Tutorial 5-6 (final project) Model calibration, ESILV, 2024-2025

Work to be submitted before 8pm on Thursday, January 23. To be done in groups of two students (with the possibility of having a group of 1 if there is an odd number of students, but not a group of 3). You will send by email a single pdf file of **20 pages maximum (appendices included)** containing your answers to each question, with the graphs that you have generated and commented on, as well as the **code appended to the pdf** (a pdf generated in LaTeX will be appreciated positively). Analyze and comment on your results for each question.

Any plagiarism, use of LLM or any software automatically generating text, code or images is dishonest for an academic work and is therefore strictly prohibited. Any violation of this rule for any part of your work may result in a grade of 0 for the entire project (and possibly a disciplinary council). The use of libraries answering the questions of this project is also prohibited.

For the entire project, we are given call prices of one-year maturity, all defined on the same underlying whose spot price is 100€:

Strike	95€	96€	97€	98€	99€	100€	101€	102€	103€	104€
Price	10.93€	9.55€	8.28€	7.40€	6.86€	6.58€	6.52€	6.49€	6.47€	6.46€

If needed, call prices of other maturities will be assumed to be such that their implied volatility is the same as the one of the call of same strike in the above table. The risk-free rate is 0.2%.

I – Local volatility

1/ Using the above dataset, calibrate the implied Black-Scholes volatility for all the options.

2/ Estimate successively the SVI and the generalized SVI models to depict in average the volatility skew.

3/ For a time of 1 year, provide the local volatility (since we freeze the time, the local volatility is only a function of the underlying price) using Dupire's approach applied to a fine grid of prices (step of 0.01€ for prices between 90 and 110) consistent with the volatility skew estimated in the previous question.

4/ Calibrate (and not estimate) the SVI model only on the three options the closest to the money.

5/ Calculate in the same grid as in question 3 Dupire's local volatility using this new SVI model. Quantify and evaluate the differences between the local volatilities in question 3 and question 5.

6/ We want to stress successively upward and downward the option price of strike 100€ (other prices remain identical to what they are in the provided dataset). Determine the maximal stress for this price so that all the one-year option prices are arbitrage-free. For such a new price, show precisely how both SVI and Dupire's models of questions 4 and 5 are changed.

II – Stochastic volatility

7/ Using the above dataset, estimate the Heston model explaining in average the price of all the options (you must be able first to price an option with this model).

8/ Calibrate (and not estimate) Heston model only on the three options the closest to the money.

9/ Applying successively the same two modifications of the at-the-money call price as in question 6, show precisely how the parameters of Heston model are affected.

10/ Using the full one-year dataset provided at the beginning of the project, estimate the fractional Heston model (the variance process is led by an fBm and, if needed, you can consider it to be independent of the Brownian motion leading the price) and compare its accuracy with the one of the standard Heston model estimated in question 7.