



# Poincaré Plots: A Mini-Review

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# Henri Poincaré (1854-1912)

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# Poincaré plot

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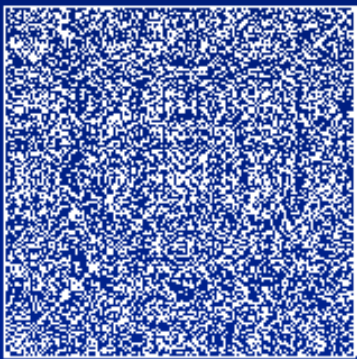
- Poincaré HRV plot is a graph in which each RR interval is plotted against next RR interval (a type of delay map)
- Synonyms
  - Scatter plot; scattergram
  - Return map; phase delay map
  - Lorenz plot



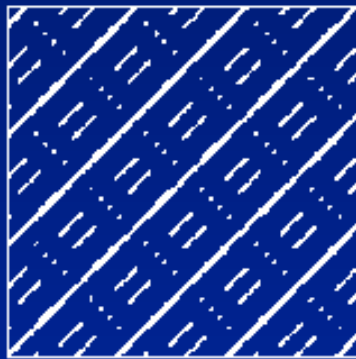
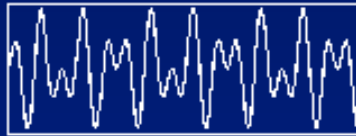
# Poincaré & related plots as nonlinear tools

Visualization of higher-dimensional phase spaces  
in two or three-dimensional sub-spaces

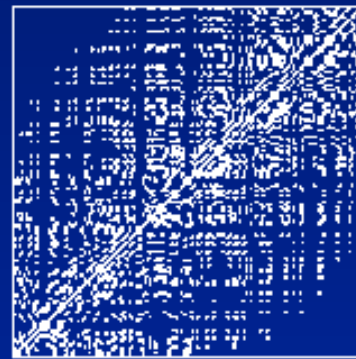
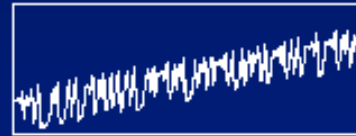
uncorrelated noise



harmonic oscillation



chaotic time series  
with linear trend



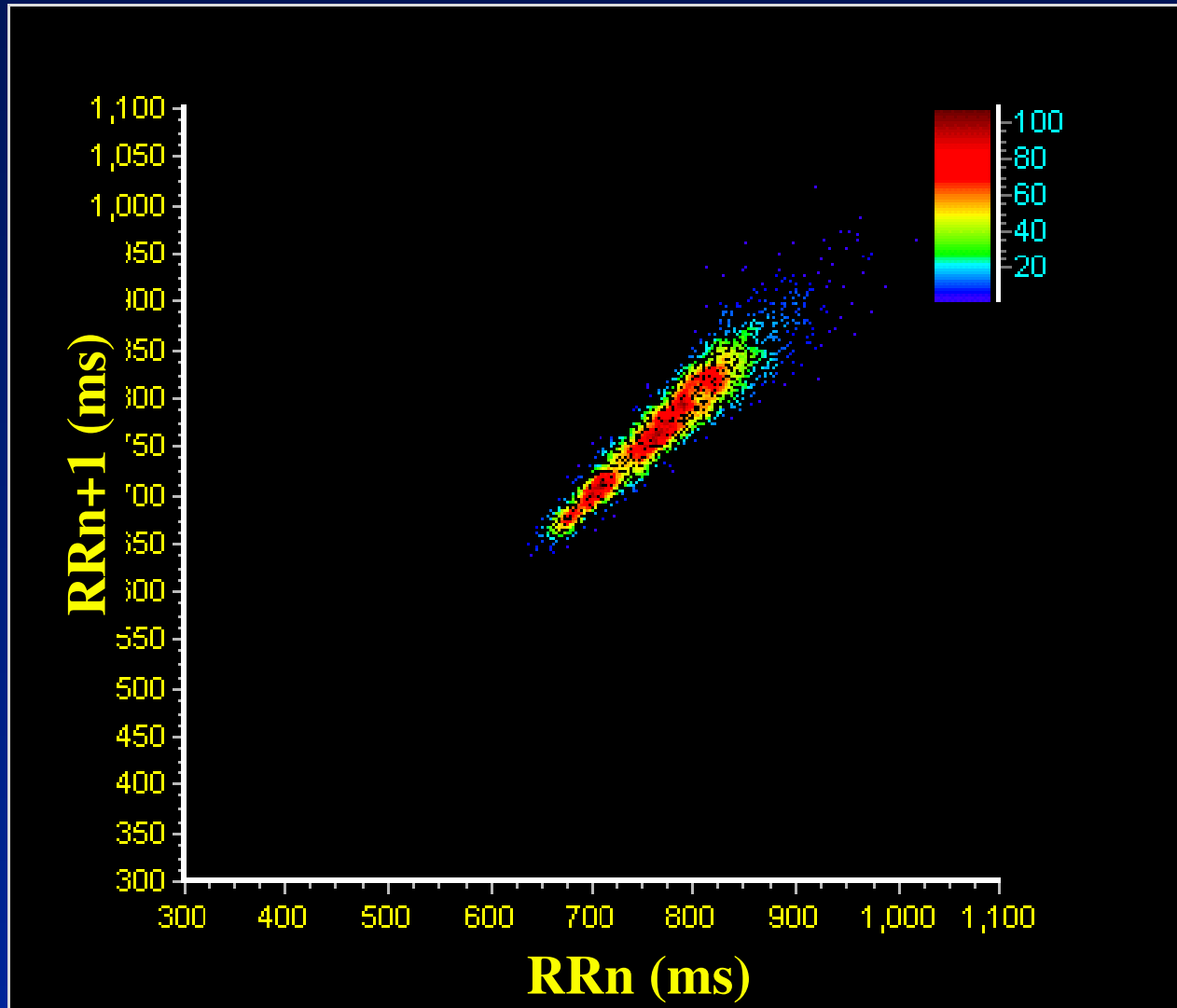
time series generated  
from an AR model



Source: [http://en.wikipedia.org/wiki/Recurrence\\_plot](http://en.wikipedia.org/wiki/Recurrence_plot)

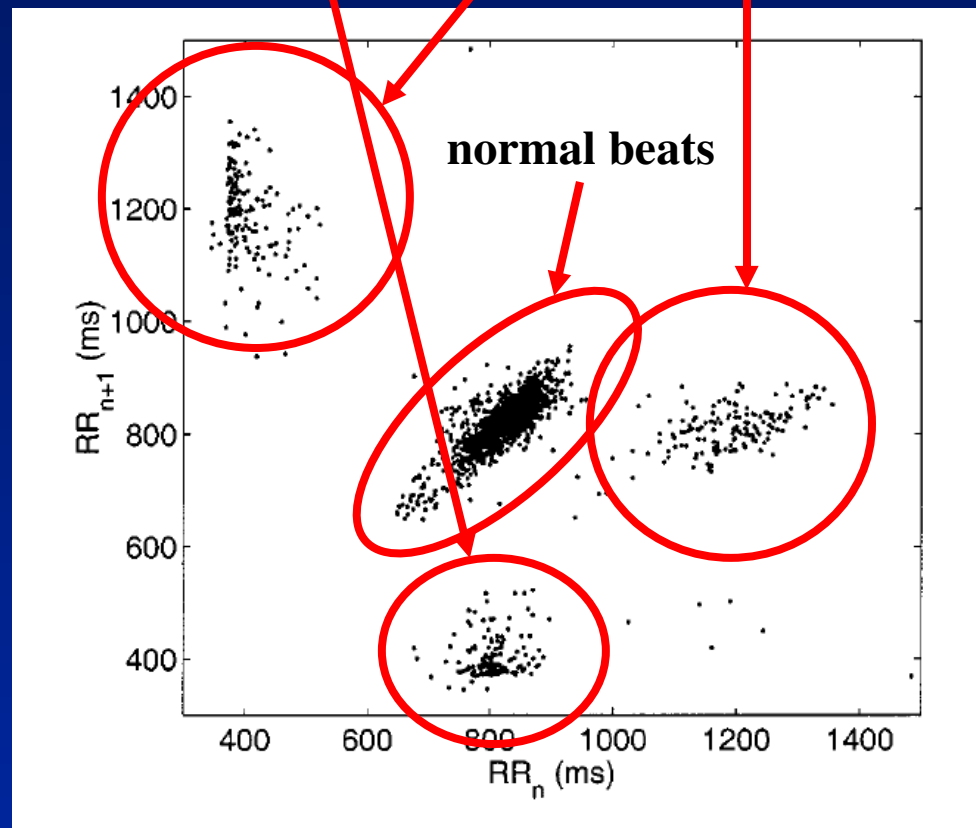
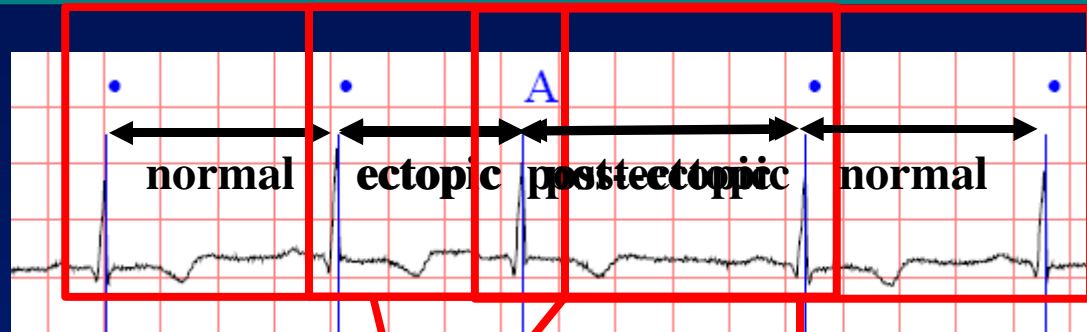


# Poincaré HRV plot: normal





# Visualization of ectopic beats

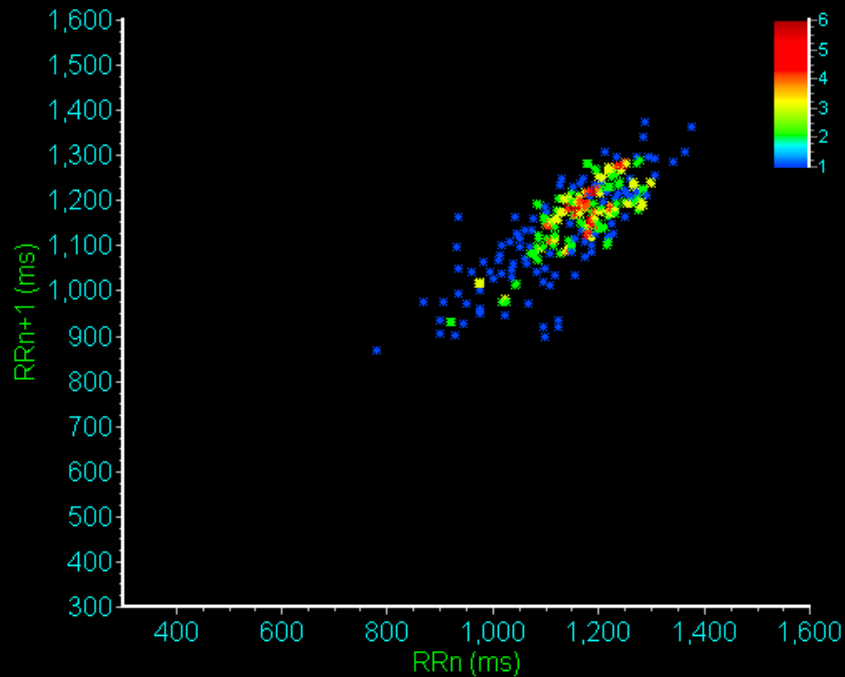




# Poincaré HRV plot: healthy vs. disease

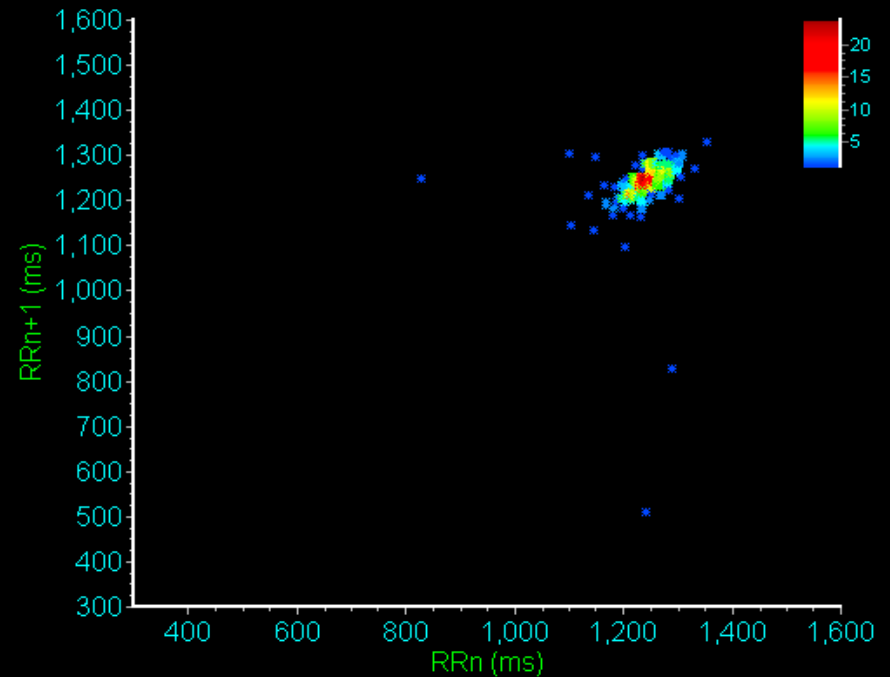
## Healthy Control

mean heart rate : 52 bpm



## Critically ill Patient

mean heart rate : 51 bpm



**Data Source: Taipei Veterans General Hospital, Taiwan**

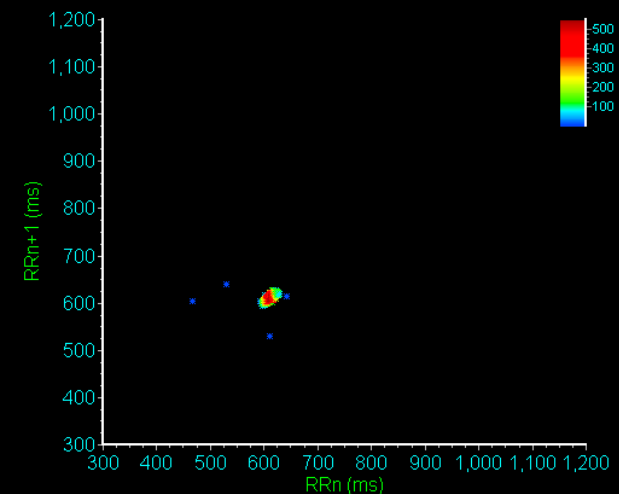
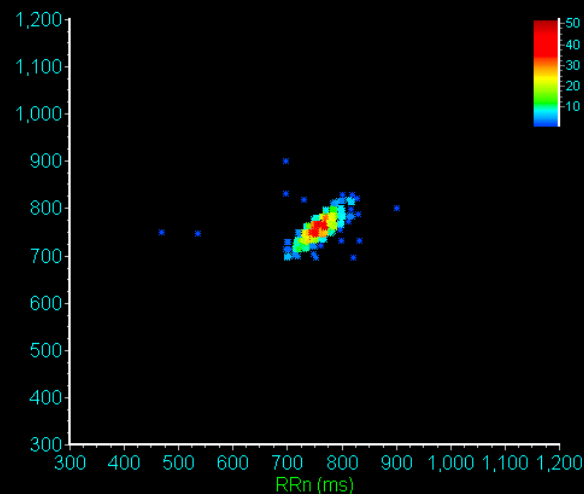
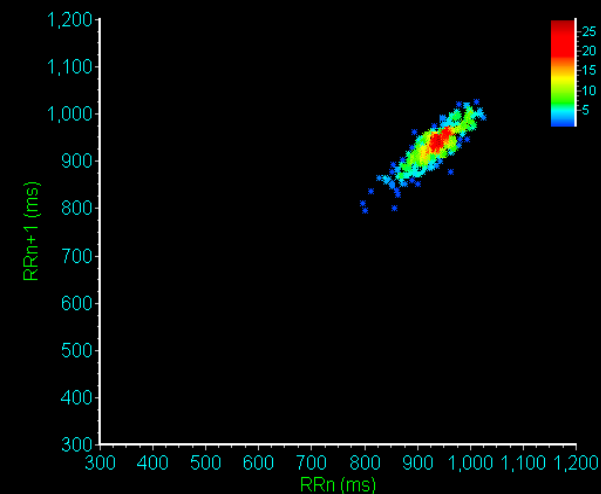


# *HRV in different stages of cancer*

Early-detected  
cancer patient

Chemotherapy  
cancer patient

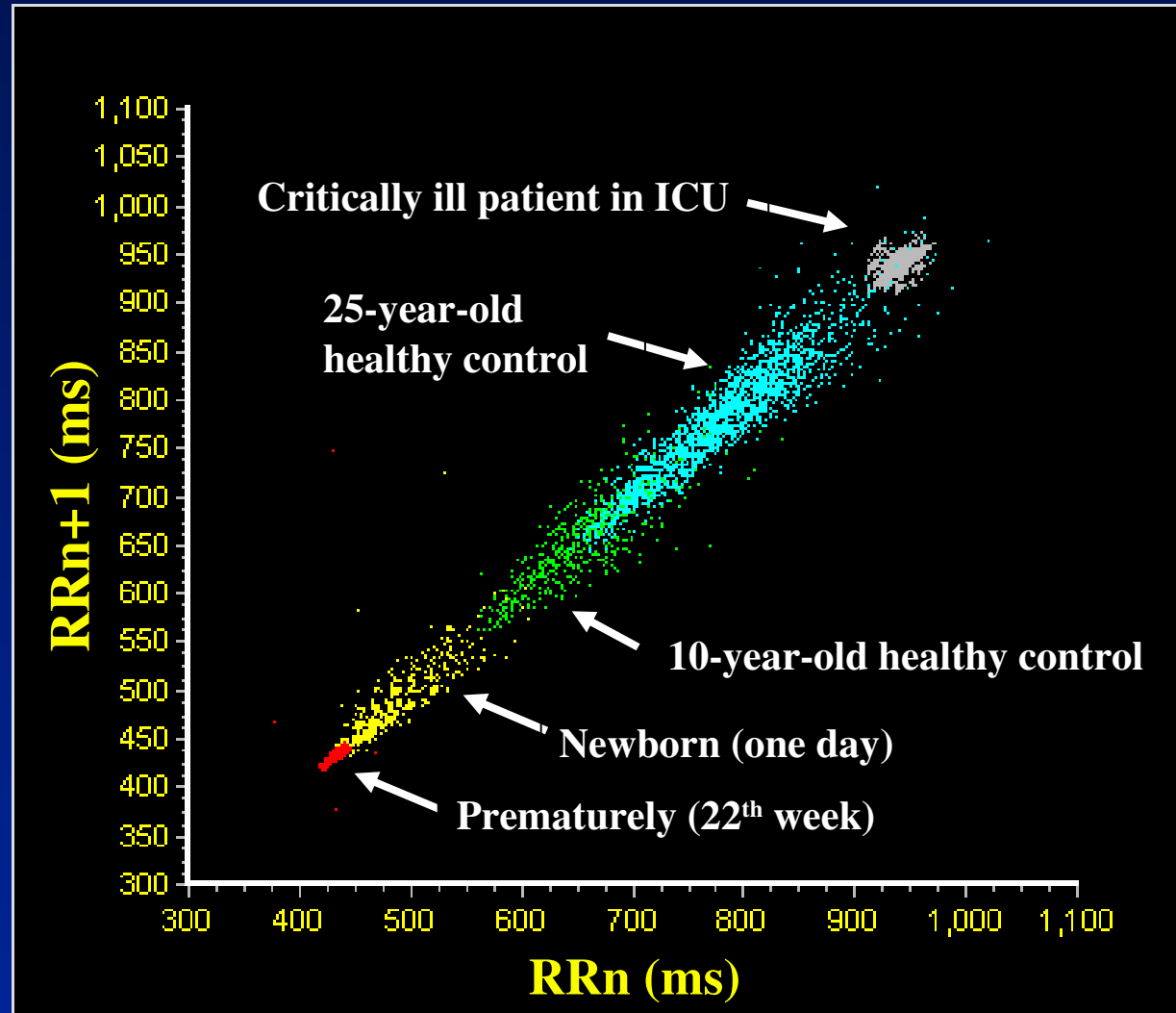
Hospice  
cancer patient



**Data Source: Taipei Veterans General Hospital, Taiwan**



# How aging and illness may affect the geometry of Poincaré HRV plot

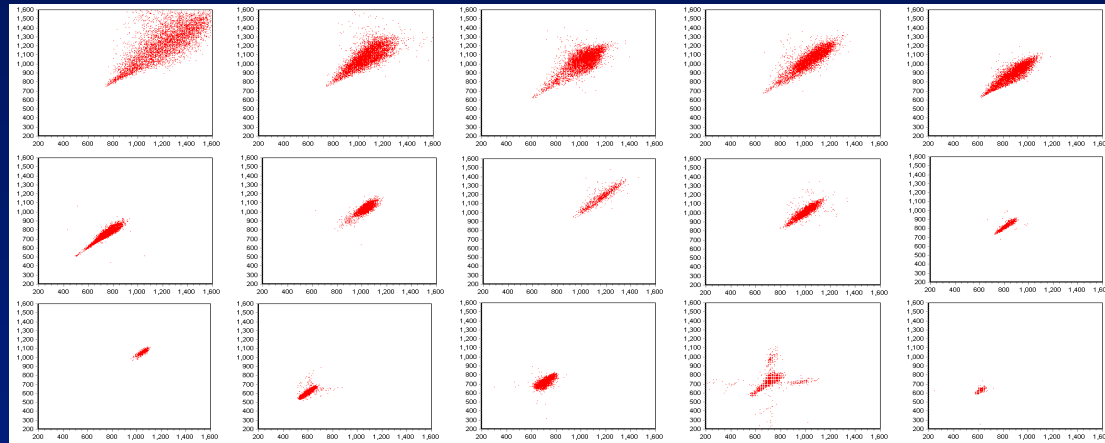


Data Source: Taipei Veterans General Hospital, Taiwan



# Quantitative analysis of Poincaré HRV plots

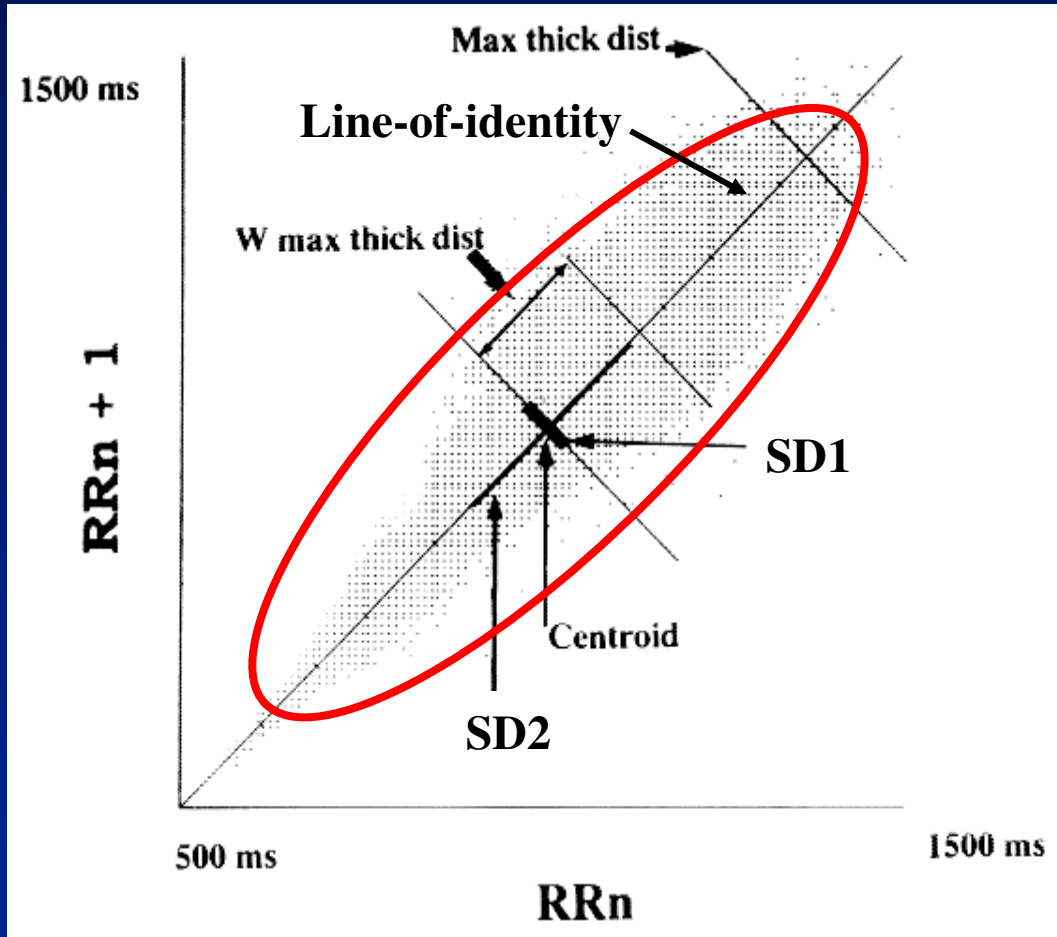
## Variable geometries of Poincaré HRV plots



- Ellipse fitting technique
- Histogram technique
- Correlation coefficient



# Ellipse fitting technique



**SD1: dispersion (standard deviation) of points perpendicular to the axis of line-of-identity**

**SD2: dispersion (standard deviation) of points along the axis of line-of-identity**



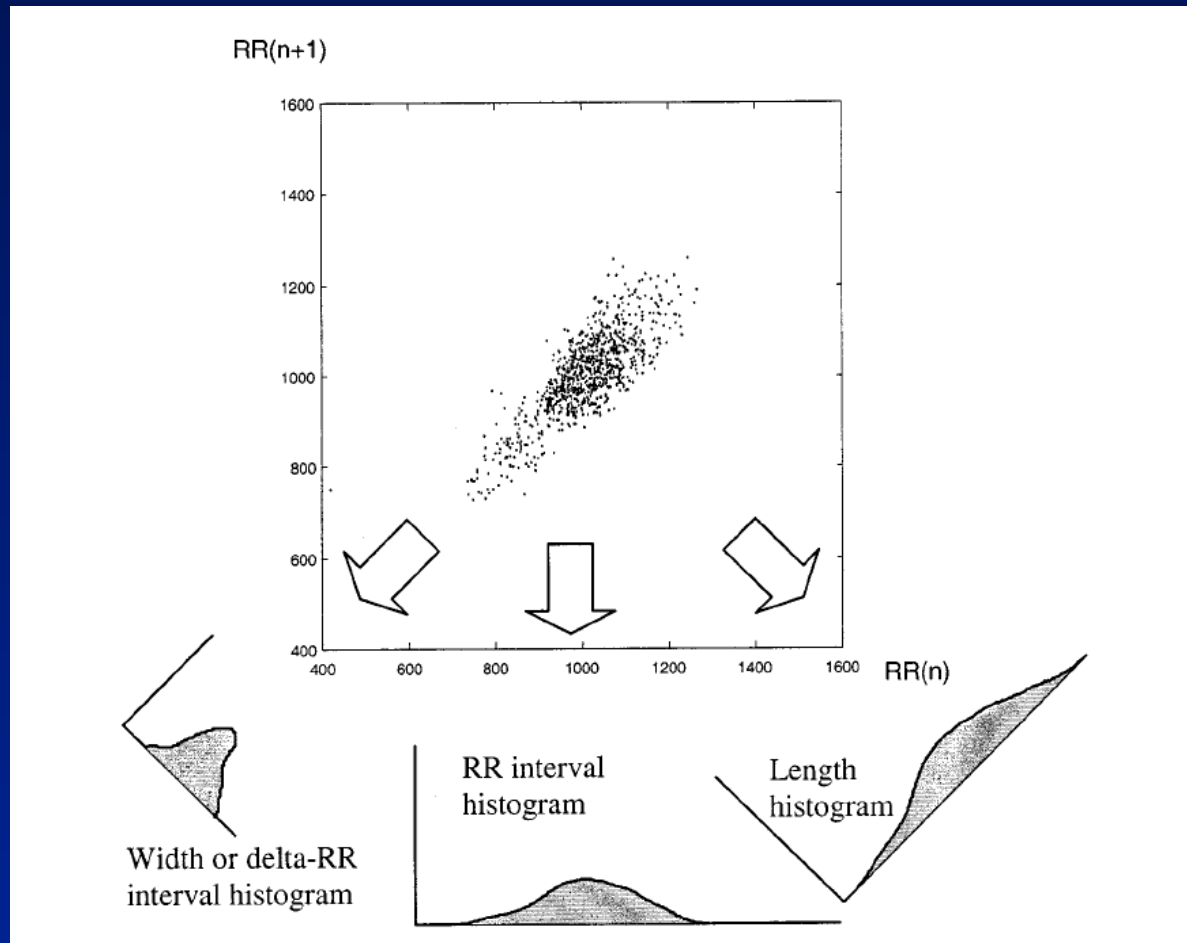
# Ellipse fitting technique – Risk Stratification in Cardiovascular Disease

**Table 1.** Studies of Non-Linear HRV in Cardiac Patients

Reference	Population	Results/Conclusions
Brouwer et al., 1996 [19]	95 pts with HF, HRV and Poincaré plots from 24 hr holter recordings (Ibopamine Multicenter Trial study group)	Shape of Poincaré plots independent prognostic value in pts with mild to moderate HF
Bigger et al., 1996 [2]	(1) 715 pts with recent MI (2) 274 healthy pts (3) 19 pts with heart transplant (Multicenter Post Infarction Program)	MI or denervation of the heart causes a steeper slope and decreased height of power law slope
Huikuri et al., 1999 [10]	446 with MI with decreased LV function (EF<35%) F/U 685±360 days	Alpha 1 is the most powerful predictor of all cause mortality
Huikuri et al., 1998 [7]	Random sample of 347 patients of >65 yrs F/U for 10 yrs	Power law slope is a more powerful predictor of death than the traditional risk markers in elderly subjects
Kamen et al, 1995 [21]	Poincaré plot pattern to display beat to beat HRV data from 23 pts with HF and compared with 20 healthy people	Poincaré plot is a semi-quantitative tool which can be applied to the analysis of R-R interval
Laitio et al., 2002 [22]	HRV and Poincaré plots of 40 pts with CABG	SD1/SD2 ratio is the most powerful independent predictor of postoperative ischaemia
Laitio et al., 2004 [13]	32 pts aged ≥60 yrs admitted to hospital for surgical repair of traumatic hip fracture	Alpha-1 predicts post operative myocardial infarction
Lombardi et al 1996 [8]	HRV in 2 groups of pts after MI (normal and reduced LVEF). Group 1: 20 pts; Group 2: 15 pts	Steeper slope of the negative regression line between power and frequency among reduced LVEF
Mäkikallio et al., 1998 [23]	38 pts with stable angina without previous MI or cardiac medication and 38 age matched healthy pts	Alpha-1 helps differentiate CAD and healthy pts



# Histogram technique



**Brennan M. et al. IEEE Trans Biomed Eng 2001;48:1342-47**

# Do these indices actually measure nonlinear properties of heart rate dynamics

Table 3. Correlation coefficients among average HR, time and frequency domain measures, and quantitative beat-to-beat analysis of HR variability at ventilatory threshold

	HR	SDANN	SDsd	SDsd/SDANN	HF	LF	LF/HF	SD1/SD2	SD2	SD1	ApEn
HR	1.0	0.63‡	0.59‡	0.16	0.47†	0.58‡	0.51†	0.18	0.61‡	0.58‡	0.54†
SDANN		1.0	0.54†	-0.51†	0.29	0.73‡	0.78‡	0.46†	0.99‡	0.52†	-0.45*
SDsd			1.0	0.33	0.70†	0.63‡	0.48†	0.27	0.49†	0.99‡	-0.47†
SDsd/SDANN				1.0	0.20	-0.18	-0.42*	0.94‡	-0.56†	0.36*	0.23
HF					1.0	0.21	0.07	0.15	0.24	0.70‡	0.27
LF						1.0	0.90‡	-0.18	0.73‡	0.62‡	-0.54†
LF/HF							1.0	-0.21	0.79‡	0.49†	-0.33
SD1/SD2								1.0	-0.52†	0.31	0.27
SD2									1.0	0.47†	-0.45*
SD1										1.0	-0.48†
ApEn											1.0

Values are Pearson's correlation coefficients;  $n = 31$ . See Table 2 for definition of abbreviations. \* $P < 0.05$ ; † $P < 0.01$ ; ‡ $P < 0.001$ .



# Do these indices actually measure nonlinear properties of heart rate dynamics

## ■ Ellipse fitting technique

$$\begin{aligned} SD1^2 &= \text{Var}(x_1) = \text{Var}\left(\frac{1}{\sqrt{2}} RR_n - \frac{1}{\sqrt{2}} RR_{n+1}\right) \\ &= \frac{1}{2} \text{Var}(RR_n - RR_{n+1}) = \frac{1}{2} SDSD^2. \end{aligned}$$

$$SD2^2 = 2SDRR^2 - \frac{1}{2}SDSD^2.$$

**SDRR : standard deviation of the RR intervals**

**SDSD: standard deviation of the successive differences of the RR intervals**



# Summary

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- Advantages
  - Simple visualization tool
  - Outlier (ectopic beat or artifact) identifier
  - Possible insights into short-term and long-term variability
- Limitations
  - Derived statistics not independent of other time domain measures