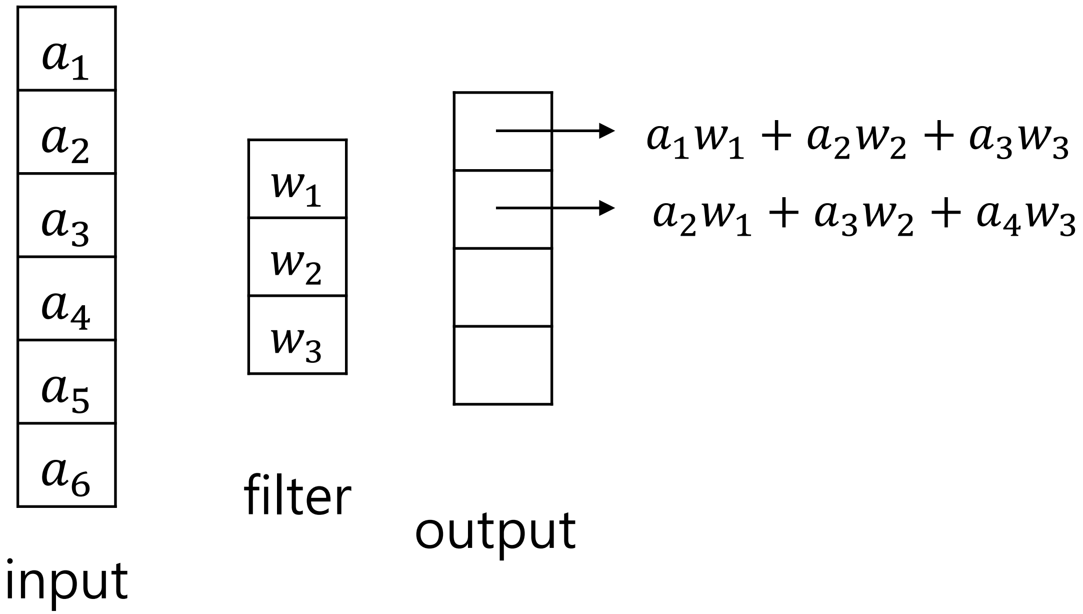
Chapter 3
Convolutional Neural Networks

3.1 Filters, Strides, and Padding

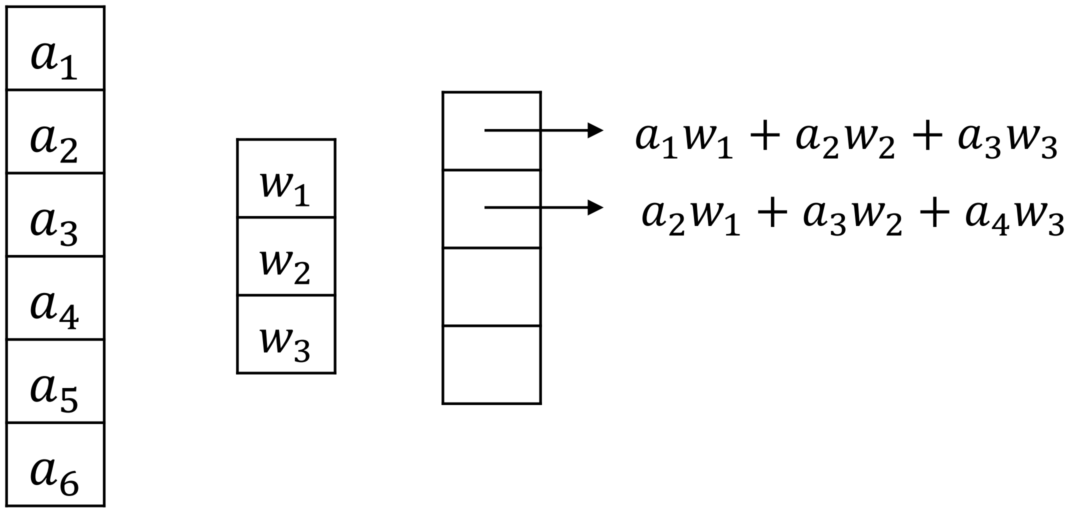
- Input I, kernel K

$$V(x,y) = (I \cdot K)(x,y) = \sum_{m} \sum_{n} I(x+m, y+n) K(m,n)$$

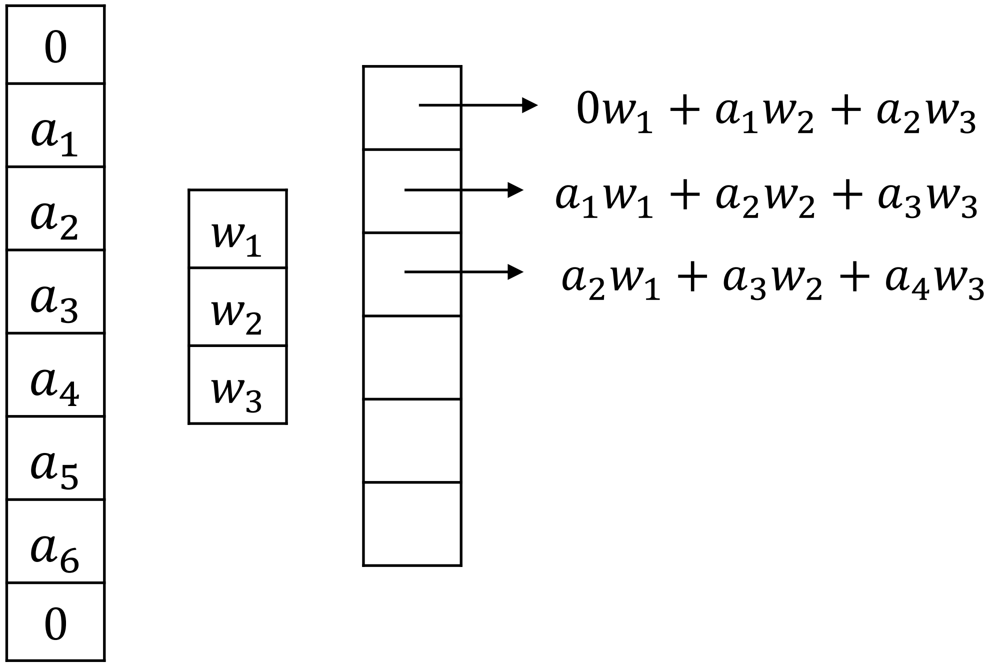
1-dimensional CNN

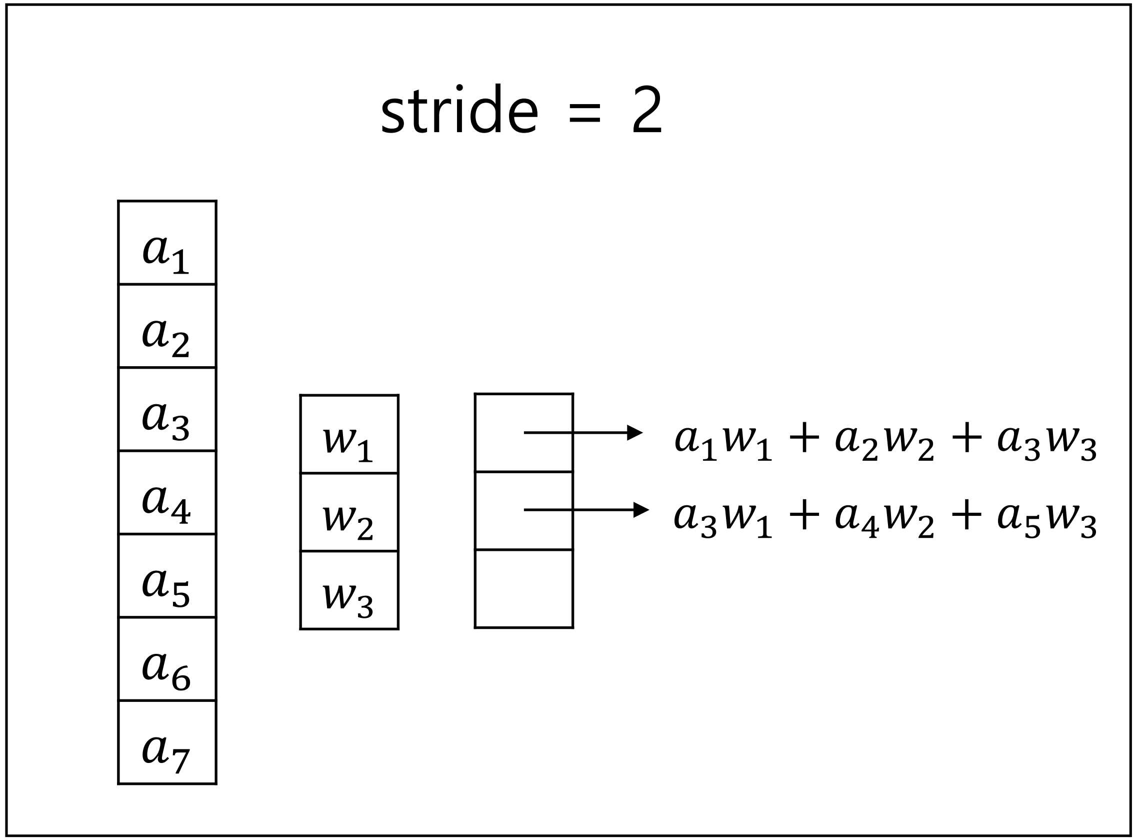


padding=‘VALID’



padding=‘SAME’





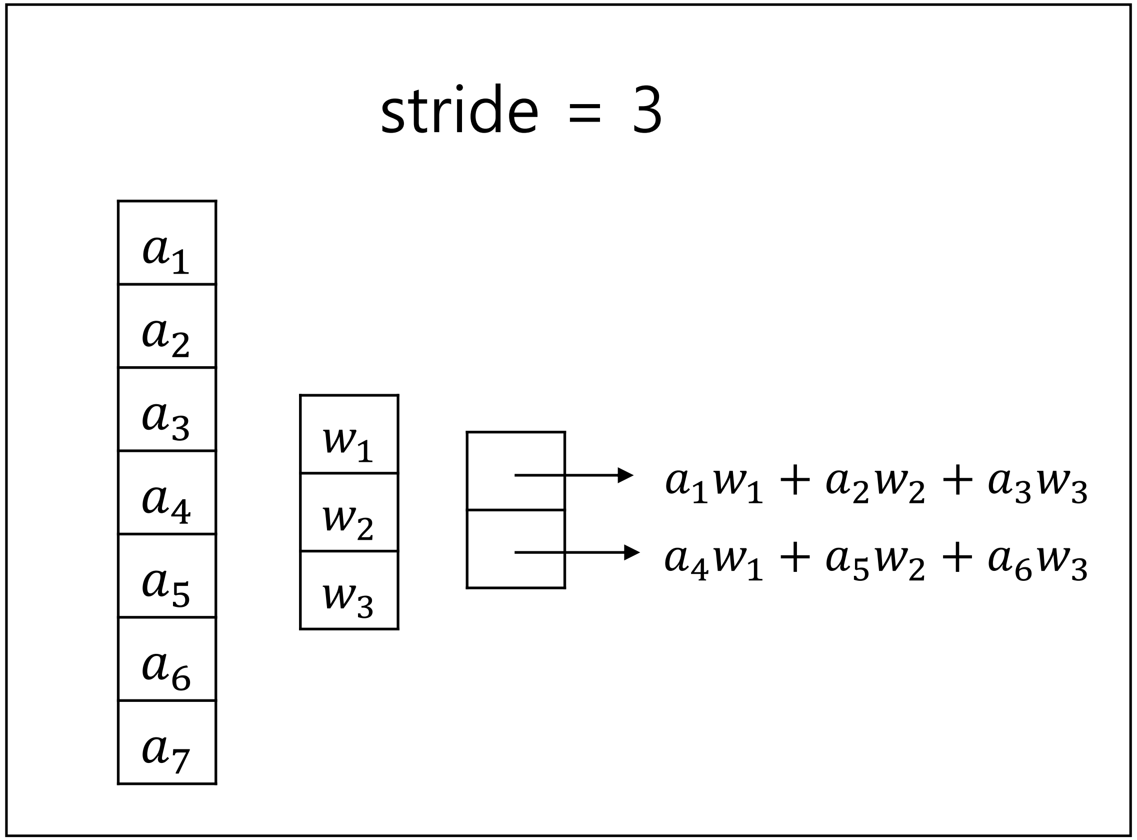
stride = 2

a_1, a_2, a_3, a_4, a_5, a_6, a_7

w_1, w_2, w_3

$a_1w_1 + a_2w_2 + a_3w_3$

$a_3w_1 + a_4w_2 + a_5w_3$



stride = 3

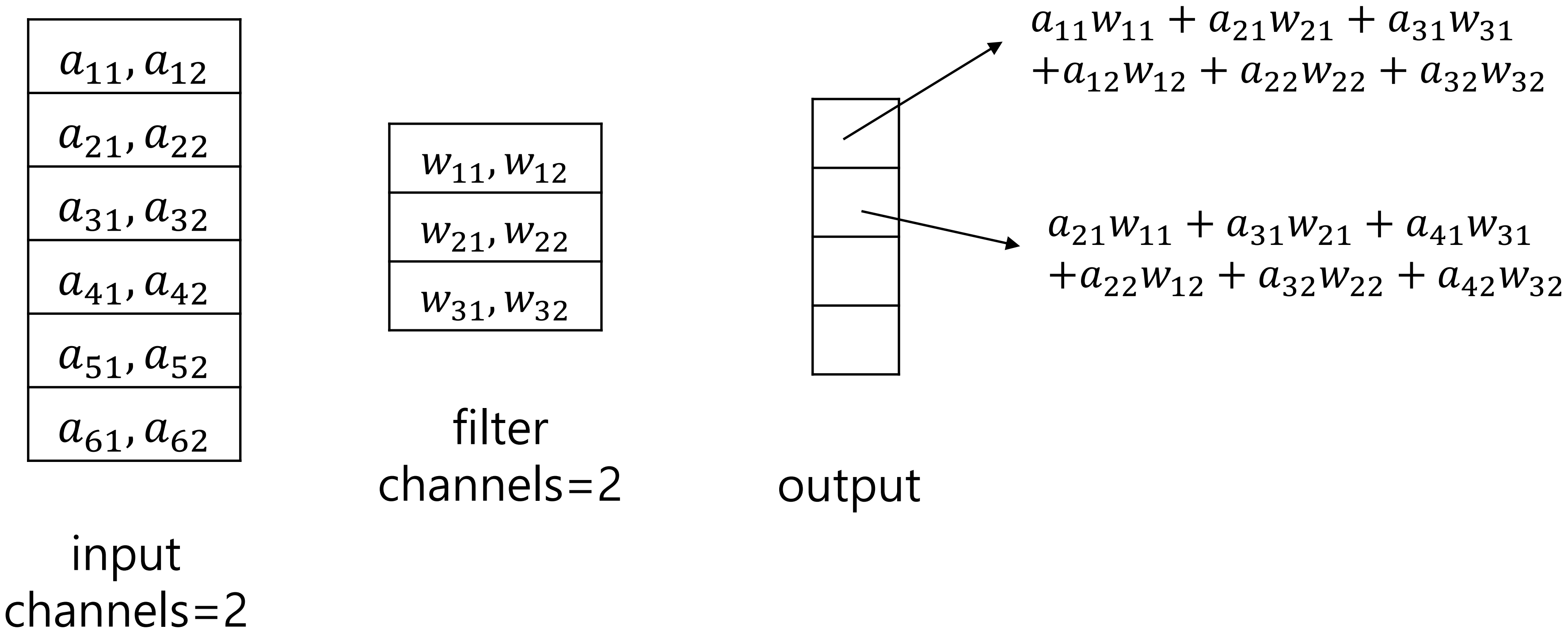
a_1, a_2, a_3, a_4, a_5, a_6, a_7

w_1, w_2, w_3

$a_1w_1 + a_2w_2 + a_3w_3$

$a_4w_1 + a_5w_2 + a_6w_3$

input channels



a_{11}, a_{12}

a_{21}, a_{22}

a_{31}, a_{32}

a_{41}, a_{42}

a_{51}, a_{52}

a_{61}, a_{62}

input channels=2

w_{11}, w_{12}

w_{21}, w_{22}

w_{31}, w_{32}

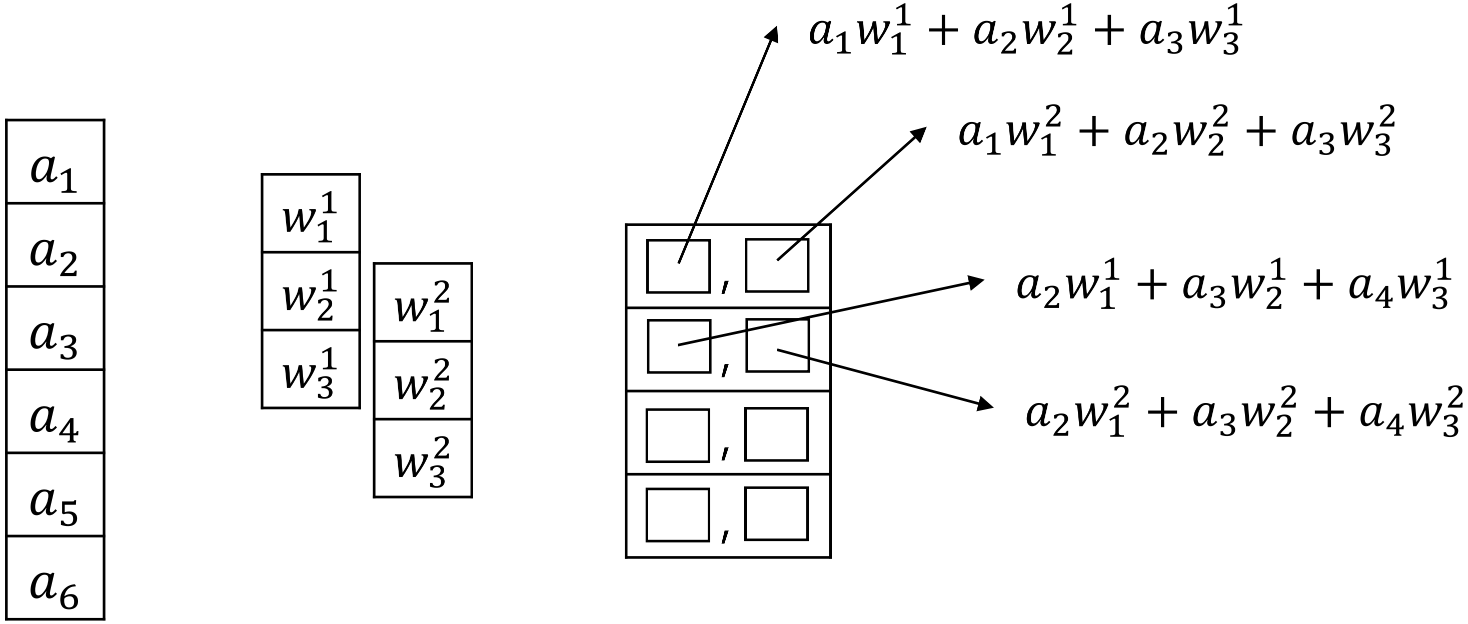
filter channels=2

output

$$a_{11}w_{11} + a_{21}w_{21} + a_{31}w_{31}$$
$$+a_{12}w_{12} + a_{22}w_{22} + a_{32}w_{32}$$

$$a_{21}w_{11} + a_{31}w_{21} + a_{41}w_{31}$$
$$+a_{22}w_{12} + a_{32}w_{22} + a_{42}w_{32}$$

output channels
=# of filters



a_1

a_2

a_3

a_4

a_5

a_6

w_1^1

w_2^1

w_3^1

w_1^2

w_2^2

w_3^2

$a_1 w_1^1 + a_2 w_2^1 + a_3 w_3^1$

$a_1 w_1^2 + a_2 w_2^2 + a_3 w_3^2$

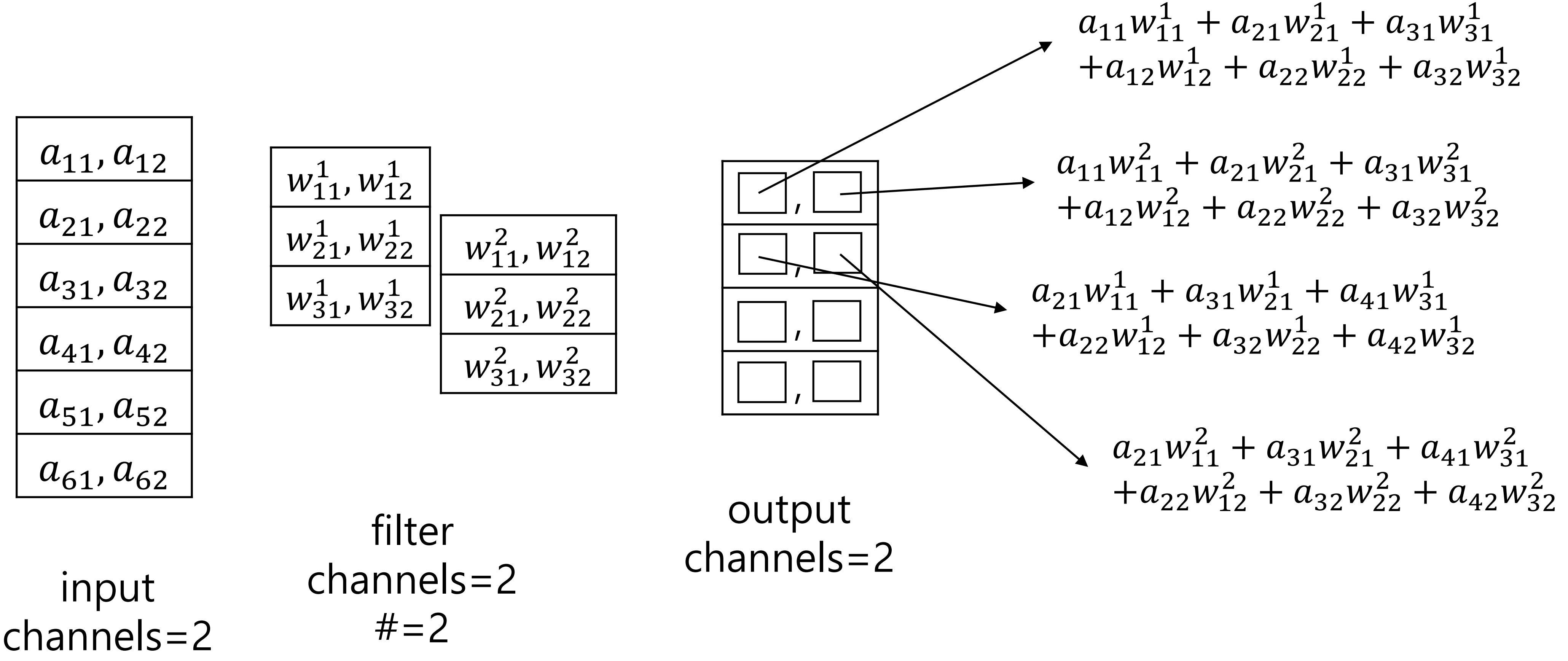
$a_2 w_1^1 + a_3 w_2^1 + a_4 w_3^1$

$a_2 w_1^2 + a_3 w_2^2 + a_4 w_3^2$

filter
#=2

output
channels=2

input channels
and
output channels



a_{11}, a_{12}

a_{21}, a_{22}

a_{31}, a_{32}

a_{41}, a_{42}

a_{51}, a_{52}

a_{61}, a_{62}

input channels=2

w^1_{11}, w^1_{12}

w^1_{21}, w^1_{22}

w^1_{31}, w^1_{32}

w^2_{11}, w^2_{12}

w^2_{21}, w^2_{22}

w^2_{31}, w^2_{32}

filter channels=2 #=2

output channels=2

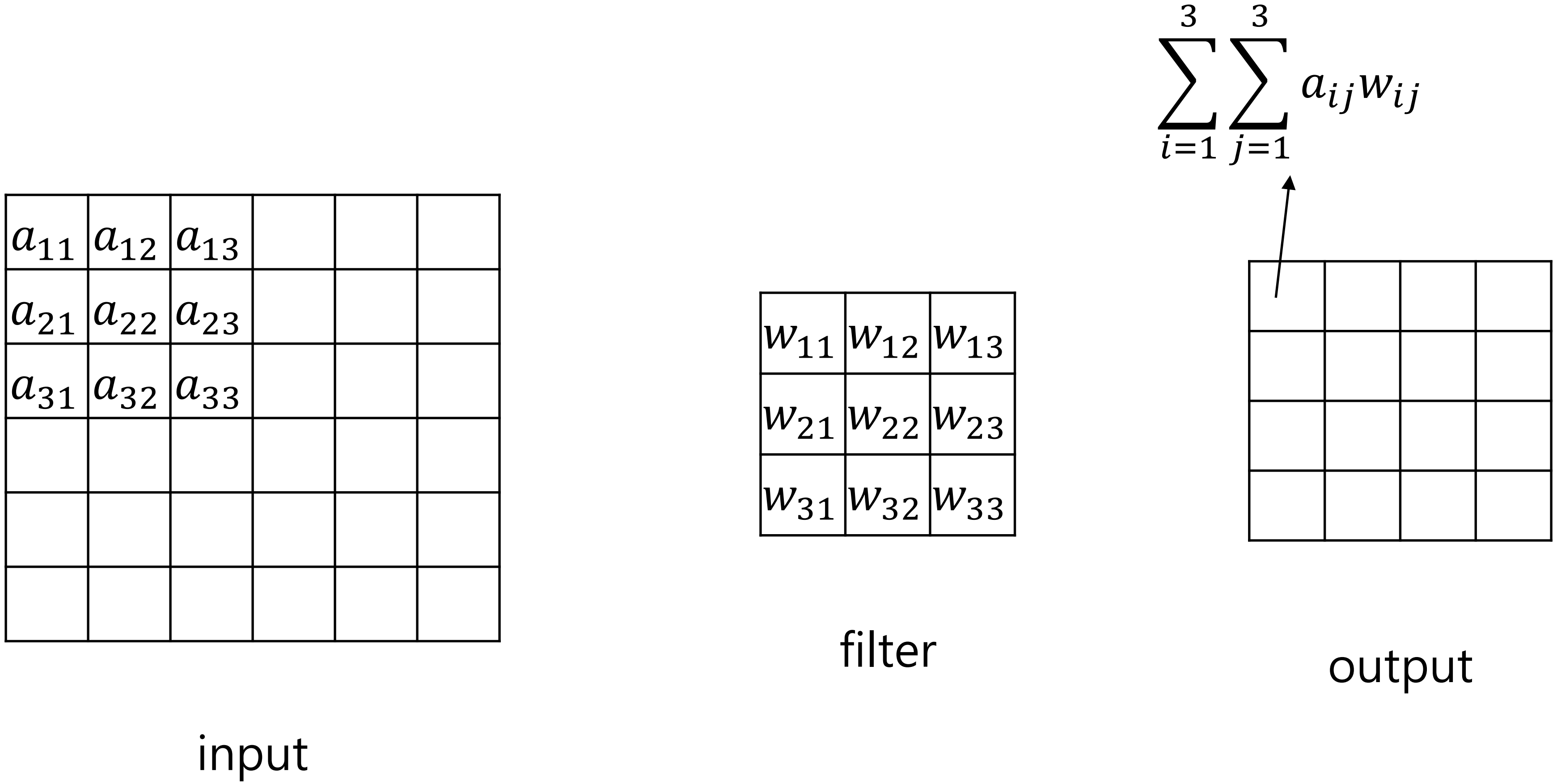
$$a_{11}w^1_{11} + a_{21}w^1_{21} + a_{31}w^1_{31}$$
$$+a_{12}w^1_{12} + a_{22}w^1_{22} + a_{32}w^1_{32}$$

$$a_{11}w^2_{11} + a_{21}w^2_{21} + a_{31}w^2_{31}$$
$$+a_{12}w^2_{12} + a_{22}w^2_{22} + a_{32}w^2_{32}$$

$$a_{21}w^1_{11} + a_{31}w^1_{21} + a_{41}w^1_{31}$$
$$+a_{22}w^1_{12} + a_{32}w^1_{22} + a_{42}w^1_{32}$$

$$a_{21}w^2_{11} + a_{31}w^2_{21} + a_{41}w^2_{31}$$
$$+a_{22}w^2_{12} + a_{32}w^2_{22} + a_{42}w^2_{32}$$

2-dimensional CNN



$$\sum_{i=1}^{3}\sum_{j=1}^{3} a_{ij}w_{ij}$$

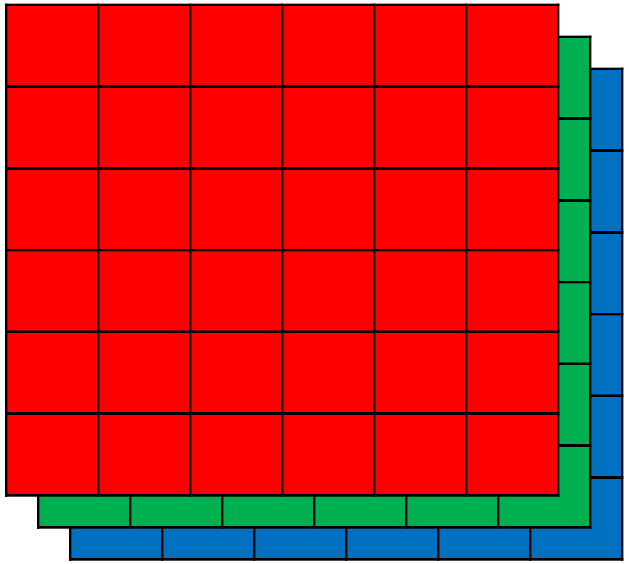
a_{11}	a_{12}	a_{13}			
a_{21}	a_{22}	a_{23}			
a_{31}	a_{32}	a_{33}			

input

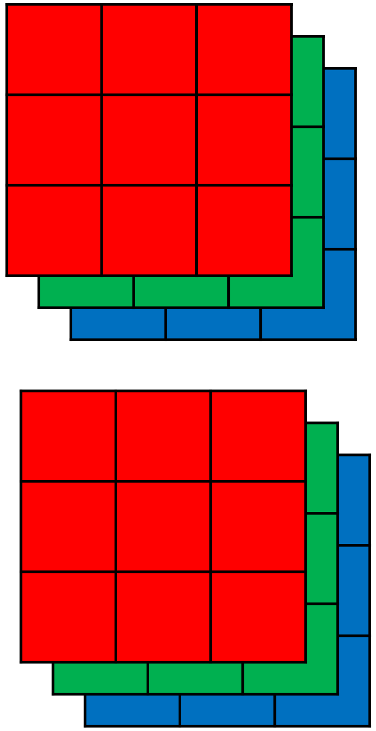
w_{11}	w_{12}	w_{13}
w_{21}	w_{22}	w_{23}
w_{31}	w_{32}	w_{33}

filter

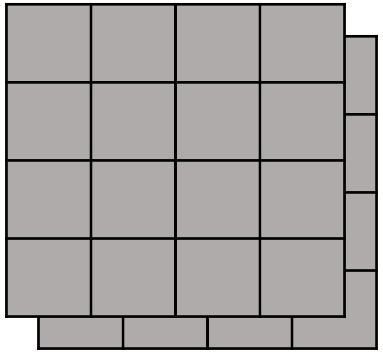
output



input
channels=3



filter
channels=3
#=2



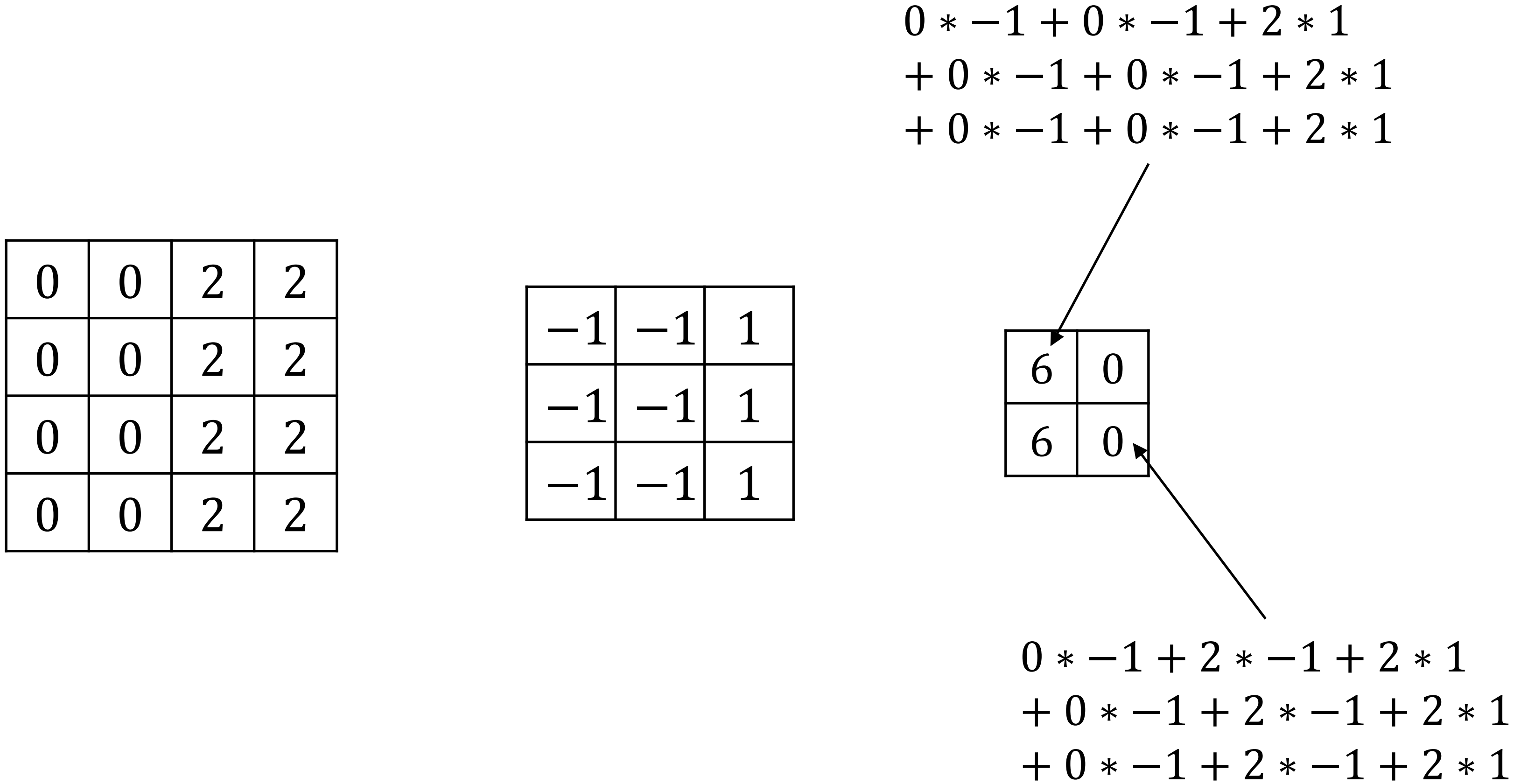
output
channels=2

- Stride s_h, s_v
- Padding ‘SAME’, ‘VALID’
- Tensorflow code

```
tf.nn.conv2d(input, filters, strides, padding)
```

3.2 A Simple TF Convolution Example

```
ii = [[[[0], [0], [2], [2]],
      [[0], [0], [2], [2]],
      [[0], [0], [2], [2]],
      [[0], [0], [2], [2]]]]
I = tf.constant(ii, tf.float32)
print(I.shape) # (1, 4, 4, 1) – batch size, width, height, input channels
ww = [[[[-1]], [[-1]], [[1]]],
      [[[-1]], [[-1]], [[1]]],
      [[[-1]], [[-1]], [[1]]]]
W = tf.constant(ww, tf.float32)
print(W.shape) # (3, 3, 1, 1) – width, height, input channels, # of filters
C = tf.nn.conv2d(I, W, strides=[1, 1, 1, 1], padding='VALID')
sess = tf.Session()
print(res.shape)
print(res) # (1, 2, 2, 1) – batch size, width, height, output channels
```



$$\begin{bmatrix} 0 & 0 & 2 & 2 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 2 & 2 \end{bmatrix} \quad \begin{bmatrix} -1 & -1 & 1 \\ -1 & -1 & 1 \\ -1 & -1 & 1 \end{bmatrix} \quad \begin{bmatrix} 6 & 0 \\ 6 & 0 \end{bmatrix}$$

$$\begin{aligned} &0 * -1 + 0 * -1 + 2 * 1 \\ &+ 0 * -1 + 0 * -1 + 2 * 1 \\ &+ 0 * -1 + 0 * -1 + 2 * 1 \end{aligned}$$

$$\begin{aligned} &0 * -1 + 2 * -1 + 2 * 1 \\ &+ 0 * -1 + 2 * -1 + 2 * 1 \\ &+ 0 * -1 + 2 * -1 + 2 * 1 \end{aligned}$$

Modified Figure 2.2

```
# ==========================================================================
image = tf.reshape(img, [batchSz, 28, 28, 1])
flts = tf.Variable(tf.truncated_normal([4, 4, 1, 4], stddev=0.1))
convOut = tf.nn.conv2d(image, flts, [1, 2, 2, 1], 'SAME’)
convOut = tf.nn.relu(convOut)
print(convOut.shape, 'batch size, width, height, numFlters=output channels')
convOut = tf.reshape(convOut, [100, 784])
prbs = tf.nn.softmax(tf.matmul(convOut, W) + b)
# prbs = tf.nn.softmax(tf.matmul(img, W) + b)
# ==========================================================================
```

Output

```
(100, 14, 14, 4) batch size, width, height, numFlters=output channels
Test Accuracy: 0.9625000046491623
```

3.3 Multilevel Convolution

```
# ================================================================
image = tf.reshape(img, [batchSz, 28, 28, 1])
flts = tf.Variable(tf.random_normal([4, 4, 1, 16], stddev=0.1))
convOut = tf.nn.conv2d(image, flts, [1, 2, 2, 1], "SAME")
convOut = tf.nn.relu(convOut)
flts2 = tf.Variable(tf.random_normal([2, 2, 16, 32], stddev=0.1))
convOut2 = tf.nn.conv2d(convOut, flts2, [1, 2, 2, 1], "SAME")
convOut2 = tf.reshape(convOut2, [100, 1568])
prbs = tf.nn.softmax(tf.matmul(convOut2, W) + b)
# prbs = tf.nn.softmax(tf.matmul(img, W) + b)
# =================================================================
```

Test Accuracy: 0.9663000059127808

3.4 Convolution Details

- 3.4.1 Biases

```
bias = tf.Variable(tf.zeros [16])
convOut += bias
```

  - Number of output filters = 16

- 3.4.2 Layers with Convolution

```
tf.contrib.layers.conv2d(inpt, numFlts, fltDim, strides, pad)
convOut = layers.conv2d(image, 16, [4,4], 2, "Same")
```

- 3.4.3 Pooling

```
convOut = tf.nn.conv2d(image, flts, [1,1,1,1], "SAME")
convOut = tf.nn.max pool(convOut, [1,2,2,1], [1,2,2,1], "SAME")
```