

The magnitude of the electric field at the photosensor is given by the sum of the electric fields of each of the waves,

$$E = E_1 + E_2 + E_3 + E_4 \quad (1)$$

The intensity is proportional to the square modulus of (1)

$$|E|^2 = (E_1 + E_2 + E_3 + E_4)(E_1^* + E_2^* + E_3^* + E_4^*) \quad (2)$$

$$= |E_1|^2 + |E_2|^2 + |E_3|^2 + |E_4|^2 + 2\text{Re}(E_1 E_2^*) + 2\text{Re}(E_1 E_3^*) \quad (3)$$

$$+ 2\text{Re}(E_1 E_4^*) + 2\text{Re}(E_2 E_3^*) + 2\text{Re}(E_2 E_4^*) + 2\text{Re}(E_3 E_4^*) \quad (4)$$

In this particular case, we can write the (time-averaged) electric fields as

$$E_1 = E_{01} e^{ik_1 x} \quad (5)$$

$$E_2 = E_{01} e^{ik_1(x+2\Delta x)} \quad (6)$$

$$E_3 = E_{02} e^{ik_2 x} \quad (7)$$

$$E_4 = E_{02} e^{ik_2(x+2\Delta x)} \quad (8)$$

Substituting into (3), and after performing some trigonometric algebra, we get

$$|E|^2 = 2 [E_{01}^2 + E_{02}^2 + E_{01}^2 \cos(2k_1 \Delta x) + E_{02}^2 \cos(2k_2 \Delta x) + 4E_{01} E_{02} \cos(k_1 \Delta x) \cos(k_2 \Delta x) \cos((k_1 - k_2)(\Delta x + 2x))] \quad (9)$$

We define  $a = \frac{E_{01}}{E_{02}}$ , and rewrite this as

$$I \propto \frac{1}{2} \left| \frac{E}{E_{02}} \right|^2 = 1 + a^2 + a \cos(2k_1 \Delta x) + \cos(2k_2 \Delta x) + 4a \cos(k_1 \Delta x) \cos(k_2 \Delta x) \cos((k_1 - k_2)(\Delta x + 2x)) \quad (10)$$

The  $2x$  factor in the last term is just an offset that does not affect the shape of the curve, so we can set  $x = 0$ . Rewriting in terms of  $\nu_0$ , the frequency of the central mode, and  $\Delta\nu$ , the frequency separation between the modes, we have that

$$I \propto 1 + a^2 + a \cos\left(\frac{4\pi\nu_0 \Delta x}{c}\right) + \cos\left(\frac{4\pi(\nu_0 + \Delta\nu) \Delta x}{c}\right) + 4a \cos\left(\frac{2\pi\nu_0 \Delta x}{c}\right) \cos\left(\frac{2\pi(\nu_0 + \Delta\nu) \Delta x}{c}\right) \cos\left(\frac{2\pi\Delta\nu \Delta x}{c}\right) \quad (11)$$

For an HeNe laser with nominal wavelength  $\lambda_0 = 632.8$  nm, and cavity length  $L = 10$  cm, we have that  $\nu_0 = 4.74 \times 10^{14}$  Hz and  $\Delta\nu = \frac{c}{2L} = 1.50 \times 10^9$  Hz. Figure 1 shows a plot of (10) after normalization, using these values, and  $a = 1$ .

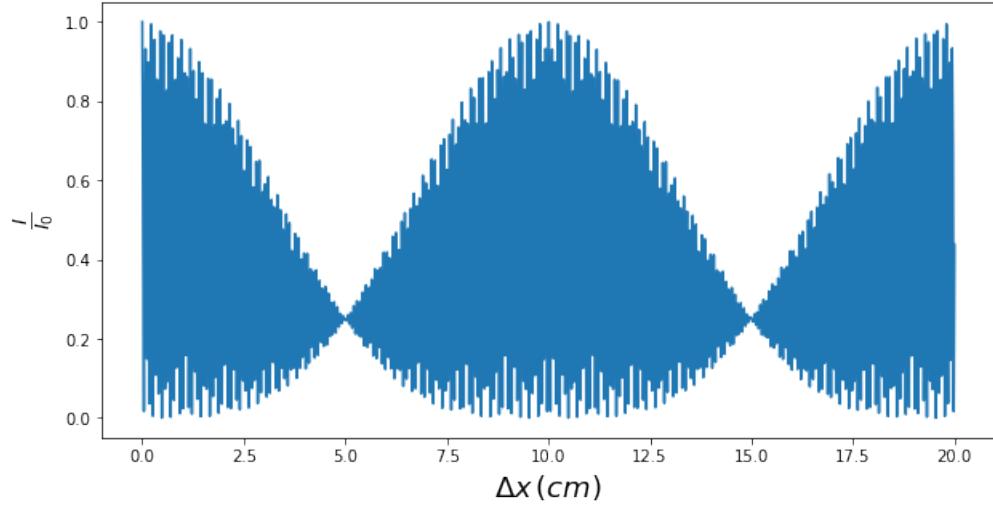


Figure 1: Normalized intensity vs  $\Delta x$  with  $\nu_0 = 4.74 \times 10^{14}$  Hz and  $\Delta\nu = 1.50 \times 10^9$  Hz.