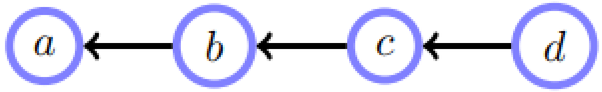
Chapter 5
Efficient Inference in Trees

5.1 Marginal Inference

- Given a distribution, inference is the process of computing functions of the distribution
- Marginal inference is concerned with the computation of the distribution of a subset of variables, possibly conditioned on another subset
- For example, given a joint distribution $p(x_1, x_2, x_3, x_4, x_5)$ and evidence $x_1 = \mathrm{tr}$, a marginal inference calculation is

$$p(x_5 \mid x_1 = \mathrm{tr}) \propto \sum_{x_2, x_3, x_4} p(x_1 = \mathrm{tr}, x_2, x_3, x_4, x_5)$$

- 5.1.1 Variable elimination in a Markov chain and message passing
  - Markov chain

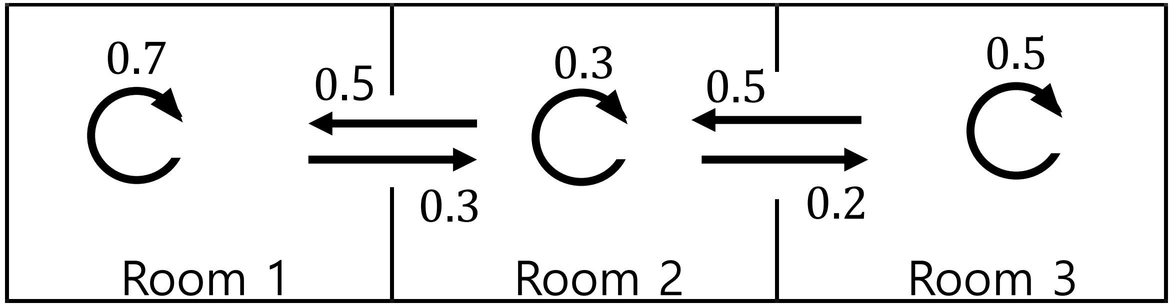


$$p(a, b, c, d) = p(a|b)p(b|c)p(c|d)p(d)$$

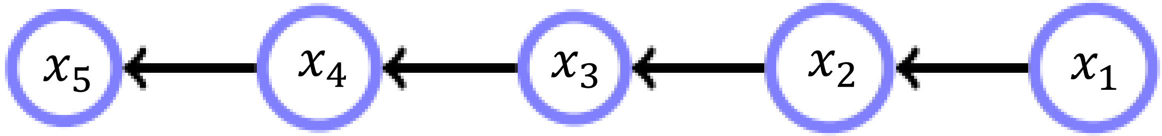
- Example 5.1 (Where will the fly be?)
  - Room 1, 2, 3
  - x_t indicates which room the fly is in at time t
  - The transition matrix

$$M = \begin{pmatrix} 0.7 & 0.5 & 0 \\ 0.3 & 0.3 & 0.5 \\ 0 & 0.2 & 0.5 \end{pmatrix}$$

where $M_{ij} = p(x_{t+1} = i \mid x_t = j)$



  - Compute $p(x_5 \mid x_1 = 1)$



$$
\begin{aligned}
p(x_5 \mid x_1 = 1) &= \sum_{x_2, x_3, x_4} \frac{p(x_1 = 1, x_2, x_3, x_4, x_5)}{p(x_1 = 1)} \\
&= \sum_{x_2, x_3, x_4} p(x_5 \mid x_4) p(x_4 \mid x_3) p(x_3 \mid x_2) p(x_2 \mid x_1 = 1) \\
&= M^4 v
\end{aligned}
$$

where $v = (1,0,0)^T$.

$$
M^4 v = \begin{pmatrix} 0.5746 \\ 0.3180 \\ 0.1074 \end{pmatrix}
$$

$$
p(x_5 = 1 \mid x_1 = 1) = 0.5746
$$

try

$$
\lim_{n \to \infty} M^n = \begin{pmatrix} 0.5435 & 0.5435 & 0.5435 \\ 0.3261 & 0.3261 & 0.3261 \\ 0.1704 & 0.1704 & 0.1704 \end{pmatrix}
$$

- 5.1.2 The sum-product algorithm on factor graphs