Some Basic Relationships between Pixels

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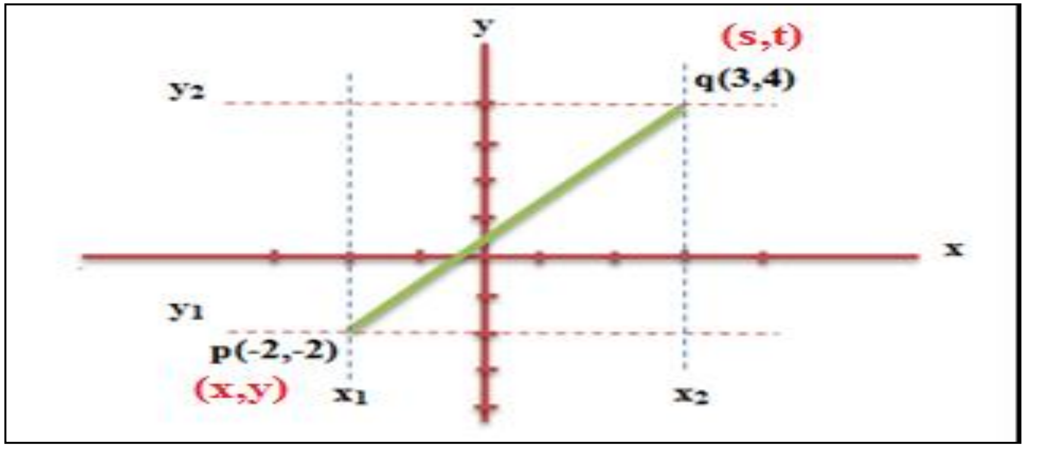

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Region

Let *R* be a subset of pixels in an image, we call *R* a region of the image if *R* is a connected set.

000000
010010
011010
010110
000000

Boundary

The boundary (also called *border* or *contour*) of a region *R* is the set of pixels in the region that have one or more neighbors that are not in *R*.

If R happens to be an entire image, then its boundary is defined as the set of pixels in the first and last rows and columns of the image.

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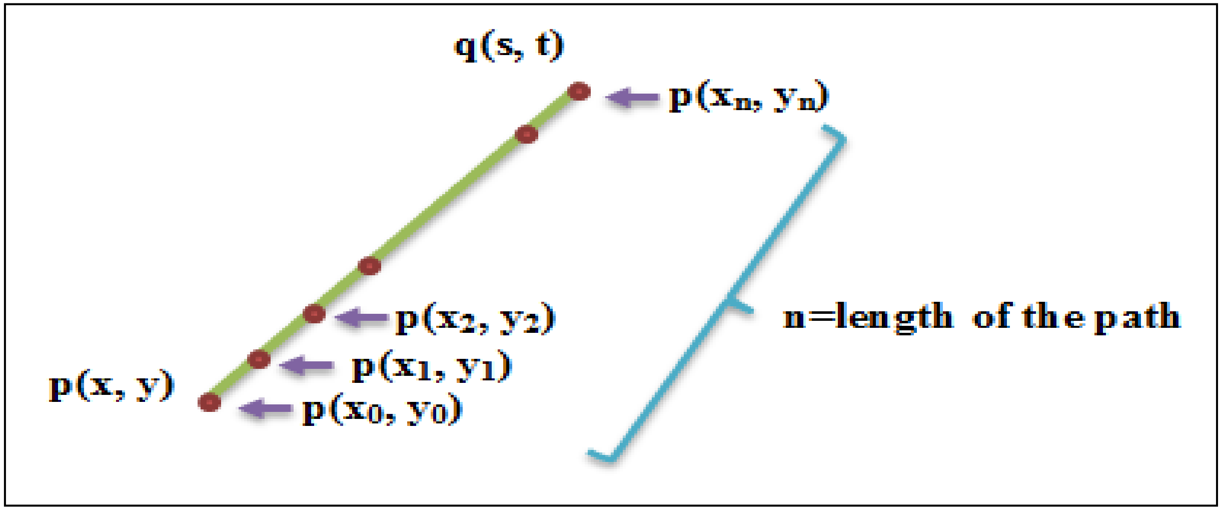
Digital Path

A (digital) path (or curve) from pixel p with coordinates (x , y) to pixel q with coordinates (s , t) is a sequence of distinct pixels with coordinates

$$(X_0 , Y_0) , (X_1 , Y_1) \ldots\ldots (X_n , Y_n)$$

Where (X_i , Y_i) and (X_{i-1} , Y_{i-1}) are adjacent for $1 \le i \le n$.

- **Here n is the length of the path.**
- **If $(X_0 , Y_0) = (X_n , Y_n)$), the path is closed path.**
- **We can define 4-, 8-, and m-paths based on the type of adjacency used.**



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Example : consider the image segment shown in figure (a). Compute length of shortest -4, shortest -8, shortest –m paths between pixels p & q where v={1,2}

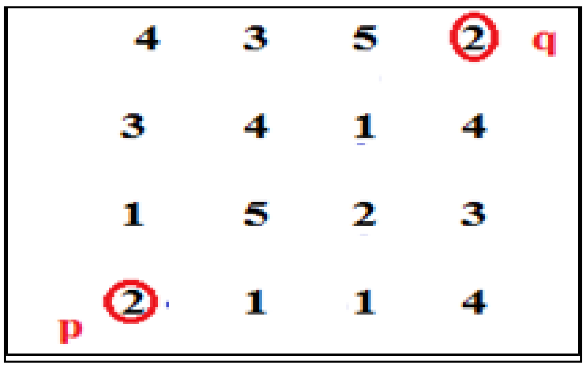
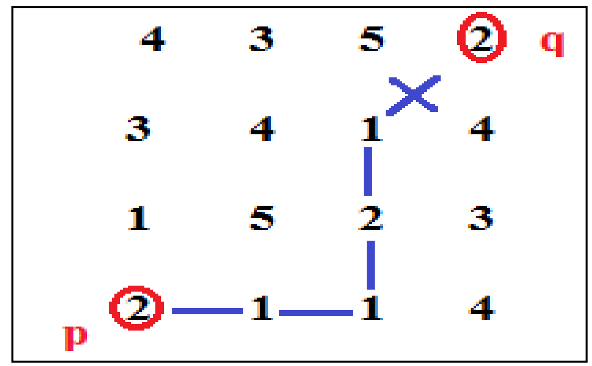
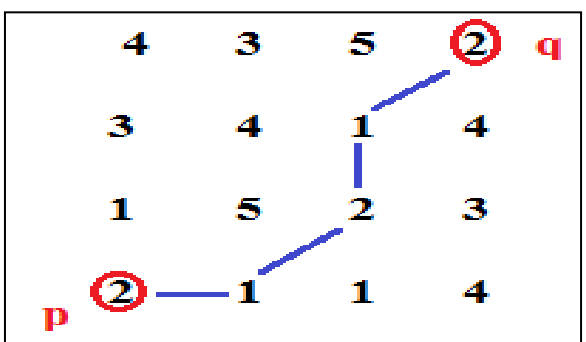


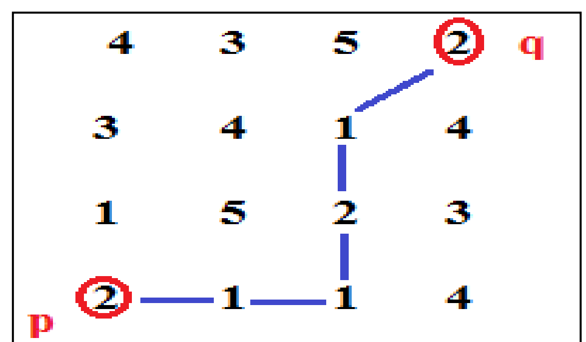
figure (a)



Shortest-4 Path does not exist



shortest-8 path = 4



shortest–m Path = 5

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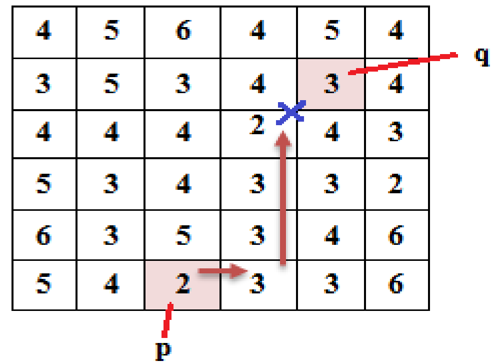
Example : **consider the image segment shown in figure (a). Compute length of shortest -4, shortest -8, shortest –m paths between pixels p & q where v={2, 3}**

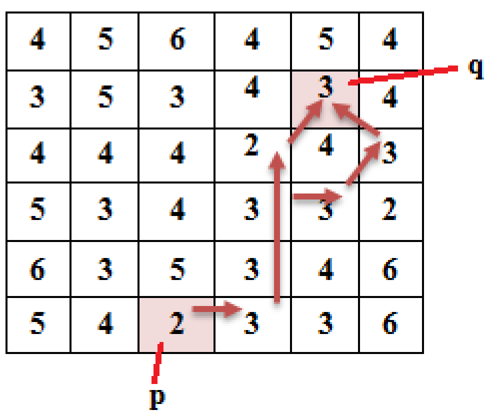
4	5	6	4	5	4
3	5	3	4	3	4
4	4	4	2	4	3
5	3	4	3	3	2
6	3	5	3	4	6
5	4	2	3	3	6

q

p **figure (a)**

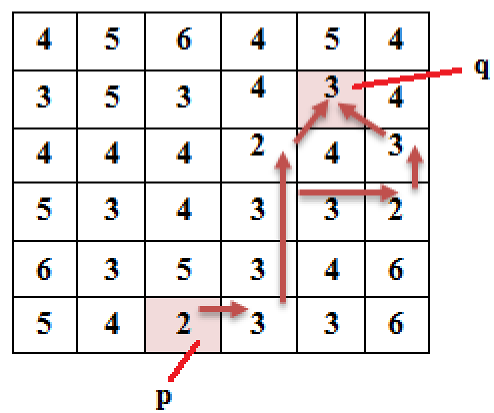


Shortest-4 Path does not exist



shortest-8 path = 5

= 6



shortest–m Path = 5

=7

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Distance Measures

Assuming there are two image points with coordinates (x, y) and (s, t). a distance measure is normally conduced for evaluating how close two these two pixels are and how they are related. A number of distance measurements have been commonly used for this purpose:

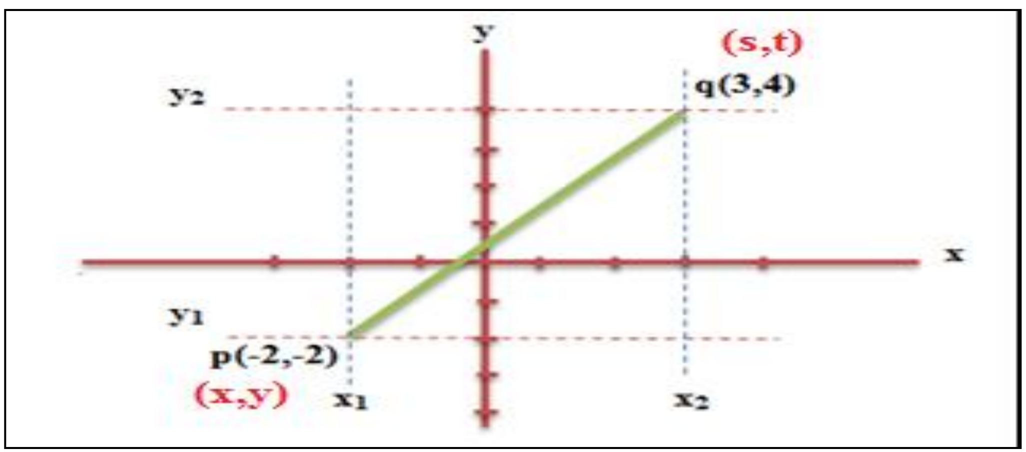
1. The Euclidean distance between two 2-D points I(x , y)and J(s , t) is defined as:

$$D_e(p,q) = \sqrt{(x-s)^2 + (y-t)^2}$$

Where:

$x = x_1$, $y = y_1$

$s = x_2$, $t = y_2$



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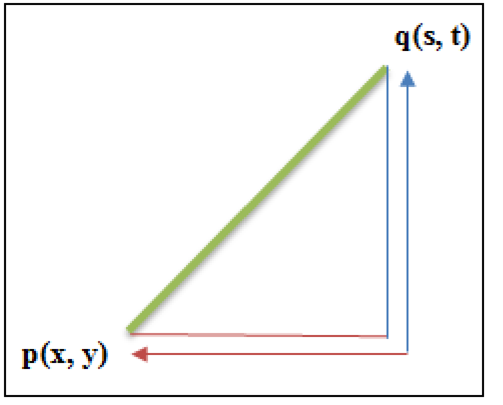
2. **The City-Block distance between two 2-D points I(x, y)and J(s, t)can be calculated as follows:**

$$D_4(p,q) = |x-s| + |y-t|$$



Example

The pixels with D_4 distance ≤ 2 from (x, y) (the center point) from the following contours of constant distance:

The pixels with D_4 =1 are the 4-nieghbors of (x, y).

2	1	2
1	1	1
2	1	2

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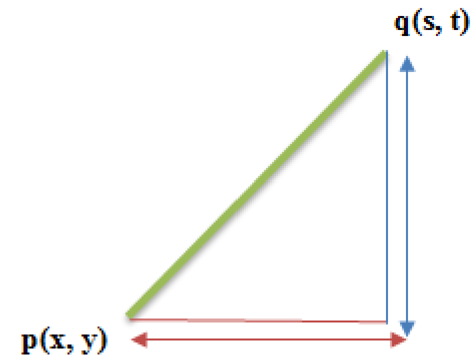
3. **The D_8 distance (called the chessboard distance)between p and q is defined as:**



$$D_8(p, q) = max\,(|x - s|, |y - t|)$$

In this case, the pixels having a D_8 distance from (x, y) less than or equal to some value r from a square centered at (x, y).

Example

The pixels with D_8 distance ≤ from (x, y) (the center point) from the following contours of constant distance

2	2	2	2	2
2	1	1	1	2
2	1	1	1	2
2	1	1	1	2
2	2	2	2	2

The pixels with D_8 =1 are the 8-nieghbors of (x, y).

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4. D_m-distance between two points is defined as the shortest m-path between the points.

In this case the distance two pixels will depend on the values of the pixels along the path as well as the values of their neighbors.

Example

Consider the following arrangement of pixels and assume that P, P_2, and P_4 have value 1 and that P_1 and P_3 can have a value of 0 or 1:

	P_3	P_4
P_1	P_2	
P		

Suppose that we consider adjacency of pixels valued 1 (i.e., v={1})

1- If P_1 and P_3 are 0, The m-path(D_m distance between P and P_4 is <u>2</u>
2- If P_1 = 1 and P_3 = 0 The D_m distance between P and P_1 P_2 P_4 is <u>3</u>
3- if P_3=1 and P_1 = 0 The D_m distance between P and P_2 P_3 P_4 is <u>3</u>
4- if P_3=1 and P_1 = 1 The D_m distance between P and P_1 P_2 P_3 P_4 is <u>4</u>

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Distance Metric	Description	Illustration
Euclidean P(x , y), q(s , t)	The Euclidean distance is the straight-line distance between two pixels. $\mathrm{D_e}(p,q) = \sqrt{(x-s)^2 + (y-t)^2}$	Image: 0 | 0 | 0 0 | 1 | 0 0 | 0 | 0 Distance transform: 1.41 | 1 | 1.41 1 | 0 | 1 1.41 | 1 | 1.41
City Block P(x , y), q(s , t)	The city block distance metric measures the path between the pixels based on a 4-connected neighborhood. Pixels whose edges touch are 1 unit apart and pixels diagonally touching are 2 units apart. $\mathrm{D_4}(p,q) = \lvert x-s\rvert + \lvert y-t\rvert$	Image: 0 | 0 | 0 0 | 1 | 0 0 | 0 | 0 Distance transform: 2 | 1 | 2 1 | 0 | 1 2 | 1 | 2
Chessboard P(x , y), q(s , t)	The chessboard distance metric measures the path between the pixels based on an 8-connected neighborhood. Pixels whose edges or corners touch are 1 unit apart. $\mathrm{D_8}(p,q) = max\,(\lvert x-s\rvert, \lvert y-t\rvert)$	Image: 0 | 0 | 0 0 | 1 | 0 0 | 0 | 0 Distance transform: 1 | 1 | 1 1 | 0 | 1 1 | 1 | 1

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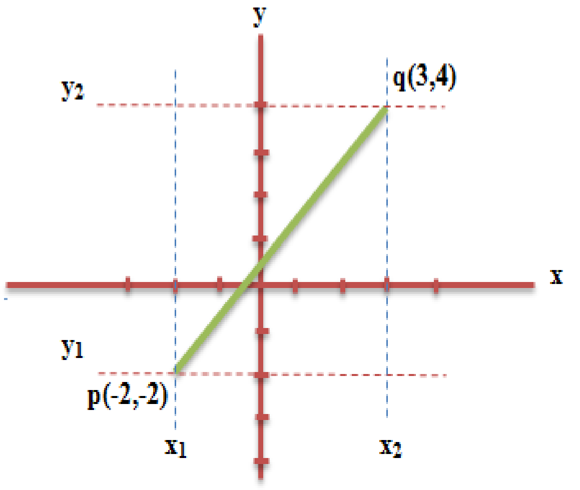
Example : **Suppose p and q are two pixels. Calculate the distance measures D_4 , D_8 and D_e**

1. $D_e(p,q) = \sqrt{(x-s)^2 + (y-t)^2}$

P(3,4) and q(-2,-2)

$$D_e(p,q) = \sqrt{((-2)-3))^2 + ((-2)-4))^2}$$
$$D_e(p,q) = \sqrt{(-5)^2 + (-6)^2}$$
$$D_e(p,q) = \sqrt{25+36}$$
$$D_e(p,q) = \sqrt{61}$$
$$D_e(p,q) = 7.8$$



2. $D_4(p,q) = |x-s| + |y-t|$

= |(-2)-3)| + | (-2)-4|

= 5+6

= 11

3. $D_8(p,q) = max\,(|(x-s)|\,,|\,(y-t)|)$

= max (|(-2)-3| , |(-2)-4|)

= max (5,6)

= 6