

# Plasma Lensing

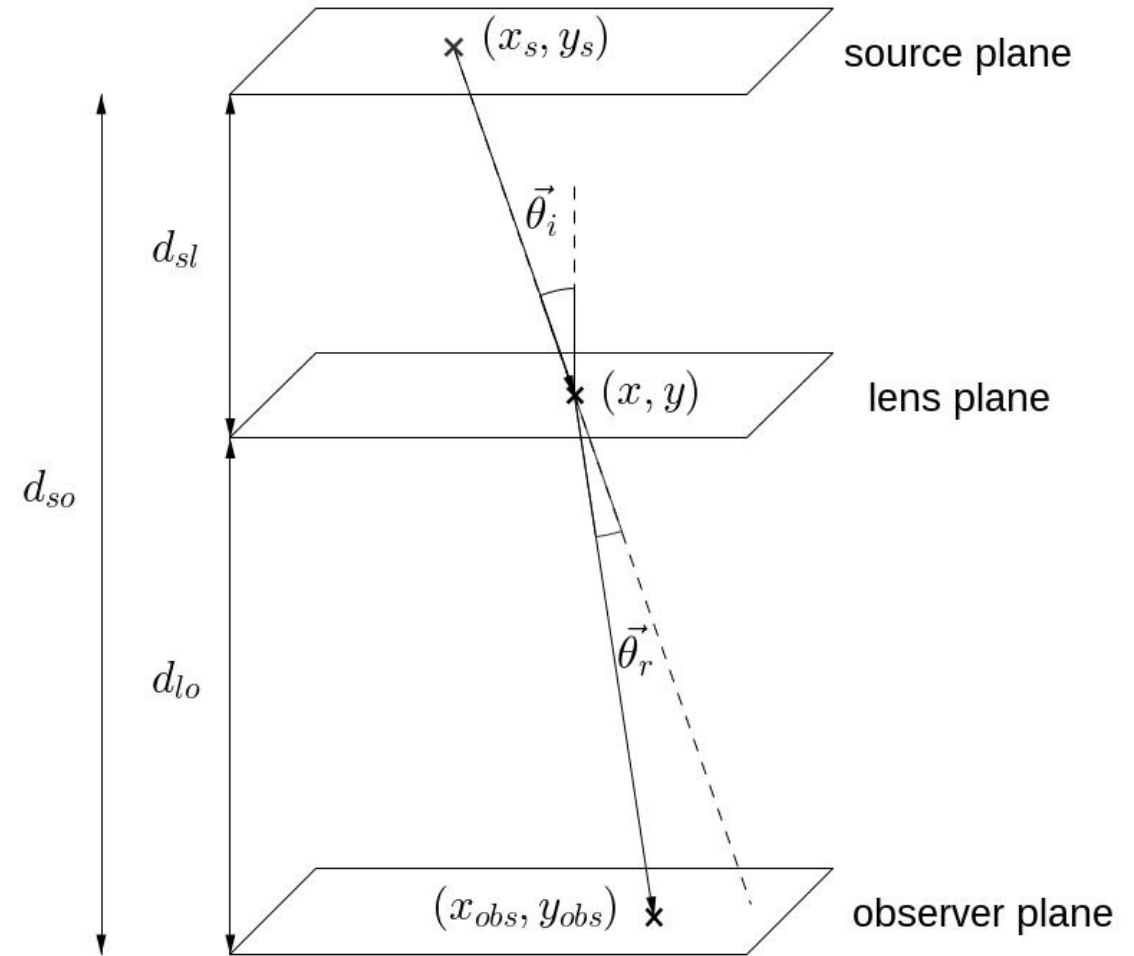


# Motivations


- Population of small lensing events might be responsible for some of our residuals.
- Big events such as the ones seen in J1713+0747 might be a result of plasma lensing.

# Geometry

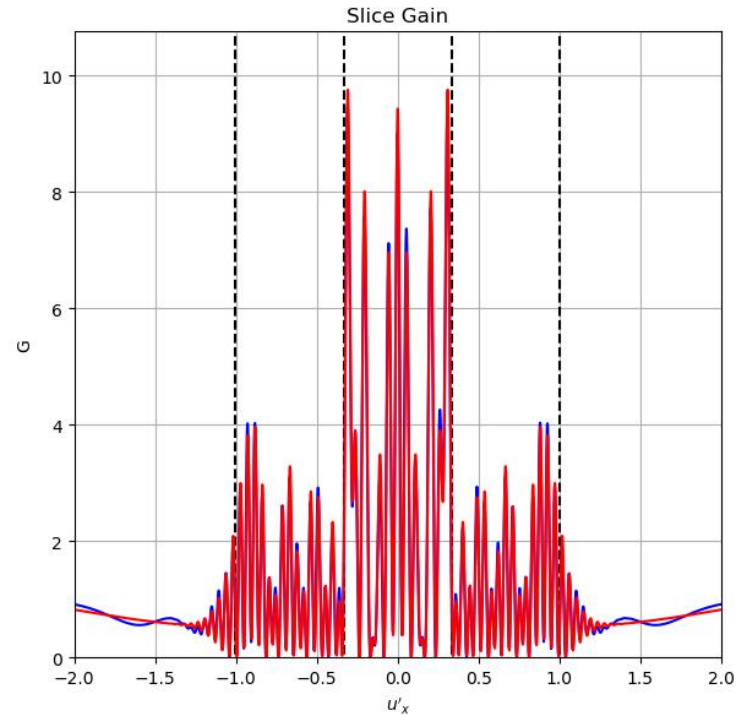
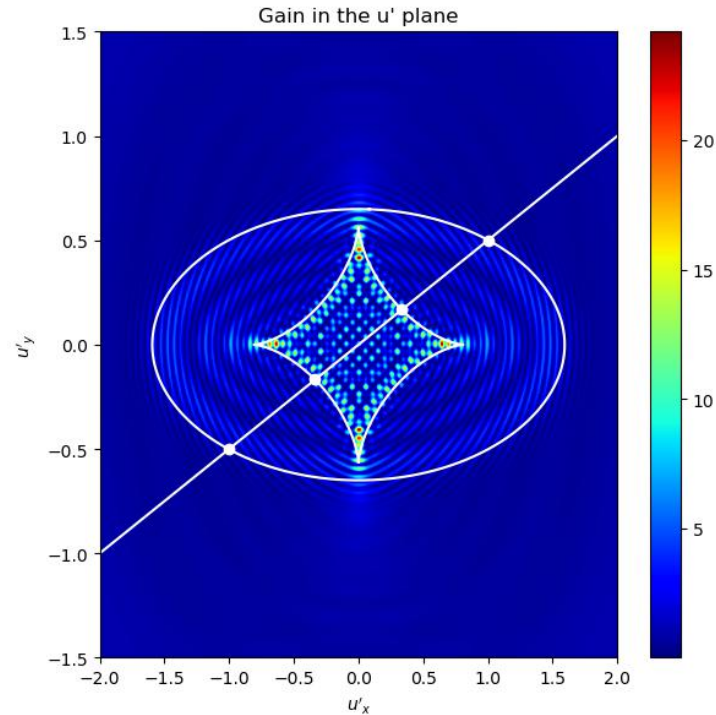
- Mapping from source plane to observer plane altered by refraction due to lens.
- Can induce positive and negative TOA perturbations, changes in pulse shapes and intensities, generation of multiple images.



# Methods

- Enhanced algebraic ray tracing, based on real and complex solutions to the lens equation, allow one to reconstruct the field and TOA perturbations due to a variety of lens shapes and parameters.
  - Fast Fourier Transform techniques enable construction of the field for weak, small lenses of arbitrary shape.
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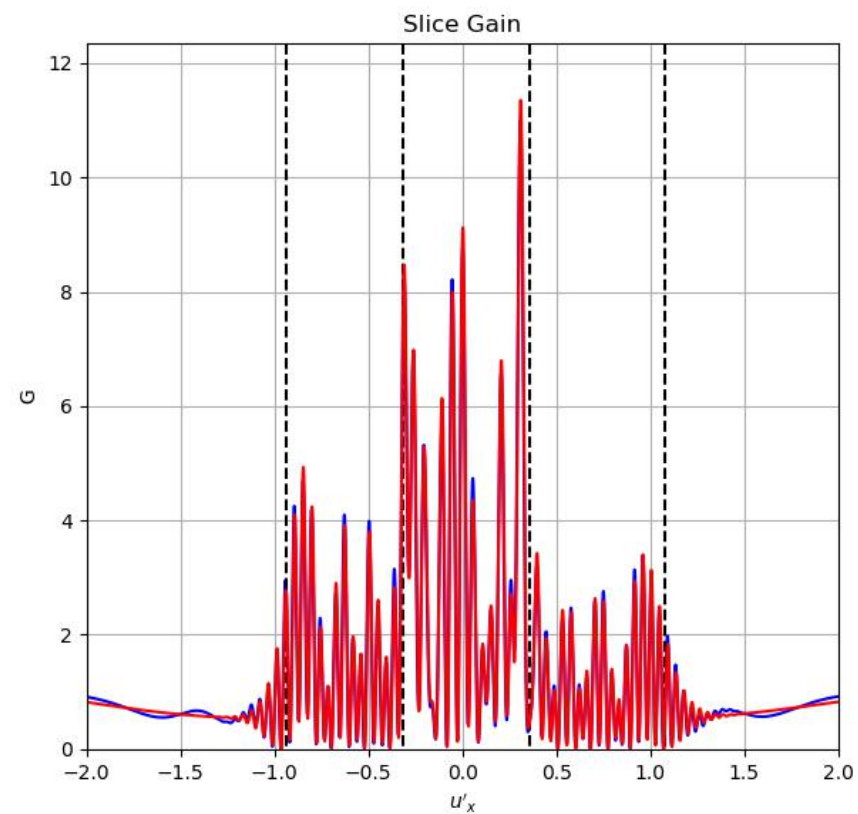
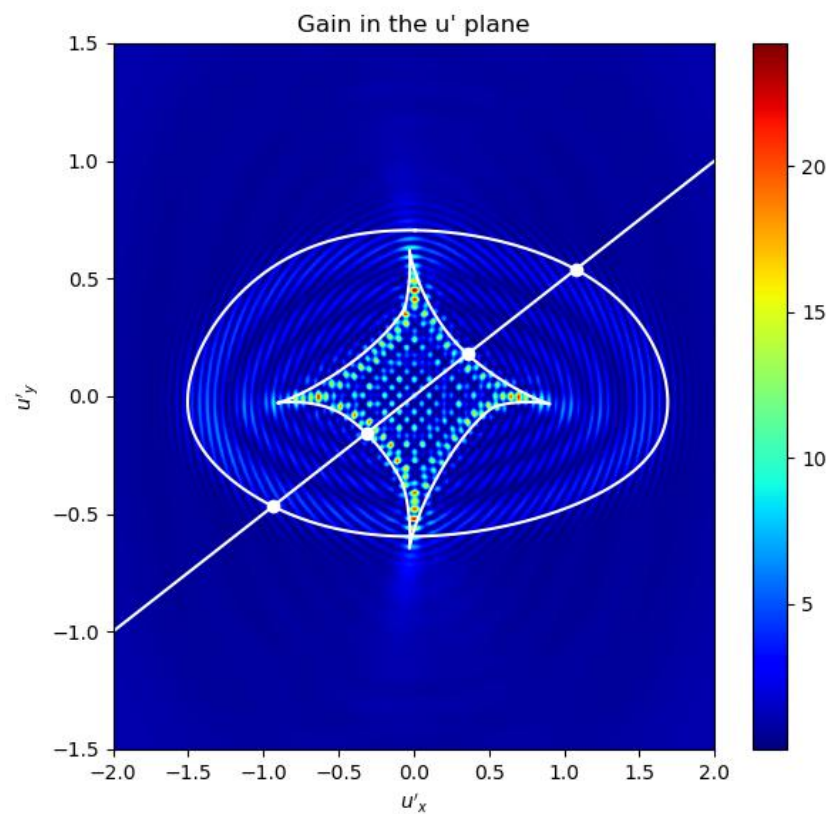
# Some Results



Blue line: Exact solution using FFT.  
Red line: Solution using enhanced ray tracing.

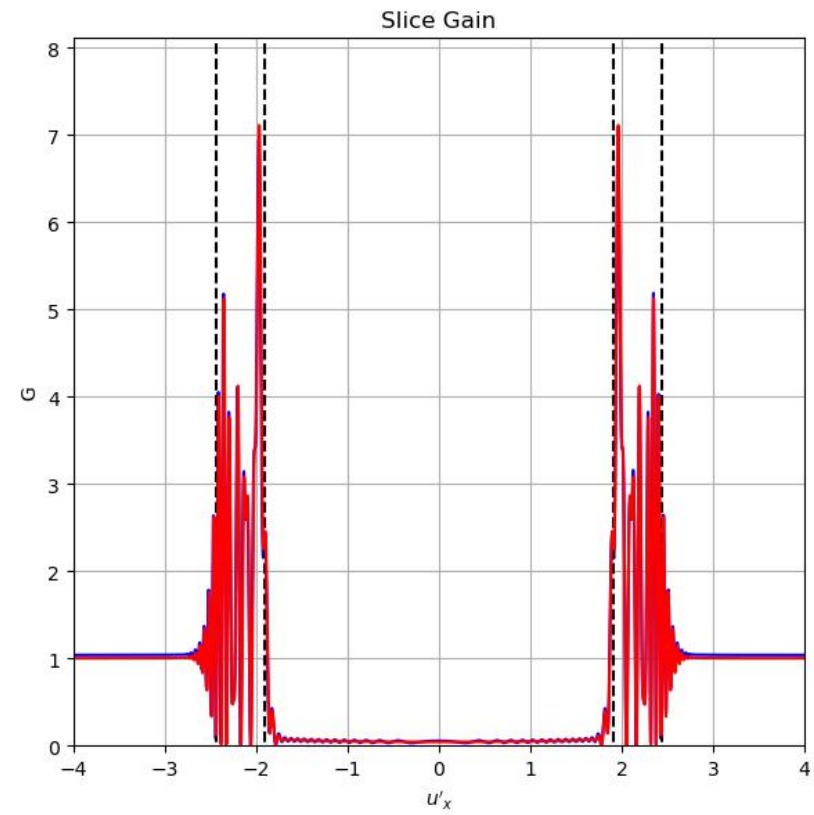
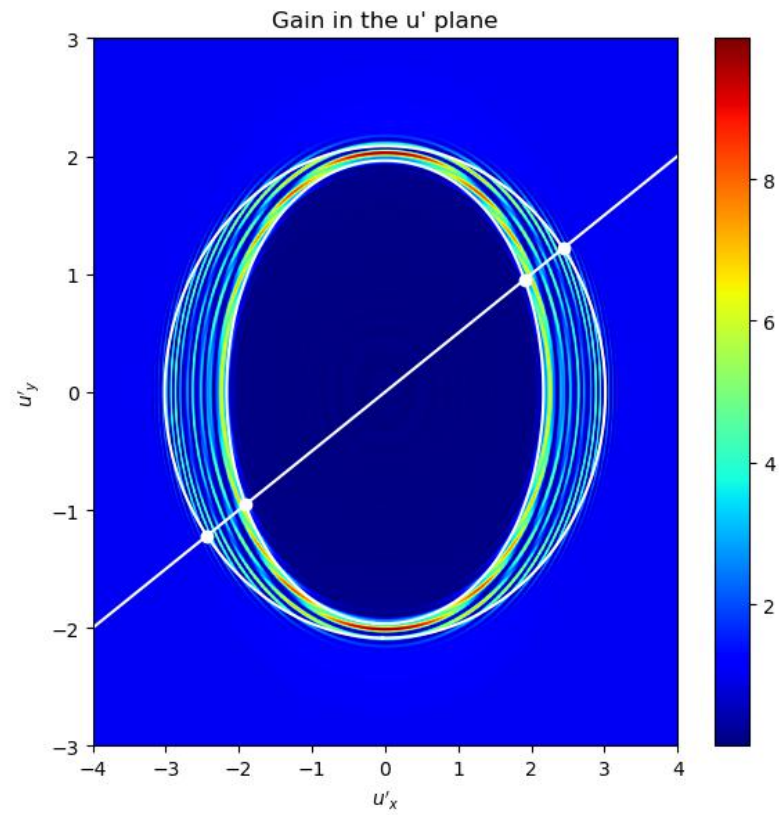
Parameter	$d_{so} (kpc)$	$d_{sl} (kpc)$	$a_x (AU)$	$a_y (AU)$	$DM_l (pc\ cm^{-3})$	$\nu$ (GHz)	Slope	Offset
Value	1.1	0.55	0.03	0.04	-5.00E-6	1.0	0.5	0.0

Lens shape	$e^{-u_x^2 - u_y^2}$
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Parameter	$d_{so}$ (kpc)	$d_{sl}$ (kpc)	$a_x$ (AU)	$a_y$ (AU)	$DM_l$ (pc cm $^{-3}$ )	$\nu$ (GHz)	Slope	Offset
Value	1.1	0.55	0.03	0.04	-5.00E-6	1.0	0.5	0.0

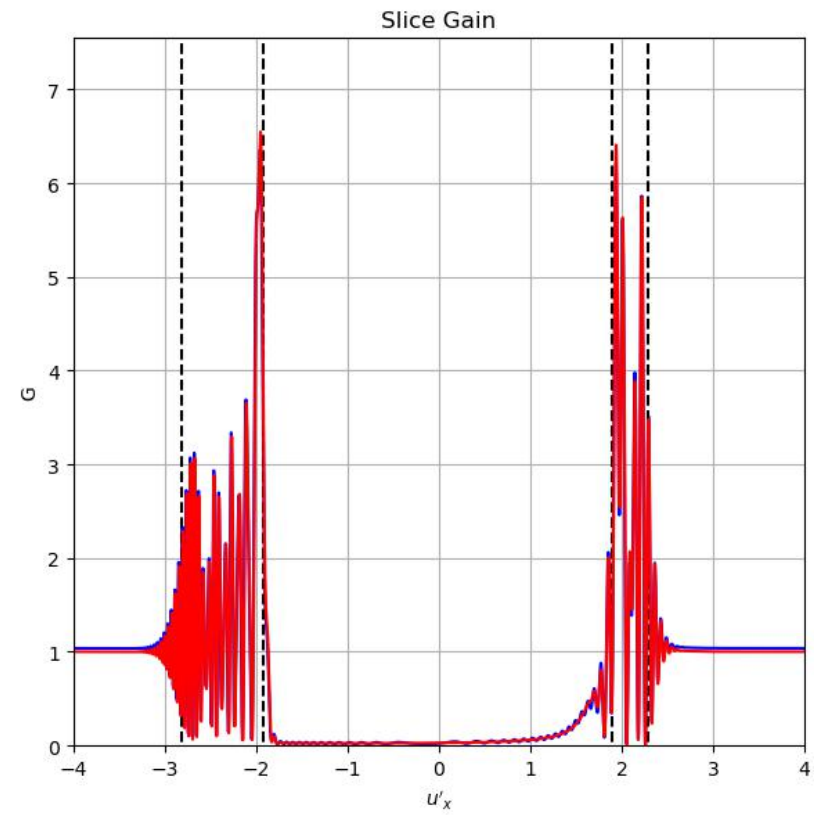
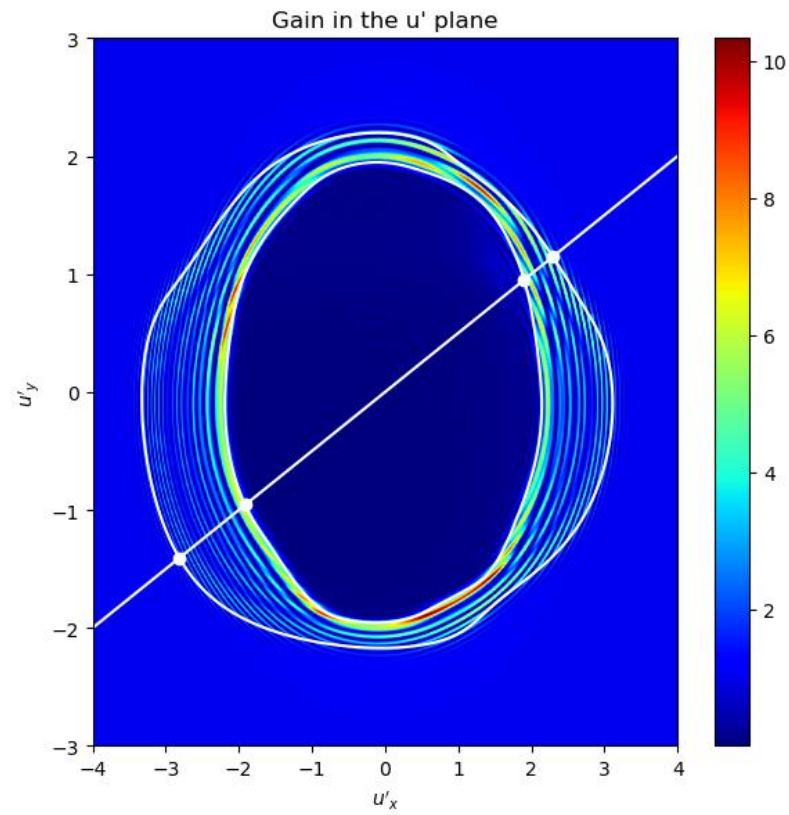
Lens shape	$(-0.01\sin(5u_x) - 0.01\sin(5u_y) + 1.0)e^{-u_x^2 - u_y^2}$
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Parameter	$d_{so}$ (kpc)	$d_{sl}$ (kpc)	$a_x$ (AU)	$a_y$ (AU)	$DM_l$ (pc cm <sup>-3</sup> )	$\nu$ (GHz)	Slope	Offset
Value	1.1	0.55	0.03	0.04	5.00E-6	1.0	0.5	0.0

Lens shape	$e^{-u_x^2 - u_y^2}$
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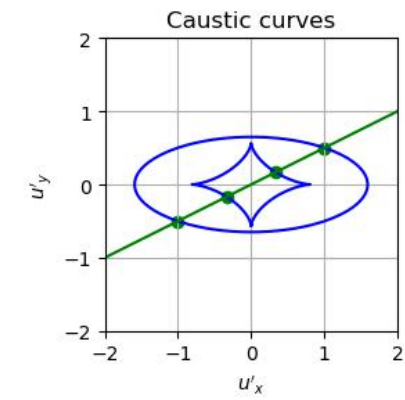
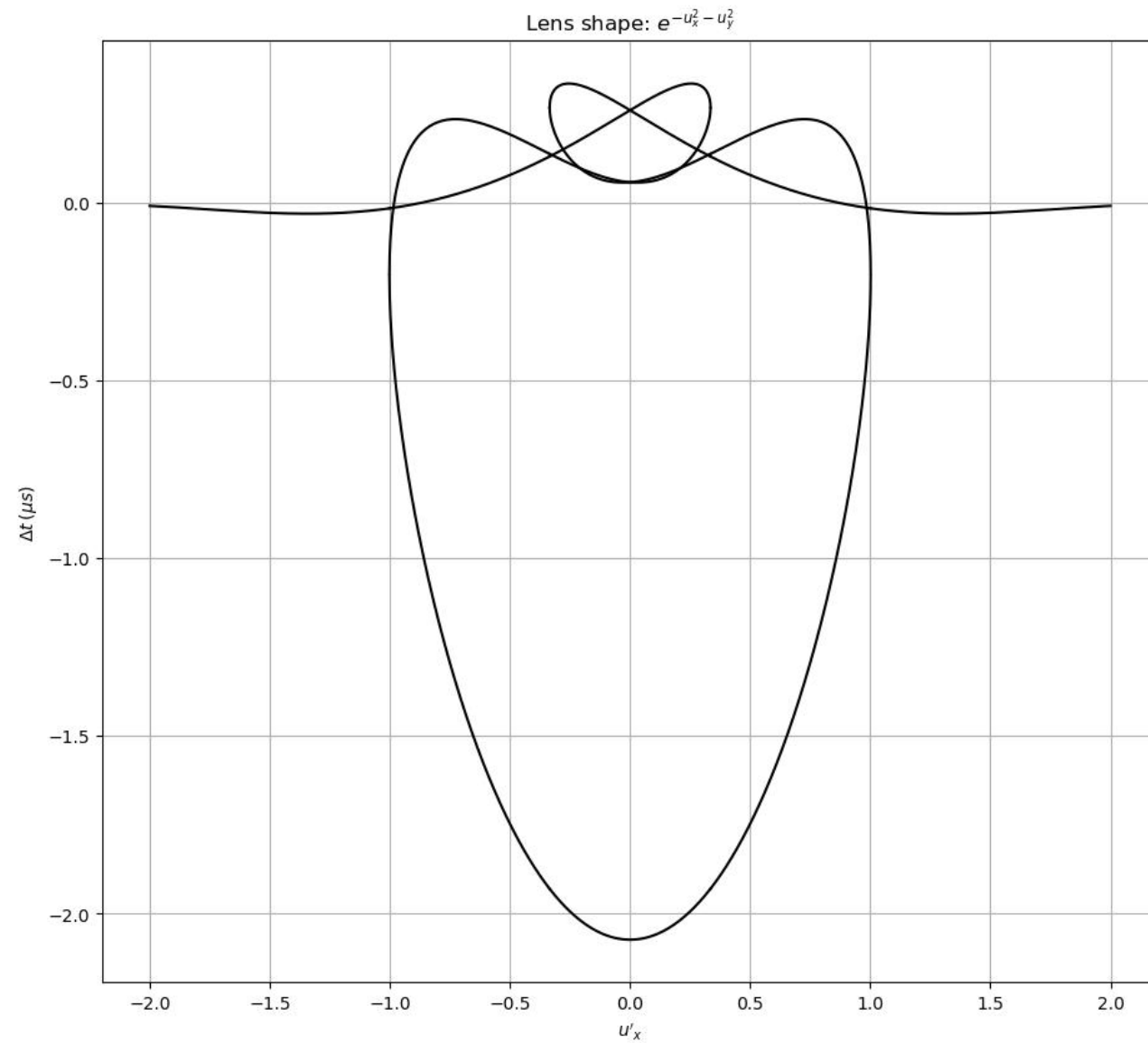




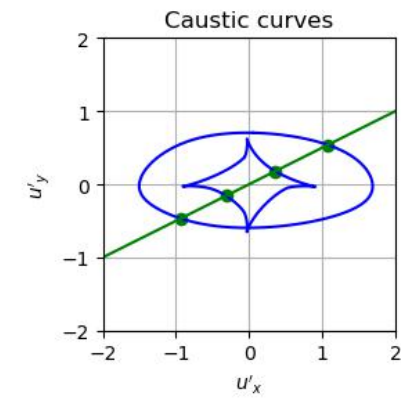
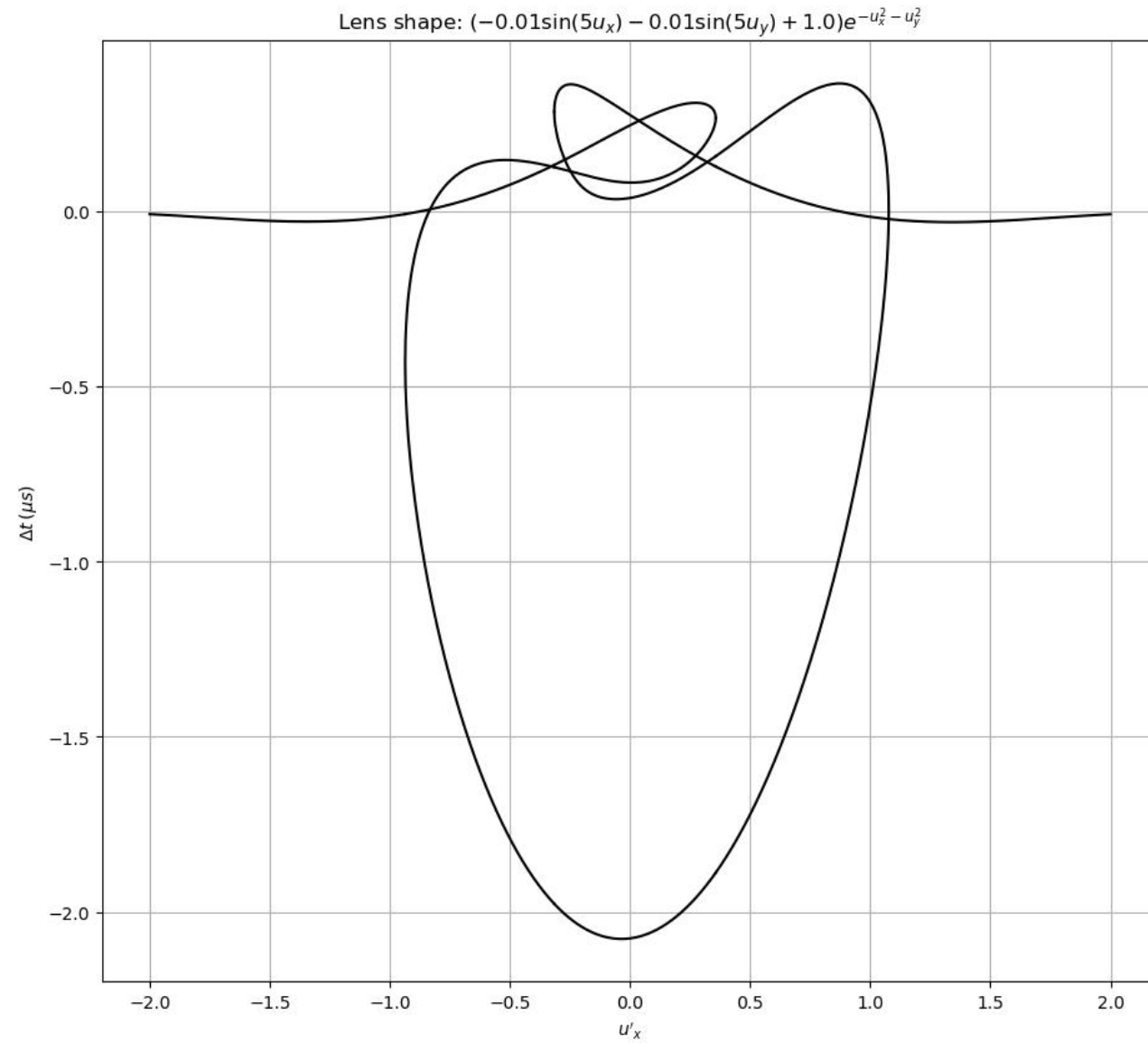
Parameter	$d_{so}$ (kpc)	$d_{sl}$ (kpc)	$a_x$ (AU)	$a_y$ (AU)	$DM_l$ (pc cm $^{-3}$ )	$\nu$ (GHz)	Slope	Offset
Value	1.1	0.55	0.03	0.04	5.00E-6	1.0	0.5	0.0

Lens shape	$(-0.05\sin(5u_x) - 0.05\sin(5u_y) + 1.0)e^{-u_x^2 - u_y^2}$
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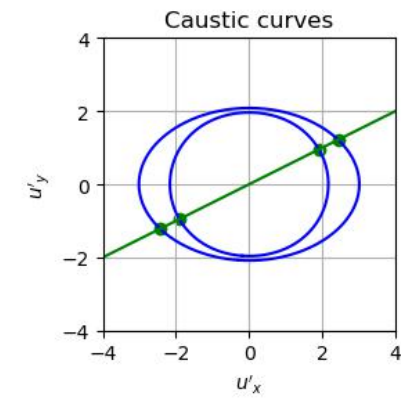
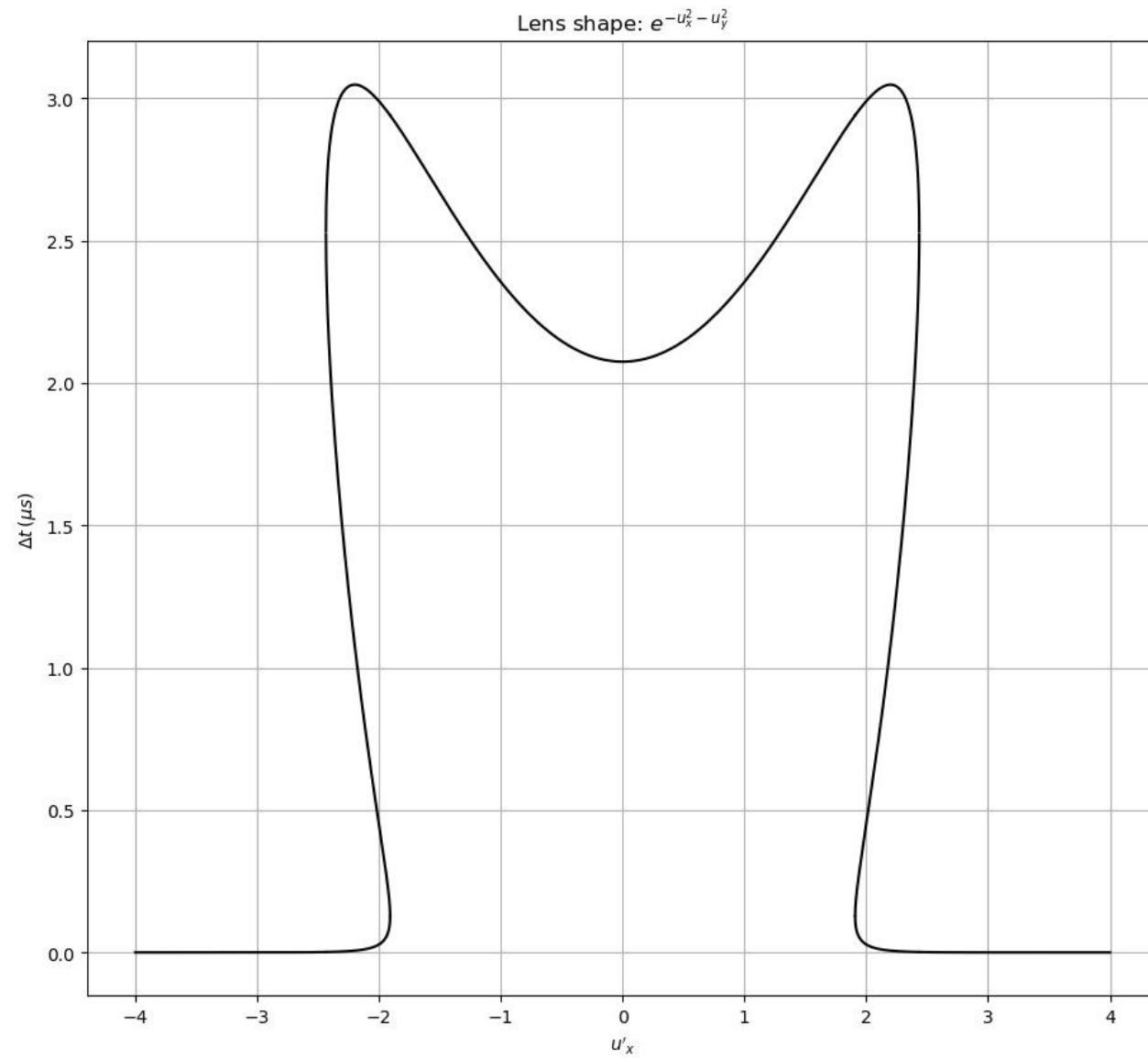




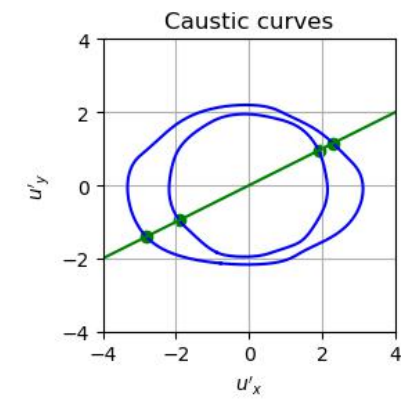
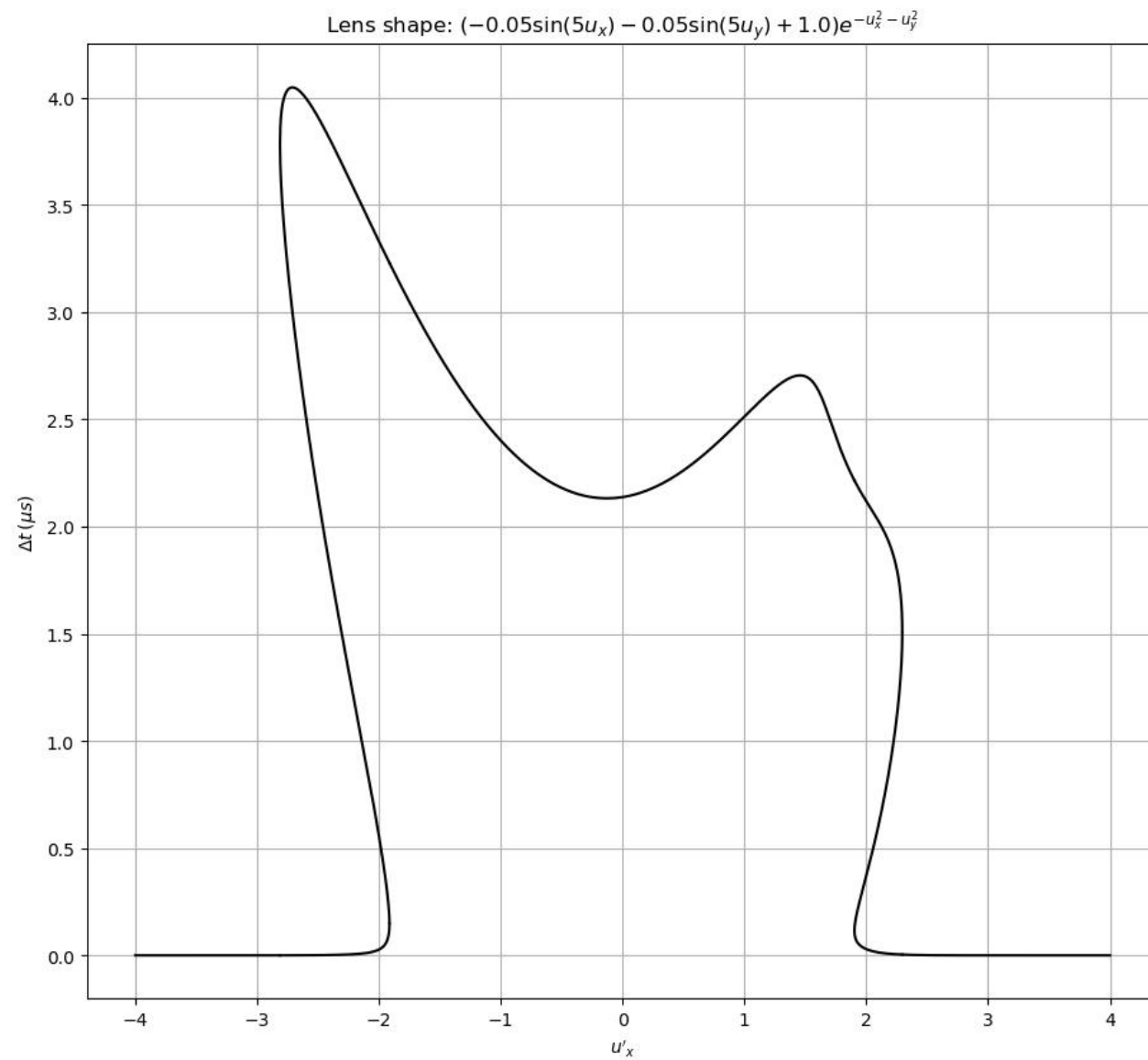
Parameter	Value
$d_{so}$ (kpc)	1.1
$d_{sl}$ (kpc)	0.55
$a_x$ (AU)	0.3
$a_y$ (AU)	0.4
$DM_I$ ( $\text{pc cm}^{-3}$ )	-5.00E-4
$\nu$ (GHz)	1.0
Slope	0.5
Offset	0.0



Parameter	Value
$d_{so} (kpc)$	1.1
$d_{sl} (kpc)$	0.55
$a_x (AU)$	0.3
$a_y (AU)$	0.4
$DM_l (pc\ cm^{-3})$	-5.00E-4
$\nu$ (GHz)	1.0
Slope	0.5
Offset	0.0

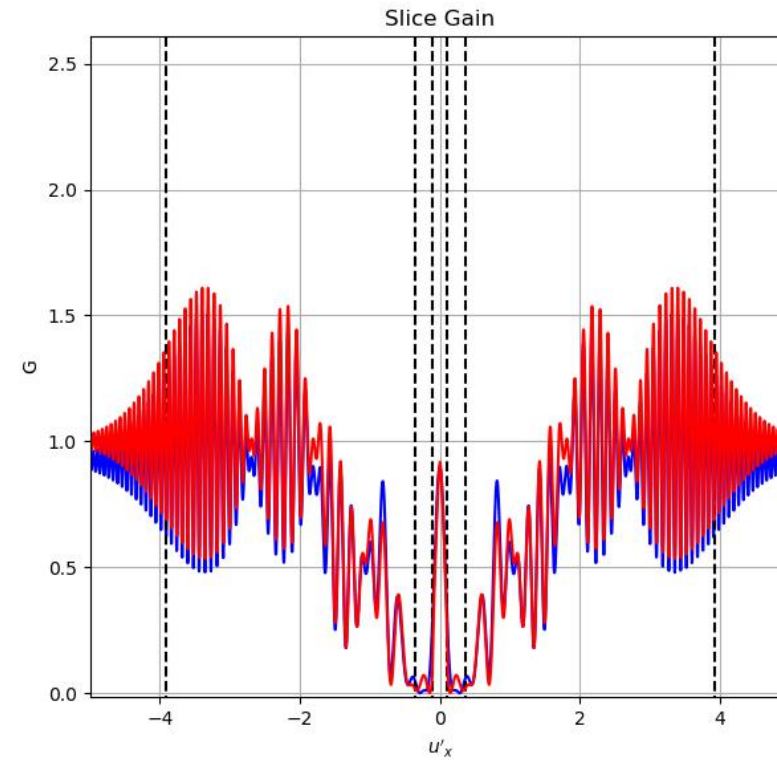
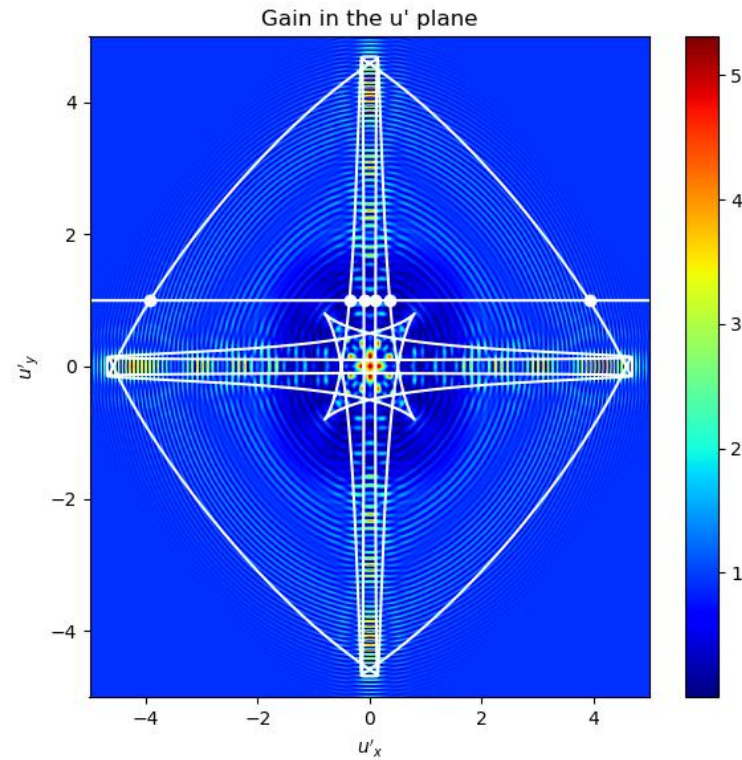


Parameter	Value
$d_{so}$ (kpc)	1.1
$d_{sl}$ (kpc)	0.55
$a_x$ (AU)	0.3
$a_y$ (AU)	0.4
$DM_I$ ( $\text{pc cm}^{-3}$ )	5.00E-4
$\nu$ (GHz)	1.0
Slope	0.5
Offset	0.0

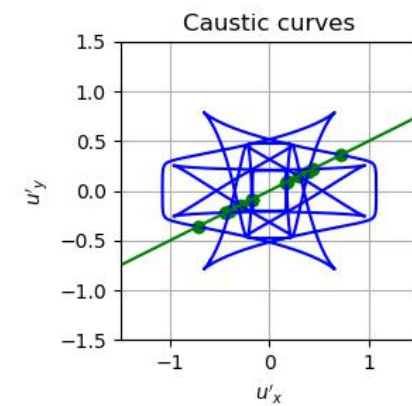
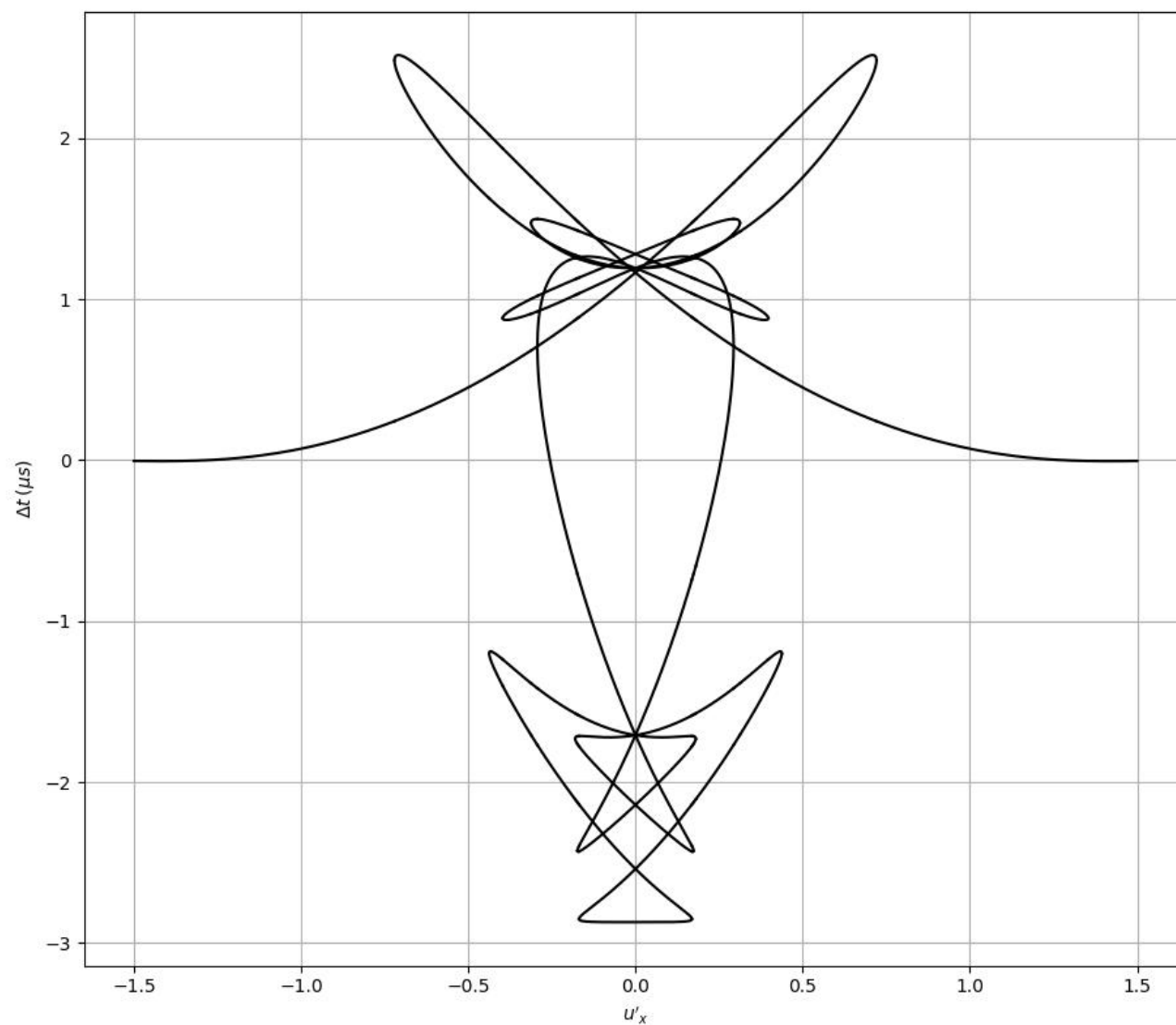


Parameter	Value
$d_{so} (kpc)$	1.1
$d_{sl} (kpc)$	0.55
$a_x (AU)$	0.3
$a_y (AU)$	0.4
$DM_I (pc\ cm^{-3})$	5.00E-4
$\nu$ (GHz)	1.0
Slope	0.5
Offset	0.0

# A fancy one...



Parameter	$d_{so}$ (kpc)	$d_{sl}$ (kpc)	$a_x$ (AU)	$a_y$ (AU)	$DM_l$ (pc cm <sup>-3</sup> )	$\nu$ (GHz)	Slope	Offset	Lens shape
Value	1.1	0.55	0.02	0.02	-2.00E-6	0.8	0.0	1.0	$e^{-ux^4 - uy^4}$



Parameter	Value
$d_{so}$ (kpc)	1.1
$d_{sl}$ (kpc)	0.55
$a_x$ (AU)	0.5
$a_y$ (AU)	0.6
$DM_l$ (pc cm <sup>-3</sup> )	-5.00E-4
$\nu$ (GHz)	0.85
Slope	0.5
Offset	0.0
Lens shape	$e^{-u_x^4 - u_y^4}$

# Future work

- Combine multiple image TOAs to predict altered pulse shapes, TOAs as measured by the telescope.
- Find lens parameters that can reproduce J1713+0747 event observations.
- Average over frequency bands.