Exercise with Tempo2

1 Introduction

Tempo2 is a pulsar timing software that takes as input a .par file and a .tim file. The .par file contains parameters detailing the properties of the pulsar that was observed, while the .tim file contains the times of arrival (TOA) of the radiation pulses coming from the target pulsar over several days of observation and across several frequency channels. Tempo2 takes these two files and calculates the residuals for the pulses, based on the expected TOAs for the different frequencies as given by parameters in the .par file. It can also recalculate the parameters in the .par file in order to minimize the residuals.

One of the parameters in the .par file is the dispersion measure (DM) parameter, which is a measure of the radiation dispersion due to the interstellar medium. This is an important parameter because not all radiation frequencies are dispersed equally, so our model has to take this into account. Formally, the DM is defined as the integrated column density of electrons n_e at distance d, and is given by

$$DM = \int_0^d n_e \, dl \tag{1}$$

The relationship between the timing residuals t, the frequency f, and DM is given by

$$t = \frac{k_{DM} DM}{f^2} \tag{2}$$

Where the dispersion constant $k_{DM}$is given by

$$k_{DM} = \frac{e^2}{2\pi m_e c} \approx 4.149 \, GHz^2 \, pc^{-1} \, cm^3 \, ms \tag{3}$$

The objective of this exercise is to use pulsar observation files and Tempo2 in order to better understand this relationship.

2 Prerequisites

In order to complete this exercise, you'll need an installation of the latest version of Tempo2, a .par and a .tim file corresponding to the same pulsar, and plotting software such as Python's matplotlib. You will also need software that is able to do symbolic math, like Sage or Mathematica. A collection of .par and .tim files are available from the NANOGrav collaboration's webpage at https://data.nanograv.org/.

3 Modifying the .par file and obtaining the residuals

Make a copy of your pulsar .par and .tim files and place them in your working directory. Open the .par file with a text editor, and locate the DM parameter. Change its value by 0.01, and save the file. Now run Tempo2 using the modified .par file and the copy of the .tim file, and output the results into a text file. We are only interested in the values of the frequency channel, the pre-fit residuals, the barycentric TOA, and the TOA uncertainty, so a good way to do this is by entering the terminal and typing

tempo2 -output general2 -f pulsarfile.par pulsarfile.tim -s "{freq} {pre} {bat} {err}\n" > file.txt

This will create a text file called file.txt containing four columns of data, corresponding to the frequency channel (in MHz), the pre-fit residuals (in seconds), the barycentric TOA (in Julian days), and the TOA uncertainty (in microseconds).

4 Plotting without error bars

4.1 Residuals vs TOAs

The text file you created contains observations for multiple frequency channels over several observation days. Your first task is to plot the residuals for two particular frequency channels as a function of the TOAs. Make sure the frequency channels are well separated. Ignore error bars for now. You should be able to get something resembling Figure 1.

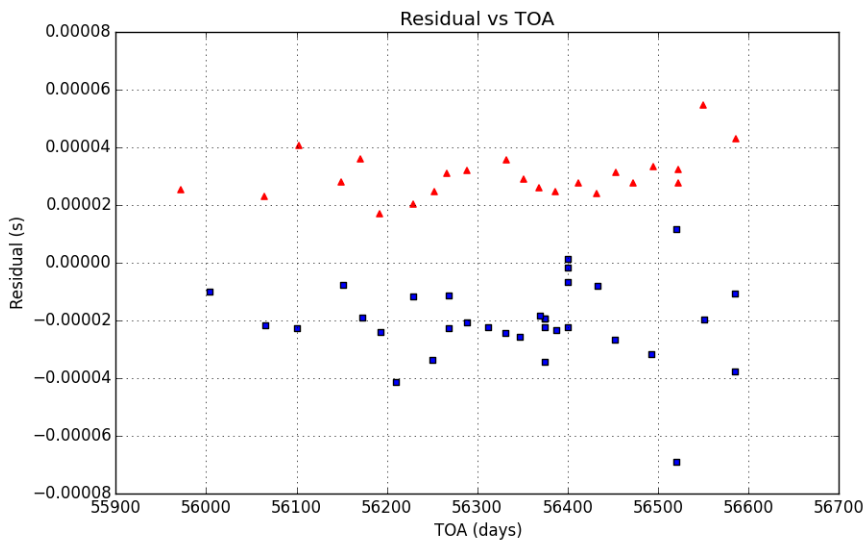

Figure 1: Residuals vs TOAs for J2302+4442

4.2 Residuals vs Frequency

Use the same text file created in the previous step in order to plot the residuals as a function of frequency. Make sure you use different colors for both frequency bands. You should get something akin to Figure 2. Again, ignore error bars.

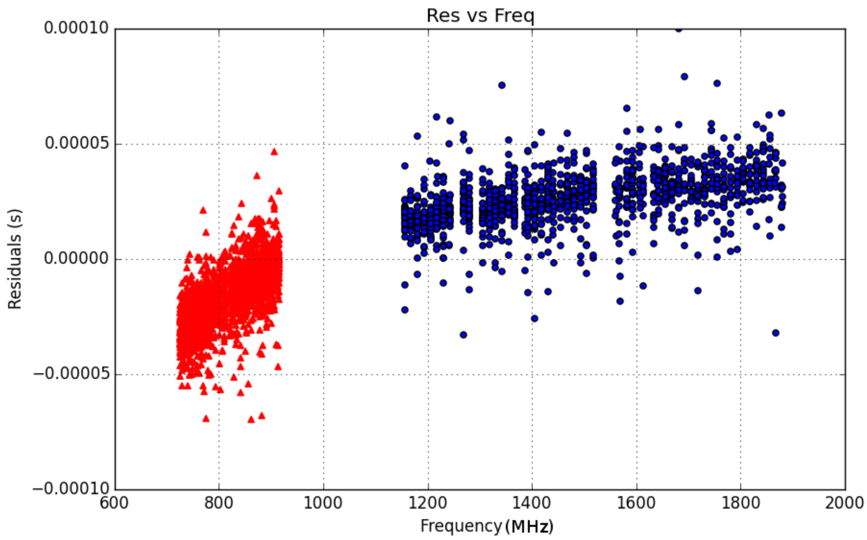

Figure 2: Residuals vs frequency for J2302+4442

4.2.1 Obtaining ΔDM

Equation (2) implies that the relationship between ΔDM, the difference between the original DM in the .par file and the one you are using in order to make this plot (which in this case we manually set to be equal to 0.01) and Δt, the time delay between at high frequency f_{hi} and a low frequency f_{low} is given by

$$\Delta t = k_{DM} \Delta DM \left(\frac{1}{f_{lo}^2} - \frac{1}{f_{hi}^2} \right) \tag{4}$$

This means that by plotting t vs $1/f^2$and making a linear fit, the slope m of the line that would be obtained will correspond to the quantity

$$m = k_{DM} \Delta DM \tag{5}$$

Use your plotting package to calculate the best fit line and include it in the plot (Figure 3), and then use the calculated slope in order to extract the value of ΔDM. You should be able to obtain a value of ± 0.01 to better than 1% accuracy. Remember that the frequency units for the value given for k_{DM} in (3) is GHz, and the unit of the frequency output from Tempo2 is MHz and adjust accordingly.

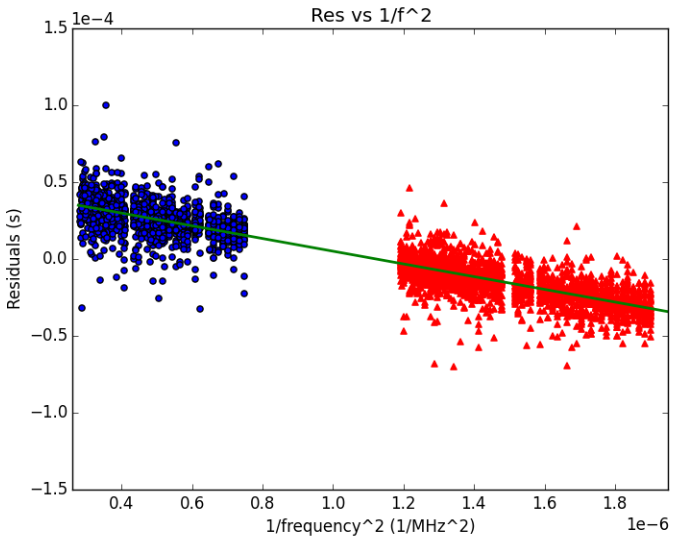

Figure 3: Residuals vs 1/frequency^2 for J2302+4442

5 Plotting with errors

5.1 Residuals vs $1/f^2$

Repeat the procedure from 4.2.1 but this time include the error bars, based on the TOA uncertainties given in the fourth column of the text file written by Tempo2. Remember that these uncertainties are given in microseconds, while the residuals are given in seconds, and adjust accordingly. You should get something similar to Figure 4.

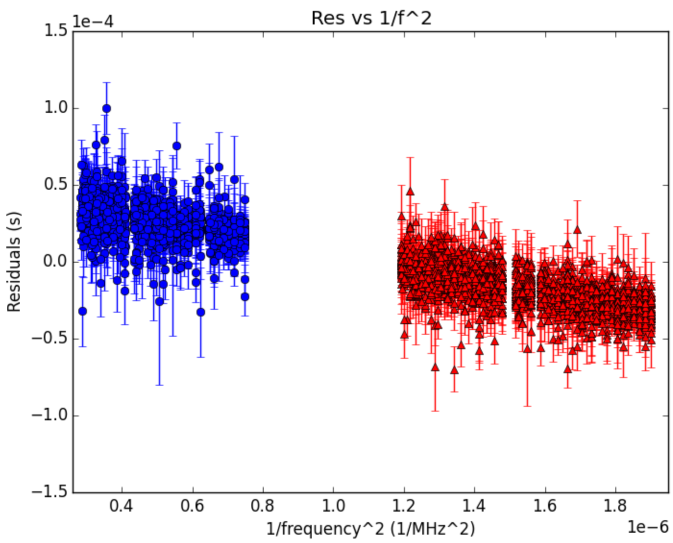

Figure 4: Residuals vs 1/frequency^2 for J2302+4442 (with uncertainties)

5.2 Calculating a weighted fit

Your plotting package probably has algorithms designed to produce a weighted fit, that is, a fit that takes into account the error bars. However, it is also possible to do this manually, and this is what we will do. We want to find the parameters β_1 and β_2 for a straight line that minimizes the quantity

S, given by

$$S = \sum_{i=1}^{n} \left(\frac{t_i - \frac{k_{DM} \Delta DM}{f_i^2}}{\sigma_i} \right)^2 \tag{6}$$

Where σ_i corresponds to the TOA uncertainty for the corresponding t_i, f_i pair of measurements. In other words, we want to find the values of β_1 and β_2 that will minimize the quantity

$$S = \sum_{i=1}^{n} \left(\frac{t_i - \beta_1 - \frac{\beta_2}{f_i^2}}{\sigma_i} \right)^2 \tag{7}$$

What we need to do then is calculate the sum in (7) in terms of β_1 and β_2, take the partial derivatives of β_1 and β_2 and set them equal to 0 in order to get a pair of equations (the normal equations), and solve them in order to get the values of β_1 and β_2 that minimize S. β_1 will then correspond to the y-intercept of your best fit line, and β_2 will correspond to the slope. Use this slope in order to extract ΔDM as we did in 4.2.1, and add that best fit line to your plot, in order to get something like Figure 5. The value of ΔDM obtained should be very similar to the one obtained previously.

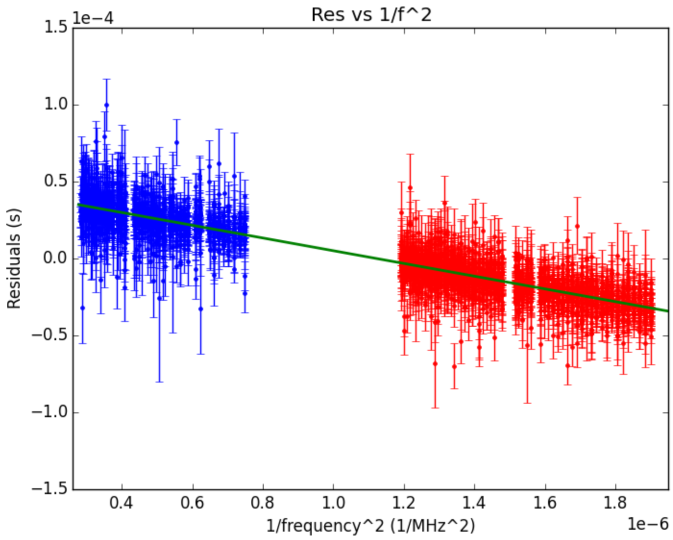

Figure 5: Residuals vs 1/frequency^2 for J2302+4442 (weighted fit)