PHI251 - Logic

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# Chapter 1: Fundamental Concepts and Truth-Functional Analysis

## Lesson 1.1: Introduction

- Argument: When reasons are given to justify a belief
- Argument Indicator Words: indicate that an conclusion is being presented
  - So
  - Hence
  - Thus
  - Therefore
  - It must be that
- Premise Indicator Words: indicate that a premise is being presented
  - For
  - Since
  - Because
  - Due to the fact that

#### Validity

- In a valid argument, if the premises were all true, the conclusion would also be true
- In a valid argument, it is not possible for the conclusion to be false when all the premises are true

### • Invalidity

- When the premises are true and the conclusion is false.
- When an argument's premises do not truly prove it's conclusion.
- Soundness: A sound argument is an argument with both of the following features:
  - It is valid
  - All of its premises are true

#### • Validity and Consistency

- Any argument is valid if and only if it would be inconsistent to assert all of it's premises but deny it's conclusion.

#### • Considering the possibilities

- To establish that an argument is valid, it seems that we must somehow show that no possible situation exists in which the premises are true with the conclusion false.
- Need just one possible situation to show invalidity

## Lesson 1.2: Truth-Functional Representation

### • Conjunction

- First conjunct, Conjunct, Second Conjunct
- $-A \wedge B = AND$

### • Disjunction

- First Disjunct, Disjunct, Second Disjunct
- $-A \vee B = OR$

#### • Formulas

- We use formulas to represent sentences
- if P is any formula, then so is P
- if P and Q are formulas, then  $P \wedge Q$  is a formula
- if P and Q are any formulas, then  $P \vee Q$  is a formula

### • Advantages of Formulas

- Formulas can be very complex
- They can express complex claims clearly and compactly
- English sentences can be long

### • Major connective

- The logical connector used last in the process of constructing the formula out of its parts

#### • Sentence Forms

- Patterns or structural frameworks, for sentences
- Look like formulas

## Lesson 1.3: Truth-Functional Analysis

	Shown by Considering Every Possible Case	Shown by Example
Argument	Valid: In every possible case, if premises are true, conclusion is also true.	<b>Invalid</b> : In at least one possible case, premises are true, conclusion is false.
Set of Sentences	<b>Inconsistent</b> : In every possible case, at least one is false.	Consistent: In at least one possible case, all are true
Pair of Sentences	<b>Equivalent</b> : In every possible case, they have the same truth-value.	Not equivalent: In at least one possible case, they have different truth-values.
Sentence	Tautologous: In every possible case, true Contradictory: In every possible case, false.	Contingent: True in at least one possible case and false in at least one possible case

## Lesson 1.4: Conditionals

- False ONLY when the antecedent is true and the consequent is false.
- For this outline, I've represented false as 0 and true as 1.

## Lesson 1.5: Truth Tables

• Conjunction (AND)

P	Q	$\mathbf{P}\wedge\mathbf{Q}$
0	0	0
0	1	0
1	0	0
1	1	1

• Disjunction (OR)

P	Q	$\mathbf{P} ee \mathbf{Q}$
0	0	0
0	1	1
1	0	1
1	1	1

• Conditional (If/then)

P	Q	$\mathbf{P}\Rightarrow\mathbf{Q}$
0	0	1
0	1	1
1	0	0
1	1	1

## Chapter 2: Derivations and Models

Lesson 2.1: Inference Rules

Lesson 2.2: Equivalence Rules

Lesson 2.3: Conditional Proof and Indirect Proof

# Chapter 3: Quantificational Logic

Lesson 3.1: The Language

Lesson 3.2: Derivations

Lesson 3.3: Predicate Logic Truth-Trees

Lesson 3.4: Unrestricted Quantification

Lesson 3.5: English Arguments

Lesson 3.6: Identity, Definite Descriptions, and Function Symbols

Lesson 3.7: Quantifier Semantics

# Bibliography

Book used: Modern Formal Logic (Second Edition) Professor: Notes from Dr. Mark Brown's Spring 2012 course