Bus lines, multilines approach

notes GG

March 28, 2022

1 Formalism

1.1 Network definition

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a oriented graph and $\mathbf{A}$ its adjacency matrix, representing a multimodal transportation network between $|\mathcal{V}| = n$ stops and possessing l lines. Each stop belong to only one line, i.e. $\mathcal{V} = \bigcup_{k=1}^{l} \mathcal{V}_k$ and $\bigcap_{k=1}^{l} \mathcal{V}_k = \emptyset$, where $\mathcal{V}_k$ represents the set of nodes in line k. The edge set $\mathcal{E}$, can also be decomposed with $\mathcal{E} = \left(\bigcup_{k=1}^{l} \mathcal{E}_k \right) \cup \mathcal{E}_{\text{trsf}}$, where $\mathcal{E}_k$ contains edges connecting node inside transportation line k, and $\mathcal{E}_{\text{trsf}}$ contains edges permitting transfer between different transportation lines. We suppose that edges belong to only a unique set, i.e. $\mathbf{A} = \sum_{k=1}^{l} \mathbf{A}_k + \mathbf{A}_{\text{trsf}}$, and there are an uniquely define route inside lines, i.e. $a^k_{i\bullet} \leq 1$ and $a^k_{\bullet i} \leq 1 \ \forall i, k$. Transfer edges also define a subset of nodes $\mathcal{V}_{\text{f}} \subset \mathcal{V}$, containing *free nodes*, i.e. nodes connected to a transfer edge. We have $i \in \mathcal{V}_{\text{f}} \iff a^{\text{trsf}}_{i\bullet} + a^{\text{trsf}}_{\bullet i} > 0$.

1.2 Problem definition

Suppose that we possess two n-length vectors $\mathbf{x}_{\text{in}} = (x^{\text{in}}_i)$ and $\mathbf{y}_{\text{out}} = (y^{\text{out}}_i)$, representing respectively the flow entering the network at stop i and going out of the network at stop i.