Chapter 7
n-step Bootstrapping

- TD(0), 1-step TD method, is 1-step bootstrapping
- TD($n-1$) , n-step TD method, is n-step bootstrapping

Bootstrapping: 주어진 상황에서 어떻게든 해결한다

7.1 n-step TD Prediction

- Monte Carlo methods
  use the entire sequence of episodes
- 1-step TD methods
  use the one next reward and the value of the state one step later
- 2-step TD methods
  use the first two rewards and the value of the state two step later
- And so on

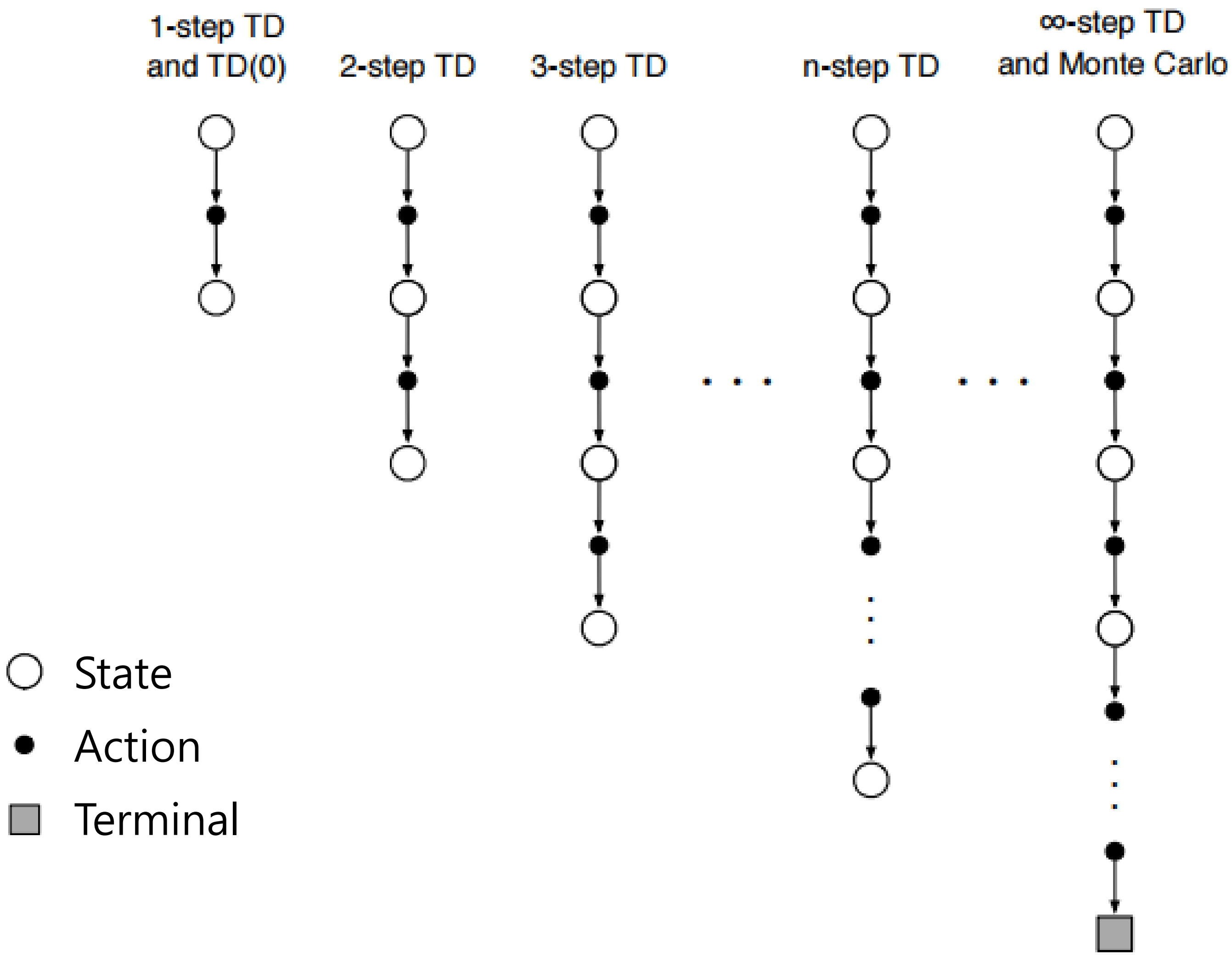


Figure 7.1: The backup diagrams of n-step methods. These methods form a spectrum ranging from one-step TD methods to Monte Carlo methods.

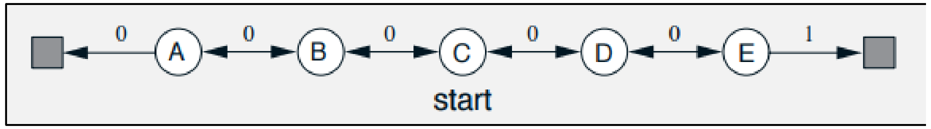
- Notation
  - V_t is the estimate at time t of v_π
  - return
$$G_t = R_{t+1} + \gamma R_{t+2} + \cdots + \gamma^{T-t-1} R_T$$
  - 1-step return
$$G_{t:t+1} = R_{t+1} + \gamma V_t(S_{t+1})$$
  - 2-step return
$$G_{t:t+1} = R_{t+1} + \gamma R_{t+2} + \gamma^2 V_{t+1}(S_{t+2})$$
  - n-step return
$$G_{t:t+n} = R_{t+1} + \gamma R_{t+2} + \cdots + \gamma^{n-1} R_{t+n} + \gamma^n V_{t+n-1}(S_{t+n})$$
- Update of n-step TD
$$V_{t+n}(S_t) = V_{t+n-1}(S_t) + \alpha[G_{t:t+n} - V_{t+n-1}(S_t)]$$

n-step TD for estimating $V \approx v_\pi$

Input: a policy π
Algorithm parameters: step size $\alpha \in (0, 1]$, a positive integer n
Initialize $V(s)$ arbitrarily, for all $s \in \mathcal{S}$
All store and access operations (for S_t and R_t) can take their index mod $n+1$

Loop for each episode:
  Initialize and store $S_0 \neq$ terminal
  $T \leftarrow \infty$
  Loop for $t = 0, 1, 2, \ldots$:
  | If $t < T$, then:
  | Take an action according to $\pi(\cdot|S_t)$
  | Observe and store the next reward as R_{t+1} and the next state as S_{t+1}
  | If S_{t+1} is terminal, then $T \leftarrow t+1$
  | $\tau \leftarrow t - n + 1$ (τ is the time whose state's estimate is being updated)
  | If $\tau \geq 0$:
  | $G \leftarrow \sum_{i=\tau+1}^{\min(\tau+n, T)} \gamma^{i-\tau-1} R_i$
  | If $\tau + n < T$, then: $G \leftarrow G + \gamma^n V(S_{\tau+n})$ $(G_{\tau:\tau+n})$
  | $V(S_\tau) \leftarrow V(S_\tau) + \alpha \left[G - V(S_\tau)\right]$
  Until $\tau = T - 1$

Example 7.1: n-step TD Methods on the Random Walk



5-state random walk

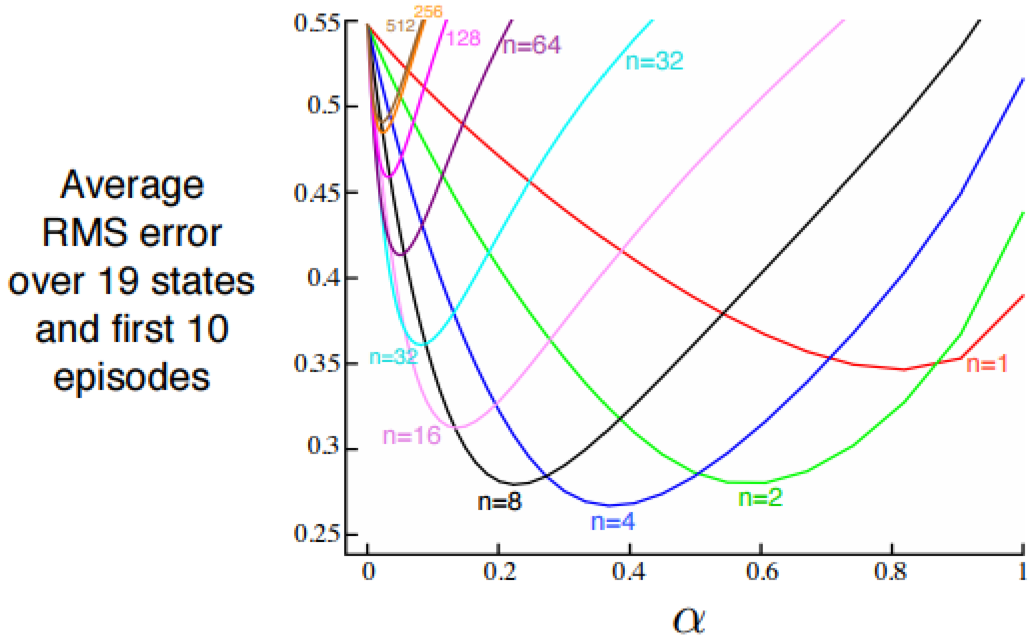


Figure 7.2: Performance of n-step TD methods as a function of α, for various values of n, on a 19-state random walk task (Example 7.1). ■

7.2 n-step Sarsa

- Notation
  - n-step return

    $$G_{t:t+n} = R_{t+1} + \gamma R_{t+2} + \cdots + \gamma^{n-1} R_{t+n} + \gamma^n Q_{t+n-1}(S_{t+n}, A_{t+n})$$
- n-step Sarsa, Sarsa(n-1)

  $$Q_{t+n}(S_t, A_t) = Q_{t+n-1}(S_t, A_t) + \alpha[G_{t:t+n} - Q_{t+n-1}(S_t, A_t)]$$

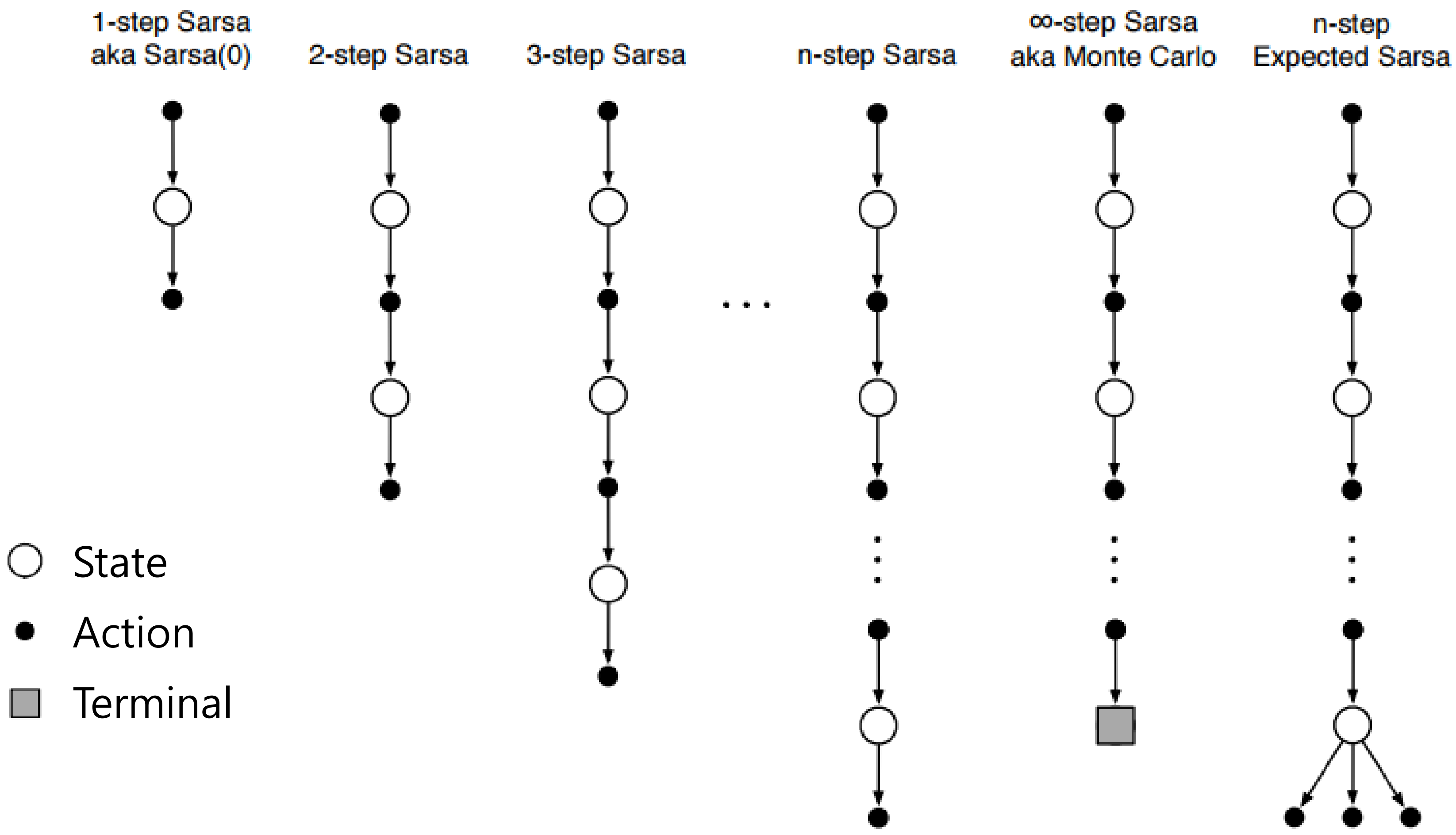


Figure 7.3: The backup diagrams for the spectrum of n-step methods for state–action values. They range from the one-step update of Sarsa(0) to the up-until-termination update of the Monte Carlo method. In between are the n-step updates, based on n steps of real rewards and the estimated value of the nth next state–action pair, all appropriately discounted. On the far right is the backup diagram for n-step Expected Sarsa.

n-step Sarsa for estimating $Q \approx q_*$ or q_π

Initialize $Q(s, a)$ arbitrarily, for all $s \in \mathcal{S}, a \in \mathcal{A}$
Initialize π to be ε-greedy with respect to Q, or to a fixed given policy
Algorithm parameters: step size $\alpha \in (0, 1]$, small $\varepsilon > 0$, a positive integer n
All store and access operations (for S_t, A_t, and R_t) can take their index mod $n + 1$

Loop for each episode:
    Initialize and store $S_0 \neq$ terminal
    Select and store an action $A_0 \sim \pi(\cdot|S_0)$
    $T \leftarrow \infty$
    Loop for $t = 0, 1, 2, \ldots$:
    | If $t < T$, then:
    | Take action A_t
    | Observe and store the next reward as R_{t+1} and the next state as S_{t+1}
    | If S_{t+1} is terminal, then:
    | $T \leftarrow t + 1$
    | else:
    | Select and store an action $A_{t+1} \sim \pi(\cdot|S_{t+1})$
    | $\tau \leftarrow t - n + 1$ (τ is the time whose estimate is being updated)
    | If $\tau \geq 0$:
    | $G \leftarrow \sum_{i=\tau+1}^{\min(\tau+n,T)} \gamma^{i-\tau-1} R_i$
    | If $\tau + n < T$, then $G \leftarrow G + \gamma^n Q(S_{\tau+n}, A_{\tau+n})$ $(G_{\tau:\tau+n})$
    | $Q(S_\tau, A_\tau) \leftarrow Q(S_\tau, A_\tau) + \alpha \left[G - Q(S_\tau, A_\tau)\right]$
    | If π is being learned, then ensure that $\pi(\cdot|S_\tau)$ is ε-greedy wrt Q
    Until $\tau = T - 1$

7.3 n-step Off-policy Learning

- Recall
  - Off-policy learning is learning the value function for one policy π, while following another policy b
  - Target policy π – the greedy policy for the current action-value function
  - Behavior policy b – exploratory policy
- Off-policy version of n-step TD
  - Update

$$V_{t+n}(S_t) = V_{t+n-1}(S_t) + \alpha\rho_{t:t+n-1}[G_{t:t+n} - V_{t+n-1}(S_t)]$$

where $\rho_{t:h} = \prod_{k=t}^{h} \frac{\pi(A_k|S_k)}{b(A_k|S_k)}$ is the importance sampling ratio

- Off-policy version of n-step TD
  - Update
    $Q_{t+n}(S_t, A_t) = Q_{t+n-1}(S_t, A_t) + \alpha\rho_{t+1:t+n}[G_{t:t+n} - Q_{t+n-1}(S_t, A_t)]$

Off-policy n-step Sarsa for estimating $Q \approx q_* \text{ or } q_\pi$

Input: an arbitrary behavior policy b such that $b(a|s) > 0$, for all $s \in \mathcal{S}, a \in \mathcal{A}$
Initialize $Q(s,a)$ arbitrarily, for all $s \in \mathcal{S}, a \in \mathcal{A}$
Initialize π to be greedy with respect to Q, or as a fixed given policy
Algorithm parameters: step size $\alpha \in (0,1]$, a positive integer n
All store and access operations (for S_t, A_t, and R_t) can take their index mod $n+1$

Loop for each episode:
    Initialize and store $S_0 \neq$ terminal
    Select and store an action $A_0 \sim b(\cdot|S_0)$
    $T \leftarrow \infty$
    Loop for $t = 0, 1, 2, \ldots$:
    | If $t < T$, then:
    | Take action A_t
    | Observe and store the next reward as R_{t+1} and the next state as S_{t+1}
    | If S_{t+1} is terminal, then:
    | $T \leftarrow t+1$
    | else:
    | Select and store an action $A_{t+1} \sim b(\cdot|S_{t+1})$
    | $\tau \leftarrow t - n + 1$ (τ is the time whose estimate is being updated)
    | If $\tau \geq 0$:
    | $\rho \leftarrow \prod_{i=\tau+1}^{\min(\tau+n-1, T-1)} \frac{\pi(A_i|S_i)}{b(A_i|S_i)}$ $(\rho_{\tau+1:t+n-1})$
    | $G \leftarrow \sum_{i=\tau+1}^{\min(\tau+n, T)} \gamma^{i-\tau-1} R_i$
    | If $\tau + n < T$, then: $G \leftarrow G + \gamma^n Q(S_{\tau+n}, A_{\tau+n})$ $(G_{\tau:\tau+n})$
    | $Q(S_\tau, A_\tau) \leftarrow Q(S_\tau, A_\tau) + \alpha\rho\,[G - Q(S_\tau, A_\tau)]$
    | If π is being learned, then ensure that $\pi(\cdot|S_\tau)$ is greedy wrt Q
    Until $\tau = T - 1$

7.4 *Per-decision Methods with Control Variates

7.5 Off-policy Learning Without Importance Sampling: The n-step Tree Backup Algorithm

7.6 *A Unifying Algorithm: n-step $Q(\sigma)$