Chapter 3
Belief Networks

3.1 The Benefits of Structure

- Belief network is a way to depict the independence of variables

- 3.1.1 Modelling independencies

One morning Tracey leaves her house and realises that her grass is wet.
Is it due to overnight rain?
Or did she forget to turn off the sprinkler last night?

She notices that the grass of her neighbour, Jack, is also wet.
And she concludes that it has probably been raining.

$R \in \{0,1\}$ – rain
$S \in \{0,1\}$ – sprinkler
$J \in \{0,1\}$ – Jack's grass
$T \in \{0,1\}$ – Tracey's grass

$$
\begin{aligned}
p(T, J, R, S) &= p(T|J, R, S)\, p(J, R, S) \\
&= p(T|J, R, S)\, p(J|R, S)\, p(R, S) \\
&= p(T|J, R, S)\, p(J|R, S)\, p(R|S)\, p(S)
\end{aligned}
$$

$R \in \{0,1\}$ – rain
$S \in \{0,1\}$ – sprinkler
$J \in \{0,1\}$ – Jack's grass
$T \in \{0,1\}$ – Tracey's grass

$p(T|J, R, S)$ - $2^3 = 8$ values are needed
$p(J|R, S)$ - 4 values are needed
$p(R|S)$ - 2 values are needed
$p(S)$ - 1 value is needed
Total – 15 values are needs

- Conditional independence
  - Tracey's grass is wet
    - Depends on rain and her sprinkler.
    - Does not depend on Jack's grass

$$p(T|J,R,S) = p(T|R,S)$$

  - Jack's grass is wet
    - Depends on raining

$$p(J|R,S) = p(J|R)$$

  - The rain is not influenced by the sprinkler

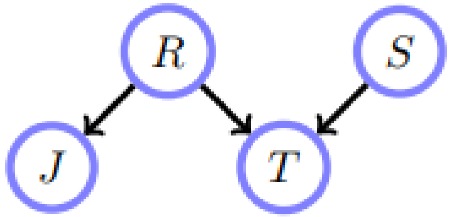
$$p(R|S) = p(R)$$

$$\begin{aligned} p(T,J,R,S) &= p(T|J,R,S)\,p(J,R,S) \\ &= p(T|J,R,S)\,p(J|R,S)\,p(R,S) \\ &= p(T|J,R,S)\,p(J|R,S)\,p(R|S)\,p(S \end{aligned}$$

$$p(T,J,R,S) = p(T|R,S)p(J|R)p(R)p(S)$$

$$p(T, J, R, S) = p(T|R, S)p(J|R)p(R)p(S)$$

$4 + 2 + 1 + 1 = 8$ values are needed



Belief network structure for the 'wet grass' example

- conditional probability table(CPT)
  - $p(R = 1) = 0.2$
  - $p(S = 1) = 0.1$
  - $p(J = 1|R = 1) = 1$
    $p(J = 1|R = 0) = 0.2$
  - $p(T = 1|R = 1, S = 0) = 1$
    $p(T = 1|R = 1, S = 1) = 1$
    $p(T = 1|R = 0, S = 1) = 0.9$
    $p(T = 1|R = 0, S = 0) = 0$

$$
\begin{aligned}
p(S=1|T=1) &= \frac{p(S=1,T=1)}{p(T=1)} = \frac{\sum_{J,R} p(T=1,J,R,S=1)}{\sum_{J,R,S} p(T=1,J,R,S)} \\
&= \frac{\sum_{J,R} p(J|R)p(T=1|R,S=1)p(R)p(S=1)}{\sum_{J,R,S} p(J|R)p(T=1|R,S)p(R)p(S)} \\
&= \frac{\sum_{R} p(T=1|R,S=1)p(R)p(S=1)}{\sum_{R,S} p(T=1|R,S)p(R)p(S)} \\
&= \frac{0.9 \times 0.8 \times 0.1 + 1 \times 0.2 \times 0.1}{0.9 \times 0.8 \times 0.1 + 1 \times 0.2 \times 0.1 + 0 \times 0.8 \times 0.9 + 1 \times 0.2 \times 0.9} = 0.3382
\end{aligned}
$$

$$
\begin{aligned}
p(S=1|T=1,J=1) &= \frac{p(S=1,T=1,J=1)}{p(T=1,J=1)} \\
&= \frac{\sum_{R} p(T=1,J=1,R,S=1)}{\sum_{R,S} p(T=1,J=1,R,S)} \\
&= \frac{\sum_{R} p(J=1|R)p(T=1|R,S=1)p(R)p(S=1)}{\sum_{R,S} p(J=1|R)p(T=1|R,S)p(R)p(S)} \\
&= \frac{0.0344}{0.2144} = 0.1604
\end{aligned}
$$