

## Interview talk

**Candidate** Guanxiong Luo

**Title** Development of Advanced Generative Priors for MRI Reconstruction

**Date** Nov 21, 2023

**Place** online via Teams

## Outline

- **MRI reconstruction as inverse problems**
  - Bayesian MRI reconstruction using diffusion priors
  - Phase augmentation for training priors
  - Software
  - Summary and outlook

**MR scans have revolutionized clinical practice in numerous ways**

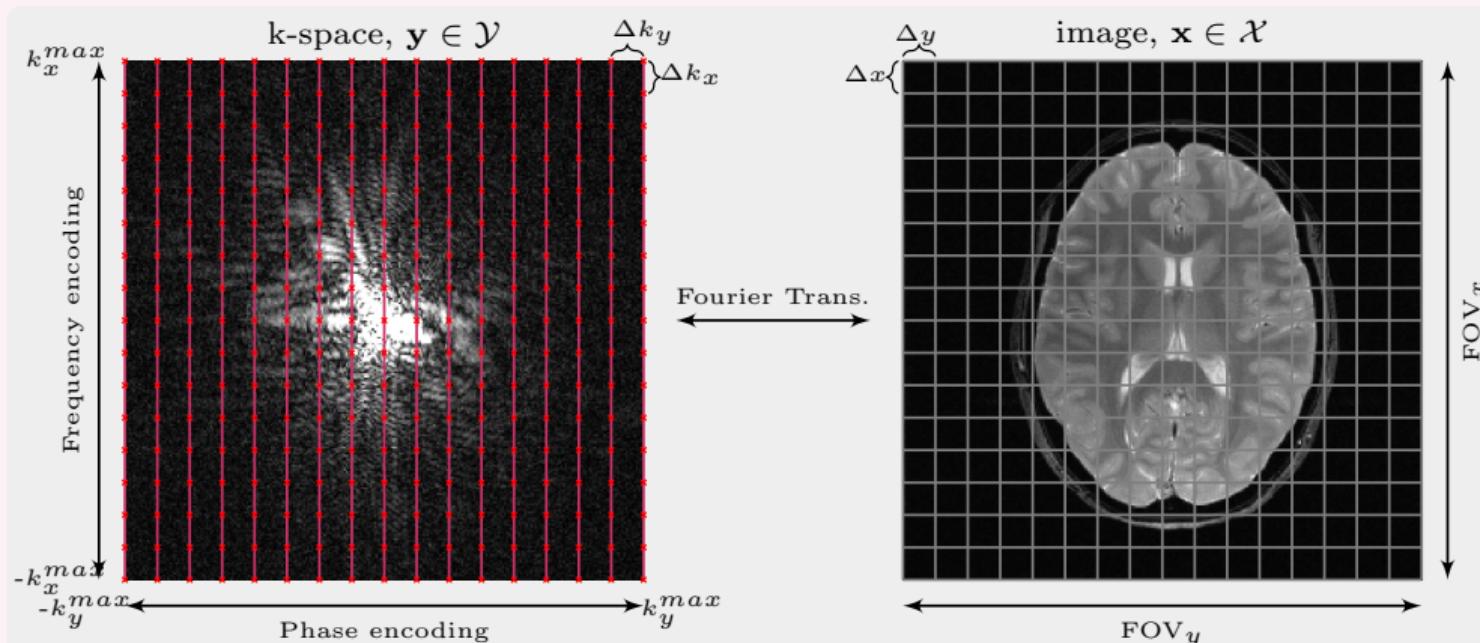
- non-invasive, radioactive-free
  - organs, tissues and skeletal system
  - multi-contrast, high-resolution
  - functional imaging
  - ...
  - **intrinsically slow**



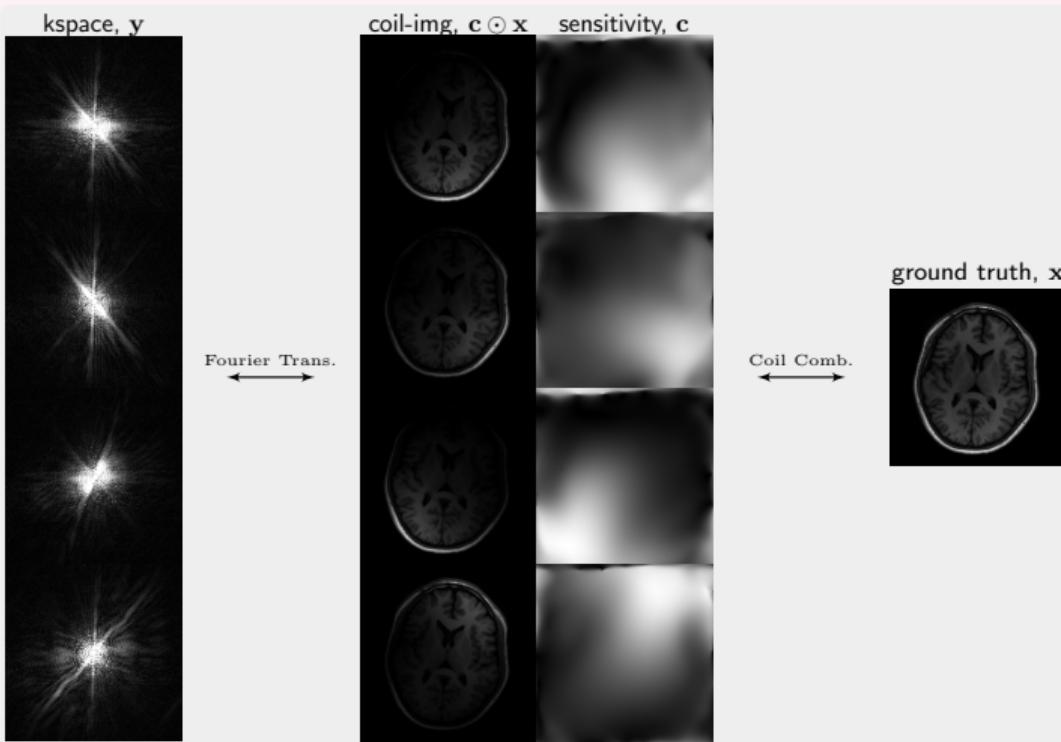
Created by DALL-E-3

## The acquisition in k-space, full-sampling

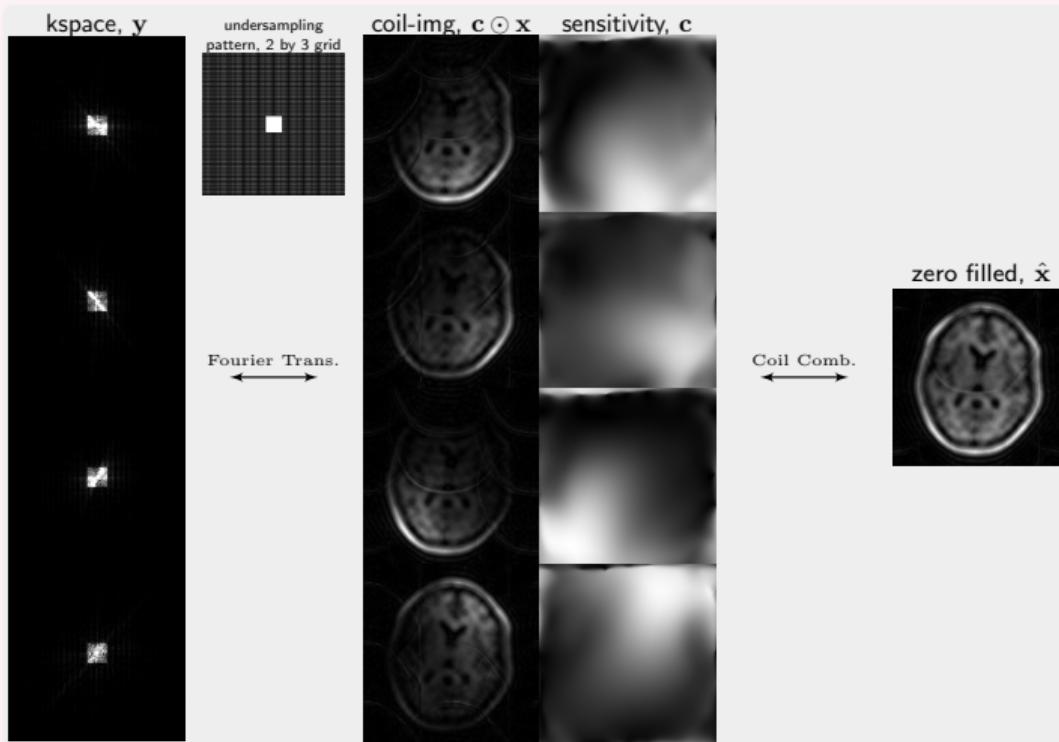
Nyquist Theorem at least requires:  $\text{FOV}_x = 1/\Delta k_x$ ,  $\Delta x = 1/(2 * k_x^{max})$ ,  $\text{FOV}_x = N * \Delta x$ .



### Parallel imaging, multiple coils imaging, full-sampling



## Parallel imaging, multiple coils imaging, under-sampling



## Linear and non-linear reconstruction

Parallel MRI is formulated as an inverse problem

$$(\mathcal{F}_S(\mathbf{x} \cdot \mathbf{c}_1), \dots, \mathcal{F}_S(\mathbf{x} \cdot \mathbf{c}_N)) = \mathbf{y}.$$

- $\mathcal{F}_S$ , undersampling Fourier operator.  $\mathbf{y} = (y_1, \dots, y_N)$ , measured k-space data
- $\mathbf{x}$ , image.  $N$ , number of coils.  $\mathbf{c} = (c_1, \dots, c_N)$ , coil sensitivities

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With pre-computed sensitivities, the forward operator is  $\mathcal{A} : \mathbf{x} \rightarrow \mathbf{y}$ . The reconstruction is achieved by minimizing

$$\min_{\mathbf{x}} \frac{1}{2} \|\mathcal{A}\mathbf{x} - \mathbf{y}\|^2 + \alpha R(\mathbf{x}).$$

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- Compressed sensing: L1 prior in wavelet domain  $R(\mathbf{x}) = \|\Phi\mathbf{x}\|_1$ , and random sampling pattern.
- Coil sensitivities estimation<sup>1</sup>
- Joint estimation of  $\mathbf{c}$  and  $\mathbf{x}$  via nonlinear inversion<sup>2</sup>

<sup>1</sup>Uecker et al. MRM 2014. <sup>2</sup>Uecker et al. MRM 2008.

## The application of deep learning in fast MRI

- Many unrolling neural networks<sup>1,2,3</sup> aim to predict the reconstruction from the undersampled k-space  $\mathbf{y}$ .
- A dataset  $D = \{(\mathbf{x}_i, \mathbf{y}_i) | i = 1, \dots, N\}$ , consisting of paired undersampled k-space data  $\mathbf{y}$  and ground truth images  $\mathbf{x}$ .
- The network  $F_\theta^\dagger$  parametrized by  $\theta$  is trained by minimizing the error

$$\hat{\theta} = \arg \min_{\theta} \mathbb{E}_D [\|F_\theta^\dagger(\mathbf{y}) - \mathbf{x}\|_2^2] . \quad (1)$$

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- The network depends on the pre-definition of the forward operator  $\mathcal{A}$

<sup>1</sup>Yang et al. NIPS 2016. <sup>2</sup>Hammernik et al. MRM 2018. <sup>3</sup>Aggarwal et al. TMI(2019).

## Challenges 1: Generalizability

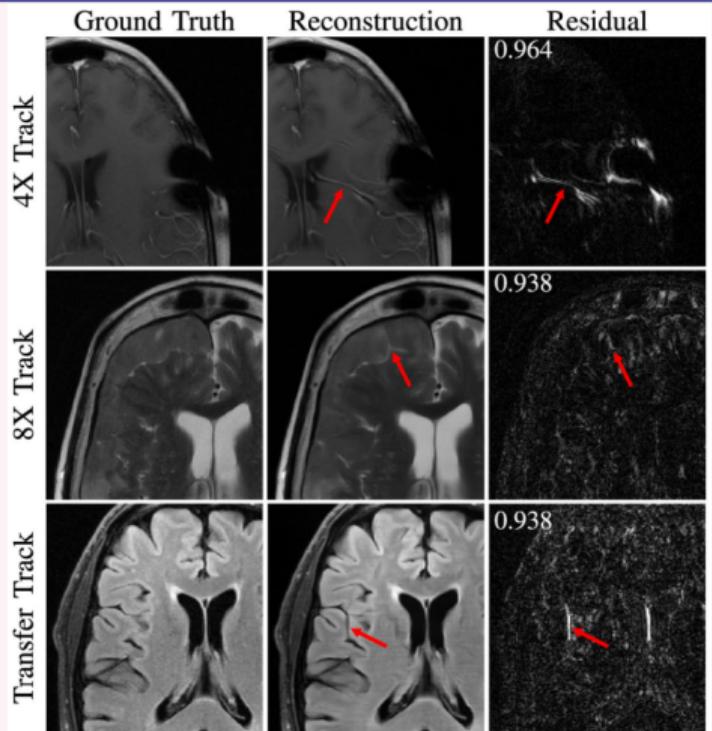
Study<sup>1</sup> reported that tiny perturbations, contrast changes, and sampling pattern variations can fail most unrolling neural networks  $F_{\theta}^{\dagger}$ .

<sup>1</sup>Knoll et al. MRM 2019.

## Challenges 2: Hallucination

Studies<sup>1,2</sup> reported hallucinations generated by an unrolling neural network  $F_\theta^\dagger$ .

- (top) a false-generated vessel
- (middle) a linear bright signal mimicking a cleft of cerebrospinal fluid
- (bottom) a false-generated sulcus or prominent vessel.



<sup>1</sup>Muckley et al. IEEE TMI 2021. <sup>2</sup>Bhadra et al. IEEE TMI 2021.

## Challenges 3: Data availability

- FastMRI<sup>1</sup> includes full-sampled k-space data and four types of contrast. Limited organ types
- but to collect more is expensive and takes time

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<sup>1</sup>Zbontar et al. Radiology 2020.

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- **Bayesian MRI reconstruction using diffusion priors**
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# Reconstruction from Bayesian Perspective

- **posterior:** the image  $\mathbf{x}$  conditioned on the acquired k-space data  $\mathbf{y}$  is

$$p(\mathbf{x}|\mathbf{y})_{\text{posterior}} = \frac{p(\mathbf{y} \mid \mathbf{x})p(\mathbf{x})}{p(\mathbf{y})} \propto p(\mathbf{y} \mid \mathbf{x})_{\text{likelihood}} \cdot p(\mathbf{x})_{\text{prior}}, \quad (2)$$

- **likelihood:**  $p(\mathbf{y} \mid \mathbf{x}) \rightarrow$  the probability of the acquired k-space data  $\mathbf{y}$  for a given image  $\mathbf{x}$
  - **prior:**  $p(\mathbf{x}) \rightarrow$  the prior model of image  $\mathbf{x}$

## The likelihood for k-space

- The likelihood  $p(\mathbf{y}|\mathbf{x})$  for observing the  $\mathbf{y}$  determined by  $\mathbf{y} = \mathcal{A}\mathbf{x} + \boldsymbol{\eta}$
- The noise  $\boldsymbol{\eta}$  is zero mean and normal distributed with covariance matrix  $\sigma_{\boldsymbol{\eta}}^2 \mathbf{I}$

$$p(\mathbf{y}|\mathbf{x}) = \mathcal{CN}(\mathbf{y}; \mathcal{A}\mathbf{x}, \sigma_{\boldsymbol{\eta}}^2 \mathbf{I}). \quad (3)$$

then, we have

$$p(\mathbf{x}|\mathbf{y}) \propto \frac{1}{\sqrt{2\pi\sigma_{\boldsymbol{\eta}}^2}} \exp\left(-\frac{1}{2\sigma_{\boldsymbol{\eta}}^2} \|\mathbf{y} - \mathcal{A}\mathbf{x}\|_2^2\right) \cdot p(\mathbf{x}) \quad (4)$$

$$\log p(\mathbf{x}|\mathbf{y}) \propto -\frac{1}{2\sigma_{\boldsymbol{\eta}}^2} \|\mathbf{y} - \mathcal{A}\mathbf{x}\|_2^2 + \underbrace{\log p(\mathbf{x})}_{\text{prior}} \quad (5)$$

## Non-learned and learned priors

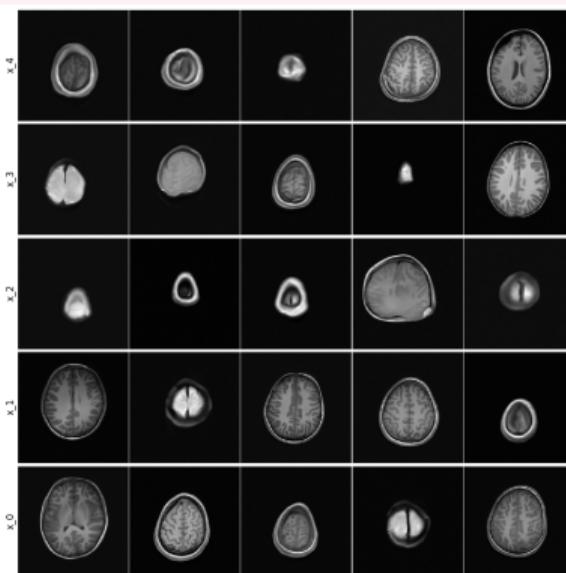
- Laplace distribution,  $p(\mathbf{x} | \mu, b) = \frac{1}{2b} \exp\left(-\frac{|\mathbf{x}-\mu|}{b}\right)$ , can promote sparsity in the wavelet domain. (Compressed sensing)
- We can learn an empirical prior  $p_{\theta}(\mathbf{x})$  from the i.i.d. samples  $D_n = \{\mathbf{x}^1, \dots, \mathbf{x}^{(n)}\}$ , using generative modeling.

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- **Generative priors for MRI reconstruction**

## Generative model/prior

A generative model is a statistical model  $p_{\theta}(X = \mathbf{x})$  that is used to approximate distribution of a high-dimensional random variable  $p_D(X = \mathbf{x})$  from an observed dataset  $D$ .



## MAP and MMSE

With the posterior  $p(\mathbf{x}|\mathbf{y})$ , the reconstruction can be achieved by

- Maximize posterior (**MAP**)

$$\hat{\mathbf{x}}_{\text{MAP}}(\mathbf{y}) = \arg \max_{\mathbf{x}} \log p(\mathbf{x}|\mathbf{y})$$

- Minimum mean square error (**MMSE**)

$$\hat{\mathbf{x}} = \arg \min_{\tilde{\mathbf{x}}} \int \|\tilde{\mathbf{x}} - \mathbf{x}\|^2 p(\mathbf{x}|\mathbf{y}) d\mathbf{x} = \mathbb{E}[x|y]$$

- Explore the posterior to estimate the uncertainty of reconstruction

## Sample the posterior

- We generate samples from  $p(\mathbf{x}|\mathbf{y})^{1,2,3}$ , using **Langevin dynamics**

$$\mathbf{x}^{k+1} \leftarrow \mathbf{x}^k + \frac{\gamma}{2} \nabla_{\mathbf{x}} \log p_{\theta}(\mathbf{x}^k | \mathbf{y}) + \sqrt{\gamma} \mathbf{z}, \quad z \sim \mathcal{CN}(0, \mathbf{I}). \quad (6)$$

## Sample the posterior

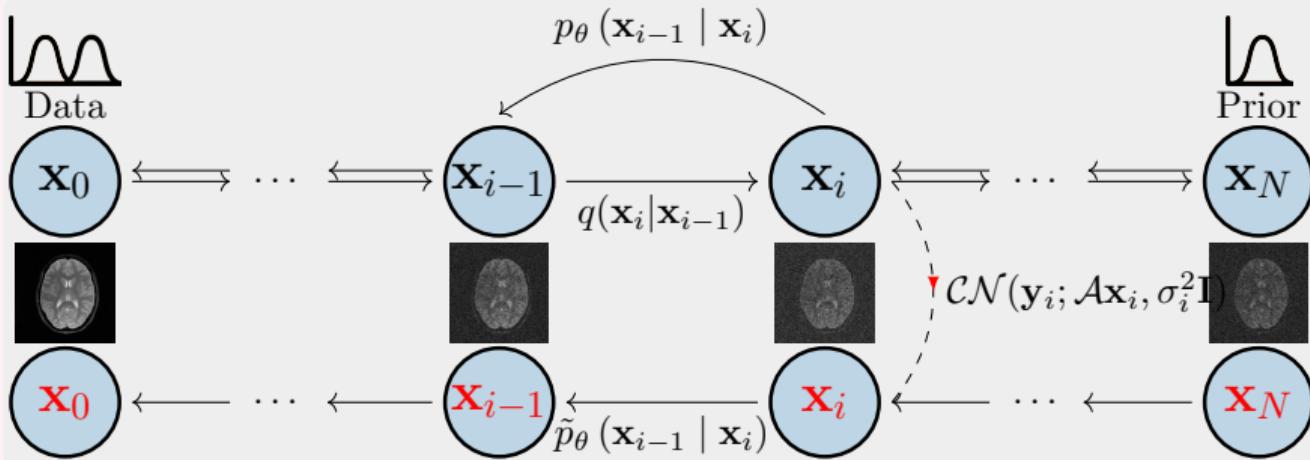
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- The variance computed pixelwise over samples is an indicator of **uncertainty**.
- The mean computed pixelwise over samples is the estimation for  $\mathbf{x}_{\text{MMSE}}$ .
- **The prior distribution of image is difficult to determine.**

<sup>1</sup>Jalal et al. NIPS (2021). <sup>2</sup>Chung et al. MIA (2022). <sup>3</sup>Luo et al. ISMRM (2022).

## Denoising score matching



- $\mathbf{x}_i = \mathbf{x}_0 + \sigma_i \mathbf{z}$ . Learn **score function**  $\nabla_{\mathbf{x}_i} \log p_\theta^{\text{data}}(\mathbf{x}_i)$  at different noise scales  $\sigma_i$ , using a deep neural network<sup>1,2,3</sup>
- Incorporate the k-space likelihood term

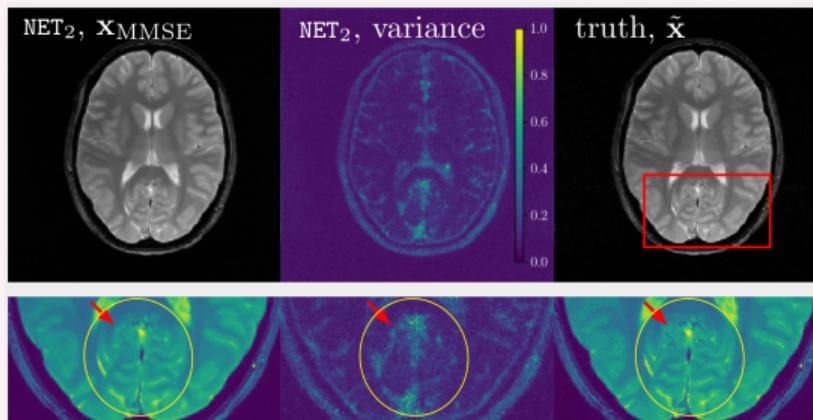
<sup>1</sup>Sohl-Dickstein et al. ICML (2015). <sup>2</sup>Song et al. NIPS (2019). <sup>3</sup>Song et al. ICLR (2021).

## Experiments

- $\mathbf{x}_{\text{MMSE}}$  and variance
- Uncertainty estimation
- Transferability of a learned prior

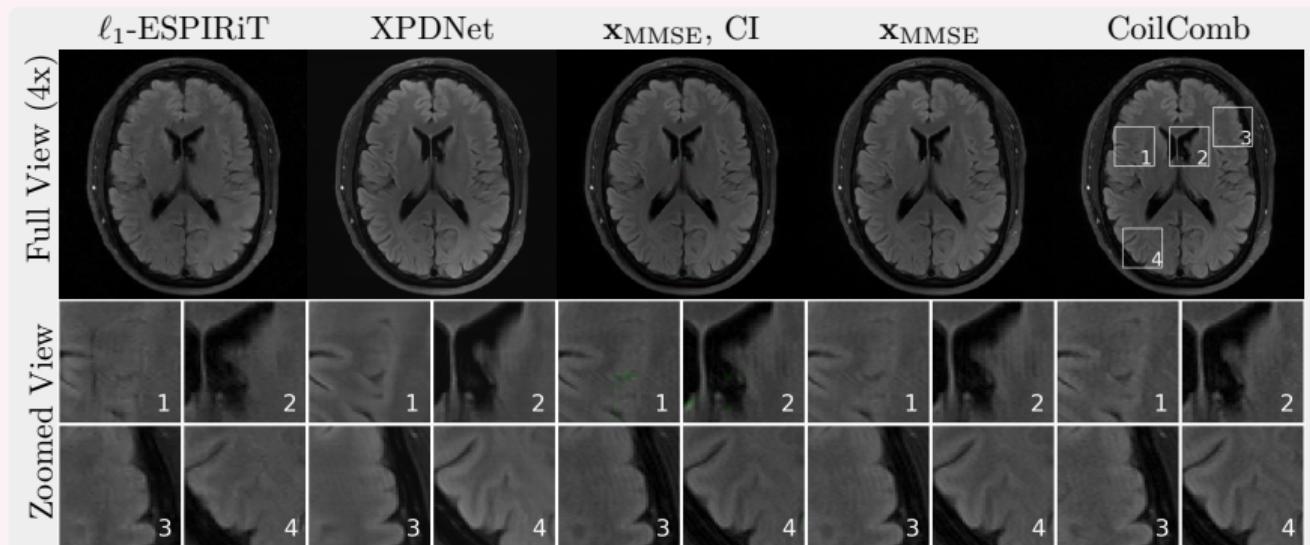
## MMSE and variance map

- The undersampling pattern covers 11.8% k-space using variable density poisson disc. The central  $20 \times 20$  region is fully acquired.
- 10 images were drawn from  $p(\mathbf{x}|\mathbf{y})$ .



## Uncertainty and hallucination

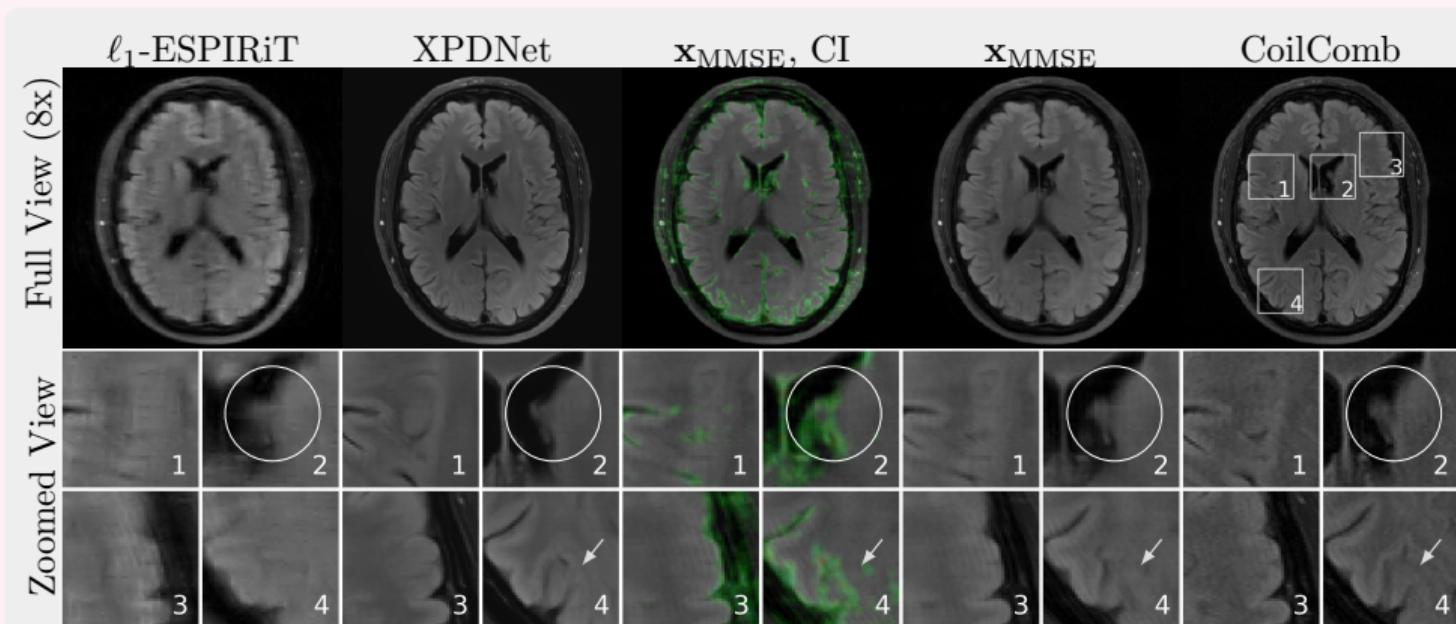
- 4x undersampling along phase direction in k-space.
- Reconstructions are  $\ell_1$ -wavelet, XPDNet<sup>1</sup>,  $\mathbf{x}_{\text{MMSE}}$  highlighted with thresholding credibility interval (CI),  $\mathbf{x}_{\text{MMSE}}$  and a fully-sampled coil-combined image (ground truth).



<sup>1</sup>Ramzi et al. arXiv (2020)

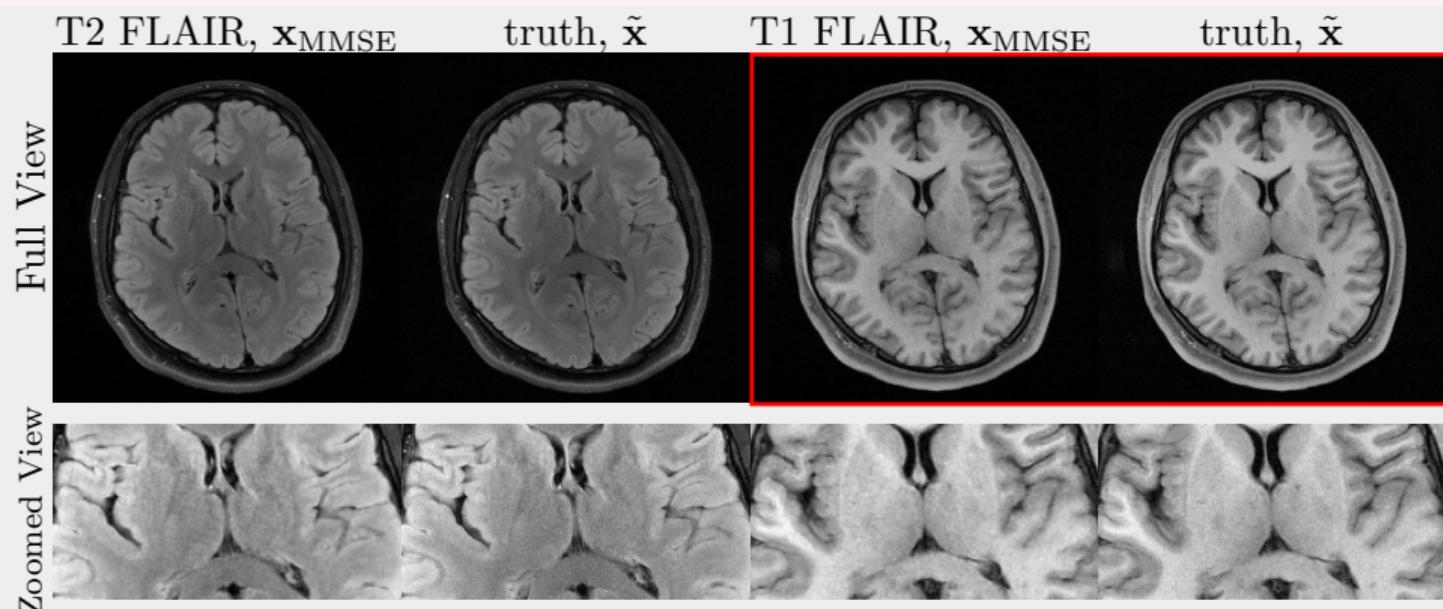
## Uncertainty and hallucination

- Hallucinations appear when using **8-fold** acceleration and are highlighted with the CI after thresholding. Selected regions of interests are presented in a zoomed view.



## Transferability of learned information

- Reconstruction of T2 and T1 FLAIR images with a prior trained on T2 FLAIR.
- Use a Poisson-disk pattern of 8.2x undersampling in k-space, 320x320.



## Recap 1

- We combine concepts from machine learning, Bayesian inference and image reconstruction.
- The image reconstruction is realized by drawing samples from the posterior term  $p(\mathbf{x}|\mathbf{y})$  using a learned prior.
- This method provides a minimum mean square reconstruction and uncertainty estimation.
- This method shows good performance and transferability to different contrasts and sampling patterns.

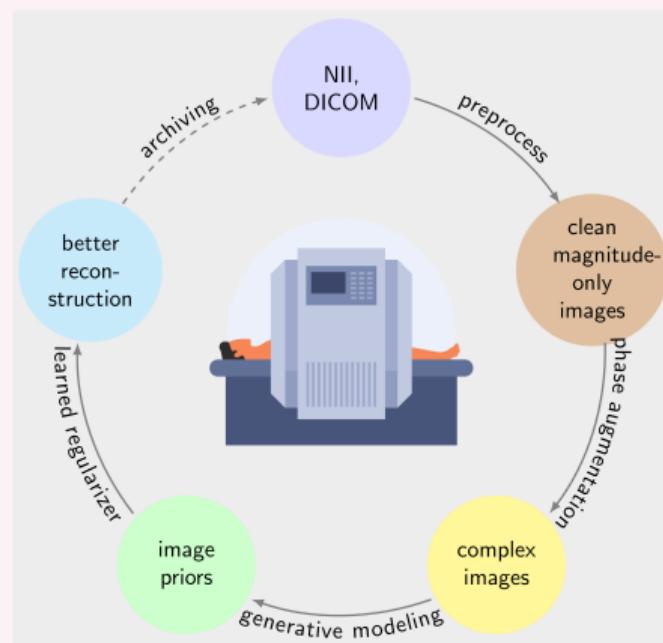
[1] G. Luo, M. Blumenthal, M. Heide, M. Uecker. “Bayesian MRI reconstruction with joint uncertainty estimation using diffusion models”. Magn Reson Med. March 2023;1-17. DOI: 10.1002/mrm.29624

## Outline

- MRI reconstruction as inverse problems
- Bayesian MRI reconstruction using diffusion priors
- **Phase augmentation for training priors**
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## Construct dataset from magnitude-only images

- MR images are complex-valued
- Easy to get magnitude-only images in hospital
- Especially pathological data



## Phase augmentation using Langevin sampling

- **prior:** a prior for complex-valued images  $p(\mathbf{x})$ .
- **likelihood:** the likelihood term of the magnitude

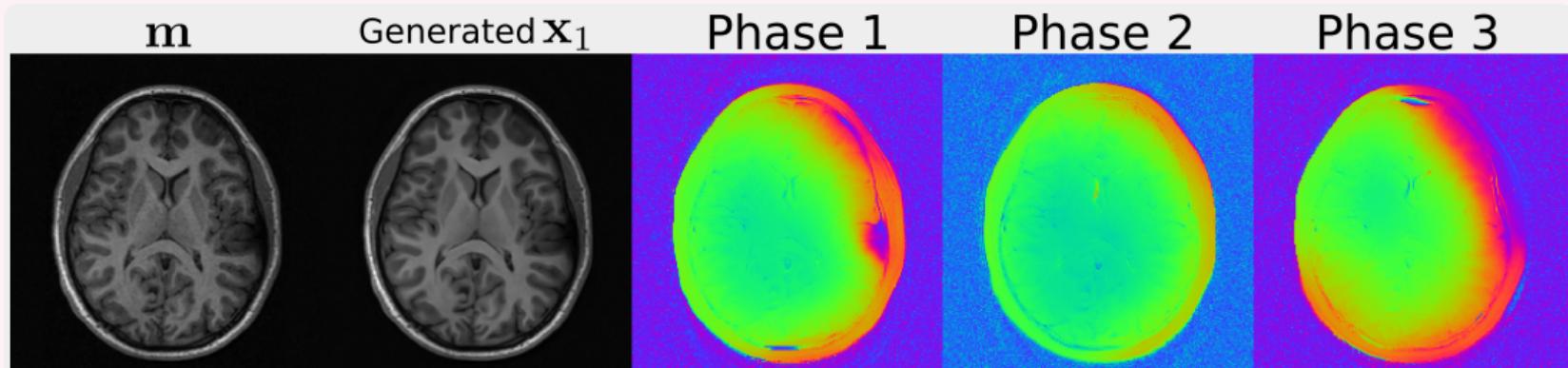
$$p(\mathbf{m}|\mathbf{x}) \propto \exp\left(-\epsilon \|\mathbf{m} - \sqrt{\mathbf{x}_r^2 + \mathbf{x}_i^2}\|_2^2\right).$$

- **posterior:** the complex image is proportional to

$$p(\mathbf{x}|\mathbf{m}) \propto p(\mathbf{x}) \cdot p(\mathbf{m}|\mathbf{x}) .$$

## Examples of generated phase maps

- The magnitude and its corresponding phase maps of the generated complex-valued images



## MRI Image Priors

When training a prior,

- what is the effect of phase?
- how does the dataset size affect the prior?

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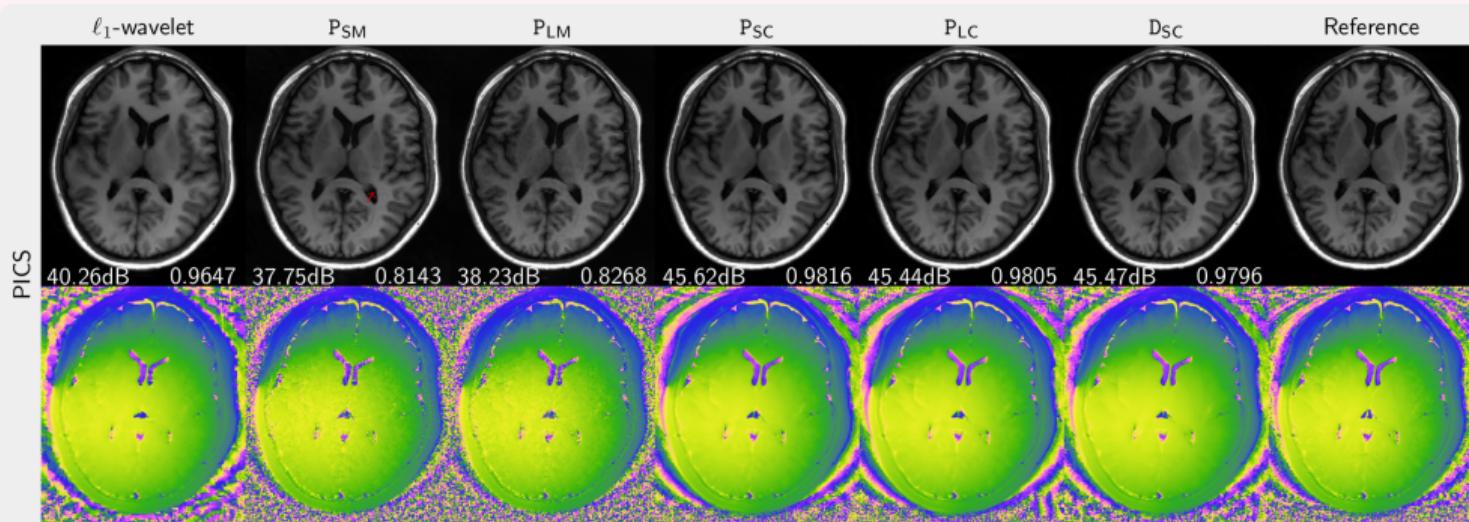
**Table:** Datasets and computational resources used to train the six different priors used in this work.

Prior		Model	Phase	Nr. of Images	MR Contrasts	GPUs	Parameters	Time × epochs
P <sub>SC</sub>	(small, complex)	PixelCNN <sup>1</sup>	preserved	1000	T <sub>1</sub> , T <sub>2</sub> , T <sub>2</sub> -FLAIR, T <sub>2</sub> <sup>*</sup>	4×A100, 80G	~22M	~40s × 500
P <sub>SM</sub>	(small, magnitude)	PixelCNN	not available	1000	T <sub>1</sub> , T <sub>2</sub> , T <sub>2</sub> -FLAIR, T <sub>2</sub> <sup>*</sup>	4×V100, 32G	~22M	~144s × 500
P <sub>LM</sub>	(large, magnitude)	PixelCNN	not available	23078	MPRAGE	4×A100, 80G	~22M	~748s × 100
P <sub>LC</sub>	(large, complex)	PixelCNN	generated	23078	MPRAGE	3×A100, 80G	~22M	~1058s × 100
D <sub>SC</sub>	(SMLD, complex)	Diffusion	generated	79058	MPRAGE	4×A100, 80G	~8M	~2330s × 50
D <sub>PC</sub>	(DDPM, complex)	Diffusion	generated	79058	MPRAGE	8×V100, 32G	~8M	~1430s × 200

<sup>1</sup>Salimans et al. ICLR 2016.

## Influence of phase maps

learned priors	P <sub>SC</sub>	P <sub>SM</sub>	P <sub>LM</sub>	P <sub>LC</sub>	D <sub>SC</sub>
Phase	preserved	not available	not available	generated	generated
Nr. Images	1000	1000	23078	23078	79058

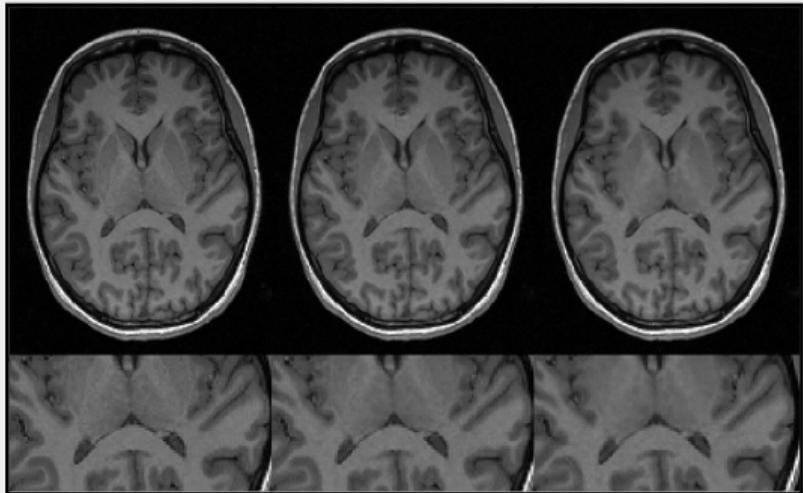


### 8.2x undersampling in k-space with Poisson disc

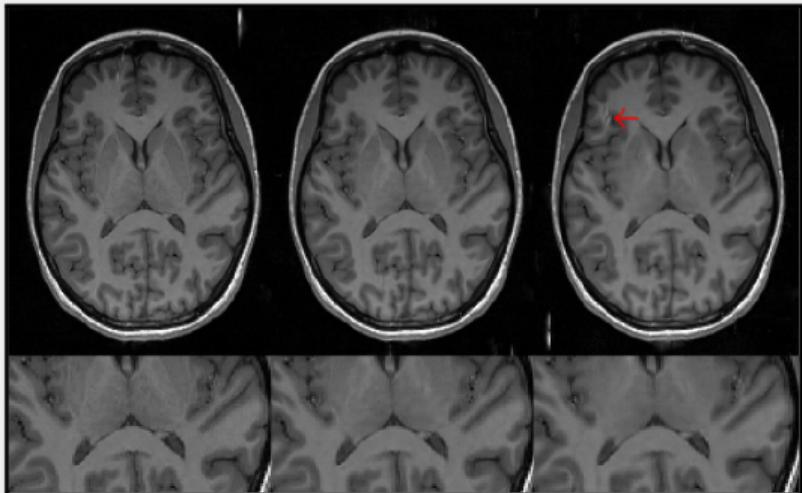
## Small prior Psc vs Large prior PLC

- MPRAGE (Magnetization Prepared-RApid Gradient Echo) sequence
- 2-fold acceleration with grappa and 4/5 partial parallel Fourier imaging
- Dimensions (256, 256, 224) and voxel sizes (1mm, 1mm, 1mm).

$P_{LC}$  (x2, x4, x6)

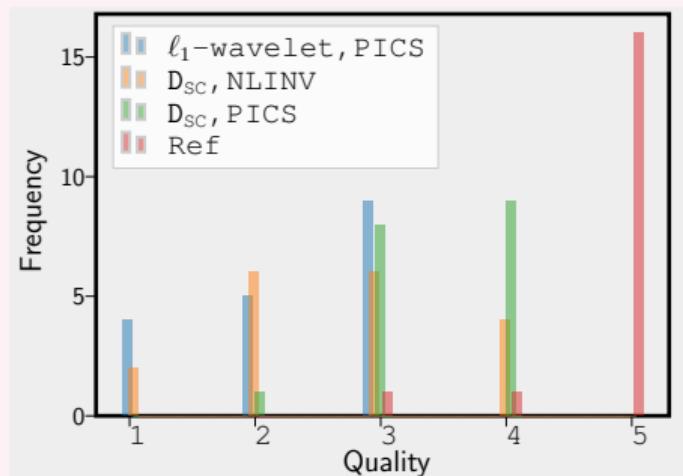


$P_{SC}$  (x2, x4, x6)

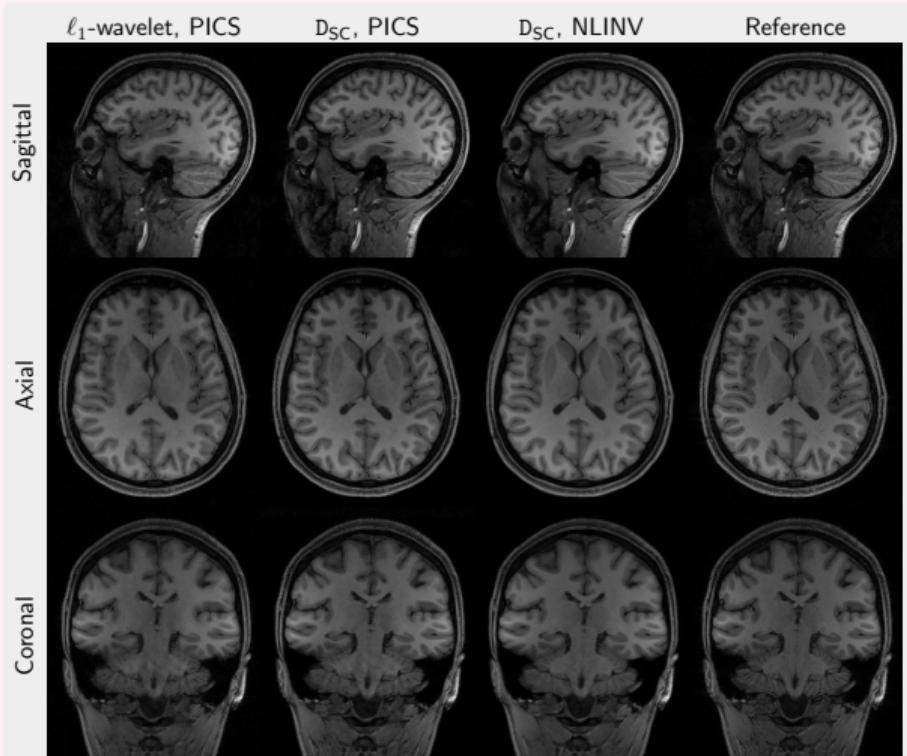


## Clinical results

- 5-Excellent, 1-Bad
- 3D acquisition using Turbo-FLASH
- 8.2x undersampling



Collaborative work with Prof. Niels Focke at UMG



## Recap 2

- We demonstrate that priors trained on complex images are superior to priors trained on magnitude-only images
- We leverage a diffusion model trained on a small dataset of complex images to augment a much larger dataset
- We integrate priors at regularization term into reconstruction.

G. Luo, X. Wang, M. Blumenthal, M. Schilling, EHU Rauf, R. Kotikalapudi, N. K. Focke, M. Uecker. “Generative Image Priors for MRI Reconstruction Constructed from Magnitude-Only Images Using Phase Augmentation” arXiv: 2308.02340

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# A tool to develop priors: spreco

- **spreco**<sup>1</sup>
- **image-priors**<sup>2</sup>

### Navigation

- [Project description](#)
- [Release history](#)
- [Download files](#)

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### Project links

- [Bug Tracker](#)
- [Homepage](#)

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### Statistics

GitHub statistics:

- [Stars: 15](#)
- [Forks: 1](#)
- [Open issues: 0](#)
- [Open PRs: 0](#)

View statistics for this project via [Libraries.io](#), or by using [our public dataset on Google BigQuery](#)

### Project description

#### Speed up MR scanner with generative priors for image reconstruction (SPRECO)

This package is to help you train generative image priors of workflow MRI images and then use them in image reconstruction. It has the following features:

1. Distributed training
2. Interruptible training
3. Efficient dataloader for medical images
4. Customizable with a configuration file
5. Seamless deployment with [BART](#)

**Installation:** Clone this repository and use [conda](#) to set up the environment.

```
$ git clone https://github.com/mrirecon/spreco.git
$ cd spreco
$ pip install .
```

### Reference

We would appreciate it if you tried our codes and cited our work.

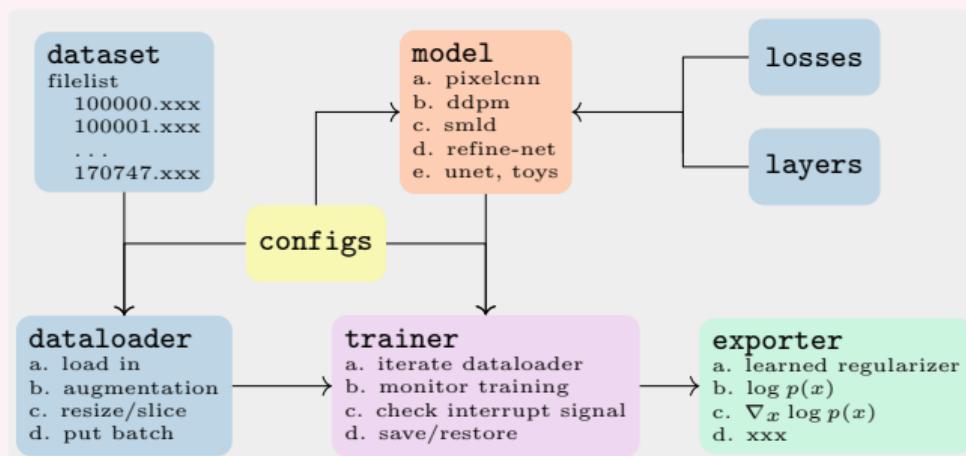
[1] G. Luo, X. Wang, M. Blumenthal, M. Schilling, EHU. Rauf, R. Kotikalapudi, NK. Focke, M. Uecker. Generative image priors for MRI reconstruction trained from magnitude-only images. arXiv preprint arXiv:2308.02340 (2023)

[2] G. Luo, M. Blumenthal, M. Heide, M. Uecker. Bayesian MRI reconstruction with joint uncertainty estimation using diffusion models. Magn Reson Med. 2023; 1-17

<sup>1</sup><https://github.com/mrirecon/spreco>   <sup>2</sup><https://github.com/mrirecon/image-priors>

## Overview

- The loss and layer functions are used to create models.
- The trainer is fed by the dataloader and trains the model on multiple GPUs.
- The exporter is used to customize trained models for deployment.



It is implemented with : 1) tensorflow; 2) numpy; 3) pyyaml; 4)matplotlib;  
5)scikit-image; 6) tqdm.

## Hands-on

1. Generative prior trained on a large public dataset

 Open in Colab

2. Sample the posterior

 Open in Colab

3. Train an image prior

 Open in Colab

4. Using Prior with BART

 Open in Colab

Background  
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Challenges  
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Bayesian MRI  
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Phase augmentation  
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SPRECO  
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Summary  
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## Summary

- What are the advantages of a prior?

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- How to use a prior for reconstruction?

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- What are the advantages of a prior? Decoupled from the forward operator, which leads to better generalizability to many acquisition scenarios.
- How to use a prior for reconstruction? Sample the posterior or use it as regularization term, which permits uncertainty estimation and flexibility.
- How to get more data for training?

## Summary

- What are the advantages of a prior? Decoupled from the forward operator, which leads to better generalizability to many acquisition scenarios.
  - How to use a prior for reconstruction? Sample the posterior or use it as regularization term, which permits uncertainty estimation and flexibility.
  - How to get more data for training? Phase augmentation of a large magnitude-only dataset using diffusion priors trained on small complex-valued images.
  - How to develop and deploy a prior? spreco

## Outlook

- How to determine the performance bound of a prior?
- How does it relate to the uncertainties of the reconstructed image?
- How many samples need to be acquired to keep the uncertainty within an acceptable limit?
- How effectively can the prior information transfer to a broader range of scenarios?

**Background**  
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**Challenges**  
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**Bayesian MRI**  
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**Phase augmentation**  
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**SPRECO**  
○○○○

**Summary**  
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a figure for outlook a backup for pics and nlinv, phase maps

## Q&A

## Algorithm

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### Algorithm 1 Sampling the posterior via Markov chain Monte Carlo

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- 1: Give the acquired k-space  $\mathbf{y}_0$
  - 2: Construct the forward operator  $\mathcal{A}$  with sampling pattern  $\mathcal{P}$  and coil sensitivities  $\mathcal{S}$
  - 3: Set the Langevin steps K, Langevin step coefficient  $\beta$ , for the tuning factor  $\lambda$ , the start noise level index N.
  - 4: Initialize  $\mathbf{x}_N^0$  with samples from a uniform distribution  $\mathcal{U}_{[-1,1]}$
  - 5: **for**  $i$  in  $\{N - 1, \dots, 0\}$  **do**
  - 5.1: Draw samples from  $\tilde{p}(\mathbf{x}_i | \mathbf{x}_{i+1})$  by running K Langevin steps.
  - 6: **end for**
-

## Network, Data, Training

- Refine-Net. 1. conditional instance normalization layer 2. Gaussian position embedding.
- Self-attention modules are added to model high resolution images (320x320).
- Small dataset (1300 images) is used for development. FastMRI dataset is used for performance evaluation.
- $\text{NET}_1$  and  $\text{NET}_2$  are trained with small dataset.  $\text{NET}_3$  is trained with fastMRI dataset
- $\text{NET}_1$  uses conditional instance normalization layer (10).  $\text{NET}_2$  and  $\text{NET}_3$  use Gaussian position embedding.
- MCMC sampling algorithm is implemented with TensorFlow and Numpy.

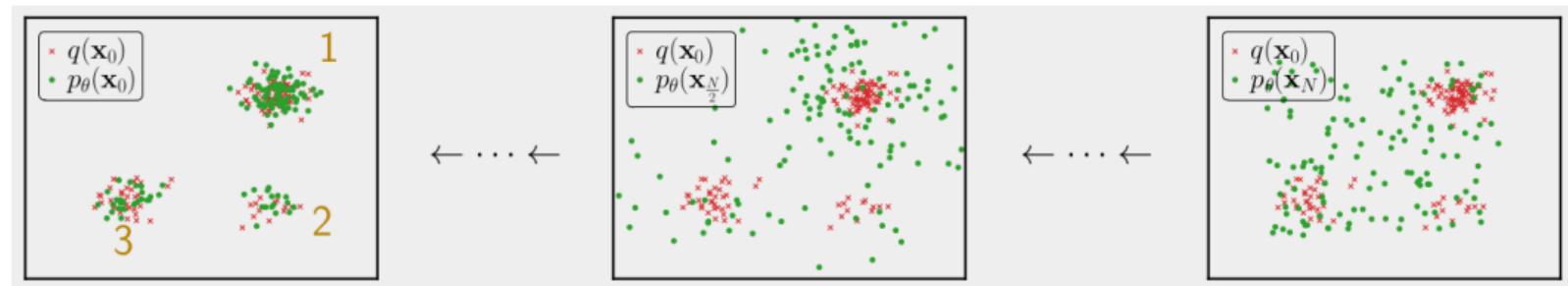
## Generate samples

For a multi-modal distribution, use annealed Langevin dynamics. For  $\sigma_i$  in  $\{\sigma_N, \dots, \sigma_1\}$ ,

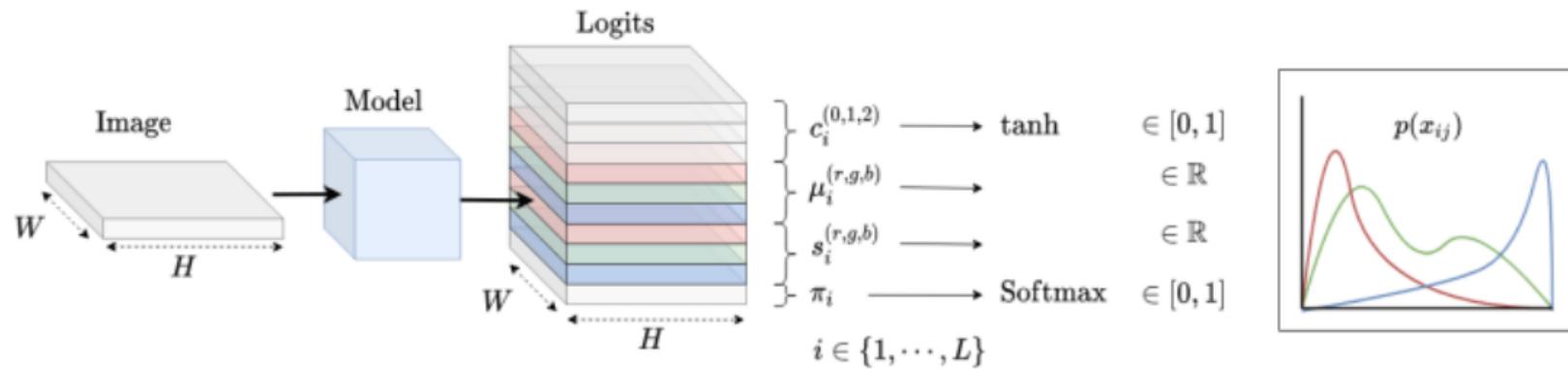
$$\mathbf{x}_i^k = \mathbf{x}_i^{k-1} - \frac{\lambda}{2} \nabla_{\mathbf{x}} \log p(\mathbf{x}_i^{k-1}) + \omega_i^k$$

A mixture of bivariate Gaussian as prior distribution

- $p(\mathbf{x}) = \sum_{i=1}^K \phi_i \mathcal{CN}(\boldsymbol{\mu}_i, \boldsymbol{\Sigma}_i)$
- $\phi_i$  is the mixture indicator,  $\phi_i = \{0.7, 0.2, 0.1\}$
- $\boldsymbol{\mu}_i = \{[5, 5], [-5, -5], [5, -5]\}$ ,  $\boldsymbol{\Sigma}_i = \mathbf{I}$ .



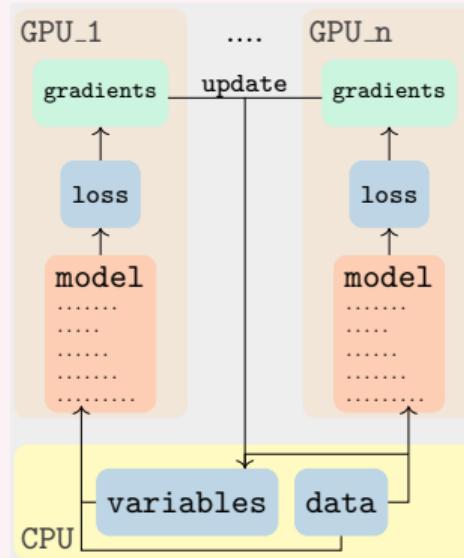
# PixelCNN



- $\pi_i \in (0, 1)$  the probability of each logistic distribution
- $\mu_{i,j} \in (-\infty, \infty)$  the centre of the  $i$ th logistic distribution for the  $j$ th color
- $s_{i,j} \in (0, +\infty)$  the width of the  $i$ th logistic distribution for the  $j$ th color
- $c_{k,i}$  the  $k = 0, 1, 2$  linear coefficients for  $r \rightarrow g$  and  $r, g \rightarrow b$ , for each logistic function  $i$

# Spreco

- The trainable variables are shared across GPUs
- Dataloader feeds data to every GPU worker.
- Config file for training models



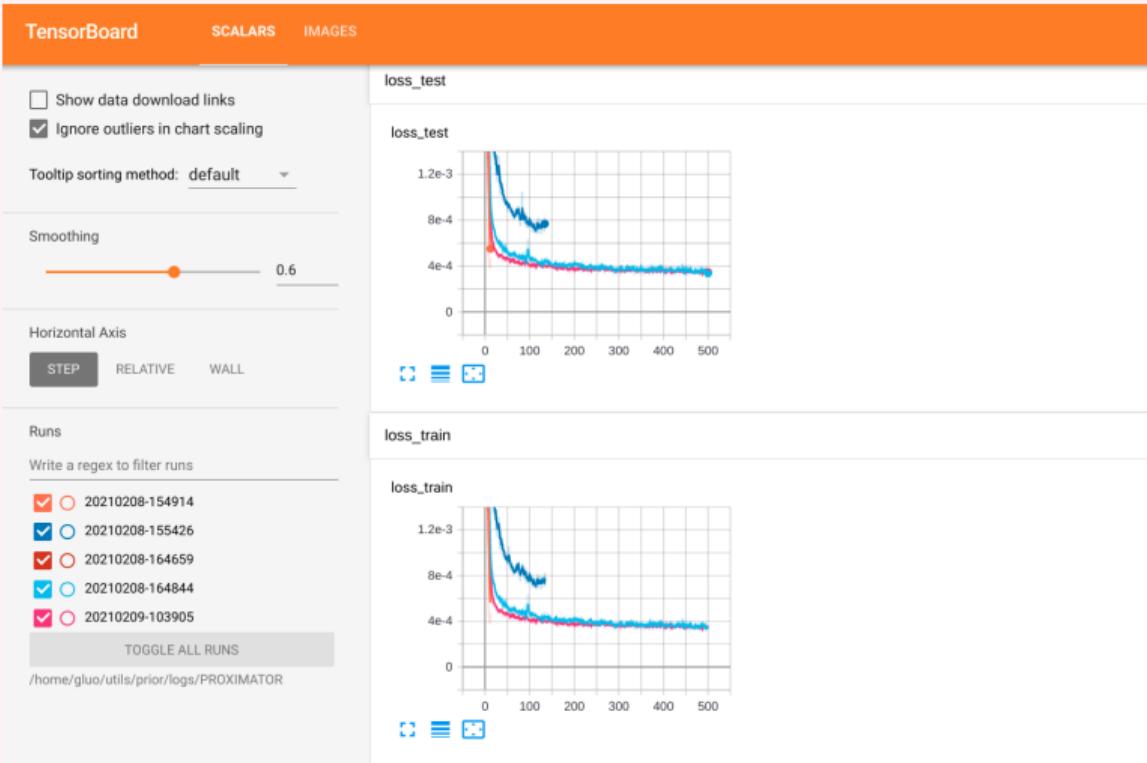
```

smld.yaml
1## SMLD
2model: "SDE"
3batch_size: 15
4input_shape: [256, 256, 2]
5data_chns: 'CPLX'
6
7sigma_max: 5,
8sigma_min: 0.0085
9reduce_mean: True
10
11lr_warm_up_steps: 100
12lr_start: 0.0001
13lr_min: 0.0003
14lr_max: 0.0005
15lr_max_decay_steps: 200
16
17seed: 1234
18net: 'refine'
19body: small
20nr_filters: 64
21nonlinearity: 'elu'
22fourier_scale: 16
23affine_X: False
24attention: True
25
26max_keep: 100
27max_epochs: 2000
28save_interval: 50
29saved_name: smld
30log_folder: /home/gluo/workspace/nlinv_prior/logs
31#restore_path: /home/gluo/workspace/nlinv_prior/logs/20230410-093726/sd6_hku_1000
32num_thread: 30
33print_loss: true
34train_list: /home/gluo/workspace/nlinv_prior/data/hku/hku_train
35test_list: /home/gluo/workspace/nlinv_prior/data/hku/hku_test
36
37nr_gpu: 2
38gpu_id: '1,2'

```

YAML Tab Width: 8 Ln 28, Col 18 INS

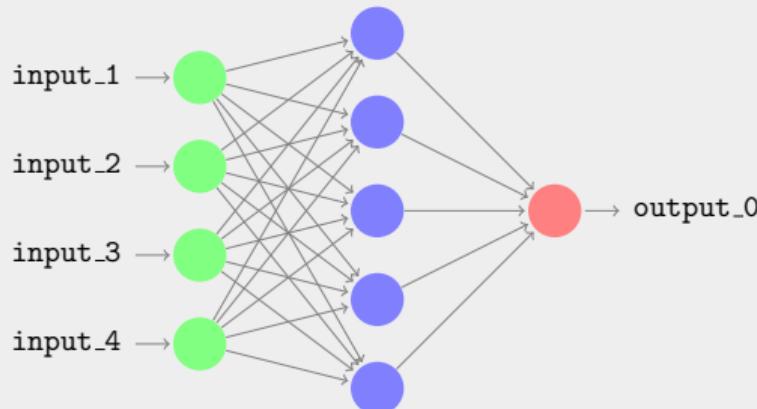
# Spreco



## BART with tensorflow computation graph

- Export the trained model with customized labels for inputs  $\mathbf{x}$  and outputs  $\mathbf{y} = \text{Net}_\theta(\mathbf{x})$ ;
- Initialize an exported graph, the restoration of a saved model using C API;
- Wrap the exported computation graph into BART's non-linear operator (nlop)

(a) Export the trained model as computation graph



(b) Use the graph as regularization in BART

```
$ bart pics -R TF:<graph_path>:λ <kspace> <coils> <reco>
```