Final Project Outline: Solving the Equations of Stellar Structure

Gianfranco Grillo

November 18, 2016

The structure of stars can be described by a set of five differential equations:

$$
\begin{aligned}
\frac{\partial r}{\partial m} &= \frac{1}{4\pi r^2 \rho} && (1)\\
\frac{\partial P}{\partial m} &= -\frac{Gm}{4\pi r^4} && (2)\\
\frac{\partial L}{\partial m} &= \epsilon_n - \epsilon_\nu - c_P \frac{\partial T}{\partial t} + \frac{\delta}{\rho}\frac{\partial P}{\partial t} && (3)\\
\frac{\partial T}{\partial m} &= -\frac{GmT}{4\pi r^4 P}\nabla && (4)\\
\frac{\partial X_i}{\partial t} &= \frac{m_i}{\rho}\left(\sum_j r_{ji} - \sum_k r_{ik}\right),\ i = 1, ..., I\,. && (5)
\end{aligned}
$$

Equation (1) is a consequence of conservation of mass, equation (2) is the equation of hydrostatic equilibrium, equation (3) is conservation of energy, (4) describes energy transport, and (5) describes the star's change in composition as a function of time, as it burns certain elements via nuclear fusion. A few other definitions are in order:

$$
\begin{aligned}
\delta &= -\left(\frac{\partial \ln \rho}{\partial \ln T}\right)_P && (6)\\
\nabla &= \frac{d \ln T}{d \ln P} && (7)
\end{aligned}
$$

When the energy transport is due to radiation,

$$\nabla = \nabla_{rad} = \frac{3}{16\pi ac}\frac{\kappa L P}{m T^4} \tag{8}$$

When the energy is carried by convection, we say that $\nabla = \nabla_{ad}$, which is a function of P, T, and X_i. In general, we can find equations for the other parameters (ρ, ϵ_n, ϵ_ν, κ, and r_{ij}), that also depend on P, T, and X_i, the latter of which is a function of m and t. This gives the same number of equations as there are unknown variables.

There is more than one choice of boundary conditions for this problem. In general, however, we have that at the center, $m = L = r = 0$. A common approximation for the surface conditions is the so-called "radiative zero" conditions, where we have that at the surface, $m = M$, $P = 0$, and $T = 0$. In any case, since we only know two boundary conditions at the surface, and two at the center, this problem is a two-point boundary value problem, and needs to solved using numerical methods. The most popular way for solving this equation is known as the Henyey method, after American astronomer Louis G. Henyey, which involves using relaxation to solve the equations after these have been transformed into a set of difference equations. Section 18.3 of [1] describes relaxation methods for solving two-point boundary value problems in general, and [2], [3], and [4] go deeper on the physics of stellar structure and evolution and the Henyey method in particular. My goal would be to implement Henyey's method and eventually time evolve a star of a given mass across a certain phase of its lifetime, tracing its evolution on an HR diagram. This would be a joint project with ASTRO 6560, taught by Prof. David Chernoff.

References

[1] W. H. Press, S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery *Numerical Recipes, The Art of Scientific Computing, 3rd edit.*, Camb. Univ. Press, 2007.

[2] R. Kippenhahn, A. Weigert, and A. Weiss *Stellar Structure and Evolution, 2nd edit.,* Springer-Verlag Berlin Heidelberg, 2012.

[3] R. Kippenhahn, A. Weigert, and E. Hofmeister: "Methods for Calculating Stellar Evolution," *Methods in Computational Physics*, vol. 7 (Eds.: B. Adler, S. Fernbach, and M. Rotenberg), Academic, New York, 1967, pp. 129-190.

[4] G. W. Collins, *The Fundamentals of Stellar Astrophysics,* W. H. Freeman and Company, 1989.