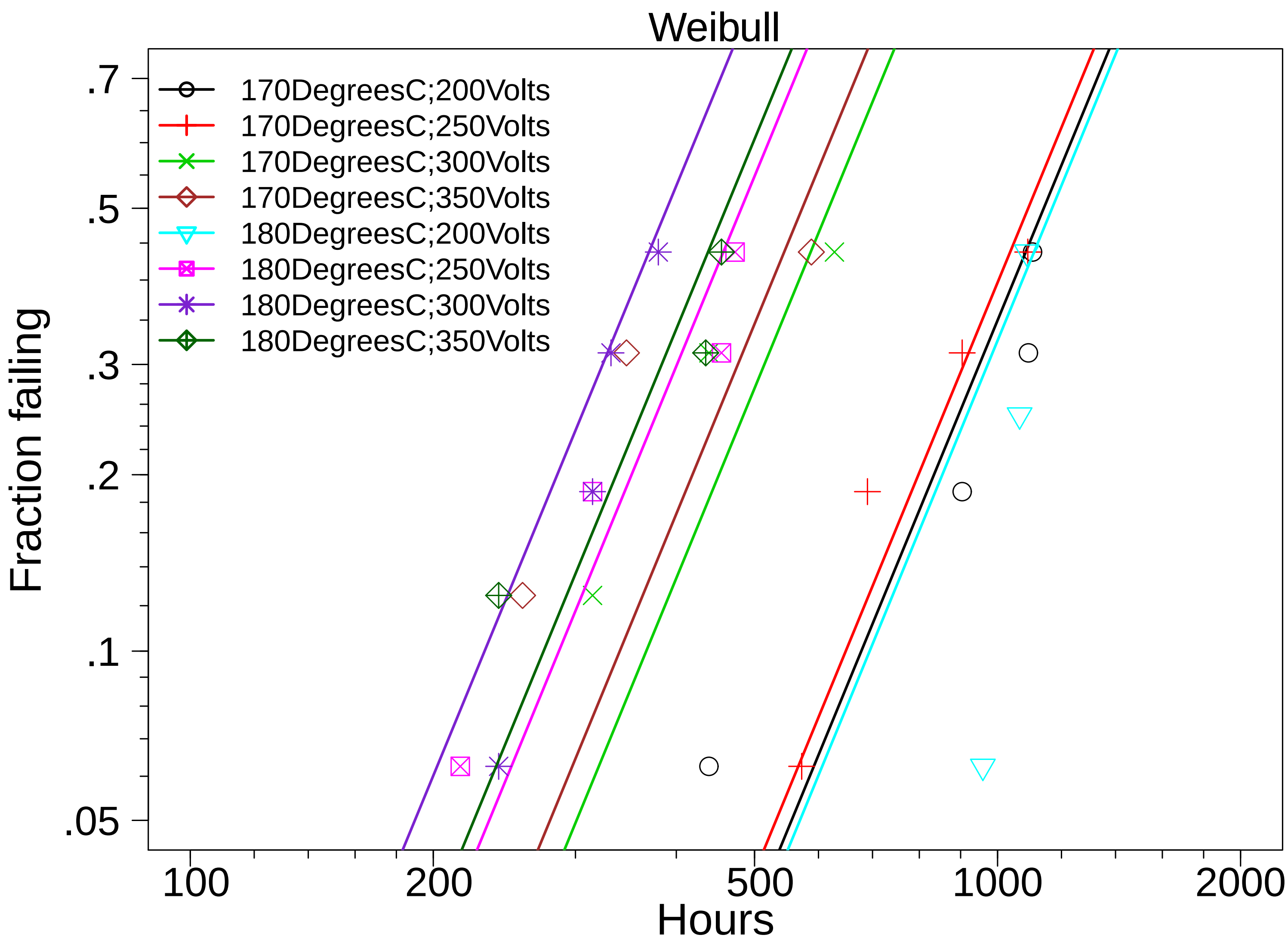
$$-2 \times (\mathcal{L}_1 - \mathcal{L}_2) = -2 \times (-244.24 + 244.17) = 0.14$$

  which is small relative to $\chi^2_{(0.95,1)} = 3.84$.

Weibull Probability Plot
Glass Capacitor Life Test Results
Interaction Model



Weibull Probability Plot
Glass Capacitor Life Test Results
Equal Shape Parameter



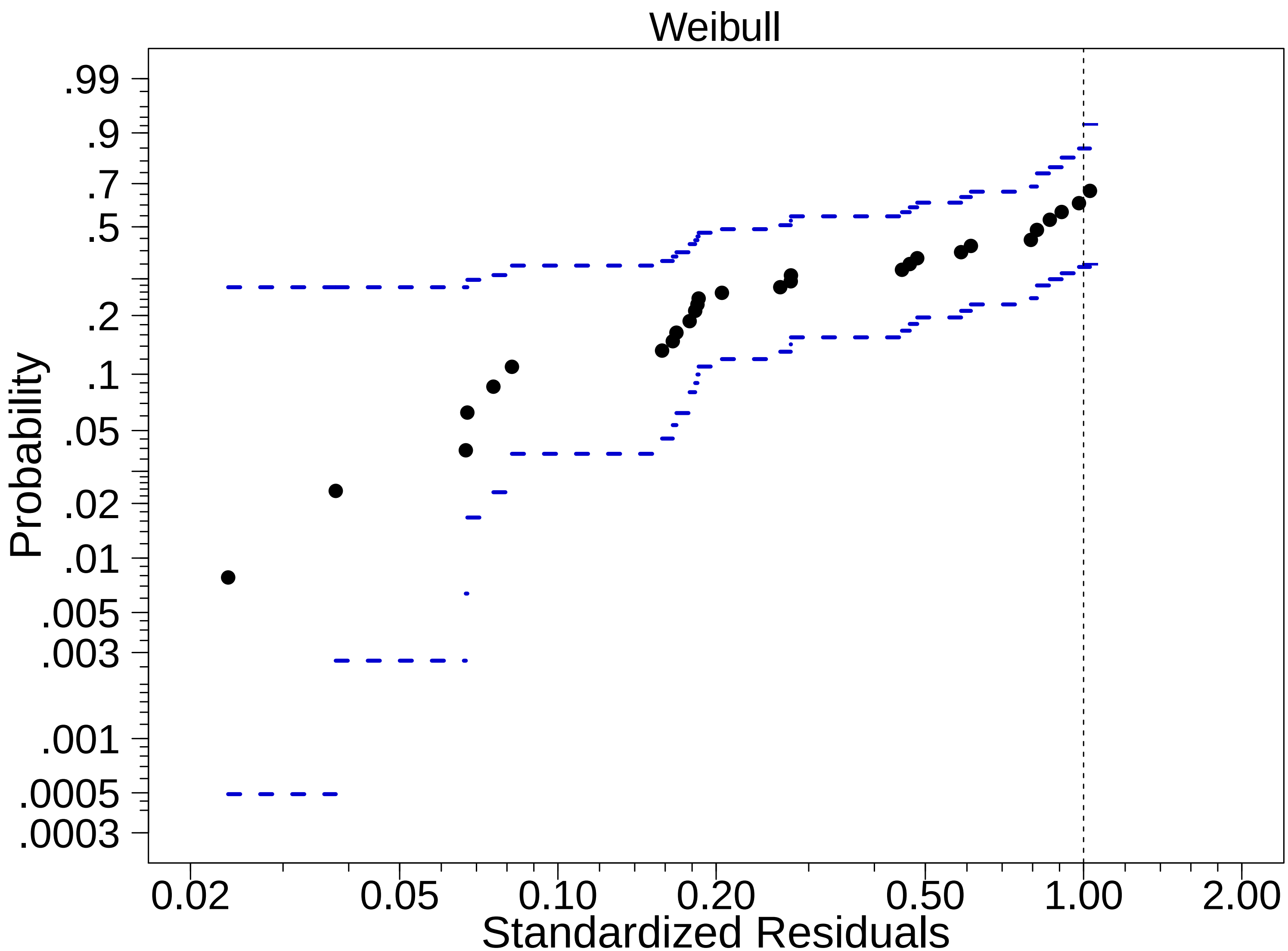
Glass Capacitor Life Test Results
Weibull Distribution-Fitting Summary

Model	-2LogLike	AIC	# Param
SepDists	463.3	495.3	16
EqualSig	476.3	494.3	9
RegrModel	488.5	496.5	4
Pooled	509.1	513.1	2

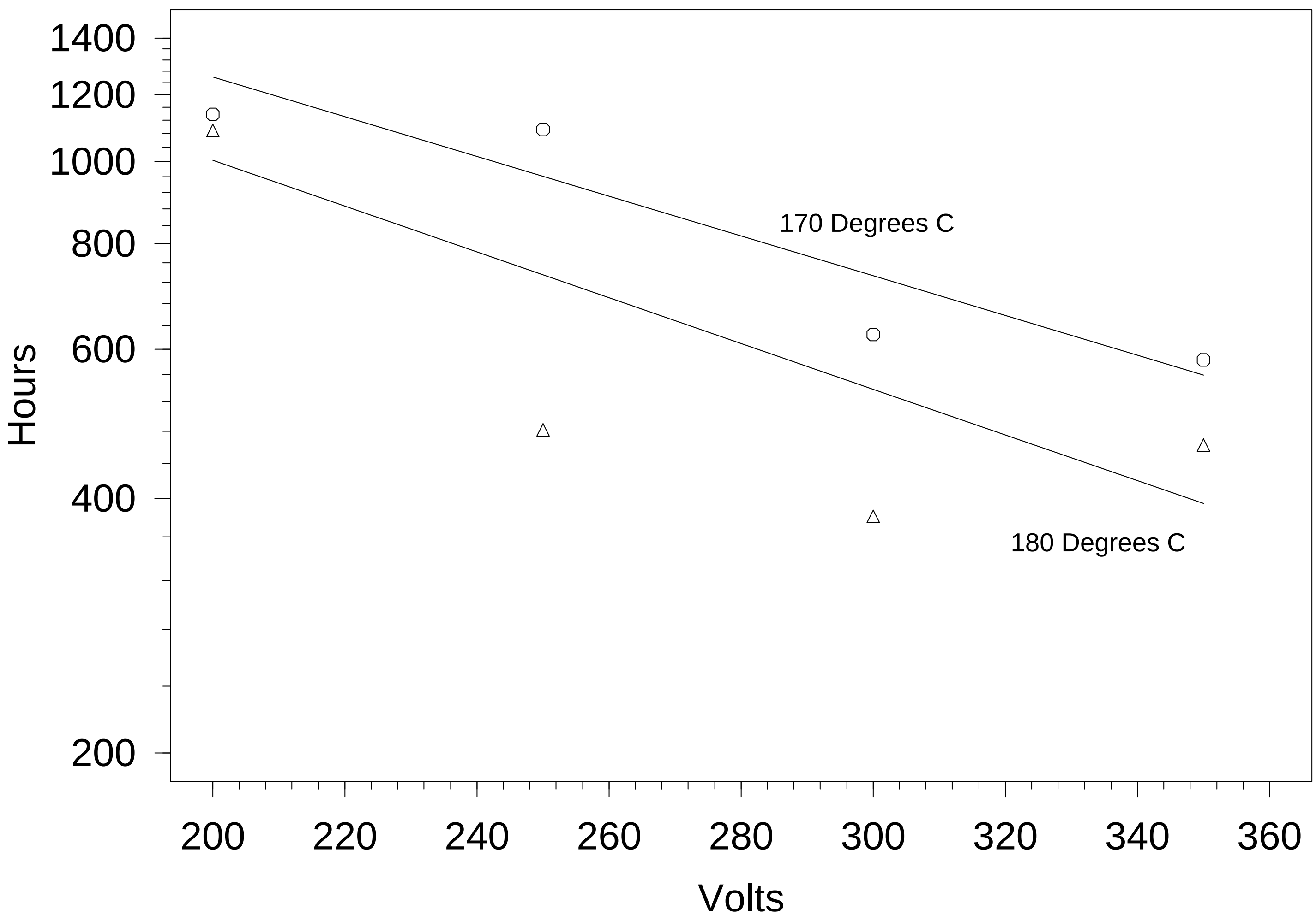
Glass Capacitor Life Test Results
Weibull Distribution-Fitting Summary

Comparison	LR Statistic	dof	p-value
SepDists vs EqualSig	12.96	7	0.073
EqualSig vs RegrModel	12.19	5	0.032
RegrModel vs Pooled	20.57	2	< 0.001

Weibull Probability Plot of the Interaction-Model Residuals
Glass Capacitor Life Test Results



Estimates of Weibull $t_{0.5}$ Plotted for each Combination of the Glass Capacitor Test Conditions Model with Interaction Points are Regression-Model-Free Estimates



Glass Capacitor Failure Data Analysis Excluding Data at 180°C and 200 Voltage

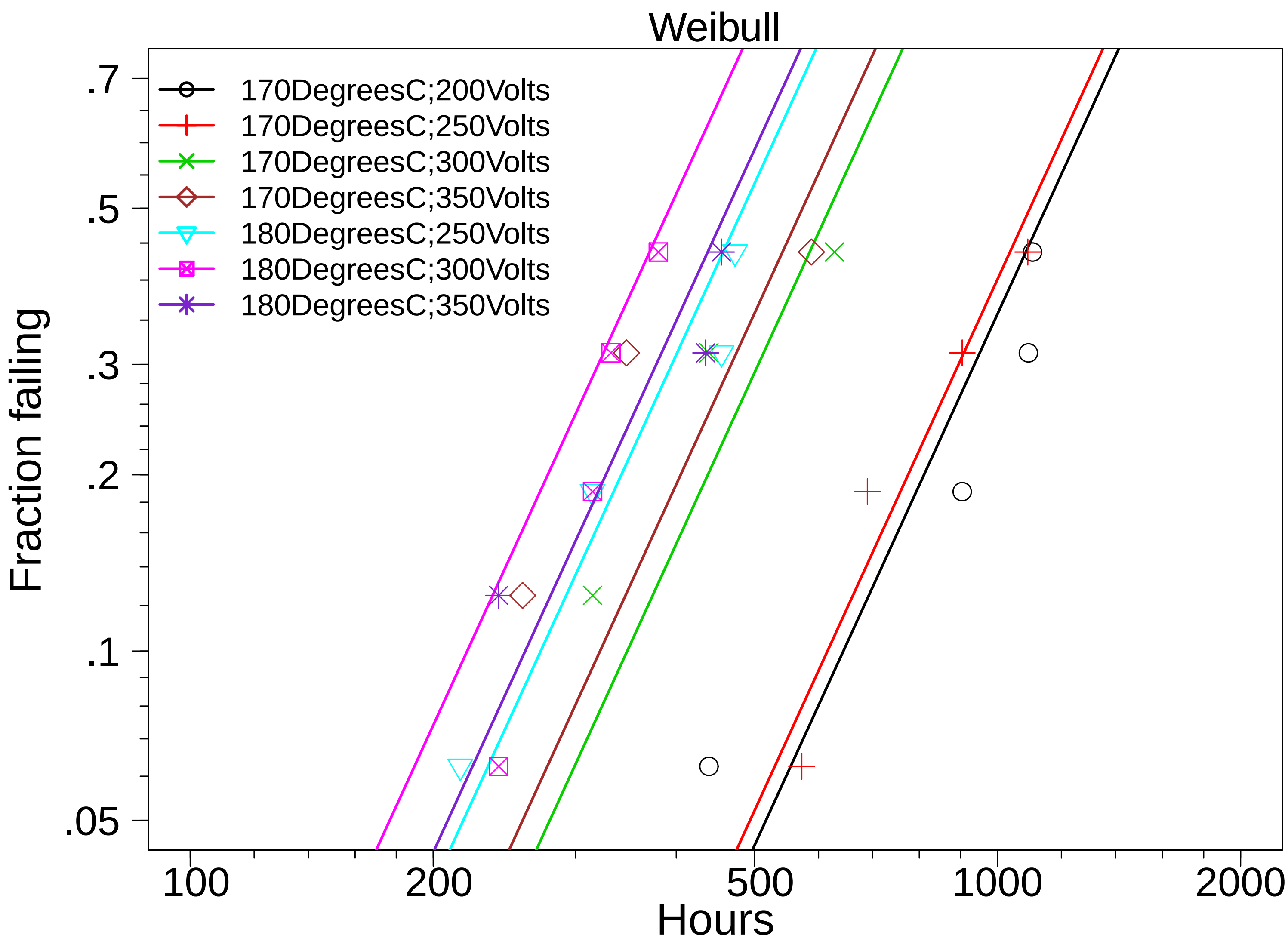
- Model fits indicate strong evidence of lack of fit due to data at 180°C and 200 Voltage.

- There is less spread in the data at that condition.

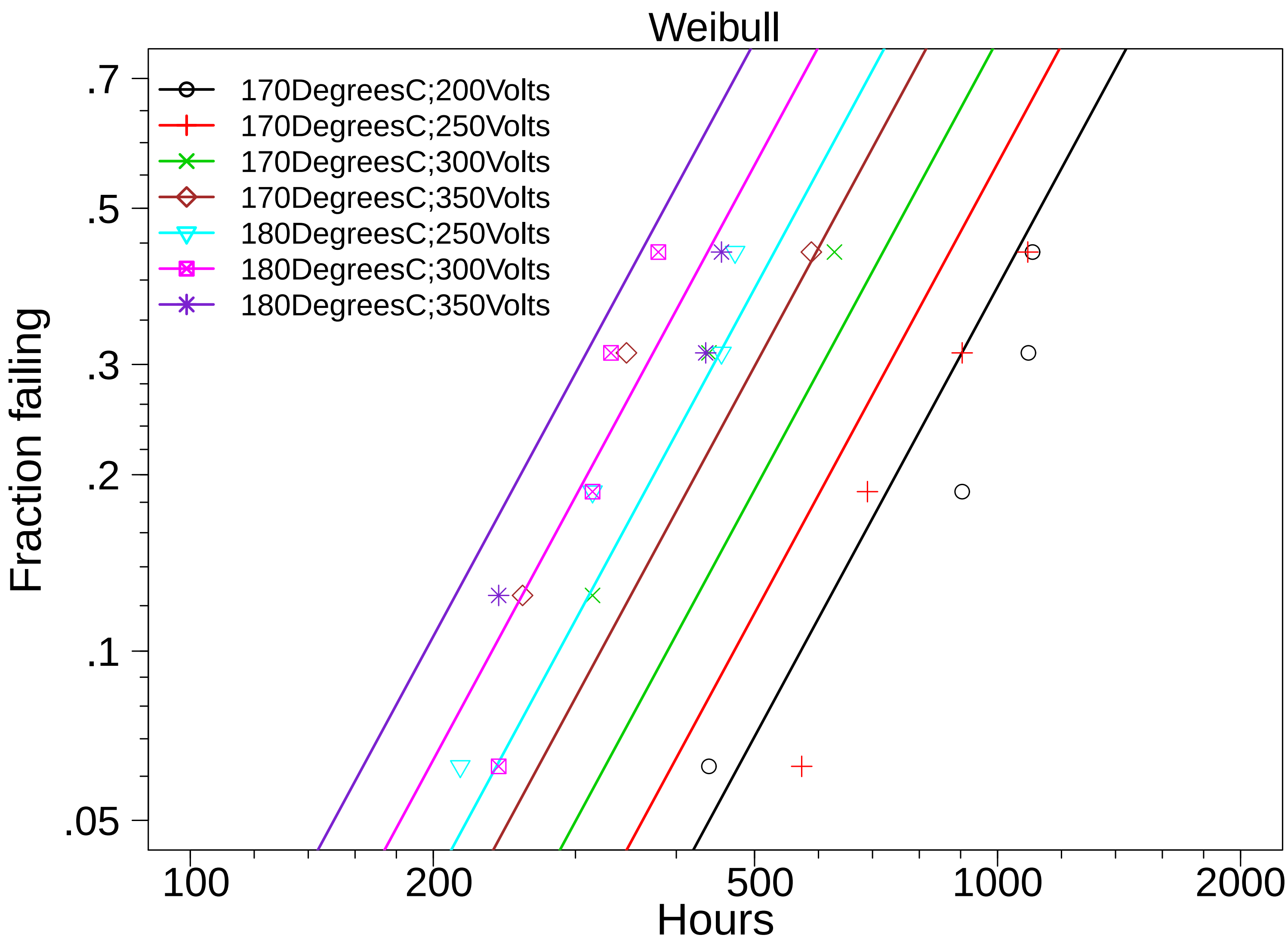
- Failure times at 180°C tend to be larger than those at 170°C. In particular, ML estimates suggest longer lifetime at the higher temperature.

- Refit the Weibull no-interaction regression with data at 180°C and 200 Voltage excluded.

Weibull Probability Plot
Glass Capacitor Subset Data Life Test Results
Equal Shape Parameter



Weibull Probability Plot
Glass Capacitor Subset Data Life Test Results
No-Interaction Model



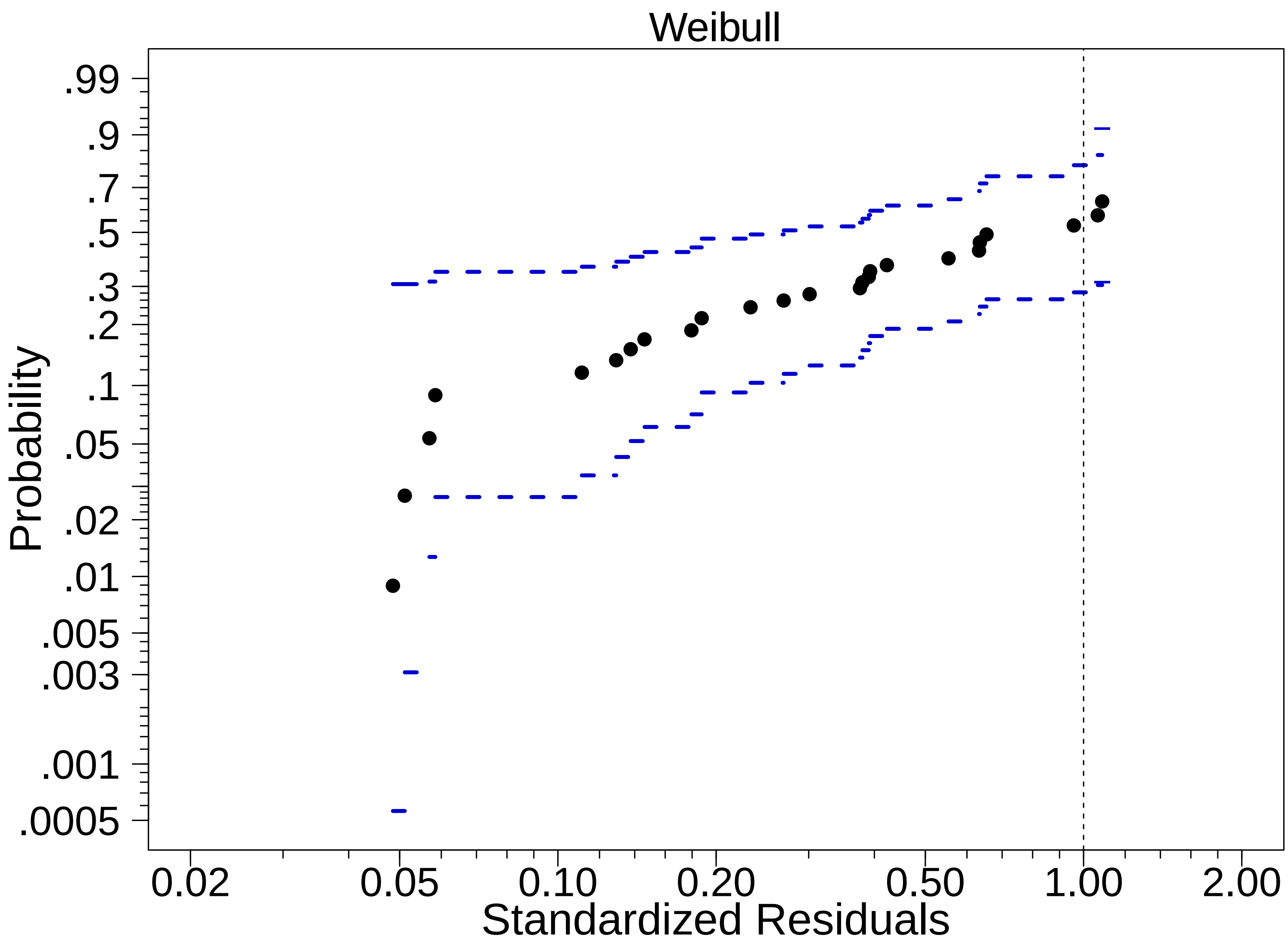
Glass Capacitor Subset Data Life Test Results
Weibull Distribution-Fitting Summary

Model	-2LogLike	AIC	# Param
SepDists	414	442	14
EqualSig	416	432	8
RegrModel	422	430	4
Pooled	441	445	2

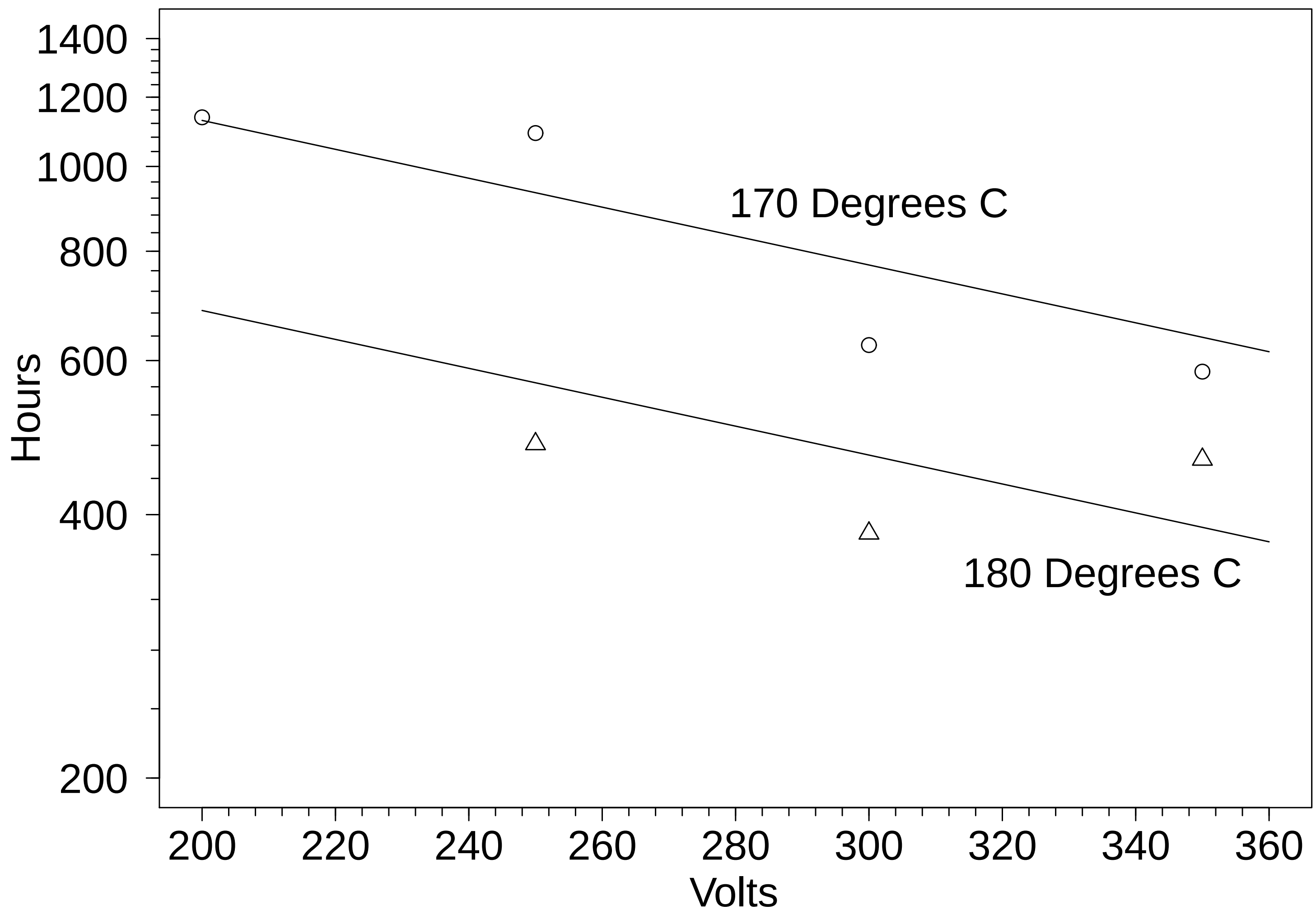
Glass Capacitor Subset Data Life Test Results
Weibull Distribution-Fitting Summary

Comparison	LR Statistic	dof	p-value
SepDists versus EqualSig	2.75	6	0.840
EqualSig versus RegrModel	5.96	4	0.201
RegrModel versus Pooled	18.30	2	< 0.001

Weibull Probability Plot of the No-Interaction Model Residuals
Glass Capacitor Subset Data Life Test Results



Estimates of Weibull $t_{0.5}$ Plotted for each Combination of the Glass Capacitor Subset Data Test Conditions Model with No Interaction Points are Regression-Model-Free Estimates



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