

Numerical Analysis of Solutions to Equations

University of California Davis

Winter 2023

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Math 128b: Coding Report 2

Due May 5

1 Problems (100 pts)

For this chapter, you will write scripts to compute the solution to the linear equation $Ax = b$. First, write a script to perform Gaussian elimination **without pivoting** and a separate script to perform back substitution. Next, write a third function to determine the LU factorization of a matrix and a fourth to solve $LUx = b$ via forward and backwards solves (you should re-use your backsolve algorithm in this).

First, apply these scripts to the matrix you studied in problem 2 in the second problem set to confirm their correctness with $b = [3, 2, 1]^T$. Report the factored matrices and the x you find with each method.

Next, define a $(n \times n)$ matrix $A = (5\sqrt{n})I + R$ where R is a matrix with random entries from the normal distribution centered at 0 with a standard deviation of 1 and I is the identity matrix. Define b as a $(n \times 1)$ vector with random entries drawn from the same random distribution. For $n = 50, 100, 250, 500$, solve the system $Ax = b$ using your code. Then fill in the table below with the residual error $\|Ax - b\|_2$. Please report these errors using scientific notation. Finally, plot the time needed to solve the system vs. n for each method (please include both methods on the same graph). Do not include the time to create the matrix A in your experiments.

Describe the pattern you observe in the errors and wallclock time graph and how that relates to the theory which we've developed in class.

Lastly, try a few experiments with the matrix family $\hat{A} = R$ (with R defined as above) and write a sentence or two on your observations. You do not need to submit any code or figures for this, but you should explain what you observe and why this happens.

	GE	LU
50		
100		
250		
500		

Table 1: Error for $\|Ax - b\|_2$ for different sized matrices GE, and LU factorizations

Hint 1: You can time a block of code in Matlab with

```
tic;  
#### YOUR CODE HERE ####  
total_time = toc;
```

and in python with:

```
import time
t0 = time.time()
#### YOUR CODE HERE ####
t1 = time.time()
total_time = t1-t0
```

Hint 2 To create the random matrices you can do the following in Matlab:

```
A = 5*sqrt(5)*eye(n)+randn(n,n);
```

and in python as:

```
import numpy
A = 5*np.sqrt(5)*np.eye(n) + np.random.normal(size=(n, n))
```

Attach all code you wrote to the end of the report after your results.

2 Collaboration

Please use this space to recognize any and all collaborations which assisted you on the completion of this assignment.

3 Academic Integrity

Please copy and sign the following statement of academic integrity:

On my personal integrity as a student and member of the UCD community, I have not given, nor received and unauthorized assistance on this assignment.