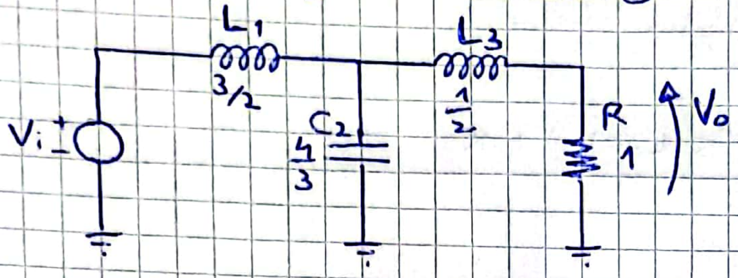
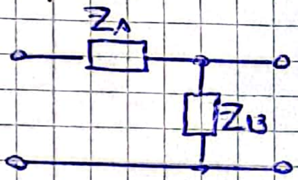
Ejercicio 2)

Dado el siguiente circuito

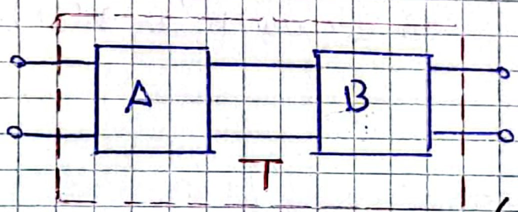


a) Obtener la transferencia de tensión $\frac{V_o}{V_i}$ por métodos de cuadripolos

El circuito puede entenderse como la conexión de dos cuadripolos L en cascada.



$$T = \begin{pmatrix} \frac{Z_A + Z_B}{Z_B} & Z_A \\ \frac{1}{Z_B} & 1 \end{pmatrix}$$



$$T_T = T_A \cdot T_B = \begin{pmatrix} \frac{SL_1 + \frac{1}{SC_2}}{1/SC_2} & SL_1 \\ SC_2 & 1 \end{pmatrix} \begin{pmatrix} \frac{SL_3 + R}{R} & SL_3 \\ \frac{1}{R} & 1 \end{pmatrix}$$

$$T_T = \begin{pmatrix} S^2C_2L_1 + 1 & SL_1 \\ SC_2 & 1 \end{pmatrix} \begin{pmatrix} \frac{SL_3}{R} + 1 & SL_3 \\ \frac{1}{R} & 1 \end{pmatrix}$$

$$T_T = \begin{pmatrix} (S^2C_2L_1+1)\left(\frac{SL_3}{R}+1\right) + \frac{SL_1}{R} & (S^2C_2L_1+1)\,SL_3 + SL_1 \\ SC_2\left(\frac{SL_3}{R}+1\right) + \frac{1}{R} & S^2L_3C_2 + 1 \end{pmatrix}$$

Notas:

Para obtener la transferencia de tensiones me interesa el parámetro A.

$$A = \frac{V_i}{V_o} = \left(s^2 C_2 L_1 + 1\right)\left(\frac{sL_3}{R} + 1\right) + \frac{sL_1}{R}$$

$$A = \frac{s^3 C_2 L_1 L_3}{R} + s^2 C_2 L_1 + \frac{sL_3}{R} + \frac{sL_1}{R} + 1$$

$$A = \frac{s^3 C_2 L_1 L_3 + s^2 C_2 L_1 R + s(L_3 + L_1) + R}{R}$$

$$\frac{V_o}{V_i} = \frac{1}{A} = \frac{R}{s^3 C_2 L_1 L_3 + s^2 C_2 L_1 R + s(L_3 + L_1) + R}$$

Reemplazando valores:

$$T(s) = \frac{1}{s^3 \cdot \frac{4}{3} \cdot \frac{3}{2} \cdot \frac{1}{2} + s^2 \, \frac{4}{3} \cdot \frac{3}{2} \cdot 1 + s\left(\frac{1}{2} + \frac{3}{2}\right) + 1}$$

$$\boxed{T(s) = \frac{1}{s^3 + s^2 \cdot 2 + s \cdot 2 + 1}}$$

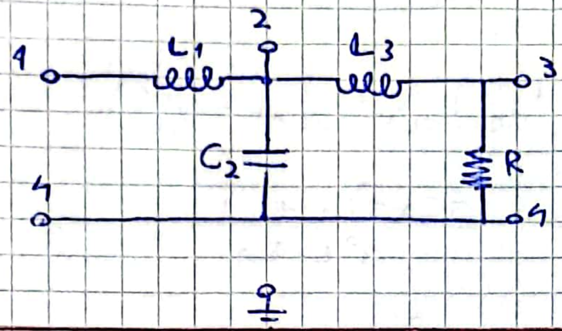
Filtro pasabajo Butterworth de 3er orden

Hoja:

Fecha:

(b) Construya la matriz de admitancia indefinida (MAI) del circuito.

Debo desreferenciar el circuito



$$MAI = \begin{bmatrix} \frac{1}{sL_1} & -\frac{1}{sL_1} & 0 & 0 \\ -\frac{1}{sL_1} & sC_2+\frac{1}{sL_1}+\frac{1}{sL_3} & -\frac{1}{sL_3} & -sC_2 \\ 0 & -\frac{1}{sL_3} & \frac{1}{sL_3}+\frac{1}{R} & -\frac{1}{R} \\ 0 & -sC_2 & -\frac{1}{R} & sC_2+\frac{1}{R} \end{bmatrix}$$

(c) Compute la transferencia de tensión con la MAI

$$A^{34}_{14} = sgn(3-4).sgn(1-4)\ \frac{Y^{14}_{34}}{Y^{14}_{14}}$$

$$A^{34}_{14} = \frac{V_{34}}{V_{14}} = sgn(-1).sgn(-3)\ .\ \frac{Y^{14}_{34}}{Y^{14}_{14}}$$

$$\frac{V_{34}}{V_{14}} = (-1).(-1).\frac{Y^{14}_{34}}{Y^{14}_{14}} = \frac{Y^{14}_{34}}{Y^{14}_{14}}$$

Notas:

Calculo los cofactores de

$$\underline{Y}^{34}_{14} = \left(-\frac{1}{sL_1}\right)\left(-\frac{1}{sL_3}\right) = \frac{1}{s^2 L_1 L_3} \,//$$

$$\underline{Y}^{14}_{14} = \left(sC_2 + \frac{1}{sL_1} + \frac{1}{sL_3}\right)\left(\frac{1}{sL_3} + \frac{1}{R}\right) - \left(\frac{1}{sL_3}\right)^2$$

$$\underline{Y}^{14}_{14} = \left(sC_2 + \frac{1}{sL_1} + \frac{1}{sL_3}\right)\frac{1}{R} + \left(sC_2 + \frac{1}{sL_1}\right)\frac{1}{sL_3} + \cancel{\left(\frac{1}{sL_3}\right)^2} - \cancel{\left(\frac{1}{sL_3}\right)^2}$$

$$\underline{Y}^{14}_{14} = \frac{sC_2 . s^2 L_1 L_3 \cancel{R} + sL_3 + sL_1}{R\; s^2 L_1 L_3} + \frac{sC_2 s^2 L_1 + 1}{s^2 L_1 L_3}$$

$$\underline{Y}^{14}_{14} = \frac{s^3 C_2 L_1 L_3 + s(L_3 + L_1)}{R\; s^2 L_1 L_3} + \frac{s^2 C_2 L_1 R + R}{R\; s^2 L_1 L_3}$$

$$\underline{Y}^{14}_{14} = \frac{s^3 C_2 L_1 L_3 + s^2 C_2 L_1 R + s(L_3 + L_1) + R}{R\; s^2 L_1 L_3} \,//$$

Entonces

$$T(s) = \frac{\dfrac{1}{s^2 L_1 L_3}}{\dfrac{s^3 C_2 L_1 L_3 + s^2 C_2 L_1 R + s(L_1 + L_3) + R}{R\; s^2 L_1 L_3}}$$

$$\boxed{T(s) = \frac{R}{s^3 C_3 L_1 L_3 + s^2 C_2 L_1 R + s(L_1 + L_3) + R}}$$

Verifico que la transferencia obtenida en el punto a).

		Hoja:
		Fecha:

Plus) Computo la impedancia de entrada con la MAI

$$Z_{11} = \frac{V_{11}}{I_{11}} = \frac{Y^{11}_{11}}{Y^{1}_{1}}$$

$$Y^{11}_{11} = \frac{s^3 C_2 L_1 L_3 + s(L_3 + L_1) + s^2 C_2 L_1 R + R}{R\, s^2 L_1 L_3} \quad //$$

$$Y^{1}_{1} = \left(sC_2 + \frac{1}{sL_1} + \frac{1}{sL_3}\right)\left[\left(\frac{1}{sL_3} + \frac{1}{R}\right)\left(sC_2 + \frac{1}{R}\right) - \frac{1}{R^2}\right]$$

$$+ \frac{1}{sL_3}\left[-\frac{1}{sL_3}\left(sC_2 + \frac{1}{R}\right) - \frac{sC_2}{R}\right]$$

$$- sC_2\left[\frac{1}{sL_3 R} + sC_2\left(\frac{1}{sL_3} + \frac{1}{R}\right)\right]$$

$$Y^{1}_{1} = \left(\cancel{sC_2} + \frac{1}{sL_1} + \cancel{\frac{1}{sL_3}}\right)\left[\frac{C_2}{L_3} + \frac{1}{sL_3 R} + \frac{sC_2}{R} + \cancel{\frac{1}{R^2}} - \cancel{\frac{1}{R^2}}\right]$$

$$\cancel{+ \frac{1}{sL_3}\left[-\frac{C_2}{L_3} - \frac{1}{sL_3 R} - \frac{sC_2}{R}\right]}$$

$$\cancel{- sC_2\left[\frac{1}{sL_3 R} + \frac{C_2}{L_3} + \frac{sC_2}{R}\right]}$$

$$Y^{1}_{1} = \frac{1}{sL_1}\left[\frac{C_2}{L_3} + \frac{1}{sL_3 R} + \frac{sC_2}{R}\right]$$

$$Y^{1}_{1} = \left[\frac{sC_2 R + 1 + s^2 C_2 L_3}{sL_3 R}\right]\frac{1}{sL_1}$$

$$Y^{1}_{1} = \frac{s^2 C_2 L_3 + sC_2 R + 1}{R\, s^2 L_1 L_3} \quad //$$

Notas:

$$Z_{in} = \frac{\dfrac{s^3 C_2 L_1 L_3 + s^2 C_2 L_1 R + s(L_1+L_3) + R}{\cancel{s^2 L_1 L_3 R}}}{\dfrac{s^2 C_2 L_3 + s C_2 R + 1}{\cancel{s^2 L_1 L_3 R}}}$$

$$\boxed{Z_{in} = \frac{s^3 C_2 L_1 L_3 + s^2 C_2 L_1 R + s(L_1+L_3) + R}{s^2 C_2 L_3 + s C_2 R + 1}}$$