② Dado la siguiente transferencia de impedancia:

$$T(s) = \frac{V_2}{I_1} = \frac{k(s^2+9)}{s^3+2s^2+2s+1}$$

a) Sintetizar un cuadripoloso pasivo sin pérdidas, que cumpla con la transimpedancia indicada

Primero debo vincular la transferencia con una función de excitación.

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \qquad ; \text{ donde } I_2 = \frac{-V_2}{R_L}$$

$$V_2\left(1 + \frac{Z_{22}}{R_L}\right) = Z_{21} I_1$$

$$\frac{V_2}{I_1} = \frac{Z_{21}}{1+\frac{Z_{22}}{R_L}} = T(s) \qquad ; \text{ Trabajo con } R_L \text{ normalizado } (R_L = 1$$

$$T(s) = \frac{V_2}{I_1} = \frac{Z_{21}}{1+Z_{22}} = \frac{k(s^2+9)}{s^3+2s^2+2s+1}$$

Saco la parte impar del denominador

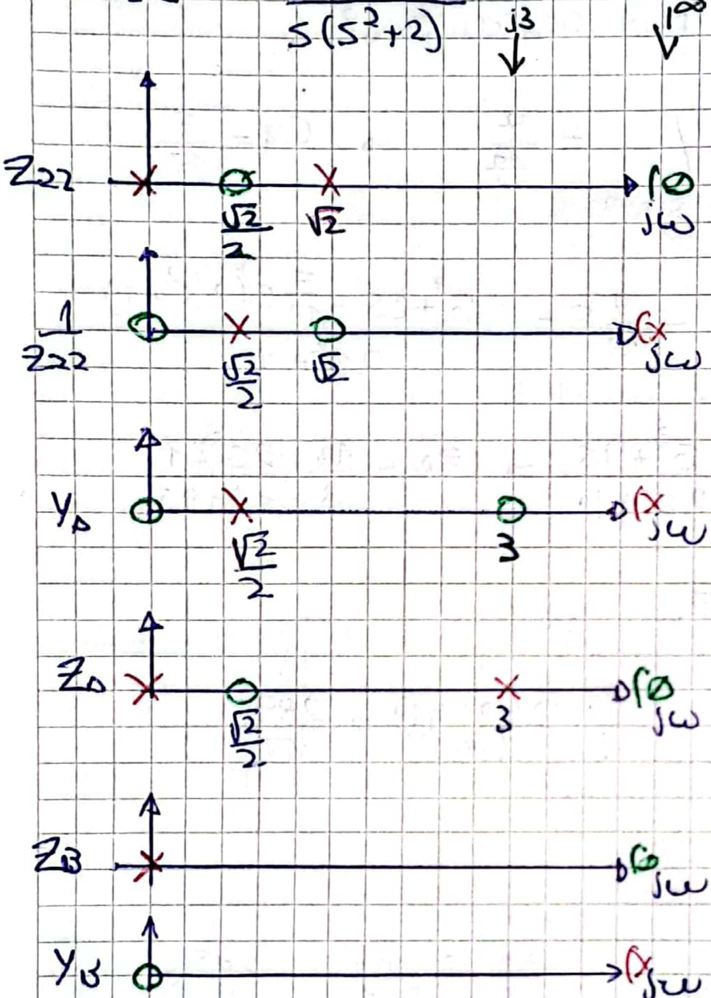
$$T(s) = \frac{k(s^2+9)}{\frac{s^3+2s^2+2s+1}{s^3+2s}} \cdot \frac{1}{s^3+2s}$$

$$T(s) = \frac{\frac{k(s^2+9)}{s^3+2s}}{\frac{2s^2+1}{s^3+2s}+1} \qquad ; \text{ con lo cual } \qquad Z_{21} = \frac{k(s^2+9)}{s^3+2s}$$

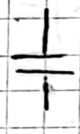
$$Z_{22} = \frac{2s^2+1}{s^3+2s}$$

Síntesis gráfica

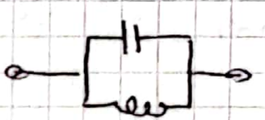
$$Z_{22} = 2\,\frac{\left(s^2+\frac{1}{2}\right)}{s\left(s^2+2\right)}$$



remoción parcial

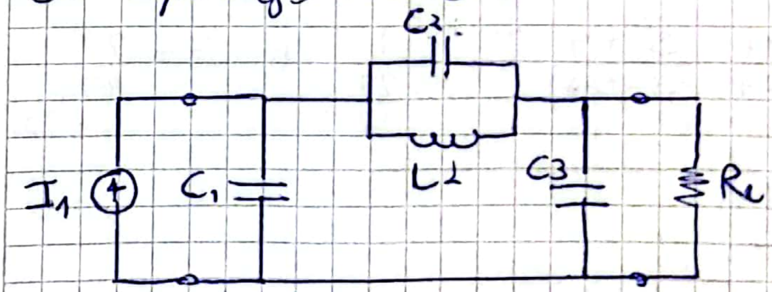


removo un tanque LC en serie



* El elemento de apertura debe estar en derivación debido a que está con un generador de corriente

La topología resulta



Síntesis analítica

$$Z_{22} = \frac{2s^2+1}{s^3+2s} \rightarrow \frac{1}{Z_{22}} = \frac{s^3+2s}{2s^2+1}$$

remuevo parcialmente en infinito para fijar un cero en $j3$

$$K'_\infty = \left.\frac{1}{s \cdot Z_{22}}\right|_{s^2=-9} = \left.\frac{1}{s}\cdot\frac{s(s^2+2)}{2s^2+1}\right|_{s^2=-9} = \frac{7}{17} \rightarrow C_3 = \frac{7}{17}$$

$$Y_A = \left(\frac{1}{Z_{22}} - K'_\infty s\right) = \frac{s^3+2s}{2s^2+1} - \frac{7}{17}s = \frac{s^3+2s-\frac{7}{17}s(2s^2+1)}{2s^2+1}$$

$$Y_A = \frac{\frac{3}{17}s^3+\frac{27}{17}s}{2s^2+1} = \frac{3}{17}\,\frac{s^3+9s}{2s^2+1} \rightarrow Z_A = \frac{17}{3}\,\frac{2s^2+1}{s^3+9s}$$

Remuevo el polo en $j3$ de Z_A

$$2K_1 = \lim_{s^2\to-9} \frac{(\cancel{s^2+9})}{s}\cdot\frac{17}{3}\,\frac{2s^2+1}{s(\cancel{s^2+9})} = \frac{17}{3}\,\frac{(2(-9)+1)}{(-9)} = \frac{289}{27}$$

$$C_2 = \frac{1}{2K_1} = \frac{27}{289} \quad ; \quad L_2 = \frac{289}{243}$$

$$Z_B = \frac{17}{3}\,\frac{2s^2+1}{s^3+9s} - \frac{\frac{289}{27}s}{(s^2+9)}\cdot\frac{s}{s} = \frac{\frac{34}{3}s^2+\frac{17}{3}-\frac{289}{27}s^2}{s^3+9s}$$

$$Z_B = \frac{\frac{17}{27}s^2+\frac{17}{3}}{s^3+9s} = \frac{17}{27}\,\frac{\cancel{s^2+9}}{s(\cancel{s^2+9})} = \frac{1}{s\,\frac{27}{17}} \Rightarrow C_1 = \frac{27}{17}$$

Lo síntesis resulta

b) Verificar la transimpedancia del circuito obtenido

$$T(S) = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{1}{C}$$

no es I_2 I_N

$$T_T = \begin{pmatrix} 1 & 0 \\ S.\frac{27}{17} & 1 \end{pmatrix} \begin{pmatrix} 1 & \dfrac{1}{S.\frac{27}{289} + \frac{1}{S\frac{289}{243}}} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ S\frac{7}{17}+1 & 1 \end{pmatrix}$$

$$T_T = \begin{pmatrix} 1 & 0 \\ S.\frac{27}{17} & 1 \end{pmatrix} \begin{pmatrix} 1 + \left(S\frac{7}{17}+1\right)\left(\dfrac{289}{S\,27 + \frac{243}{S}}\right) & - \\ S\frac{7}{17}+1 & - \end{pmatrix}$$

$$C = S\frac{27}{17}\left[1 + \left(S\frac{7}{17}+1\right)\left(\frac{289}{S\,27 + \frac{243}{S}}\right)\right] + S\frac{7}{17} + 1$$

$$C = S\frac{27}{17}\left[1 + \left(S\frac{7}{17}+1\right)\frac{S\,289}{S^2 27 + 243}\right] + S\frac{7}{17} + 1$$

$$C = S\frac{27}{17}\left[\frac{27S^2 + 243 + 119S^2 + 289S}{27S^2 + 243}\right] + S\frac{7}{17} + 1$$

$$C = \frac{S\frac{27}{17}}{27}\left[\frac{146S^2 + 289S + 243}{\left(S^2 + \frac{243}{27}\right)}\right] + S\frac{7}{17} + 1$$

$$C = \frac{\left(\frac{146}{17}S^3 + 17S^2 + \frac{243}{17}S\right) + \left(S\frac{7}{17}+1\right)\left(S^2 + \frac{243}{27}\right)}{S^2 + \frac{243}{27}}$$

$$C = \frac{\frac{146}{17}S^3 + 17S^2 + \frac{243}{17}S + \frac{7}{17}S^3 + \frac{63}{17}S + S^2 + \frac{243}{27}}{S^2 + \frac{243}{27}}$$

$$C = \frac{9S^3 + 18S^2 + 18S + 9}{S^2 + 9}$$

$$C = 9 \frac{s^3 + 2s^2 + 2s + 1}{s^2 + 9}$$

$$\boxed{T(s) = \frac{1}{C} = \frac{1}{9} \frac{s^2 + 9}{s^3 + 2s^2 + 2s + 1}}$$