ASTRO 4410 LAB 5

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Abstract

Galactic rotation curves are mysterious entities. Their shape cannot be explained by a näive application of the law of universal gravitation, as the amount and distribution of visible matter is inconsistent with observed rotation curves measured for most spiral galaxies. Instead of finding that the orbital velocity of material decreases with increasing distance from the galactic center, studies have found that this velocity is approximately constant; that is, that rotation curves are approximately flat, starting from a radius relatively close to the center. The rotation curve of our own galaxy, the Milky Way, has been shown to be consistent with this model. This lab is an attempt to derive the magnitude of the rotational velocity of the galaxy under the assumption that the curve is perfectly flat, based on observations of hydrogen gas made with a Cornell University's radio telescope. The results will show that the assumption of a completely flat rotation curve leads to some errors, at least in the case of the longitudes that were scanned. I will proceed to argue that nevertheless, the results make complete sense, and will illustrate this by plotting the rotation curve of the galaxy assuming that the Sun's rotational velocity is known. I will also use the obtained data to estimate the radius of the galaxy.

1 Introduction

Newton's Universal Law of Gravitation, one of the most impressive achievements of human ingenuity, has been used with brilliant accuracy in order to predict the motions of celestial bodies. It allows us to make sense of the dynamics of our solar system, and can be used to make predictions about the movements of everything from asteroids and comets to moons and planets. Newtonian dynamics and gravitation were responsible for taking us to the moon. Nonetheless, its predictions regarding some specific phenomena have been proven incorrect. Some of these incorrect predictions, like the perihelion precession of Mercury, or the quantitative description of gravitational lensing, were solved by Einstein's General Relativity, which in effect superseded Newtonian gravity as the correct theory of gravitation. However, not even General Relativity is able to explain, without the help of certain untested hypotheses, the shape of the Milky Way's rotation curve or, for that matter, the shape of the rotation curve of most galaxies that we can observe. That is, we expect the rotational velocity of the material in the galaxy to decrease, after a certain radius, as a function of distance from the center, but instead we see that it is more or less constant as a function of increasing radius, as illustrated in Figure 1. This lab reproduces this perplexing result, and explores the limitations of treating the rotational velocity of objects in the galaxy as a constant, using the radio telescope mounted on the roof of Cornell University's Space Sciences Building.

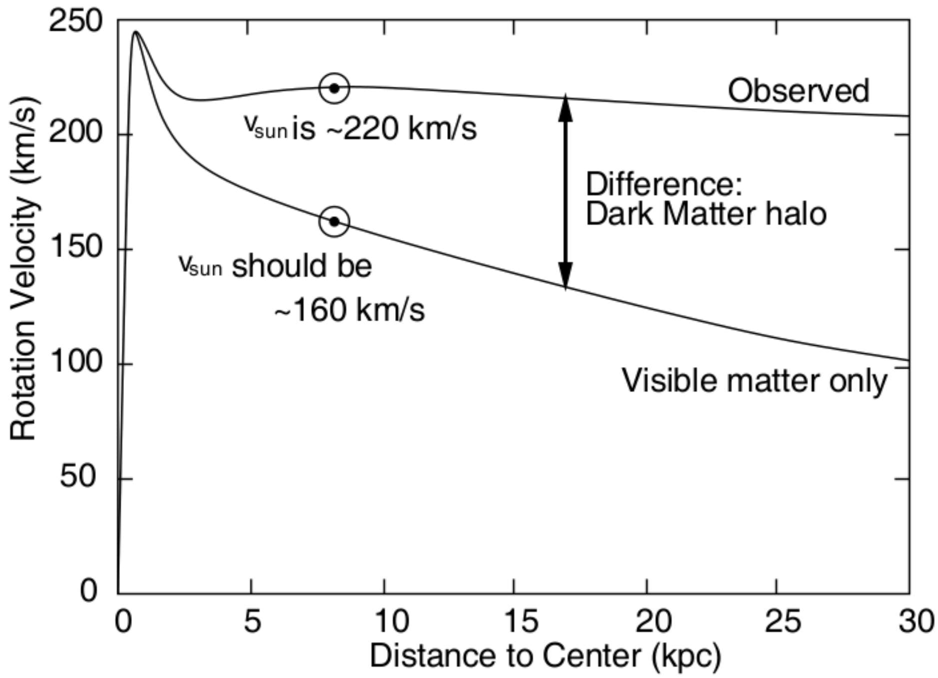

Figure 1: Expected vs observed rotation curve of Milky way. Obtained from [1], p. 5.

2 Objectives

1. Obtain spectra centered at 1420 MHz for different galactic longitudes.
2. Calculate the rotational velocity of objects in the galaxy assuming a flat rotation curve.
3. Calculate the rotation velocity of the observed gas as a function of galactic radius.
4. Plot a portion of the Milky Way's rotation curve.
5. Estimate the radius of the galaxy.

3 Setup

3.1 Date of observations and telescope characteristics

The data for this lab was acquired on Monday, November 23rd, from approximately 14:30 to 15:30, using the radio telescope mounted in the roof of Cornell University's Space Sciences Building. The antenna's reflector is a 3.8 m paraboloid with a helical feed at the prime focus that accepts circularly polarized radiation between 1 and 2 GHz. Behind the helix there is a circular ground plane where the antenna terminals connect to a receiver box that contains a low-noise amplifier and a noise diode that can be used for calibration purposes. This antenna is mounted on a king post that allows it to move in azimuth and elevation. The signal produced by the low-noise amplifier is sent to an analog receiver in a lab room in the floor directly below the roof. The signal goes through a bandpass filter that selects a 57 MHz band around 1420 MHz, and is then mixed into an intermediate frequency, and a second bandpass filter centered at 260 MHz is used to eliminate aliasing. The antenna's movement and orientation is controlled by a computer in the same lab room, which is also used for producing the data files generated by the observations.

3.2 Procedures for data acquisition

The data acquisition consisted in having the antenna point towards nine different galactic longitudes in the galactic plane, plus seven different latitudes ranging from 0 to 12 degrees at a longitude of 55 degrees (in galactic coordinates). Sets of three or more spectra were obtained for each of the directions at which the antenna was pointed. Each of the spectra consists of 787 channels spaced over a 10 Mhz band centered at 1420 MHz, the 21 cm hydrogen line. The galaxy is composed mostly of hydrogen gas, so we expect to detect this line near its rest frequency, but redshifted or blueshifted depending on the velocity of the gas relative to us. Finding this Doppler shift is central to achieving the lab's objectives, as will be seen below.

4 Data reduction

4.1 Rotational velocity

Figure 2 shows a diagram of the positions and velocities of the Sun and a certain star at galactic latitude b=0. From the diagram, it becomes apparent that V, the rotational velocity of the star, is related to its galactic latitude l and the velocity of the Sun V_0 by the following equation:

$$V = V_{max} + V_0 sin(l) \tag{1}$$

Here, V_{max} corresponds to the maximum observed velocity, the velocity when the angle $\alpha = 0$. Since we are looking at galactic hydrogen, and not at a particular star, we will always observe some hydrogen that has $\alpha = 0$ with respect to us. Assuming a flat rotation curve implies that $V = V_0$, and equation (1) becomes

$$V_0 = V_{max} + V_0 sin(l) \tag{2}$$

Solving for V_0 gives

$$V_0 = \frac{V_{max}}{1 - sin(l)} \tag{3}$$

The quantity V_{max} can be obtained from the spectra by looking at the minimum frequency at which hydrogen is detected, and then calculating the velocity of hydrogen at that point, which corresponds to the maximum velocity of the hydrogen cloud. This can be done via the equation

$$V_{max} = c\left(\frac{f_{rest} - f_{obs}}{f_{rest}}\right) \tag{4}$$

Where f_{rest} corresponds to the rest frequency of the 21 cm HI line, 1420.4 MHz, c corresponds to the speed of light, $3 \cdot 10^5 \mathrm{kms}^{-1}$, and f_{obs} corresponds to the minimum frequency of the detected hydrogen. In practice, the value of f_{obs} will be determined by looking at the point at which the value of the hydrogen line's signal reaches 20% of it's maximum value.

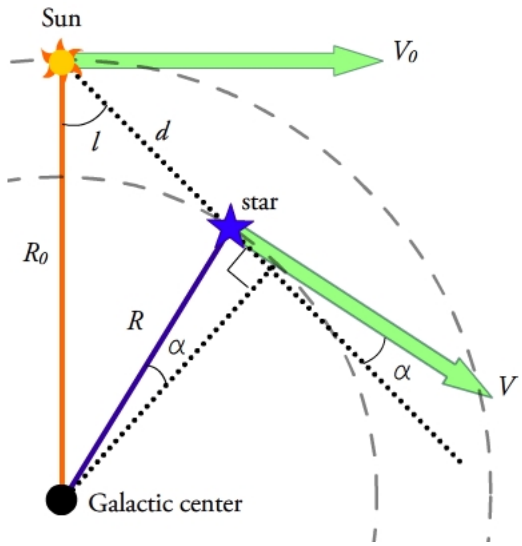

Figure 2: Geometric illustration of galactic rotation. Obtained from Wikimedia Commons.

4.2 Radius

Assuming a flat rotation curve, the relationship between the maximum distance from the center of the galaxy for the observed hydrogen cloud, or its maximum radius, and its minimum velocity as per the spectrum obtained with the antenna is given by

$$R_{max} = \frac{V_0 sin(l)\, R_0}{V_{min} + V_0 sin(l)} \tag{5}$$

Where $R_{\max}$ is the hydrogen's cloud maximum distance from the galactic center, R_0 is the distance of the Sun from the galactic center ($\approx 8.5\, kpc$) and V_{min} is the minimum velocity, which can be obtained from the spectrum in the same way as the maximum velocity was in 4.1, via equation (4), but instead of finding the point at which the value of the signal reaches 20% of it's maximum value, the point to be found corresponds to the frequency at which the signal falls to 20% of it's maximum value after passing through the maximum. In other words, f_{obs} here will correspond to the maximum frequency at which hydrogen can be detected, while in 4.1 f_{obs} corresponds to the minimum frequency at which hydrogen can be detected. The value of $R_{\max}$ can be regarded as the point at which there is no more hydrogen to be found, and provides a good estimate for the radius of the galaxy.

4.3 Corrections

The fact that the Sun has a certain peculiar velocity with respect to the other objects in our neighborhood implies that a certain correction must be made to the velocities obtained in parts 4.1 and 4.2 above. This is given by

$$V_{\odot} = 19.5\,kms^{-1}(cos(b)cos(23°)cos(l - 57°) + sin(b)sin23°) \quad (6)$$

Where the 23° and 57° correspond to the Sun's galactic latitude and longitude, respectively. Subtracting this from the obtained V_{max} and V_{min} accounts for this motion and eliminates this source of error. In addition to this, the Earth's orbital velocity and its rotation about its own axis also affect the velocities obtained, to a degree dependent on the time of the day and year, and the galactic longitude and latitude being observed. All of this was accounted for during the data reduction process via use of JPL's Solar System ephemeris DE405.

5 Results and analysis

5.1 Determination of V_0assuming a flat rotation curve

Some of the spectra obtained during the data acquisition process proved to be faulty, so not all of the spectra was used. Thus the analysis was done using six of the spectral sets: the ones corresponding to l=25°, l=30°, l=40°, l=55°, and l=60°. In all cases, b=0. Figures 3 and 4 show the spectrum obtained for l=25° and l=55°, respectively, after a bandpass fit. Table 1 shows the results obtained for V_0 as function of galactic longitude/distance from the galactic center.[1]

Table 1: Values for V_0obtained assuming a completely flat rotation curve.

V_0(km/s)	l (°)	R (kpc)
197.6	25	3.6
212.6	30	4.2
243.1	35	4.9
245.8	40	5.5
340.8	55	7.0
341.8	60	7.4
263.6 ± 72.1		

As shown in the table, the obtained value for the rotational velocity is

$$V_0 = 263.6 \pm 72.1\,kms^{-1} \quad (7)$$

Comparing this to Figure 1, which gives $V_{sun} = V_0 \approx 220\,kms^{-1}$, the obtained value is consistent with expectations since the expected value is within the range allowed by the error bounds, and it is relatively close to the average value found. However, an honest assessment appears to suggest that V_0 is directly proportional to l, as illustrated in Figure 5. Thus it seems that assuming a flat rotation curve leads to a contradiction, since calculating the rotational velocities at different longitudes under this assumption leads to the conclusion that the rotation curve is not, in fact, perfectly flat, but instead is proportional to the longitude. A better result could be obtained by taking into account measurements only between the longitude range $30° < l < 40°$, but justifying this sort of range choice does not appear to be possible without resorting to some sort of *ad hoc* argument.

[1]The distance from the galactic center R is given by $R = R_0 sin(l)$, where $R_0$is the distance from the Sun to the galactic center, about 8.5 kpc.

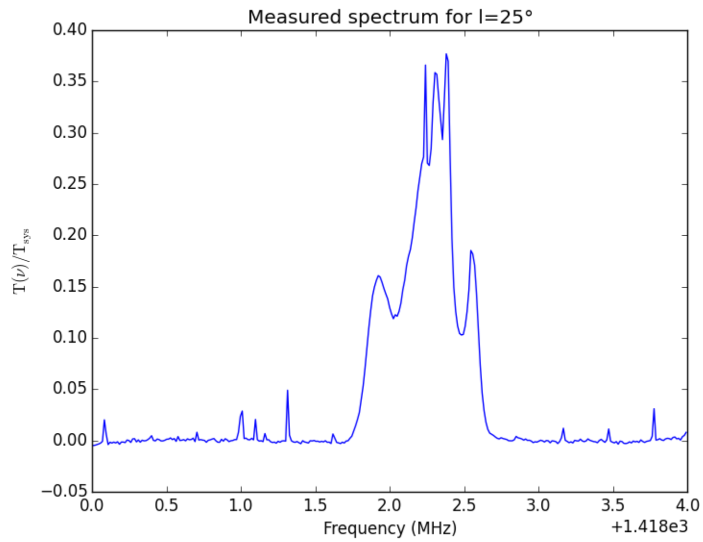

Figure 3: Spectrum for l=25°

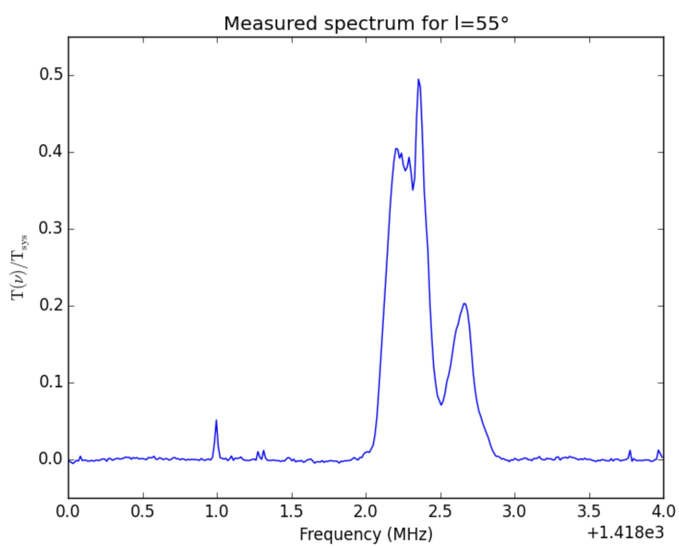

Figure 4: Spectrum for l=55°

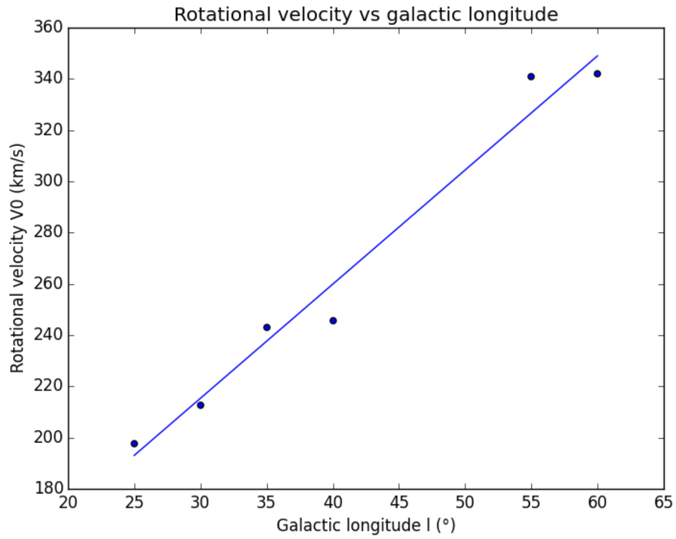

Figure 5: Rotational velocity as a function of galactic longitude (assuming $V = V_0$ in equation(1))

5.2 Plotting the galactic rotation curve

A better way of determining whether the results obtained are accurate is to use them in order to plot a portion of the Milky Way's rotation curve and compare it with accepted models. Instead of assuming that the rotation curve is perfectly flat as was done in 5.1, here it will be assumed that equation (1) holds in its original form and that the value of V_0, corresponding to the Sun's rotational velocity, is known. With this assumptions, and with $V_0 = 220\,kms^{-1}$, and using the same values for V_{max} as before, we obtain the rotational velocities shown in Table 2. Plotting with respect to distance from the center yields Figure 6.

Table 2: Values for rotational velocity V assuming $V_0 = 220\,kms^{-1}$

V (km/s)	l (°)	R (kpc)
207.0	25	3.6
216.3	30	4.2
229.9	35	4.9
229.2	40	5.5
241.8	55	7.0
236.3	60	7.4
226.8 ± 17.4		

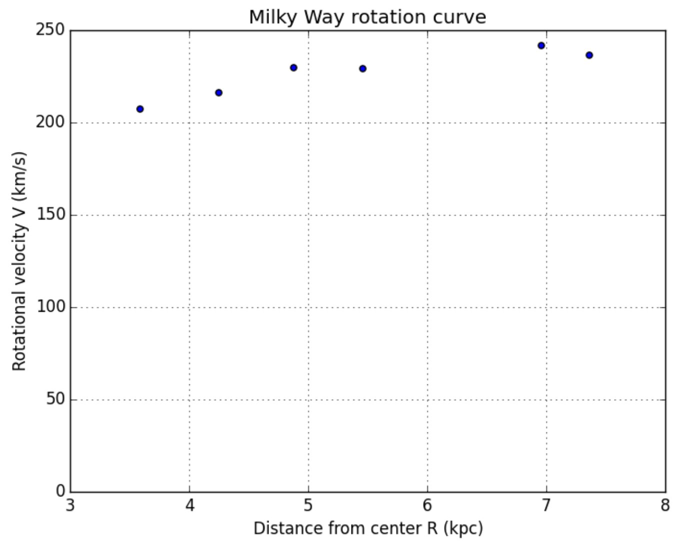

Figure 6: Obtained galactic rotation curve

The velocities fall within a reasonable range of values given the expectations, and the shape of the curve is approximately flat. Clearly, this method produces much better results than the one used in 5.1, and confirms that the data obtained is reasonably accurate. Even more accurate results might be obtained by utilization of more sophisticated code in order to do the analysis, as well as by performing more closely spaced measurements over a wider range of longitudes. Nonetheless, these results show that a respectable galactic rotation curve for the Milky Way galaxy can be obtained by using a 3.8 m telescope and a relatively crude data reduction method.

5.3 Galactic radius

Using equation (5), with $V_0 = 220\, kms^{-1}$and V_{min} as determined by the data, an estimate for the galactic radius R_{max} for each of the longitudes of observation was obtained, as shown in Table 3.

Table 3: Values for $R_{\text{max}}$for different longitudes of observation

R_{max}(kpc)	l (°)
17.1	25
17.7	30
16.2	35
15.7	40
14.2	55
14.7	60
15.9 ± 1.8	

The data obtained indicates that $R_{max} = 15.9{\pm}1.8\, kpc$. Since the Milky Way's diameter is supposed to be $\approx 30\, kpc$, it can be said that the measurements are in reasonable agreement with the accepted value. There is not much dispersion in the values obtained for each of the longitudes, and there does not appear to be a clear relationship between longitude and R_{max}, so it seems fair to conclude that the data obtained is of good quality. As in 5.2, more accurate results might be obtained by utilization of more sophisticated code during the data reduction process, and by performing more closely spaced measurements over a wider range of longitudes.

6 Conclusion

The purpose of this lab was to use spectra of hydrogen gas captured by Cornell's Space Sciences radio telescope at different galactic longitudes in order to study the rotation curve of the Milky Way galaxy. This was done by first attempting to obtain a single value for the rotational velocity of hydrogen assuming that the rotation curve was perfectly flat, then plotting a rotation curve dropping this assumption, and finally by estimating the galactic radius on the basis of the observations. It was found that assuming a perfectly flat rotation curve leads to a contradiction, but that plotting the curve without assuming flatness results in an approximately flat curve, so averaging the values found using the second method leads to a better estimate of the rotational velocity of objects orbiting the center of the galaxy than attempting to find this velocity by assuming flatness from the start. The values for R_{max} obtained were consistent with expectations and serve as a further indication that the quality of data obtained was good. Finally, it was suggested that better results might be obtained for all of the parameters via utilization of a more sophisticated code for data reduction and by performing more closely spaced measurements over a wider range of galactic longitudes.

References

[1] Schneider, Peter. *Extragalactic Astronomy and Cosmology: An Introduction.* Berlin: Springer, 2006. Print.