Bus lines, multilines approach

notes GG

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1 Formalism

1.1 Network definition

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a oriented graph and $\mathbf{A}$ its adjacency matrix, representing a multimodal transportation network between $|\mathcal{V}| = n$ stops and possessing l lines. Each stop belong to only one line, i.e. $\mathcal{V} = \bigcup_{k=1}^{l} \mathcal{V}_k$ and $\bigcap_{k=1}^{l} \mathcal{V}_k = \emptyset$, where $\mathcal{V}_k$ represents the set of nodes in line k. The edge set $\mathcal{E}$, can also be decomposed with $\mathcal{E} = \left(\bigcup_{k=1}^{l} \mathcal{E}_k\right) \cup \mathcal{E}_{\mathrm{trsf}}$, where $\mathcal{E}_k$ contains edges connecting node inside transportation line k, and $\mathcal{E}_{\mathrm{trsf}}$ contains edges permitting transfer between different transportation lines. We suppose that edges belong to only a unique set, i.e. $\mathbf{A} = \sum_{k=1}^{l} \mathbf{A}_k + \mathbf{A}_{\mathrm{trsf}}$, and there are an uniquely define route inside lines, i.e. $a^k_{i\bullet} \leq 1$ and $a^k_{\bullet i} \leq 1$ $\forall i, k$. Transfer edges also define a subset of nodes $\mathcal{V}_{\mathrm{f}} \subset \mathcal{V}$, containing *free nodes*, i.e. nodes connected to a transfer edge. We have $i \in \mathcal{V}_{\mathrm{f}} \iff a^{\mathrm{trsf}}_{i\bullet} + a^{\mathrm{trsf}}_{\bullet i} > 0$.

1.2 Flow definition

A matrix $\mathbf{N} = (n_{ij})$, verifying:

1. $n_{ij} \geq 0$
2. $a_{ij} = 0 \Rightarrow n_{ij} = 0$

is a *flow matrix* defined on $\mathcal{G}$. Again, we can decompose this matrix with

$$\mathbf{N} = \mathbf{N}_{\mathrm{W}} + \mathbf{N}_{\mathrm{B}} \qquad \mathbf{N}_{\mathrm{W}} := \sum_{k=1}^{l} \mathbf{N}_k \tag{1}$$

where the flow $\mathbf{N}_{\mathrm{W}}$ is the flow inside the lines, $\mathbf{N}_{\mathrm{B}}$ the flow between lines, and $\mathbf{N}_l$ the flow inside line l.

Suppose that we possess two n-length vectors $\mathbf{l}_{\mathrm{in}} = (l^{\mathrm{in}}_i)$ and $\mathbf{l}_{\mathrm{out}} = (l^{\mathrm{out}}_i)$, representing ,respectively, the flow entering the line associated with stop i and going out of this line at stop i. These vectors can in fact be expressed with:

$$\mathbf{l}_{\mathrm{in}} = \boldsymbol{\sigma}_{\mathrm{in}} + \tag{2}$$