CR-1

May 6, 2023

1 Coding Report 1

2 Problem Description

Many equations of models have roots that cannot be explicitly solved for using calculus, algebra, etc. For example, a population growth model where the initial population is 1,000,000 can be written 1,564,000 = 1,000,000 e^(lambda) + (435000)/(lambda) (e^(lambda) - 1) In this case there are numerical methods that can be used to approximate solutions to these equations to an arbitrarily high degree.

3 Results

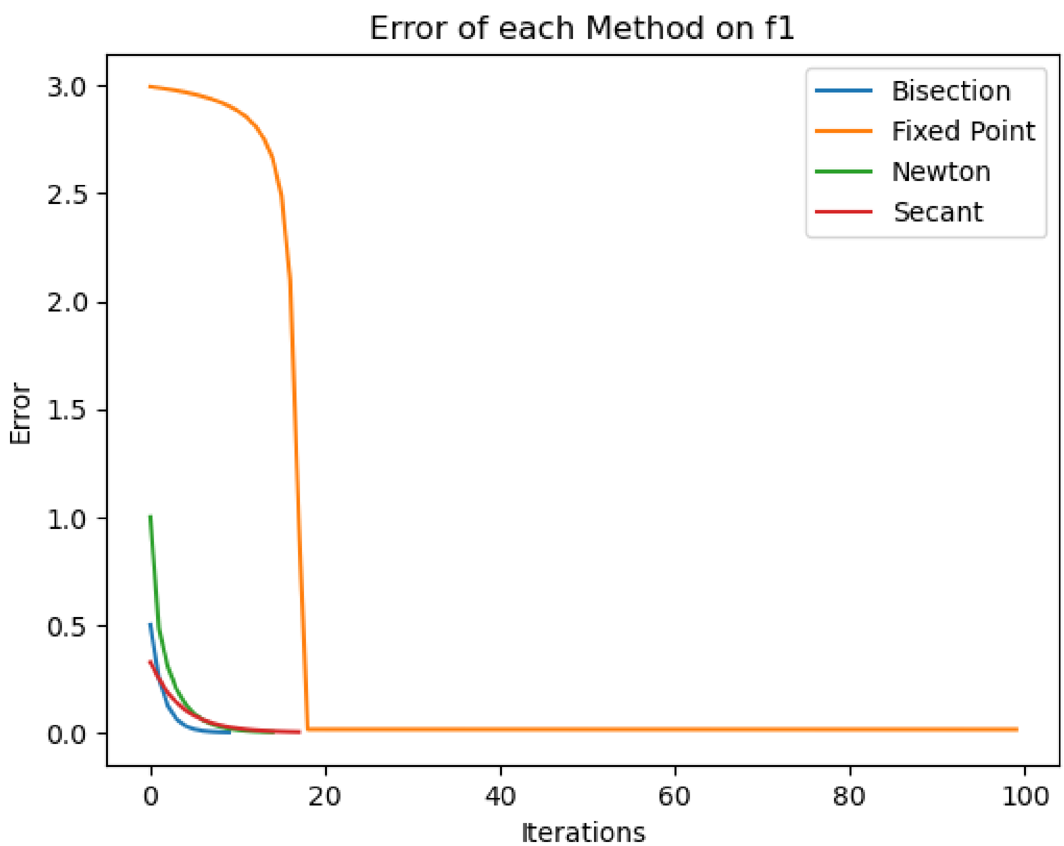
In this report I use the Bisection method, the Fixed-Point method, Newton's method, and the Secant method to determine the roots of two functions: f_1(x) = (1)/(2) sin(x-1)^3 f_2(x) = 6^(-x) - 2 Using each of these methods I constructed a plot showing the covergence rate of each method using the same margin for error for each method, on each function. I also include a table of values used in the construction of the plots. Every method showed a different rate of convergence, although in all cases the rate can be affected by how good your inital guess for where a root is. Still, my conclusion for this small sample size is that the Bisection, Newton, and Secant methods all had similarly fast convergence rates, while Fixed Point iteration had the clear slowest convergence. However, this could be a result of incorrect code for the method.

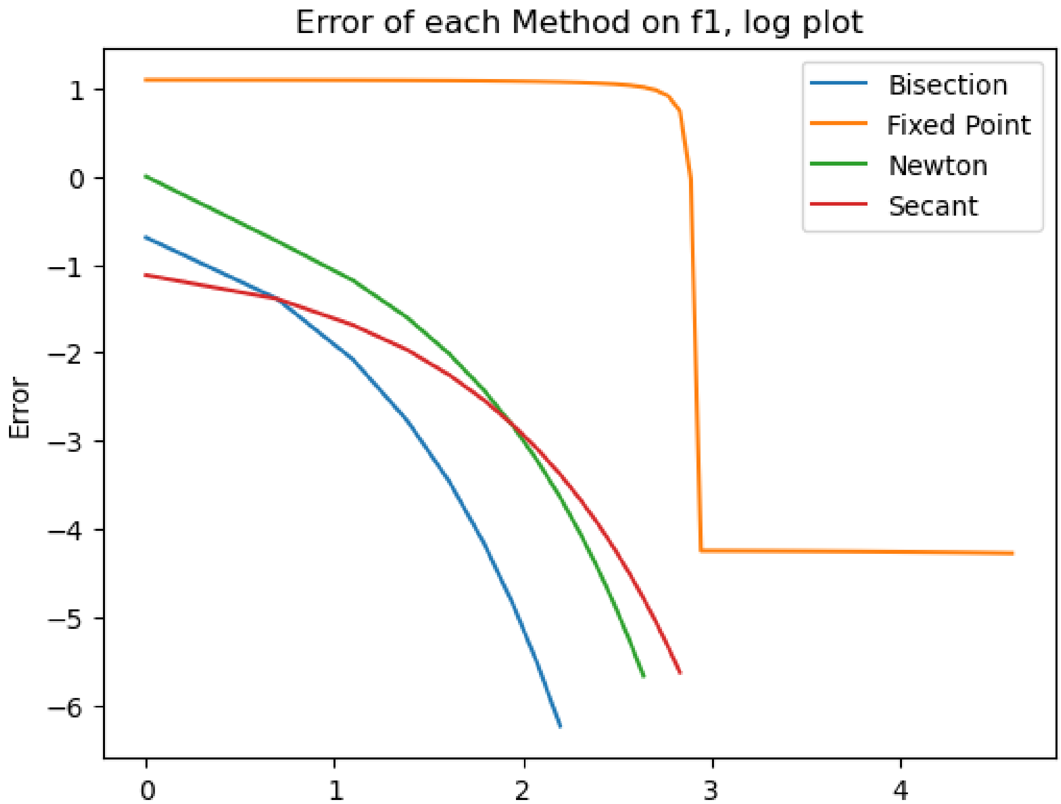
Below are the plots and table:

3.1 Plots & Table

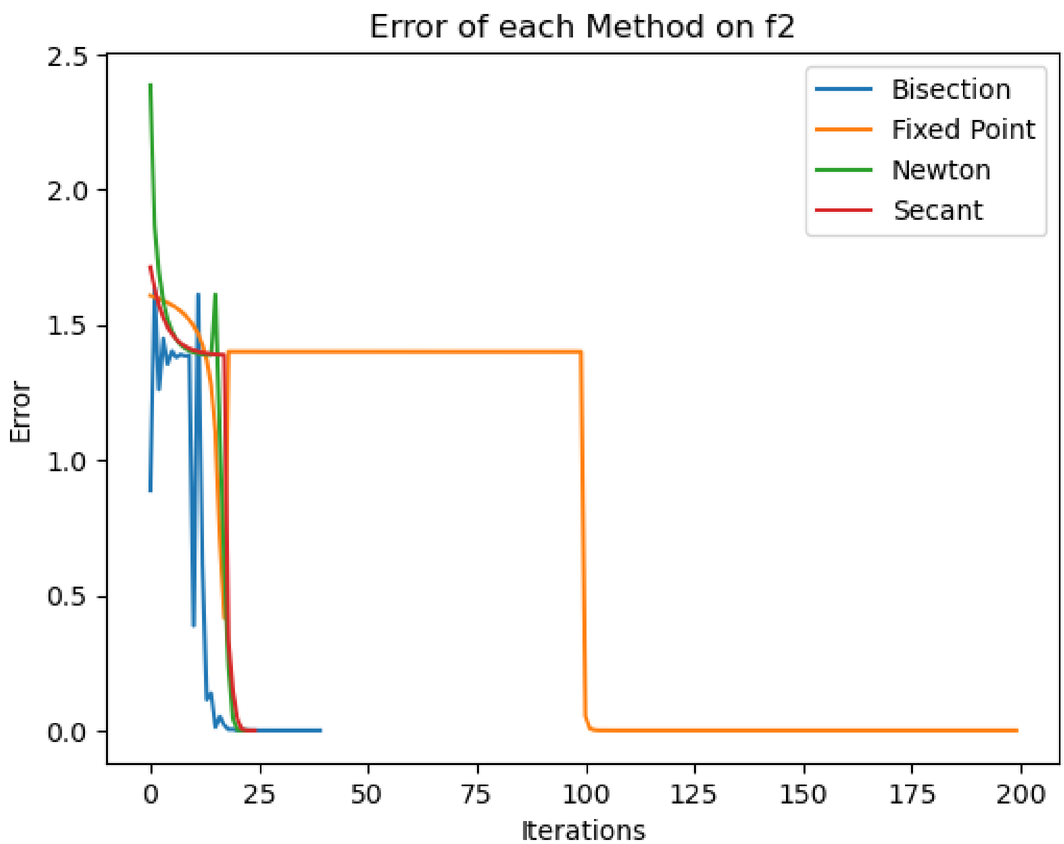
```
Iteration: 1, Approximation = 1.326360
Iteration: 2, Approximation = 1.249644
Iteration: 3, Approximation = 1.184912
Iteration: 4, Approximation = 1.139527
Iteration: 5, Approximation = 1.104897
Iteration: 6, Approximation = 1.079119
Iteration: 7, Approximation = 1.059663
Iteration: 8, Approximation = 1.045022
Iteration: 9, Approximation = 1.033976
Iteration: 10, Approximation = 1.025644
Iteration: 11, Approximation = 1.019356
Iteration: 12, Approximation = 1.014611
```

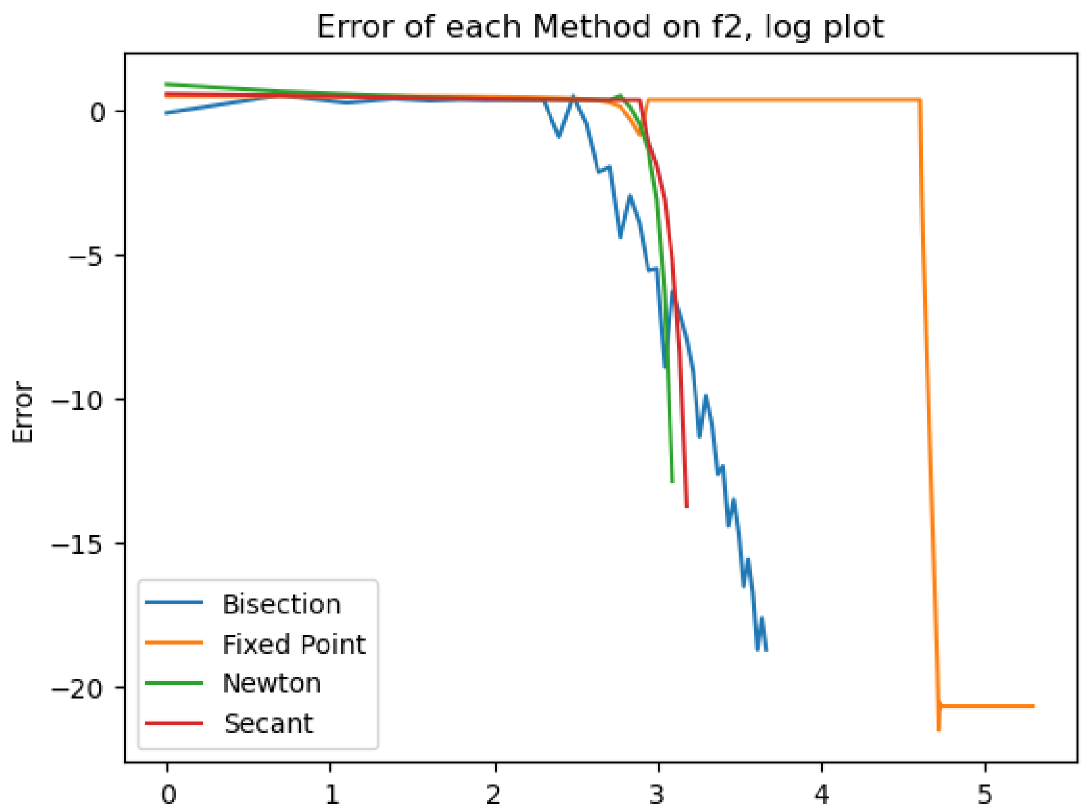
```
Iteration: 13, Approximation = 1.011029
Iteration: 14, Approximation = 1.008326
Iteration: 15, Approximation = 1.006285
Iteration: 16, Approximation = 1.004744
Iteration: 17, Approximation = 1.003581
Iteration: 18, Approximation = 1.002703

 Approximation: 1.002703
Iteration: 1, Approximation = -0.057143
Iteration: 2, Approximation = -0.530027
Iteration: 3, Approximation = -0.342777
Iteration: 4, Approximation = -0.381376
Iteration: 5, Approximation = -0.387072
Iteration: 6, Approximation = -0.386852
Iteration: 7, Approximation = -0.386853

 Approximation: -0.386853
```





Error of each Method on f1, log plot
Bisection
Fixed Point
Newton
Secant
Error
1
0
−1
−2
−3
−4
−5
−6
0
1
2
3
4


Error of each Method on f2
Bisection
Fixed Point
Newton
Secant
Error
Iterations
0.0
0.5
1.0
1.5
2.0
2.5
0
25
50
75
100
125
150
175
200



f1 - Bisection	f2 - Bisection	f1 - Fixed Point	f2 - Fixed Point	f1 - Newton	f2 - Newton	f1 - Secant	f2 - Secant
0.5	0.886853	2.99508	1.60823	1	2.38685	0.32636	1.71321
0.25	1.63685	2.98964	1.60278	0.480864	1.86772	0.249644	1.6365
0.125	1.26185	2.98357	1.59671	0.306961	1.69381	0.184912	1.57176
0.0625	1.44935	2.97675	1.58989	0.201301	1.58815	0.139527	1.52638

0.03125	1.3556	2.96901	1.58216
0.133279	1.52013	0.104897	1.49175
0.015625	1.40248	2.96015	1.5733
0.088588	1.47544	0.0791187	1.46597
0.0078125	1.37904	2.94987	1.56302
0.0589812	1.44583	0.0596631	1.44652
0.00390625	1.39076	2.93776	1.55091
0.0392979	1.42615	0.0450217	1.43187
0.00195312	1.3849	2.92325	1.53639
0.0261919	1.41304	0.0339759	1.42083
0.000976562	1.38783	2.90546	1.51861
0.0174593	1.40431	0.0256442	1.4125
N/A	0.386853	2.88305	1.4962
0.0116389	1.39849	0.0193565	1.40621
N/A	1.61315	2.85381	1.46695
0.0077591	1.39461	0.0146111	1.40146
N/A	0.613147	2.81379	1.42694
0.00517268	1.39203	0.0110293	1.39788
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0.00344844	1.3903	0.00832563	1.39518
N/A	0.136853	2.66186	1.27501
0.00229895	1.38915	0.00628478	1.39314
N/A	0.0118528	2.48981	1.10296
N/A	1.61315	0.00474421	1.3916

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N/A	N/A	N/A	N/A

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N/A	6.70552e-08	0.0141778	1.40103	
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N/A	N/A	N/A	N/A	
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N/A	7.45058e-09	0.0141629	1.40102	
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N/A	N/A	N/A	0.000290634	N/A	N/A	N/A	N/A
N/A	N/A	N/A	5.66218e-05	N/A	N/A	N/A	N/A
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N/A	N/A	N/A	4.15483e-07	N/A	N/A	N/A	N/A
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N/A	N/A	N/A	1.06214e-09	
N/A	N/A	N/A	N/A	
N/A	N/A	N/A	1.06214e-09	
N/A	N/A	N/A	N/A	

N/A	N/A	N/A	1.06214e-09
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N/A	N/A	N/A	1.06214e-09
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N/A	N/A	N/A	1.06214e-09
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N/A	N/A	N/A	1.06214e-09
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N/A	N/A	N/A	1.06214e-09
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N/A	N/A	N/A	1.06214e-09
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N/A	N/A	N/A	1.06214e-09
N/A	N/A	N/A	N/A

N/A	N/A	N/A	1.06214e-09	N/A	N/A	N/A	N/A
N/A	N/A	N/A	1.06214e-09	N/A	N/A	N/A	N/A
N/A	N/A	N/A	1.06214e-09	N/A	N/A	N/A	N/A
N/A	N/A	N/A	1.06214e-09	N/A	N/A	N/A	N/A
N/A	N/A	N/A	1.06214e-09	N/A	N/A	N/A	N/A
N/A	N/A	N/A	1.06214e-09	N/A	N/A	N/A	N/A
N/A	N/A	N/A	1.06214e-09	N/A	N/A	N/A	N/A
N/A	N/A	N/A	1.06214e-09	N/A	N/A	N/A	N/A
N/A	N/A	N/A	1.06214e-09	N/A	N/A	N/A	N/A
N/A	N/A	N/A	1.06214e-09	N/A	N/A	N/A	N/A
N/A	N/A	N/A	1.06214e-09	N/A	N/A	N/A	N/A
N/A	N/A	N/A	1.06214e-09	N/A	N/A	N/A	N/A

N/A	N/A	N/A	1.06214e-09
N/A	N/A	N/A	N/A
N/A	N/A	N/A	1.06214e-09
N/A	N/A	N/A	N/A
N/A	N/A	N/A	1.06214e-09
N/A	N/A	N/A	N/A
N/A	N/A	N/A	1.06214e-09
N/A	N/A	N/A	N/A
N/A	N/A	N/A	1.06214e-09
N/A	N/A	N/A	N/A
N/A	N/A	N/A	1.06214e-09
N/A	N/A	N/A	N/A
N/A	N/A	N/A	1.06214e-09
N/A	N/A	N/A	N/A
N/A	N/A	N/A	1.06214e-09
N/A	N/A	N/A	N/A
N/A	N/A	N/A	1.06214e-09
N/A	N/A	N/A	N/A
N/A	N/A	N/A	1.06214e-09
N/A	N/A	N/A	N/A
N/A	N/A	N/A	1.06214e-09
N/A	N/A	N/A	N/A
N/A	N/A	N/A	1.06214e-09
N/A	N/A	N/A	N/A

```
  N/A                N/A                     N/A                     1.06214e-09
N/A            N/A              N/A            N/A


  N/A                N/A                     N/A                     1.06214e-09
N/A            N/A              N/A            N/A


  N/A                N/A                     N/A                     1.06214e-09
N/A            N/A              N/A            N/A


  N/A                N/A                     N/A                     1.06214e-09
N/A            N/A              N/A            N/A
```

3.2 The Secant Method

The results of this method are included above in the final plot^

Given two initial guesses x0 and x1, the secant method computes the next approximation of the root by finding the point where the secant line through (x0, f(x0)) and (x1, f(x1)) intersects the x-axis. The algorithm works as follows:

Choose initial guesses x0 and x1, such that f(x0) and f(x1) have opposite signs. Compute the next approximation x2 using the formula x2 = x1 - f(x1) * (x1 - x0) / (f(x1) - f(x0)). Check if the absolute value of the difference between x2 and x1 is less than a desired tolerance. If it is, return x2 as the root. Otherwise, set x0 = x1 and x1 = x2, and repeat step 2. The secant method does not require the computation of derivatives, and is often faster than the bisection method for finding roots. However, it does not always converge and can sometimes converge to a root that is not the one intended.

3.3 Our functions f & g and their derivatives

```
f(x) = 0.5*sin(x - 1)**3
f'(x) = 1.5*sin(x - 1)**2*cos(x - 1)
g(x) = -2 + 6**(-x)
g'(x) = -log(6)/6**x
```

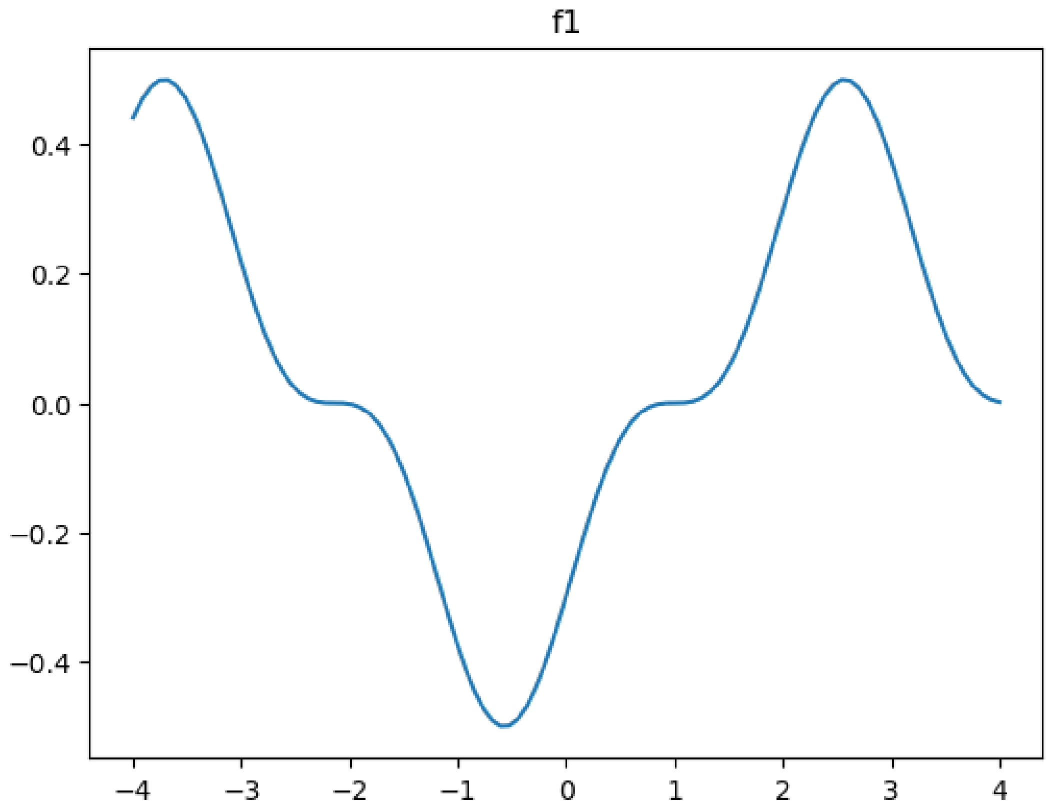
3.4 Evaluatling at user input at x for functions

```
Input x = 1
f(x) = 0.0
Input x = 2
f'(x) = 0.5738605509257195
Input x: 3
g(x) = -1.9953703703703705
```

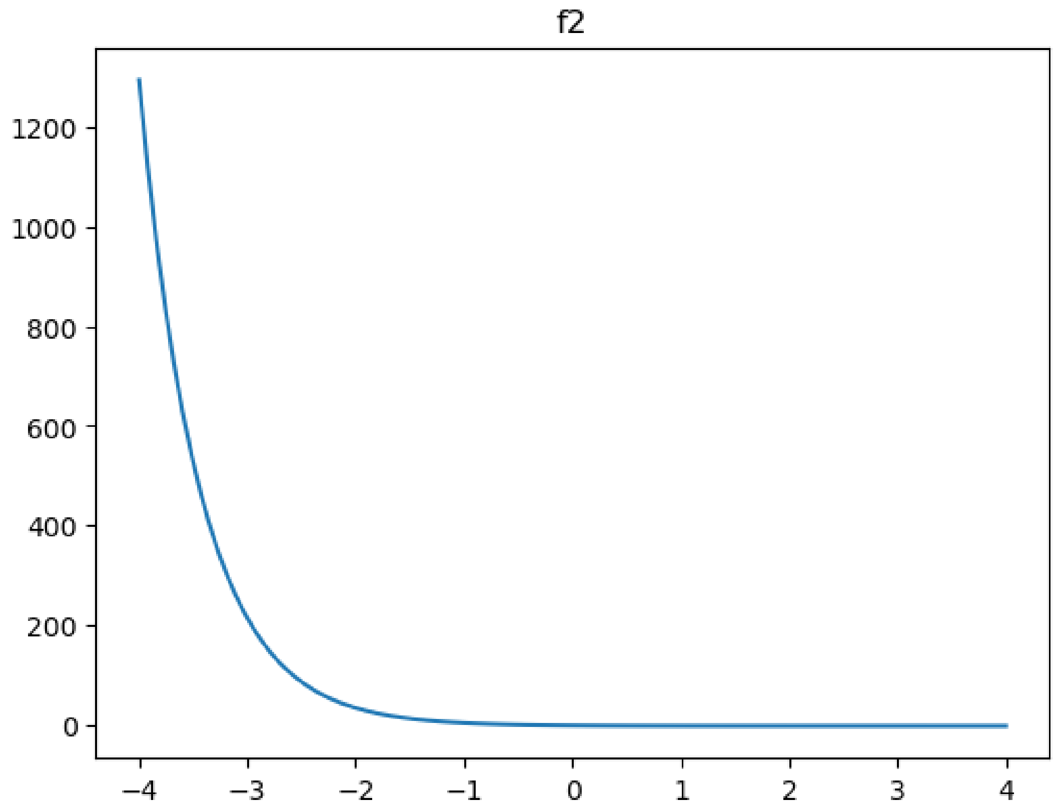
```
Input x = 4
g'(x) = -0.001382530454651277
```

3.5 Plotting f and g

[7]: <function matplotlib.pyplot.show(close=None, block=None)>



[8]: <function matplotlib.pyplot.show(close=None, block=None)>



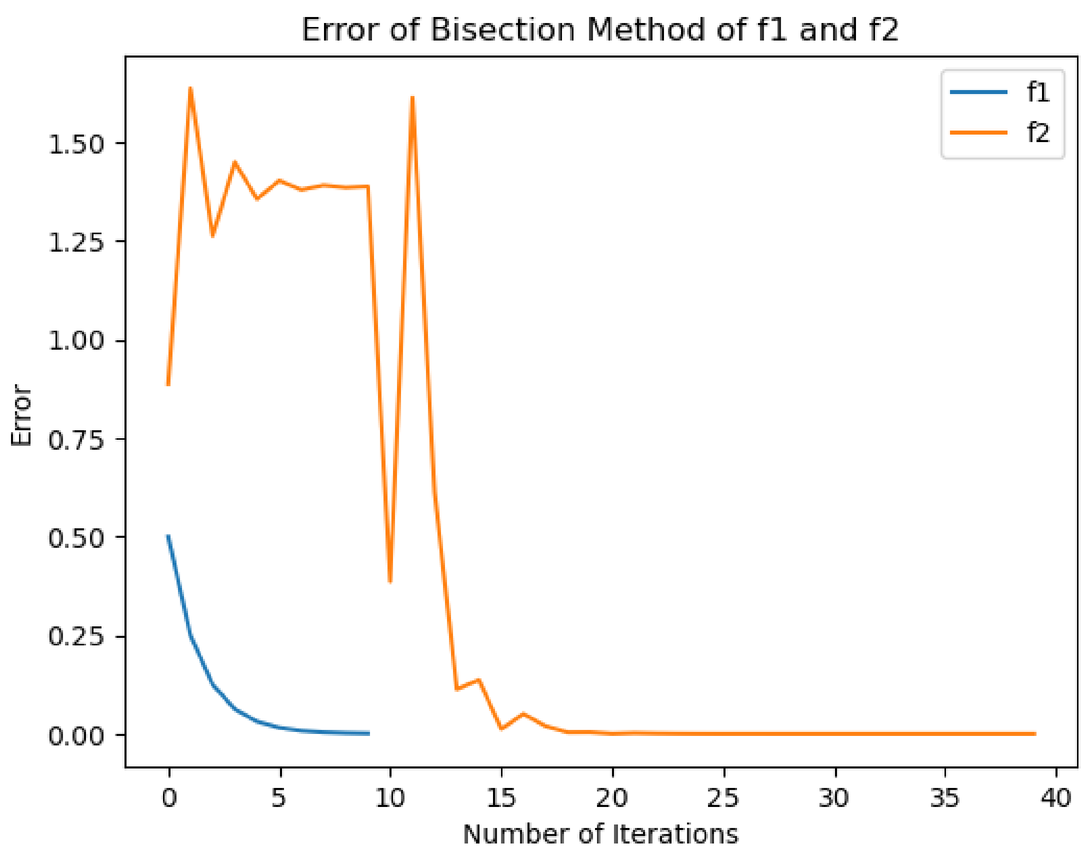
3.6 The Bisection Method

```
Approximation = 0.500000
Approximation = 1.250000
Approximation = 0.875000
Approximation = 1.062500
Approximation = 0.968750
Approximation = 1.015625
Approximation = 0.992188
Approximation = 1.003906
Approximation = 0.998047

The root of the function between these bounds is  1.0009765625
Approximation = 0.000000
Approximation = -2.000000
Approximation = -1.000000
Approximation = -0.500000
Approximation = -0.250000
Approximation = -0.375000
Approximation = -0.437500
Approximation = -0.406250
```

```
Approximation = -0.390625
Approximation = -0.382812
Approximation = -0.386719
Approximation = -0.388672
Approximation = -0.387695
Approximation = -0.387207
Approximation = -0.386963
Approximation = -0.386841
Approximation = -0.386902
Approximation = -0.386871
Approximation = -0.386856
Approximation = -0.386848
Approximation = -0.386852
Approximation = -0.386854
Approximation = -0.386853
Approximation = -0.386853
Approximation = -0.386853
Approximation = -0.386853
Approximation = -0.386853
Approximation = -0.386853
Approximation = -0.386853

The root of the function between these bounds is  -0.38685280829668045
```



Given an interval [a, b] where the function changes sign, the method repeatedly bisects the interval and selects the subinterval in which the function changes sign, until the root is located within a desired tolerance. The algorithm works as follows:

Begin with an interval [a, b] where the function changes sign. Compute the midpoint c = (a + b)/2. Evaluate the function at c. If the function at c is close enough to zero (i.e., within the desired tolerance), then return c as the root. Otherwise, determine which subinterval [a, c] or [c, b] has a sign change, and repeat the algorithm on that subinterval. The bisection method is guaranteed to converge to a root of the function as long as the function is continuous and changes sign within the interval [a, b]. However, it can be slow to converge for functions that are very flat near the root, or for intervals that are very wide.

3.7 Fixed Point Iteration

```
Iteration: 1, Approximation = -1.995082
Iteration: 2, Approximation = -1.989637
Iteration: 3, Approximation = -1.983567
Iteration: 4, Approximation = -1.976747
Iteration: 5, Approximation = -1.969014
Iteration: 6, Approximation = -1.960152
Iteration: 7, Approximation = -1.949870
Iteration: 8, Approximation = -1.937762
Iteration: 9, Approximation = -1.923247
Iteration: 10, Approximation = -1.905460
Iteration: 11, Approximation = -1.883054
Iteration: 12, Approximation = -1.853807
Iteration: 13, Approximation = -1.813794
Iteration: 14, Approximation = -1.755389
Iteration: 15, Approximation = -1.661863
Iteration: 16, Approximation = -1.489808
Iteration: 17, Approximation = -1.099186
Iteration: 18, Approximation = 0.028029
Iteration: 19, Approximation = 1.014253
Iteration: 20, Approximation = 1.014248
Iteration: 21, Approximation = 1.014243
Iteration: 22, Approximation = 1.014238
Iteration: 23, Approximation = 1.014233
Iteration: 24, Approximation = 1.014228
Iteration: 25, Approximation = 1.014223
Iteration: 26, Approximation = 1.014218
Iteration: 27, Approximation = 1.014213
Iteration: 28, Approximation = 1.014208
Iteration: 29, Approximation = 1.014203
Iteration: 30, Approximation = 1.014198
Iteration: 31, Approximation = 1.014193
Iteration: 32, Approximation = 1.014188
```

```
Iteration: 33, Approximation = 1.014183
Iteration: 34, Approximation = 1.014178
Iteration: 35, Approximation = 1.014173
Iteration: 36, Approximation = 1.014168
Iteration: 37, Approximation = 1.014163
Iteration: 38, Approximation = 1.014158
Iteration: 39, Approximation = 1.014153
Iteration: 40, Approximation = 1.014148
Iteration: 41, Approximation = 1.014143
Iteration: 42, Approximation = 1.014138
Iteration: 43, Approximation = 1.014133
Iteration: 44, Approximation = 1.014128
Iteration: 45, Approximation = 1.014123
Iteration: 46, Approximation = 1.014118
Iteration: 47, Approximation = 1.014113
Iteration: 48, Approximation = 1.014108
Iteration: 49, Approximation = 1.014104
Iteration: 50, Approximation = 1.014099
Iteration: 51, Approximation = 1.014094
Iteration: 52, Approximation = 1.014089
Iteration: 53, Approximation = 1.014084
Iteration: 54, Approximation = 1.014079
Iteration: 55, Approximation = 1.014074
Iteration: 56, Approximation = 1.014069
Iteration: 57, Approximation = 1.014064
Iteration: 58, Approximation = 1.014060
Iteration: 59, Approximation = 1.014055
Iteration: 60, Approximation = 1.014050
Iteration: 61, Approximation = 1.014045
Iteration: 62, Approximation = 1.014040
Iteration: 63, Approximation = 1.014035
Iteration: 64, Approximation = 1.014030
Iteration: 65, Approximation = 1.014026
Iteration: 66, Approximation = 1.014021
Iteration: 67, Approximation = 1.014016
Iteration: 68, Approximation = 1.014011
Iteration: 69, Approximation = 1.014006
Iteration: 70, Approximation = 1.014002
Iteration: 71, Approximation = 1.013997
Iteration: 72, Approximation = 1.013992
Iteration: 73, Approximation = 1.013987
Iteration: 74, Approximation = 1.013982
Iteration: 75, Approximation = 1.013978
Iteration: 76, Approximation = 1.013973
Iteration: 77, Approximation = 1.013968
Iteration: 78, Approximation = 1.013963
Iteration: 79, Approximation = 1.013958
Iteration: 80, Approximation = 1.013954
```

```
Iteration: 81, Approximation = 1.013949
Iteration: 82, Approximation = 1.013944
Iteration: 83, Approximation = 1.013939
Iteration: 84, Approximation = 1.013935
Iteration: 85, Approximation = 1.013930
Iteration: 86, Approximation = 1.013925
Iteration: 87, Approximation = 1.013921
Iteration: 88, Approximation = 1.013916
Iteration: 89, Approximation = 1.013911
Iteration: 90, Approximation = 1.013906
Iteration: 91, Approximation = 1.013902
Iteration: 92, Approximation = 1.013897
Iteration: 93, Approximation = 1.013892
Iteration: 94, Approximation = 1.013888
Iteration: 95, Approximation = 1.013883
Iteration: 96, Approximation = 1.013878
Iteration: 97, Approximation = 1.013874
Iteration: 98, Approximation = 1.013869
Iteration: 99, Approximation = 1.013864
Iteration: 100, Approximation = 1.013860

Not Convergent in 100 iterations.
Iteration: 1, Approximation = -0.333333
Iteration: 2, Approximation = -0.394293
Iteration: 3, Approximation = -0.385346
Iteration: 4, Approximation = -0.387143
Iteration: 5, Approximation = -0.386796
Iteration: 6, Approximation = -0.386864
Iteration: 7, Approximation = -0.386851
Iteration: 8, Approximation = -0.386853
Iteration: 9, Approximation = -0.386853
Iteration: 10, Approximation = -0.386853
Iteration: 11, Approximation = -0.386853
Iteration: 12, Approximation = -0.386853
Iteration: 13, Approximation = -0.386853
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Iteration: 15, Approximation = -0.386853
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Iteration: 18, Approximation = -0.386853
Iteration: 19, Approximation = -0.386853
Iteration: 20, Approximation = -0.386853
Iteration: 21, Approximation = -0.386853
Iteration: 22, Approximation = -0.386853
Iteration: 23, Approximation = -0.386853
Iteration: 24, Approximation = -0.386853
Iteration: 25, Approximation = -0.386853
Iteration: 26, Approximation = -0.386853
```

```
Iteration: 27, Approximation = -0.386853
Iteration: 28, Approximation = -0.386853
Iteration: 29, Approximation = -0.386853
Iteration: 30, Approximation = -0.386853
Iteration: 31, Approximation = -0.386853
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Iteration: 37, Approximation = -0.386853
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Iteration: 48, Approximation = -0.386853
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Iteration: 51, Approximation = -0.386853
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Iteration: 53, Approximation = -0.386853
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Iteration: 55, Approximation = -0.386853
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Iteration: 58, Approximation = -0.386853
Iteration: 59, Approximation = -0.386853
Iteration: 60, Approximation = -0.386853
Iteration: 61, Approximation = -0.386853
Iteration: 62, Approximation = -0.386853
Iteration: 63, Approximation = -0.386853
Iteration: 64, Approximation = -0.386853
Iteration: 65, Approximation = -0.386853
Iteration: 66, Approximation = -0.386853
Iteration: 67, Approximation = -0.386853
Iteration: 68, Approximation = -0.386853
Iteration: 69, Approximation = -0.386853
Iteration: 70, Approximation = -0.386853
Iteration: 71, Approximation = -0.386853
Iteration: 72, Approximation = -0.386853
Iteration: 73, Approximation = -0.386853
Iteration: 74, Approximation = -0.386853
```

```
Iteration: 75, Approximation = -0.386853
Iteration: 76, Approximation = -0.386853
Iteration: 77, Approximation = -0.386853
Iteration: 78, Approximation = -0.386853
Iteration: 79, Approximation = -0.386853
Iteration: 80, Approximation = -0.386853
Iteration: 81, Approximation = -0.386853
Iteration: 82, Approximation = -0.386853
Iteration: 83, Approximation = -0.386853
Iteration: 84, Approximation = -0.386853
Iteration: 85, Approximation = -0.386853
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Iteration: 88, Approximation = -0.386853
Iteration: 89, Approximation = -0.386853
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Iteration: 92, Approximation = -0.386853
Iteration: 93, Approximation = -0.386853
Iteration: 94, Approximation = -0.386853
Iteration: 95, Approximation = -0.386853
Iteration: 96, Approximation = -0.386853
Iteration: 97, Approximation = -0.386853
Iteration: 98, Approximation = -0.386853
Iteration: 99, Approximation = -0.386853
Iteration: 100, Approximation = -0.386853

Not Convergent in 100 iterations.
```
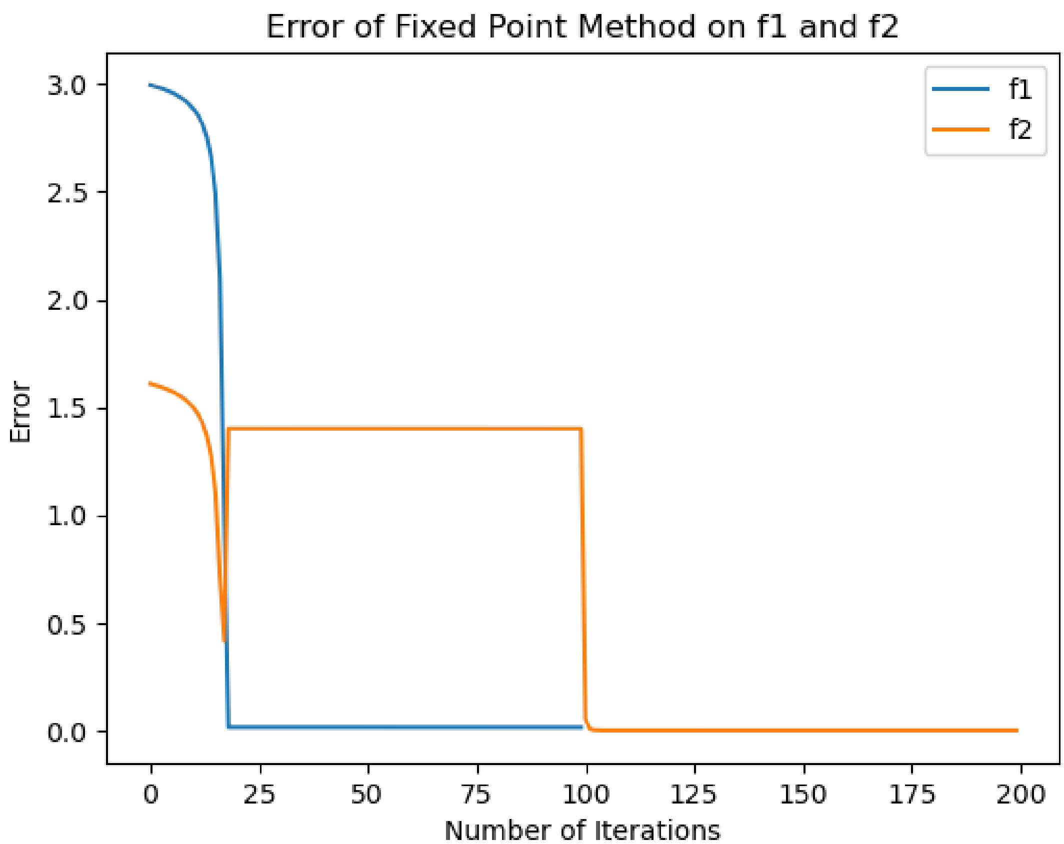



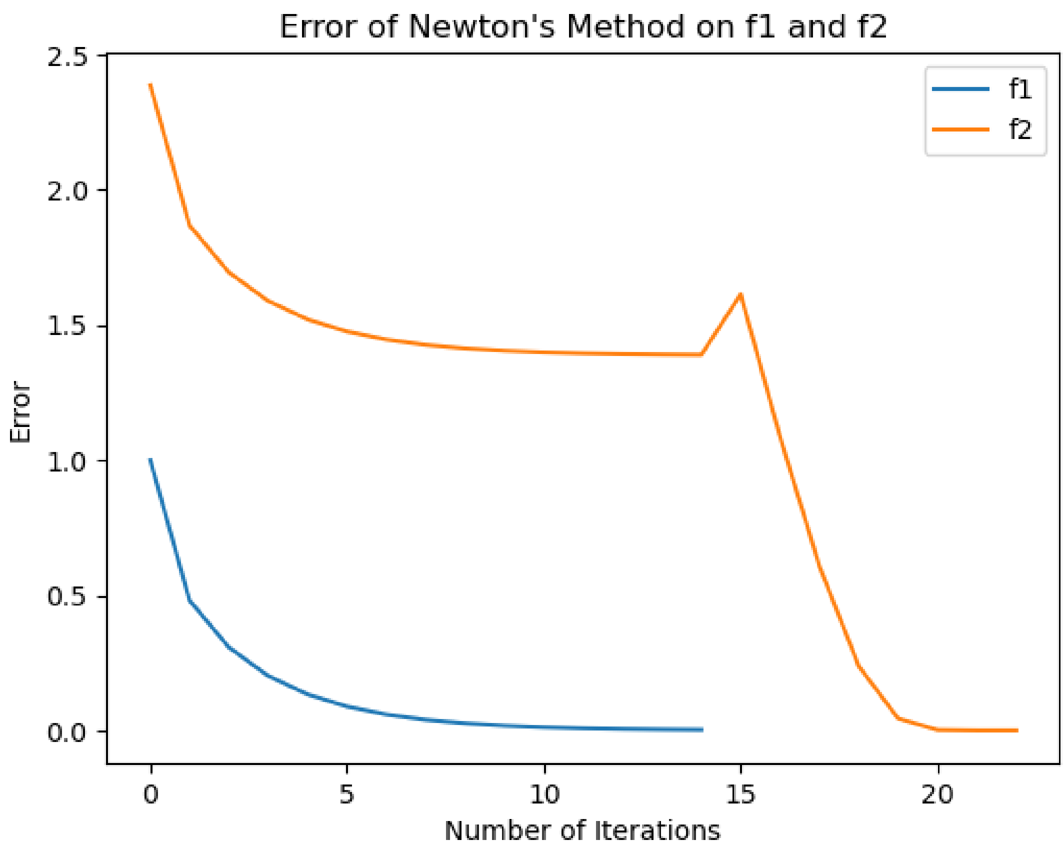
The fixed point iteration method works by repeatedly applying the function f to an initial guess x0, generating a sequence of approximations x1, x2, x3, ..., until convergence to a fixed point is achieved within a desired tolerance. The algorithm works as follows:

Choose an initial guess x0. Compute the next approximation x1 by applying the function f to x0, i.e., x1 = f(x0). Check if the absolute value of the difference between x1 and x0 is less than a desired tolerance. If it is, return x1 as the fixed point. Otherwise, set x0 = x1 and repeat step 2. The method is guaranteed to converge to a fixed point if f is a contraction mapping, which means that it satisfies a Lipschitz condition with constant less than 1. In practice, the method may converge slowly or not at all if f is not a contraction mapping or if the initial guess is far from the fixed point.

3.8 Newton's Method

```
Approximation: 2.000000
Approximation: 1.480864
Approximation: 1.306961
Approximation: 1.201301
Approximation: 1.133279
Approximation: 1.088588
Approximation: 1.058981
```

```
Approximation: 1.039298
Approximation: 1.026192
Approximation: 1.017459
Approximation: 1.011639
Approximation: 1.007759
Approximation: 1.005173
Approximation: 1.003448

Approximation: 1.002299
Approximation: -2.000000
Approximation: -1.472896
Approximation: -0.994514
Approximation: -0.624278
Approximation: -0.430894
Approximation: -0.388546
Approximation: -0.386855

Approximation: -0.386853
```



Given an initial guess x0, Newton's method computes the next approximation x1 by finding the point where the tangent line to the function at x0 intersects the x-axis. The tangent line is given by the equation y = f(x0) + f'(x0) * (x - x0), where f'(x0) is the derivative of f evaluated at x0. The algorithm works as follows:

Choose an initial guess x0. Compute the next approximation x1 using the formula x1 = x0 - f(x0) / f'(x0). Check if the absolute value of the difference between x1 and x0 is less than a desired tolerance. If it is, return x1 as the root. Otherwise, set x0 = x1 and repeat step 2. Newton's method requires the computation of the derivative of the function, which can be computationally expensive or even impossible in some cases. Additionally, Newton's method may fail to converge or converge to a non-root if the initial guess is not close enough to a root or if the function has a very flat slope near the root.