Communication Mathematics

Chapter 11: Multiple Antenna Systems

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Preview

Preview

- In this chapter, we will discuss the technology of two or more transmit or receive antennas.
- The system using several antennas for signal transmission is called the **multiple-input, multiple-output (MIMO)** system.
- The channel model used for analyzing MIMO system is called the MIMO channel.

Channels Models for Multiple Antenna Systems

Notations

- n_t is the number of transmit antennas
- n_r is the number of receive antennas
- $h_{mn}(t)$ is the impulse response between the n-th transmit antenna and the m-th receive antenna.
  - This is a time invariant channel.

Channel Model

- Let $s_n(t)$ is the signal transmitted from the n-th transmit antenna for $n = 1, 2, \ldots n_t$.
- Let $r_m(t)$ is the signal received at the m-th receive antenna for $m = 1, 2, \ldots n_r$.
- The linear MIMO channel model is

$$r_m(t) = \sum_{n=1}^{n_t} \int_{-\infty}^{\infty} h_{mn}(t-\tau) s_n(\tau) d\tau = \sum_{n=1}^{n_t} h_{mn}(t) * s_n(t)$$

- "$*$" denotes the convolution operation.

Signal Transmission in a Slow Fading Frequency Nonselective MIMO Channel

Frequency Nonselective Slow Fading Channel

Suppose that the data is modulated by PAM. For frequency nonselective slow Rayleigh fading MIMO channel, the model is represented as

$$r_m(t) = \sum_{n=1}^{n_t} h_{mn} s_n g_T(t) + z_m(t)$$

- h_{mn} is the complex zero mean Gaussian random variables.
- s_n is the symbol transmitted on the n-th antenna.
- $g_T(t)$ is the pulse shape of the modulation filter.
- $z_m(t)$ is the AWGN process on the m-th antenna.

Outputs of the demodulators

- The demodulators at the n_r receiving antennas are the matched filter to pulse shaping $g_T(t)$.
- The output of the demodulators at the m-th receiving antenna is

$$y_m = \sum_{n=1}^{n_t} s_n h_{mn} + \eta_m, \quad m = 1, 2, \ldots n_r$$

- η_m is the AWGN components.

Detection of Data Symbols

Detection Problem

The output of the demodulators at the mth receiving antenna is

$$y_m = \sum_{n=1}^{n_t} s_n h_{mn} + \eta_m, \quad m = 1, 2, \ldots n_r$$

- η_m is the AWGN components.
- The problem is to estimate s_n from y_m.
- Assume that the detector knows h_{mn}.
  - In practice, it can be estimated by using channel probe signals.

Maximally-Likelihood Detector

The maximally-likelihood (ML) detector is to find the s_n to maximize the likelihood function. This is equivalent to minimize the following Euclidean distance

$$\mu(s_1, s_2, \ldots s_n) = \sum_{m=1}^{n_r} \left| y_m - \sum_{n=1}^{n_t} h_{mn} s_n \right|$$

- exhaustive search

Minimum Mean-square-Error Detector

- The minimum mean-square-error (MMSE) detector is to form the estimation by a linear combination of y_m. The wighting coefficients are solved by minimized the mean squared error.
- Let $\boldsymbol{s} = [s_1, s_2, \ldots s_{n_t}]^T$ and $\boldsymbol{y} = [y_1, y_2, \ldots y_{n_r}]^T$. We want to solve a $n_r \times n_t$ matrix $\boldsymbol{W}$, the estimation is formed by

$$\hat{\boldsymbol{s}} = \boldsymbol{W}^H \boldsymbol{y}.$$

- The $\boldsymbol{W}$ is selected to minimize the mean squared error

$$E\left[\left\|\boldsymbol{s} - \boldsymbol{W}^H \boldsymbol{y}\right\|^2\right]$$

Inverse Channel Detector

- The inverse channel detector (ICD) is also to form the estimation by a linear combination of y_m. The correlator output can be represented in matrix form as

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{n_r} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1n_t} \\ h_{21} & h_{22} & \cdots & h_{2n_t} \\ \vdots & \vdots & \ddots & \vdots \\ h_{n_r1} & h_{n_r2} & \cdots & h_{n_rn_t} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_{n_r} \end{bmatrix} + \begin{bmatrix} \eta_1 \\ \eta_2 \\ \vdots \\ \eta_{n_r} \end{bmatrix}$$

or

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{\eta}$$

Inverse Channel Detector

- If $n_t = n_r$, the estimate is formed by

$$\hat{\boldsymbol{s}} = \boldsymbol{H}^{-1}\boldsymbol{y} = \boldsymbol{s} + \boldsymbol{H}^{-1}\boldsymbol{\eta}$$

and then quantizing it to the closet transmitted value.