

# 1 Introduction

Let  $\Omega = (a_x, b_x) \times (a_y, b_y) \times (a_z, b_z) \in \mathbb{R}^3$  with its boundary  $\Gamma = \partial\Omega$  and  $J = (0, T]$  be the time interval,  $T > 0$ . The following Initial Boundary Value Problem (IBVP) is considered:

$$u_t = \nabla \cdot (a \nabla u) + c_g \nabla \cdot (u \nabla g) + a_h \nabla \cdot (u \nabla h) + c(t, x, y)u + f, \quad t \in J, \quad (x, y, z) \in \Omega \quad (1)$$

$$u(0, x, y, z) = u_0(x, y, z) \quad (2)$$

$$u_x = u_y = u_z = 0, \quad (x, y, z) \in \Gamma \quad (3)$$

$$g_x = g_y = g_z = 0, \quad (x, y, z) \in \Gamma \quad (4)$$

$$h_x = h_y = h_z = 0, \quad (x, y, z) \in \Gamma \quad (5)$$

## 2 ADI Method implementation.

Alternating Direction Implicit method is a computationally efficient scheme, that satisfies the following properties:

- has accuracy  $O(k^2 + h^2)$ ,  $k$  is the time step and  $h$  the spatial step;
- is unconditionally stable;
- number of operations per time step is proportional to the number of unknowns,  $O(M)$  where  $M$  is the number of unknowns.

### 2.1 Douglas-Gunn method

Let us define the operators:

$$\begin{aligned} A_1 u &= -(au_x)_x - c_g(ug_x)_x - c_h(uh_x)_x - \frac{1}{3}cu, \\ A_2 u &= -(au_y)_y - c_g(ug_y)_y - c_h(uh_y)_y - \frac{1}{3}cu, \\ A_3 u &= -(au_z)_z - c_g(ug_z)_z - c_h(uh_z)_z - \frac{1}{3}cu, \end{aligned}$$

then equation can be rewritten as:

$$u_t + A_1 u + A_2 u + A_3 u = f. \quad (6)$$

Replacing the operators  $A_1$ ,  $A_2$  and  $A_3$  (6) respectively by their spatial approximations  $A_{1h}$ ,  $A_{2h}$  and  $A_{3h}$ , the proposed Douglas-Gunn method is:

$$\begin{aligned} (I + \frac{k}{2}A_{1h})u^* &= (I - \frac{k}{2}A_{1h} - kA_{2h} - kA_{3h})u^n + kf^n, & (\text{x-sweep}) \\ (I + \frac{k}{2}A_{2h})u^{**} &= u^* + \frac{k}{2}A_{2h}u^n, & (\text{y-sweep}) \\ (I + \frac{k}{2}A_{3h})u^{n+1} &= u^{**} + \frac{k}{2}A_{3h}u^n + \frac{k}{2}(f^{n+1} - f^n), & (\text{z-sweep}). \end{aligned}$$

L<sup>A</sup>T<sub>E</sub>X