

Chapter 2

Basic Graph Concepts

2.1 Graphs

- Definition 2.1

- Graph

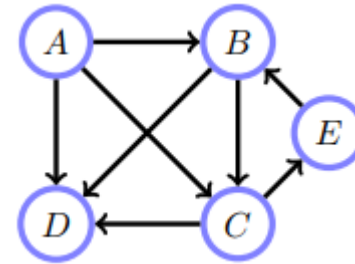
- Nodes – vertices
 - Edges – links

- Directed graph

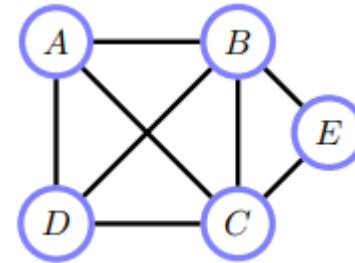
- All edges are directed

- Undirected graph

- All edges are undirected



directed graph



undirected graph

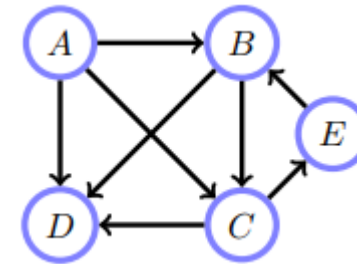
- Definition 2.2
 - Path $A \mapsto B$
 - Sequence of nodes that connects A to B
 - Directed path
 - Path with directed edges
 - A is an ancestor of B , B is a descendant of A
 - $A \mapsto B$ and $B \not\mapsto A$

- Definition 2.3

- Cycle

- a directed path
 - starts and returns to the same node

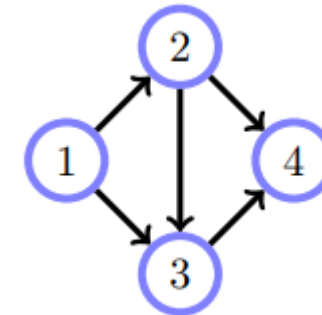
$$a \rightarrow b \rightarrow \dots \rightarrow z \rightarrow a$$



$$B \rightarrow C \rightarrow E$$

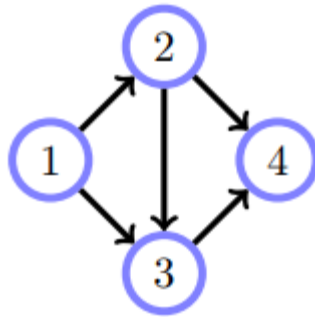
- Loop

- a path
 - containing more than two nodes
 - starts and returns to the same node
 - irrespective of edge direction

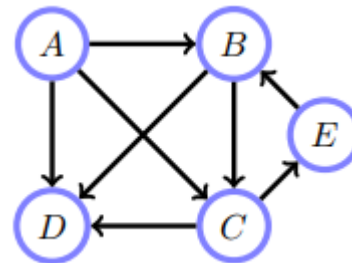


$$1 - 2 - 4 - 3 - 1$$

- Definition 2.4
 - Directed acyclic graph(DAG)
 - A directed graph
 - Without cycles



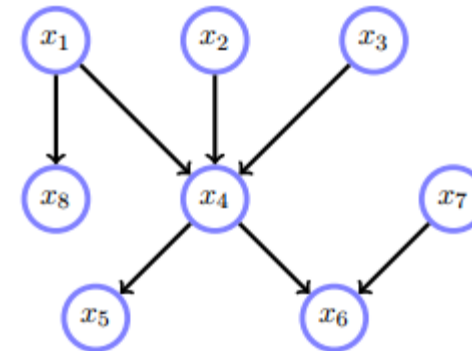
DAG



Not DAG

- Definition 2.5

- Parents of a node
- Children of a node
- Family of a node
 - Itself and its parents
- Markov blanket of a node
 - Its parents, children and the parents of its children
 - Excluding itself



$$\text{pa}(x_4) = \{x_1, x_2, x_3\}$$

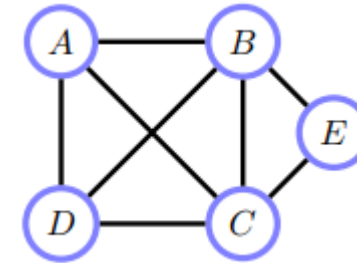
$$\text{ch}(x_4) = \{x_5, x_6\}$$

$$\text{Markov blanket of } x_4 = \{x_1, x_2, x_3, x_5, x_6, x_7\}$$

- Definition 2.6

- Neighbor

- For an undirected graph
 - The neighbours of x , $ne(x)$
 - those nodes directly connected to x



$ne(A) = ?$

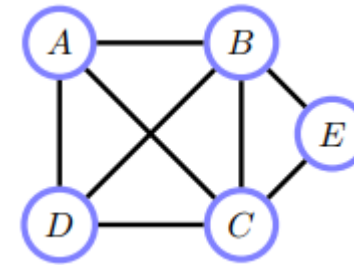
- Definition 2.7

- Clique

- Fully connected subset of nodes
i.e.
All the members are neighbours
of other member

- Maximal clique

- There is no larger clique that
contains the clique



Maximal cliques

$\{A, B, C, D\}, \{B, C, E\}$

Cliques

$\{A, B, C\}, \{A, C, D\}$

Not a clique

$\{A, B, C, E\}$

- Definition 2.8
 - Connected graph
 - An undirected graph
 - There is a path between every pair of nodes
i.e. there are no isolated islands

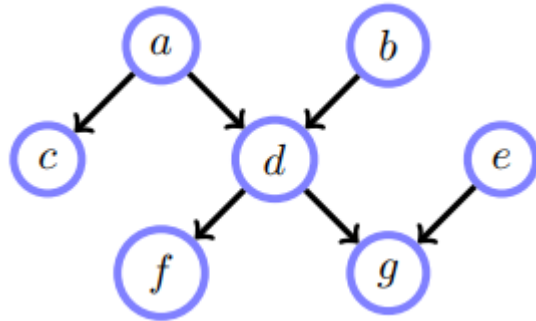
- Definition 2.9

- Singly Connected Graph

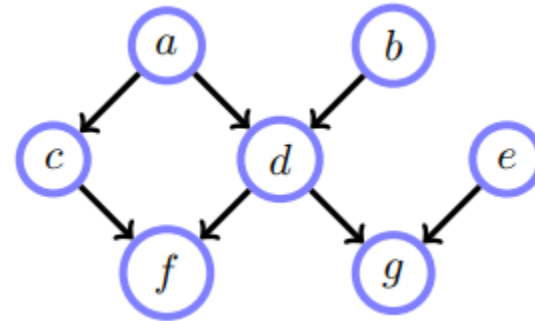
- There is only one path from any node to any other node

- Multiply connected(roopy)

- Not singly connected

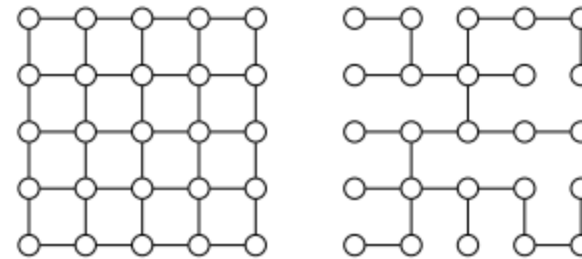


Singly connected graph



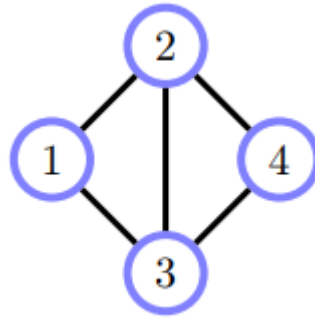
Multiply connected graph

- Definition 2.10
 - Spanning Tree
 - A singly connected subset
 - Covers all nodes



2.2 Numerically Encoding Graphs

- 2.2.1 Edge list
 - Edge list
 - All node-node pairs

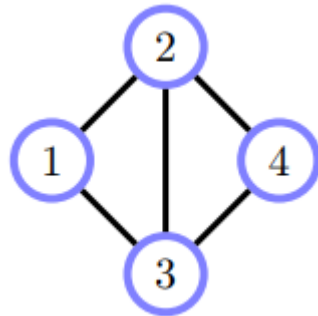


$$L = \{(1, 2), (2, 1), (1, 3), (3, 1), (2, 3), (3, 2), (2, 4), (4, 2), (3, 4), (4, 3)\}$$

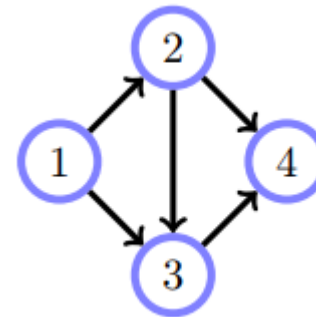
- 2.2.2 Adjacency matrix

- Adjacency matrix A

- $A_{ij} = 1$ if there is an edge from node i to node j



$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

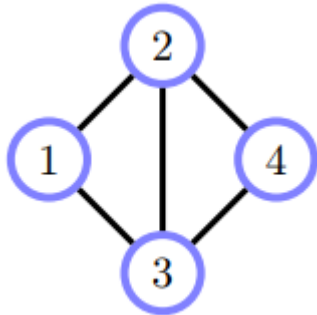


$$\mathbf{T} = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

- 2.2.3 Clique matrix

- Clique matrix

- A matrix of nodes – maximal cliques relation
 - A graph has N nodes and K maximal cliques $\rightarrow N \times K$ clique matrix



nodes = $\{1,2,3,4\}$
max clique = $\{C_1, C_2\}$
 $C_1 = \{1,2,3\}, C_2 = \{2,3,4\}$

$$\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{pmatrix}$$