Simulations of the point reactor dynamics of AP1000 using MATLAB environment

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The aim of the study was to simulate and analyze selected transient state of the AP1000 reactor core using a point kinetics and dynamics model. MATLAB and an existing codes, which were modified for the project's purposes, were utilized for this task. The obtained results were consistent with predicted model.

1 Theoretical Introduction

The power generated in a nuclear reactor heavily depends on its neutron flux and their physics. To simulate the changes in the core, kinetics and dynamics models were proposed.

The fundamental assumption is to neglect all spatial variables since they are not necessary to use in kinetic and dynamic equations. The name "point" indicates that all space dependencies are excluded and the time is the only dimension left [1].

The point kinetics model uses System of Ordinary Differential Equations presented below:

$$\frac{dn}{dt} = \frac{\rho - \beta}{\Lambda} n + \sum_{i=1}^{6} \lambda_i C_i + S \tag{1}$$

$$\frac{dC_i}{dt} = \frac{\beta_i}{\Lambda} n - \lambda_i C_i, \qquad i = 1, \ldots, 6 \tag{2}$$

where $\frac{dn}{dt}$ is the neutron population depending on time and ρ is reactivity which is the net ratio to the net neutron production. β is effective delayed neutron fraction, Λ is neutron generation time, S is the external source of neutrons and $\frac{dC}{dt}$ is the delayed neutron precursor population depending on time. The visible i index corresponds to six groups of delayed neutrons that are taken into consideration.

The point dynamics equations (PDE) were derived considering the reactivity feedbacks that are necessary in the investigation of nuclear reactor behavior [1]. The PDE model consists of Equations (1) and (2) and the fuel and coolant heat balances presented respectively in equations (3) and (4):

$$m_f c_{pF} \frac{dT_f}{dt} = n - hA_{hx}(T_f - T_c), \tag{3}$$

$$m_f c_{pF} \frac{dT_c}{dt} = hA_{hx}(T_f - T_c) - 2W_c c_{pC}(T_c - T_{Cin}), \tag{4}$$

$$\rho = \rho_0 + \rho_c(t) + \rho_f \tag{5}$$

where m_f and m_c are the fuel and coolant mass in the core, c_{pf} and c_{pc} are specific heat of fuel and coolant, h is the heat transfer coefficient between fuel and coolant, A_{hx} is the heat transfer area between fuel and coolant, W_c is the coolant flow rate, T_c and T_{cin} are respectively average coolant temperature and coolant inlet temperature and T_f is average fuel temperature.

Equation (5) describes the reactivity changes in the reactor. ρ_0 is the initial reactor reactivity with withdrawn control rods and in reference conditions, $\rho_c(t)$ is the reactivity change due to control system (control rods, chemical shim, etc.) and ρ_f is the reactivity change due to non-nuclear parameter (e.g. temperature) changes. The last term (ρ_f) of the equation (5) corresponds to behavior of fuel and coolant balances in equations (3) and (4). It can be then represented as:

$$\rho_f = \int_{P_0}^{P_F} \left(\frac{\partial \rho}{\partial T_f} \frac{dT_f}{dP} + \frac{\partial \rho}{\partial T_{AVG}} \frac{dT_{AVG}}{dP} \right) dP, \tag{6}$$

where P_0 and P_F are respectively initial and final power states and T_{AVG} is a steady-state coolant average temperature obtained from:

$$T_{AVG} = T_{Cin} + \frac{P_0}{2W_c c_{pC}} \tag{7}$$

2 Methodology

2.1 Point Kinetics Equations

PKE have significant role regarding rapid transients, when the kinetics of precursor dictates the behavior of the core. It must be obtained prior to PDE.

2.1.1 Equations and Data

The reactivity change simulated in this paper was given in four scenarios: reactivity suddenly made 0.0022 and 0.0066 positive and 0.0022 and 0.0066 negative.

In this paper, is was assumed that there is no external source of neutrons, power is proportional to flux and there is only one group of neutrons to simulate.

AP1000 is a Generation III+ Reactor and it has core heat output equal to 3400 MWth [2]. To obtain the β factor, six groups of neutrons (for U-235 only) were considered, presented in Figure 1.

Decay constants and yields of delayed-neutron precursors in thermal fission of uranium-235

$t_{1/2}$, [s]	λ_i, [s^{-1}]	β_i	β_i/λ_i
55.7	0.0124	0.000215	0.0173
22.7	0.0305	0.00142	0.0466
6.22	0.111	0.00127	0.0114
2.30	0.301	0.00257	0.0085
0.61	1.1	0.00075	0.0007
0.23	3.0	0.00027	0.0001
Total		0.0065	0.084

Figure 1: Precursor parameters [3]. β given as a total sum from β_i.

To derive the value of Λ, Eq. (8) was proposed [3].

$$\Lambda = \frac{l_p}{k}, \tag{8}$$

where l_p is prompt neutron lifetime and it is equal to 19.8 μs [4]. k is neutron multiplication factor and it can be expressed by reactivity, as a deviation from a critical state ($k = 1$).

$$k = \frac{1}{1-\rho}. \tag{9}$$

Since the simulation assumes that there is only one group of delayed neutrons, λ_i must transform into λ. From Figure (1) simple equation were proposed:

$$\lambda = \left(\sum_{i=1}^{6} \frac{1}{\beta} \frac{\beta_i}{\lambda_1} \right)^{-1} \tag{10}$$

The value of λ_i was taken from Figure (1).

Taking into consideration there is also a rough approximation of kinetics equations derived from balance equations [1]:

$$\frac{dn}{dt} = \frac{k-1}{l_d} n, \tag{11}$$

where l_d is a mean life-time of delayed neutrons given:

$$l_d = (1-\beta) l_p + \sum_{i=1}^{6} \left(\frac{1}{\lambda_i} + l_p \right) \beta_i. \tag{12}$$

2.1.2 Initial conditions

To specify the initial conditions it was assumed that reactor operates in a steady state i.e. the derivatives are zero. This leads to Eq. (13) for delayed precursor factor.

$$C_0 = \frac{\beta P}{\lambda \Lambda}. \tag{13}$$

The absence of i index indicates that only one group of neutrons is incorporated. P is a initial value for neutron flux (reactor power) as it is the core heat output stated above.

2.2 Point Dynamic Equations

The chosen transient for this simulation was "Spectrum of Rod Cluster Control Assemblies (RCCA) ejection accidents" described in DCD document [5] in Hot Full Power conditions. Then Power Range High Positive Flux Rate Reactor Trip occured [7].

2.2.1 Equations and Data

A simple control rod in worth 0.37$ [6] was rapidly ejected from the bundle. In this paper, for the simplicity, it was assumed that the rod do not return to the core − on the contrary what DCD Chapter 15.4 [6] states. The designed system detected 120% power overload and after time $T_D = 0.6\ s$ (general delay time [7]) it stops the solver. Subsequently, ramp RCCA negative reactivity was inserted to trip the reactor in a time equal $T_{new} = 2.67\ s$. This time described in Westinghouse DCD [5] indicates that the coolant pumps are still working. Overall reactivity inserted was worth 4 % [5].

The property of Λ was given from PKE as a mean value of four different reactivity change scenarios mentioned above. It was derived from Equations (8) and (9) then putted in a simple equation for mean values:

$$\Lambda = \frac{1}{4} \sum_{i=1}^{4} \Lambda_i. \tag{14}$$

λ value obtained as in PKE from Eq. (10). Some of the parameters that were directly taken from DCD are presented in Table 1.
To transition from given units simple conversion formulas were utilized:

Table 1: Parameters taken from DCD.

Name	Variable	Value	Value [SI units]	DCD
Core Inlet Temperature	T_{cin}	279.44 [°C]	552.59 [K]	[2]
Core Outlet Temperature	T_{cout}	610 [°F]	594.29 [K]	[8]
Avg. Coolant Temp. (Hot Full Power)	T_c	303.39 [°C]	576.54 [K]	[2]
Coolant Pressure	p_c	2250 [psia]	15.51 [MPa]	[8]
Coolant Flow	w_c	48443.7 [t/h]	13673 [kg/s]	[2]
Mean Coolant Velocity	v_c	4.816 [m/s]	4.816 [m/s]	[2]
Fuel Mass UO_2	m_f	211 588 [lb]	95 975 [kg]	[4]
Active Heat Transfer Area	A_{hx}	5267.6 [m^2]	5267.6 [m^2]	[2]
Average Heat Flux	q	628.7 [kW/m^2]	628.7 [kW/m^2]	[2]
Core Height	h	168 [in.]	4.27 [m]	[4]
Eff. Delayed Neutron Fraction (transient)	β_t	0.0049 [-]	0.0049 [-]	[5]

- °C $\rightarrow K$

$$K =^{\circ} C + 237.15, \tag{15}$$

- °F $\rightarrow K$

$$K = \left(\frac{^{\circ}\mathrm{F} - 32}{1.8}\right) + 273.15, \tag{16}$$

- psia $\rightarrow$ MPa

$$\mathrm{MPa} = \mathrm{psia} \times 0.00689476, \tag{17}$$

- t/h $\rightarrow$ kg/s

$$\mathrm{kg/s} = \mathrm{t/h} \times 0.27778, \tag{18}$$

- lb $\rightarrow$ kg

$$\mathrm{kg} = \mathrm{lb} \times 0.453592, \tag{19}$$

- in. $\rightarrow$ m

$$\mathrm{m} = \mathrm{in.} \times 0.0254. \tag{20}$$

Heat transfer coefficient were obtained from equation presented below:

$$h_c = \frac{q}{\Delta T}, \tag{21}$$

where ΔT was derived from difference between T_{cout} and T_{cin}. Those and q values were taken from Table 1. To calculate the coolant mass Eq. (22) was utilized.

$$m_c = \frac{w_c h}{v_c} \tag{22}$$

The Moderator temperature coefficient (r_c) was presented in DCD Chapter 4.3 [4]. The coefficient was calculated from Eq. (14) for "Best estimate" values and then converted with Eq. (16).

The Doppler temperature coefficient was taken differently due to transient properties. From DCD Chapter 15.4 [5] the Doppler (power) defect has been obtained and it equals -0.9%Δk. To estimate the proper Doppler feedback coefficient simple equation was proposed[9]:

$$r_f = \Delta\rho_D \Delta T_f, \tag{23}$$

where $\Delta\rho_D$ is power defect and ΔT_f is a difference between fuel average temperature in Hot Full Power State (T_{favg}) and fuel temperature in Hot Zero Power State (T_{fz}). T_{favg} equals 900 K taken from [10] and T_{fz} was found in DCD Chapter 4.4 [2] as a definition of a Hot State. It is given 557 °F which equals 564.82 K (see Eq. [16]).

The next element was to calculate the specific heat capacity for both coolant (cp_c) and fuel (cp_f) under constant pressure. For coolant values XSteam library for MATLAB was used. It demands pressure and temperature given in Table 1 as p_c and T_c. For fuel's property 'cp_UO2' function was utilized (see Attachment 2). It demands only T_{favg} sated above.

2.2.2 Initial conditions

For both Equations (1) and (2) methodology was identical to the one in subsection 2.1.2 with a remark that Power here equals to 102% of nominal P. It is dictated by transient nature in the beginning of operation [6].

Similar to subsection 2.1.2 initial conditions were obtained from the reactor steady-state operation. Equations (3) and (4) are given respectively:

$$T_{f0} = T_{c0} + \frac{102\%P}{hA_{hx}} \tag{24}$$

$$T_{c0} = T_{cin} + \frac{102\%P}{2w_c cp_c} \tag{25}$$

All of the calculated values are given in Appendix 1.

3 Results and Analysis

3.1 Point Kinetics

The four scenarios considered were about rapid insertion of reactivity with values of 0.0022 and 0.0066 positive and 0.0022 and 0.0066 negative. To solve the Eq. (1) and (2) MATLAB codes (Attachment 1) were utilized with 'ode45' dedicated solver. Additionally, rough approximation from Eq. (11) was compared to results from the Equations above.

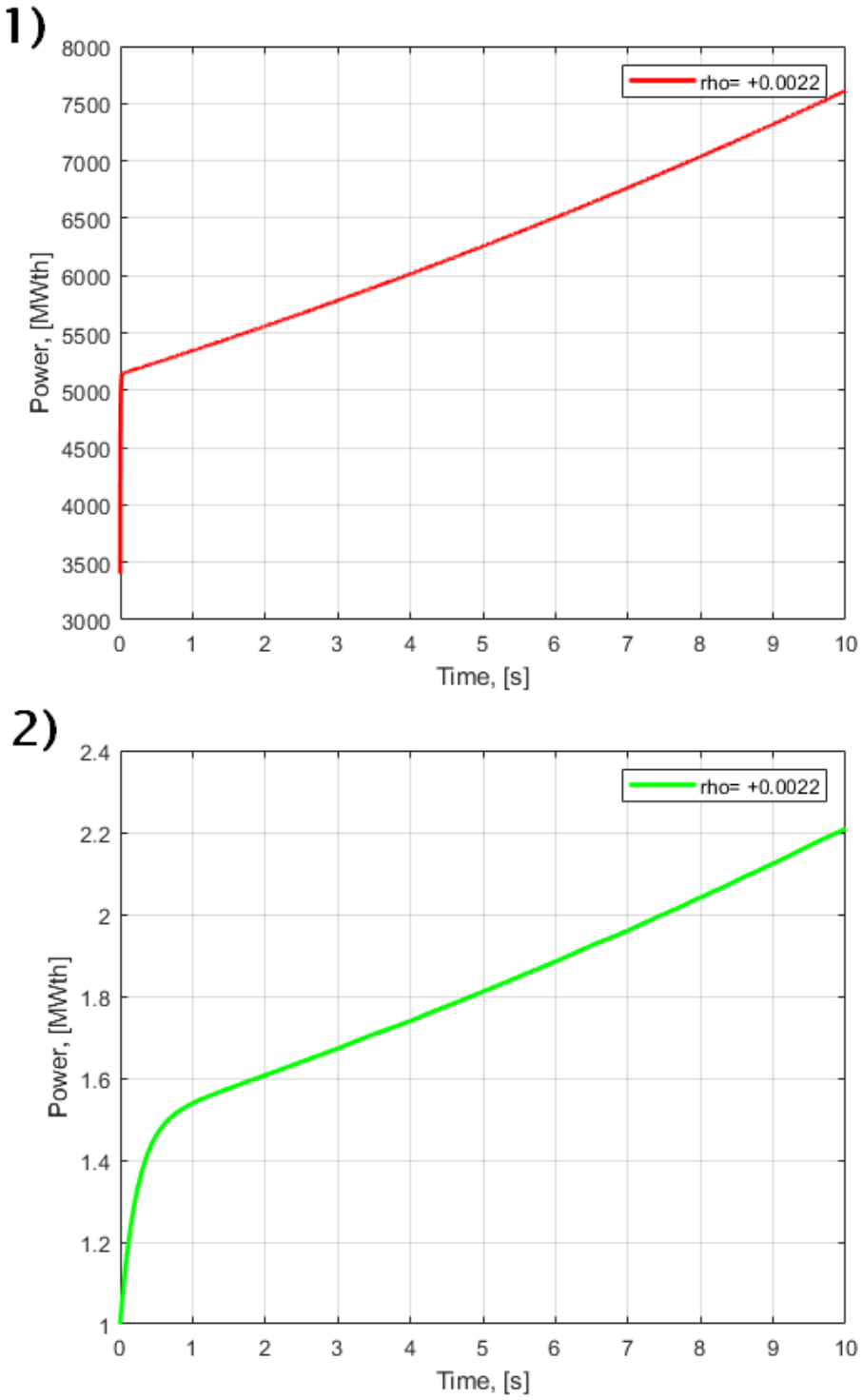

Figure 2: Power over time graph for inserted reactivities. Picture 1. depicts simulation conducted with this paper, Picture 2. presents reference simulation.

The reference simulation was taken from "Nuclear Reactor Physics Tutorials" by PhD. Piotr Darnowski [11] presented on Warsaw University of Technology. The behavior of the power in Figure 2 1. complies with the reference one. The sudden jump in simulation corresponds to different entry parameters. Here, AP1000 is taken into consideration, in "Nuclear Reactor Physics Tutorials" there is no specific reactor.
The difference lies in the parameter Λ. Based on Eqs. (8) and (9), the value for the given ρ is calculated to be 1.98×10^{-5}. However, in the "Nuclear Reactor Physics Tutorials," the value is provided as 1×10^{-3}. Neutron generation time, in conducted simulation, is two orders of

magnitude shorter than in reference with identical effective delayed neutron value. It indicates that the same amount of delayed neutrons is produced in a shorter time, hence continuity of the function is barely visible.

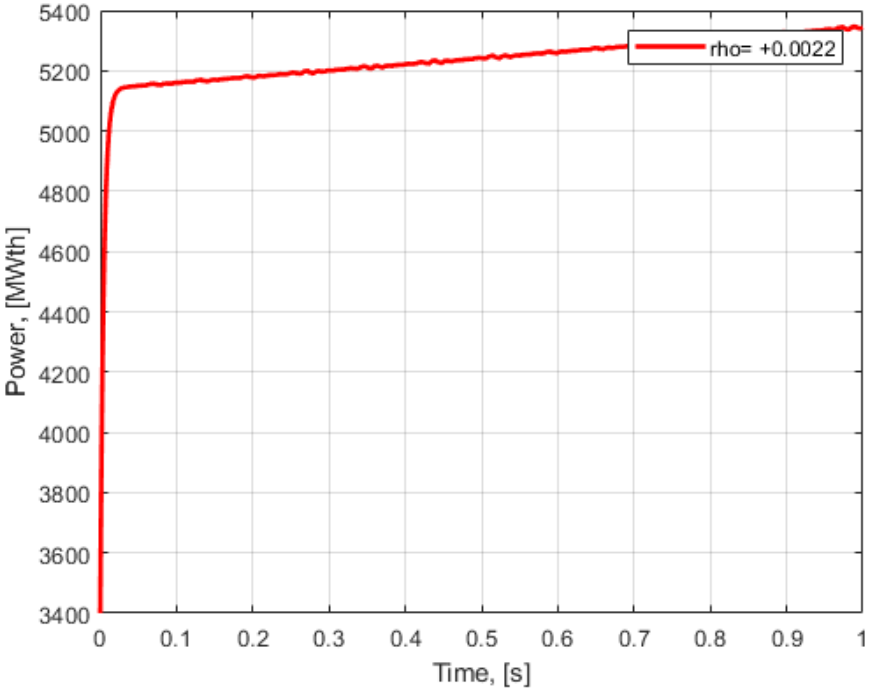

Figure 3: Power over time graph for inserted reactivity.

Figure 3 represents the same Equations as in Figure 2. 1., however time of the simulation was shortened to 1 s. Continuity here is clearly visible, which conform to the reference graph.

In the Figure 2. 1. power change occurs in the same manner as in the Figure 2. 2. It approximately rises 2.2 times after 10 s of simulation, which also indicates that the calculations were done properly.

The graph for inserted reactivity $\rho = +0.0066$ is presented in Figure 4.

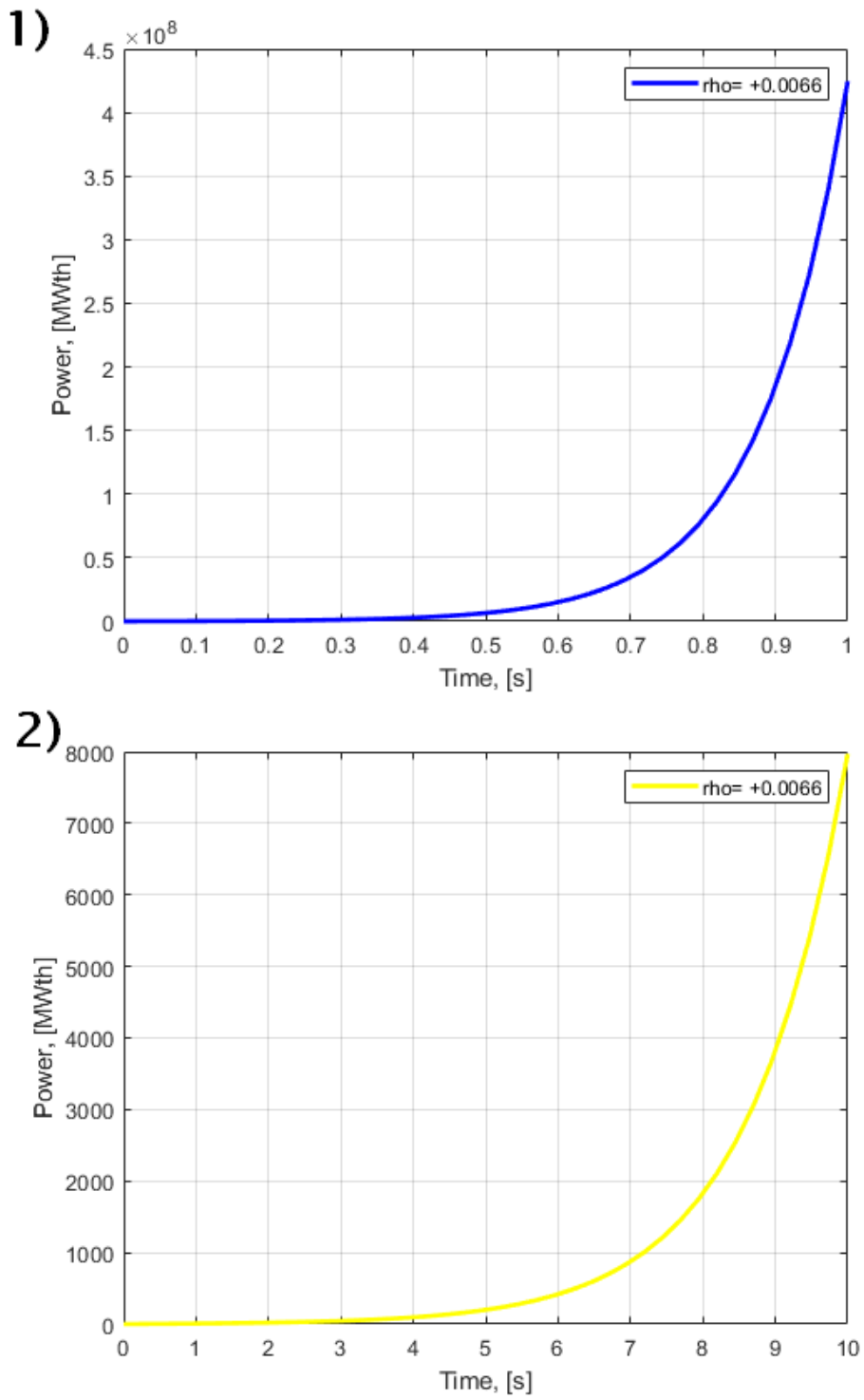

Figure 4: Power over time graph for inserted reactivities. Figure 4. 1. depicts simulation conducted with this paper, Figure 4. 2. presents reference simulation.

For the same reason as for Figure 3, simulation time in the Figure 4. 1. is shortened to 1 s. Without reactivity feedbacks, power calculated here increased dramatically. It is important to know that at such high power levels, the system rapidly disintegrates, leading to a loss of the chain reaction. It is better depicted in the Figure 5.

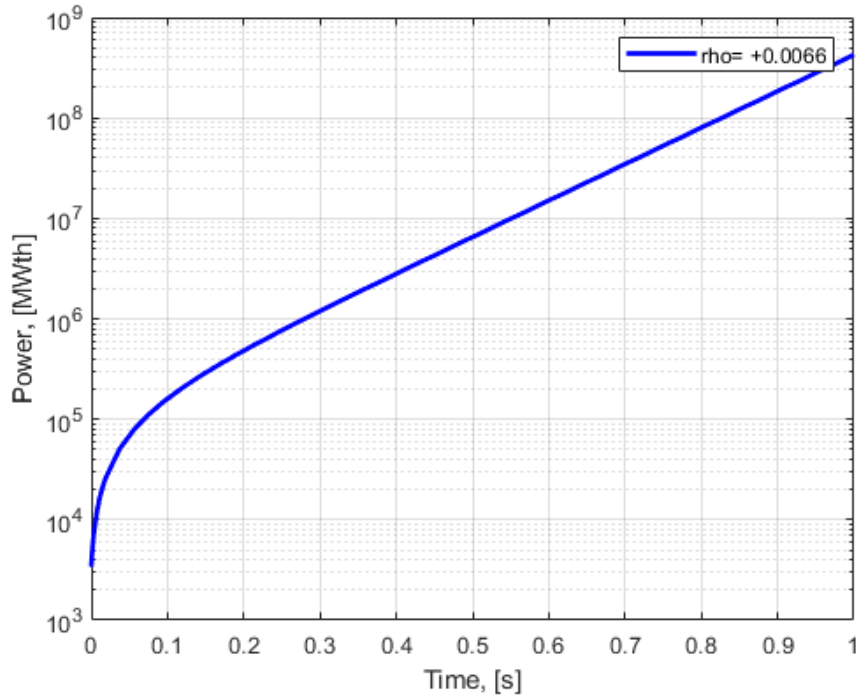

Figure 5: Power over time on logarithmic scale.

The solutions of Eq. (11) were compared with the solutions of Eqs. (1) and (2) on the Figure 6.

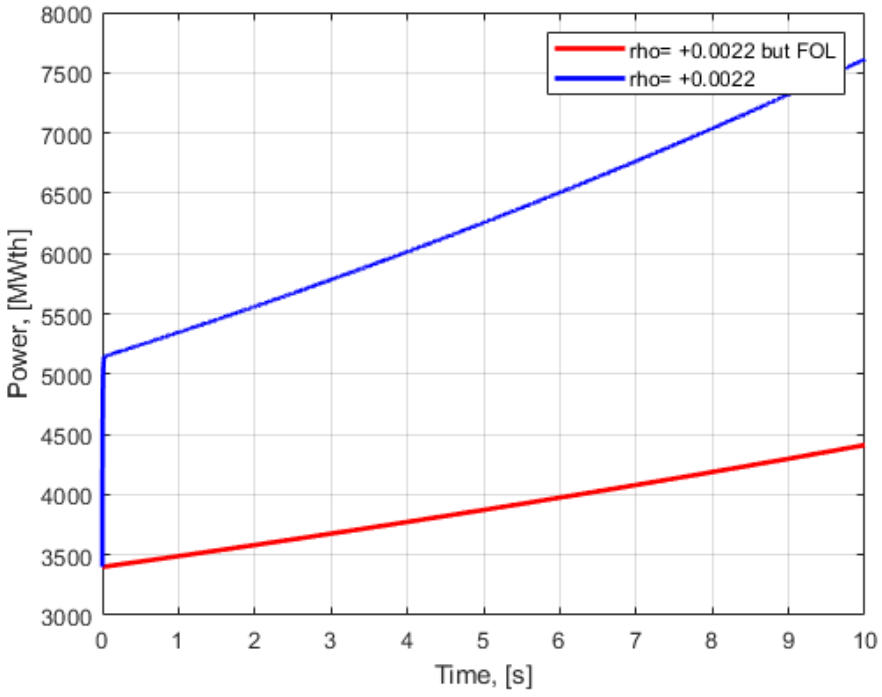

Figure 6: Power over time graph for inserted reactivities. "FOL" means first order linear differential equation which is Eq. (11).

The conclusions drawn from Figure 6 are that Eq. (11) is not suitable for approximating reactor behavior with such reactivity.

Other two scenarios are presented below.

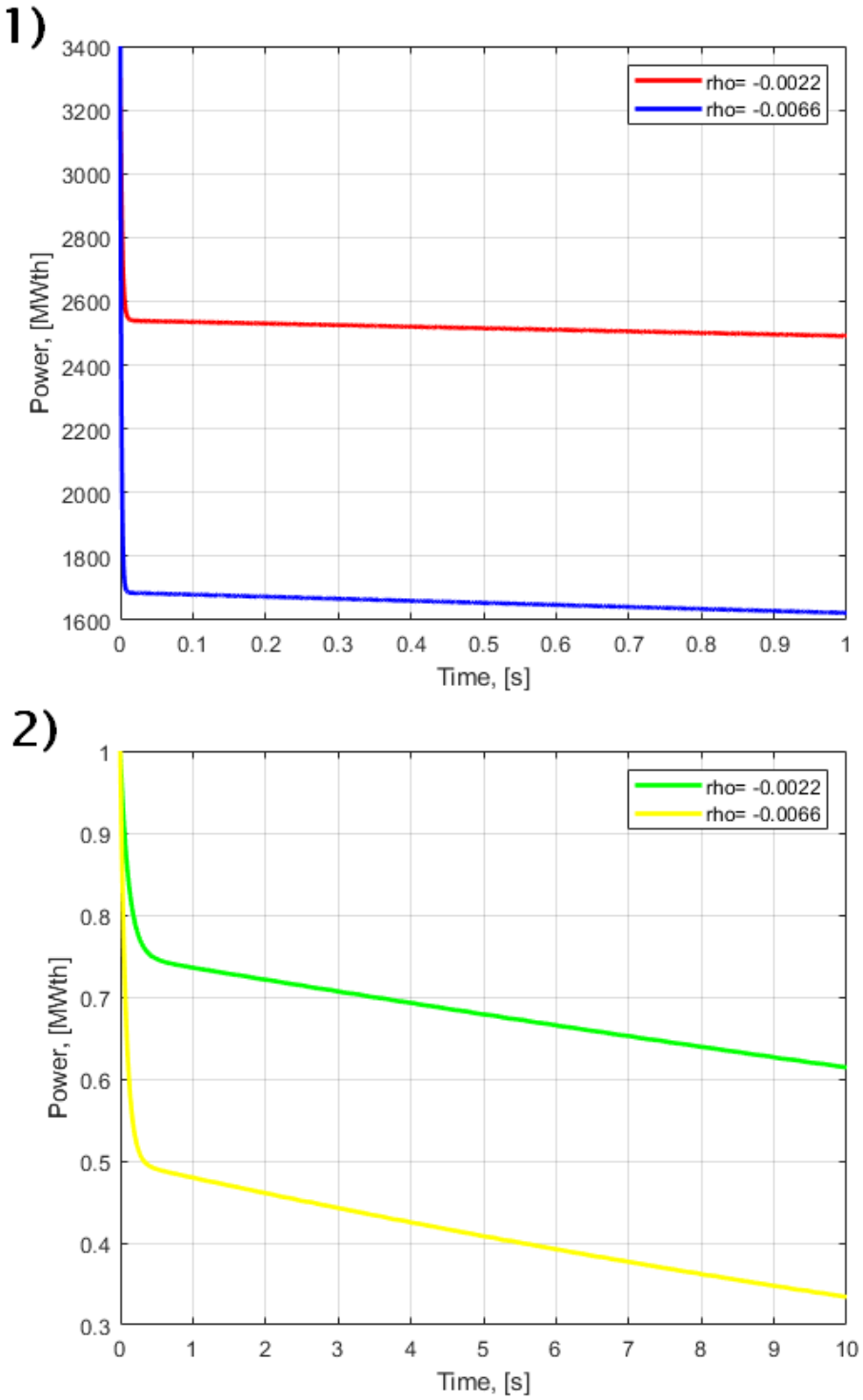

Figure 7: Power over time graph for inserted reactivities. Figure 7.1. depicts simulation conducted with this paper, Picture 7. 2. presents reference simulation [11].

Analysis for the Figure 7. 1. is identical as for the Figure 2. 1. The core behavior complies with the reference and the power coherently drops approximately by a half in the critical moment.

3.2 Point Dynamics

To solve the Eqs. (1), (2), (3) and (4) with transient "Spectrum of Rod Cluster Control Assemblies (RCCA) ejection accidents" and "Power Range High Positive Flux Rate Reactor Trip" MATLAB codes with 'ode45' solver were utilized (see Attachment 2). The simulation start was in Hot Full Power where reactor is in critical state, hence $\rho_0 = 0$.

For the purpose of this study, the special code called 'Systems.m' was developed. How the Systems code operates was shortly described in Section 2.2.1. To have a deeper view please see Attachment 2.

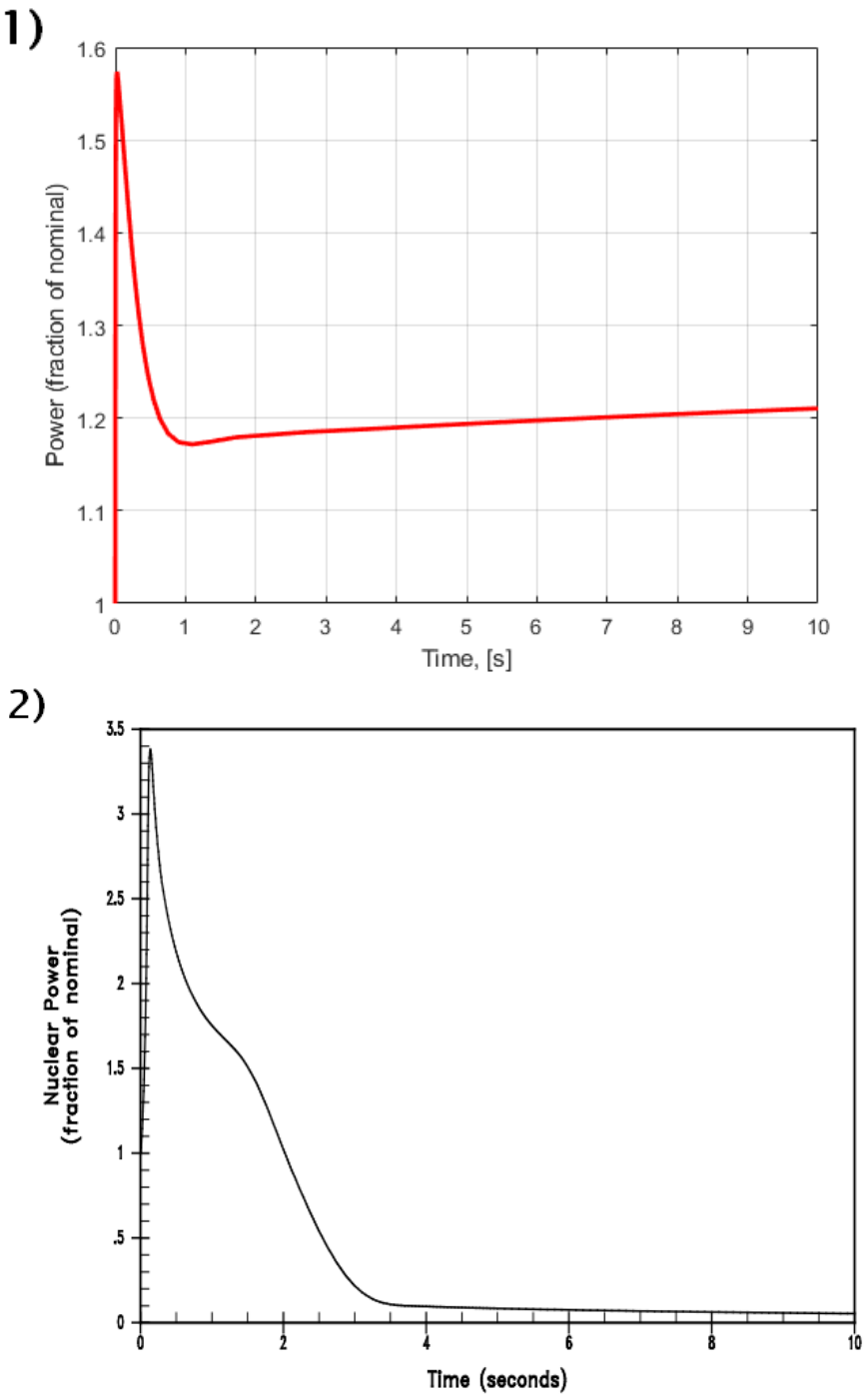

Figure 8: WITHOUT SYSTEMS CODE. Power (fraction of nominal) over time graph. Picture 1. depicts simulation conducted with this paper, Picture 2. presents reference simulation.

Firstly, the simulation was conducted without the Systems code to check If it complies with given reference. The reference simulation was taken from Westinghouse DCD [6] for this particular transient. Transient occurs in $t = 0$ and then the power increases abruptly. Later on the reactivity feedbacks throttle the flux and the reactor stabilizes. It can be noted that the behavior here with the transient is similar to the the one in the Figure 8. 2. The differences may occur due to the fact that Westinghouse was conducting spatial dynamics with dedicated reactor protection system − contrary to the assumptions presented herein. Reactivity in Figure 8. 2 also decreases in time greater than 1 s, which is longer than on the Figure 8. 1. It is due the fact that in this paper it was assumed that control rods were removed out of the core, which means the reactivity was constant and positive. Nonetheless, at this stage, it is valid to assume that with the first second presented model coheres with the reference one.

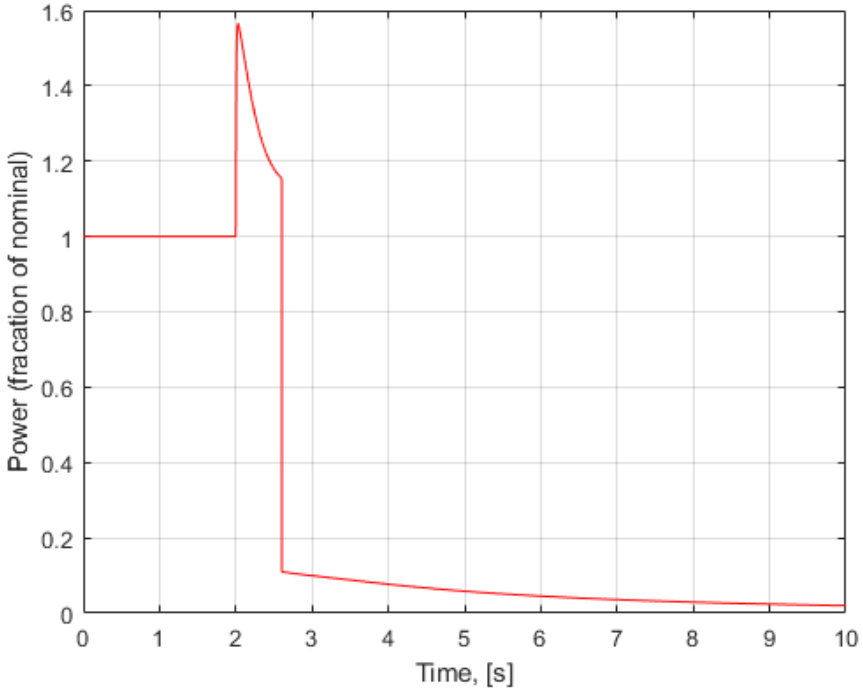

Figure 9: Power (fraction of nominal) over time graph. Systems code included.

In the Figure 9, the time from $T = 0\ s$ to $T = 2\ s$ represents that the reactor is in a steady-state, before the transient occurs. Ejection of a control rod in $T = 2\ s$ rapidly increases the neutron flux, and then in $T = 2.6\ s$ (delayed time mentioned in Section 2.2.1) Systems code starts tripping the reactor with ramp negative reactivity insertion in time $T = 2.6\ s$ to $T = 5.27\ s$. This sudden jump in $T = 2.6\ s$ is a result from rapid neutron absorption as reactivity before and after the trip varies significantly. In order to cancel the this jump more detailed analysis should be conducted (e.g. spatial analysis).

4 Conclusions

The objective of this study was to simulate and examine specific transient states of the AP1000 reactor core using a point kinetics and dynamics model. MATLAB, along with modified existing codes tailored to the project's needs, was used to complete this task. The results aligned with the predictions of the model.

References

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Appendix 1

Table 1. Calculated values.

Name	Variable	Value	Unit
Neutron multiplication factor (for ρ = +0.0022)	k_1	1.0022	[-]
Neutron multiplication factor (for ρ = +0.0066)	k_2	0.9978	[-]
Neutron multiplication factor (for ρ = -0.0022)	k_3	1.0066	[-]
Neutron multiplication factor (for ρ = -0.0066)	k_4	0.9934	[-]
Neutron generation time (for k_1)	Λ_1	1.9756e-05	[s]
Neutron generation time (for k_2)	Λ_2	1.9844e-05	[s]
Neutron generation time (for k_3)	Λ_3	1.9669e-05	[s]
Neutron generation time (for k_4)	Λ_4	1.9931e-05	[s]
Decay constant for neutron precursors	λ	0.0767	[1/s]
Mean life time of delayed neutrons	l_d	0.0847	[s]
Neutron generation time PDE	Λ_{PDE}	1.9800e-05	[s]
Sp. Heat capacity for coolant	cp_c	5545.2	$[\frac{J}{kgK}]$
Sp. Heat capacity for fuel	cp_f	309.98	$[\frac{J}{kgK}]$
Coolant mass present in the core	m_c	12122	[kg]
Doppler reactivity feedback coefficient	r_f	-2.6851e-05	$[\frac{1}{K}]$
Moderator reactivity feedback coefficient	r_c	-31.5e-5	$[\frac{1}{K}]$