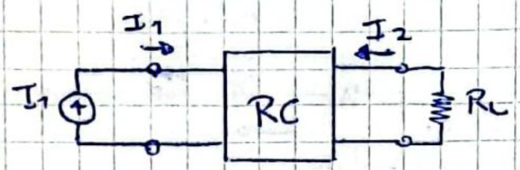
Trabajo Semanal 12

Síntesis de cuadripolos simplemente cargadas

1



$$T(s) = \frac{-I_2}{I_1} = H \cdot \frac{s^2+5s+4}{s^2+8s+12} \quad \rightarrow \text{condición: } \left.\frac{-I_2}{I_1}\right|_{V_2 = (-I_2)R_L}$$

$$Z_{21} = 6H$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$-I_2 R_L = Z_{21} I_1 + Z_{22} I_2$$

$$-I_2 (R_L + Z_{22}) = Z_{21} I_1$$

$$\frac{-I_2}{I_1} = \frac{Z_{21}}{R_L + Z_{22}}$$

Trabajo normalizado por R_L; es decir $R_L = 1$

$$T(s) = \frac{Z_{21}}{1 + Z_{22}} \Rightarrow Z_{22} = \frac{Z_{21}}{T(s)} - 1$$

$$Z_{22} = \frac{6H}{H \frac{s^2+5s+4}{s^2+8s+12}} - 1 = \frac{6s^2+48s+72 - s^2 - 5s - 4}{s^2+5s+4}$$

$$Z_{22} = \frac{5s^2+43s+68}{s^2+5s+4} = 5 \, \frac{s^2 + \frac{43}{5}s + \frac{68}{5}}{s^2+5s+4}$$

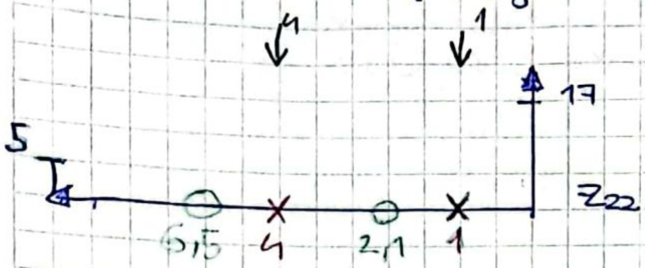
$$Z_{22} = 5 \, \frac{(s+2{,}1)(s+6{,}5)}{(s+4)(s+1)}$$

Cumple con las propiedades de una Z_{RC}?

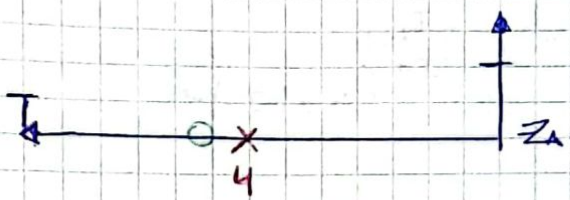
Hay alternancia ✓

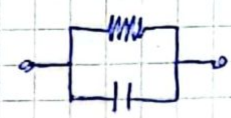
$Z_{22}(0) > Z_{22}(\infty)$ ✓

2) Obtener la topología mediante la síntesis gráfica

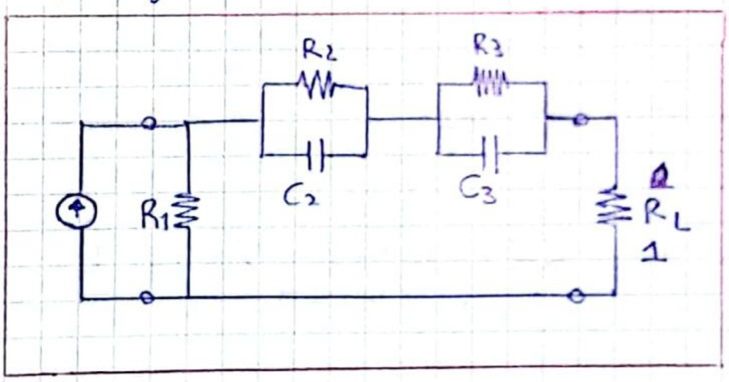


Remuevo el polo en 1 -





La topología circuital resulta



b) Calcular el valor de los componentes (síntesis analítica)

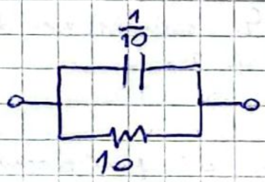
$$Z_{22} = \frac{5(S+2{,}1)(S+6{,}5)}{(S+1)(S+4)} = \frac{5S^2+43S+68}{(S+1)(S+4)}$$

Remuevo polo en $\sigma = -1$

$$K_1 = \lim_{S \to -1} (S+1)\,\frac{5(S+2{,}1)(S+6{,}5)}{(S+1)(S+4)} = \frac{5(-1)^2+43(-1)+68}{(-1+4)}$$

$K_1 = 10$

$$\frac{10}{S+1} = \frac{1}{S.\frac{1}{10}+\frac{1}{10}}$$



$$Z_A(s) = Z_{22}(s) - \frac{10}{(S+1)} = \frac{5S^2+43S+68}{(S+1)(S+4)} - \frac{10(S+4)}{(S+1)(S+4)}$$

$$Z_A(S) = \frac{5S^2+33S+28}{(S+1)(S+4)} = \frac{5.\cancel{(S+1)}\left(S+\frac{28}{5}\right)}{\cancel{(S+1)}(S+4)}$$

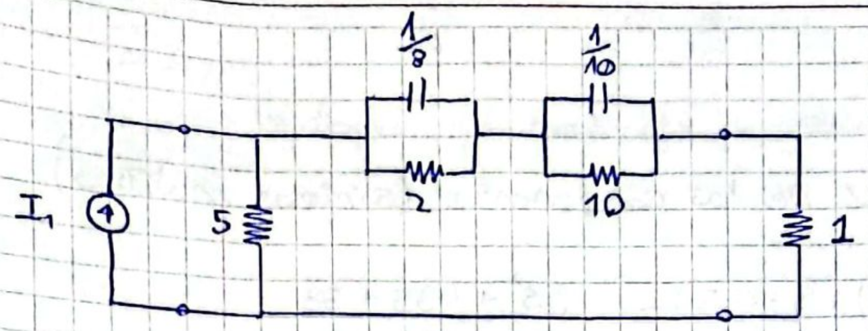
Remuevo polo en $\sigma = -4$

$$K_2 = \lim_{S \to -4} \cancel{(S+4)}\,\frac{5\left(S+\frac{28}{5}\right)}{\cancel{(S+4)}} = 5\left(-4+\frac{28}{5}\right) = 8$$

$$\frac{8}{S+4} = \frac{1}{S.\frac{1}{8}+\frac{4}{8}} = \frac{1}{S\frac{1}{8}+\frac{1}{2}} \rightarrow$$

$$Z_B(S) = Z_A(S) - \frac{8}{(S+4)} = \frac{5S+28}{(S+4)} - \frac{8}{(S+4)} = \frac{5S+20}{(S+4)} = 5\,\frac{\cancel{(S+4)}}{\cancel{(S+4)}}$$

$Z_B(S) = 5 \rightarrow$

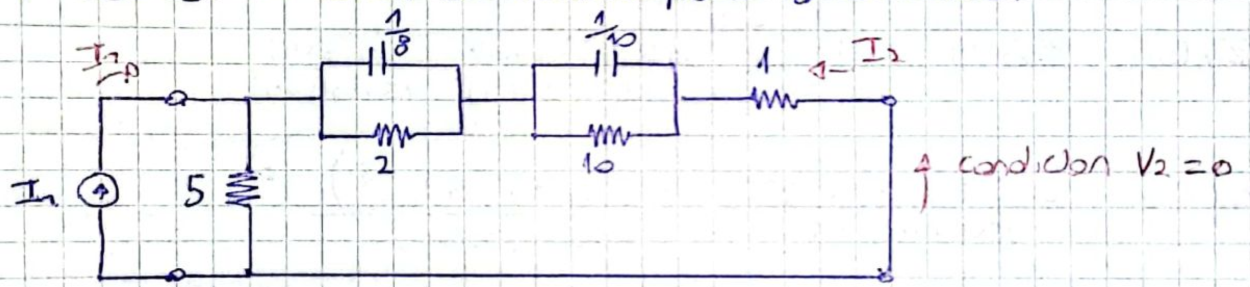


ⓒ Verificar la red hallada y averiguar el valor de H.

Por parámetros T, se tiene que

$$D = \left.\frac{I_1}{(-I_2)}\right|_{V_2=0}$$, la condición de medición me anularía la R_L.

Para poder trabajar con el parámetro D, se trata a la R_L como interno al cuadripolo y en serie.



$$T_T = \begin{pmatrix} 1 & 0 \\ \frac{1}{5} & 1 \end{pmatrix} \begin{pmatrix} 1 & \frac{8}{s+4} + \frac{10}{s+1} + 1 \\ 0 & 1 \end{pmatrix}$$

$$D = \frac{1}{5}\left(\frac{8}{s+4} + \frac{10}{s+1} + 1\right) + 1$$

$$D = \frac{8(s+1) + 10(s+4) + (s+4)(s+1)}{5(s+4)(s+1)} + 1$$

$$D = \frac{s^2 + 23s + 52}{5(s+4)(s+1)} + 1$$

$$D = \frac{s^2 + 23s + 52 + 5(s^2 + 5s + 4)}{5(s^2 + 5s + 4)}$$

$$D = \frac{6s^2 + 48s + 72}{5(s^2 + 5s + 4)} = \frac{6}{5} \, \frac{s^2 + 8s + 12}{s^2 + 5s + 4}$$

$$\boxed{T(s) = \frac{1}{D} = \frac{(-I_2)}{I_1} = \frac{5}{6} \, \frac{s^2 + 5s + 4}{s^2 + 8s + 12} \; // \qquad H = \frac{5}{6}}$$