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CURVE KICK AERODYNAMICS OF A SOCCER BALL

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ABSTRACT:

Curling trajectories are often observed in association football. Attempts are continuously being made by current generation to repeat such motions. Velocity, point of impact and angular velocity are used as model parameter and the effects of three forces, namely, the drag force, the Magnus force or the lift force and the gravitational force are considered. The model constitutes a unified theory of solid mechanics and fluid mechanics to determine the governing laws of motion for a spinning ball to predict the motion of soccer ball accurately close to the real world situations.

Keywords: Solid-Fluid Interaction, Drag Force, Magnus Force, Force Coefficients, Unified Theory

INTRODUCTION:

The trajectory of a spinning soccer ball is governed by the velocity and the spin imparted and the ambient conditions. The forces acting on an inflight spinning ball are the gravitational force, the drag force and the Magnus force or the Lift force. (Buoyant force can be neglected). The velocity and the angular velocity so imparted to the ball are in turn functions of the effective point of impact on the ball during the kick, the foot velocity, the surface texture of the ball as well as the shoe, the air pressure inside the ball and the direction in which the foot is swung during the impact. In the past several attempts have been made to determine the governing laws of motion for a spinning ball. (Ref. 1 - 4). There have also been some attempts to determine the relationship between the foot velocity and off-set distance and the ball deformation during the impact, the velocity and spin of the ball. (Ref 5 – 6).

This study however attempts to present a combination of the solid mechanics aspects during the foot-ball interaction and the fluid mechanics that dictates the laws during the flight. Some of the popular goals have been simulated on the basis of calculations and assumptions with feasible parameters. These confirm the authenticity of the study.

Ball Impact Mechanics:

Consider the football represented as a perfect sphere in the following figure.

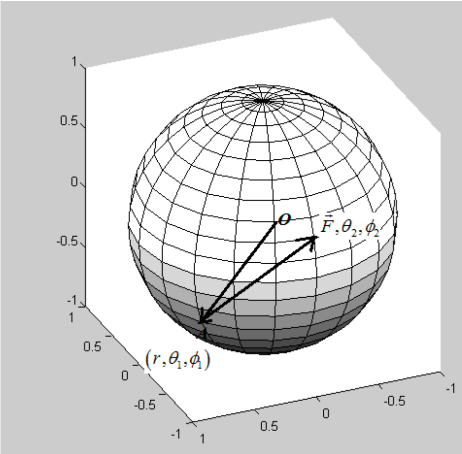

Fig1. Point of contact A and direction of force F

The force applied by a player during the impact distributes over an area, whose shape is governed by the type of kick (instep kick, outside kick, push kick and others) and the air pressure inside the ball. It is largely determined by the individual kicking the ball. This force however can be localized and treated to act at the effective point of impact. Let A be such a point. The coordinates of point A in polar coordinates are $\left(r\cos\phi_1\cos\theta_1,\ r\cos\phi_1\sin\theta_1,\ r\sin\phi_1\right)$.

The radius r of the ball used throughout the study (Adidas Teamgist) is 10.98 mm. If the ball is to be in flight after the impact, it must be hit in the lower half, i.e. $\phi_1 \le 0°$. Also, the point of impact cannot fall lower than $60°$, since it is unfeasible to further lower the impact point, $\phi_1 > -60°$.

Let the force F be applied in an arbitrary direction, represented by the angles θ_2, ϕ_2. The force vector can now be represented as,

$$\vec{F} = F\begin{pmatrix}\cos\phi_2\cos\theta_2,\ \cos\phi_2\sin\theta_2,\\ \sin\phi_2\end{pmatrix}......(1)$$

The forces acting on the ball during the impact will be the normal force F_N and the frictional force F_F. The normal force can be evaluated as the dot product of $\vec{F}$ and the vector along the normal $\vec{N}$. The normal vector $\vec{N}$ is given by,

$$\overrightarrow{N} = \cos(-\phi_1)\cos(\theta_1+\pi),\\ \cos(-\phi_1)\sin(\theta_1+\pi),\sin(-\phi_1)......(2)$$

Now, the normal force is,

$$F_N = \vec{F}\cdot\vec{N}$$
$$F_N = -F\cos\phi_1\cos\theta_1\cos\phi_2\cos\theta_2\\ -F\cos\phi_1\sin\theta_1\cos\phi_2\sin\theta_2\\ -F\sin\phi_1\sin\phi_2......(3)$$

The individual components of the normal force simplify to the following form –

$$F_{N_x} = -F_N\cos\phi_1\cos\theta_1......(4)$$
$$F_{N_y} = -F_N\cos\phi_1\sin\theta_1......(5)$$
$$F_{N_z} = -F_N\sin\phi_1......(6)$$

Since, the only other available medium of momentum transfer is the friction force.

First, the tangential component of the force acting on the ball is evaluated. This force $\vec{F}_T$ is given by,

$$\vec{F}_T = \vec{F} - \vec{F}_N......(7)$$

The individual components of the tangential force are –

$$F_{T_x} = F\cos\phi_1\cos\theta_1\left(1-\cos\phi_2\cos\theta_2\right)......(8)$$
$$F_{T_y} = F\cos\phi_1\sin\theta_1\left(1-\cos\phi_2\sin\theta_2\right)......(9)$$
$$F_{T_z} = F\left(1-\sin\phi_1\sin\phi_2\right)......(10)$$

The unit vector in the direction of the tangential force is given by,

$$\hat{F}_T = \frac{F_{T_x}, F_{T_y}, F_{T_z}}{\sqrt{{F_{T_x}}^2+{F_{T_y}}^2+{F_{T_z}}^2}}......(11)$$

The friction force will be as much as the applied force until it reaches the limiting value, i.e. μF_N. Therefore, two subcases arise –

a. $F_T \le \mu F_N$

In this case, the frictional force counters all the tangential force; the value of the individual components is as follows –

$$F_{F_x} = F\cos\phi_1\cos\theta_1\left(1-\cos\phi_2\cos\theta_2\right)......(12)$$

$$F_{F_y} = F\cos\phi_1\sin\theta_1\left(1-\cos\phi_2\sin\theta_2\right)......(13)$$

$$F_{F_z} = F\left(1-\sin\phi_1\sin\phi_2\right)......(14)$$

b. $F_T > \mu F_N$

Since, the tangential force exceeds the limiting value; the magnitude of the force acting is limited to μF_N. The direction would be along the unit vector of the tangential force. The individual components of the frictional force are given as –

$$F_{F_x} = \mu F_N \frac{F_{T_x}}{\sqrt{{F_{T_x}}^2 + {F_{T_y}}^2 + {F_{T_z}}^2}}......(15)$$

$$F_{F_y} = \mu F_N \frac{F_{T_y}}{\sqrt{{F_{T_x}}^2 + {F_{T_y}}^2 + {F_{T_z}}^2}}......(16)$$

$$F_{F_z} = \mu F_N \frac{F_{T_z}}{\sqrt{{F_{T_x}}^2 + {F_{T_y}}^2 + {F_{T_z}}^2}}......(17)$$

Velocity and Angular Velocity:

Various studies conducted on the ball impact mechanics (Ref 5-6) suggest a contact time t_c close to 9.5 ms. The mass m of the ball referred to in the study is 0.442 kg.

The individual components of the ball velocity can now be expressed as follows –

$$v_x = \frac{F_{N_x} + F_{F_x}}{m} t_c......(18)$$

$$v_y = \frac{F_{N_y} + F_{F_y}}{m} t_c......(19)$$

$$v_z = \frac{F_{N_z} + F_{F_z}}{m} t_c......(20)$$

The torque produced by the normal force is zero; therefore, the spin is imparted to the ball is largely due to the frictional force. The torque $\vec{\tau}$ produced by this friction can be evaluated as the cross product of the position vector $\vec{r}$ and the force itself.

$$\vec{\tau} = \vec{r} \times \vec{F}_F......(21)$$

The components of the position vector are as follows –

$$r_x = r\cos\phi_1\cos\theta_1......(22)$$

$$r_y = r\cos\phi_1\sin\theta_1......(23)$$

$$r_z = \ \sin\phi_1......(24)$$

This torque can be expressed in its individual components as –

$$\tau_x = r_y F_{F_z} - r_z F_{F_y}......(25)$$

$$\tau_y = r_z F_{F_x} - r_x F_{F_z}......(26)$$

$$\tau_z = r_x F_{F_y} - r_y F_{F_x}......(27)$$

These represent the mean value of the torque spanned over the impact time. The football is assumed to be a hollow sphere with uniformly distributed mass, the moment of inertia I of the ball is –

$$I = \frac{2}{3} mr^2......(28)$$

The angular velocity of the ball is then represented as –

$$w_x = \frac{\tau_x t_c}{I} = ...(29)$$

$$w_y = \frac{\tau_y t_c}{I} = ...(30)$$

$$w_z = \frac{\tau_z t_c}{I} = ...(31)$$

Inflight Dynamics –

Once the ball is in flight, it is acted upon by three forces – Gravitational Force, Drag Force and the Magnus Force. These forces are illustrated in Fig. 2.

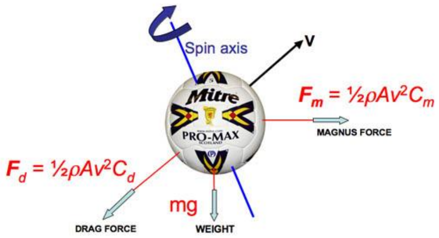

Fig.2 the forces acting on the spinning ball during the flight

The drag force opposes the velocity, the gravity acts along negative $z-$axis.

The spin factor Sp of such a spinning ball is defines as –

$$Sp=\frac{rw}{v}......(32)$$

Studies conducted by Asai et al and Goff-Carre (Ref 1, 7), attempts to determine the force coefficients as a function of Spin factor and the Reynolds number, Re. Using those results, a best fit function of the drag coefficient C_d was evaluated as (Fig. 3) –

$$C_d=c\left(Sp\right)^d......(33)$$

Here c and d are constant, with $c=0.4127$ and $d=0.3056$.

Typical soccer kicks lie in the range of $25<v<30$. Due to this reason, the lift coefficient C_l (or the Magnus coefficient C_m) was evaluated as a function of the spin factor using quadratic regression technique at a Reynolds number of $\mathrm{Re}=394,482$, i.e. $v\sim27$ m/s. The lift coefficient thus obtained was –

$$\mathrm{C_l}=7.46Sp^2+3.46Sp+0.101......(34)$$

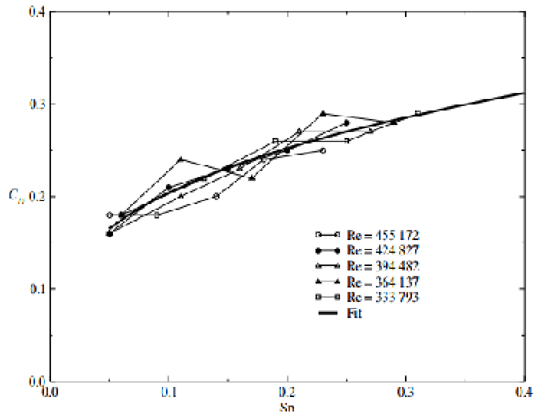

Fig.3 Experimental wind-tunnel data from [5] for C_D as a function of Sp

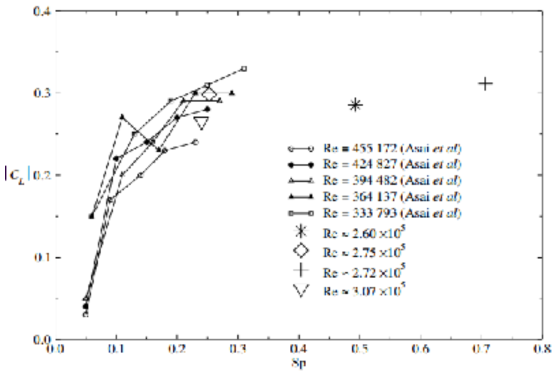

Fig.4 Experimental wind-tunnel data for C_L from [5] as a function of Sp.

The drag force not only slows down the ball but it also damps the spin. The viscous torque on a steady rotating sphere in a fluid with viscosity μ according to Lamb (Ref 8) is –

$$\tau=-8\pi\mu r^3w......(35)$$

This torque can also be expressed as $I\frac{dw}{dt}$, the equation can then be solved for w as –

$$w=w_oe^{-\lambda t}......(36)$$

$$\lambda=8\pi\mu r^3/I......(37)$$

The drag force $\vec{F}_D$ on the ball can be expressed as –

$$\vec{F}_D=-\frac{1}{2}\rho AC_Dv^2\hat{v}.....(38)$$

Here $\rho = 1.2$ kg/m^3 is the air density and $A = \pi r^2$ is the cross sectional area.

The individual components of the drag force are as follows –

$$F_{D_x} = -\frac{1}{2}\rho A C_D v v_x .....(39)$$

$$F_{D_y} = -\frac{1}{2}\rho A C_D v v_y .....(40)$$

$$F_{D_z} = -\frac{1}{2}\rho A C_D v v_z .....(41)$$

The lift force on the spinning ball is perpendicular to both the velocity as well as the angular velocity. Its direction is therefore determined by the vector $\hat{w} \times \hat{v}$. The components of Magnus force $\vec{F}_M$ are thus given by,

$$F_{M_x} = \frac{1}{2}\rho A C_M \frac{v}{w}\left(w_y v_z - w_z v_y\right)......(42)$$

$$F_{M_y} = \frac{1}{2}\rho A C_M \frac{v}{w}\left(w_z v_x - w_x v_z\right)......(43)$$

$$F_{M_z} = \frac{1}{2}\rho A C_M \frac{v}{w}\left(w_x v_y - w_y v_x\right)......(44)$$

The gravitational force $\vec{F}_g$ acts uniformly downwards and can be expressed as –

$$\vec{F}_g = -mg\hat{z}......(43)$$

Taking all the forces into account, the momentum equation for the ball can be summarized by the following equations –

$$m\frac{d^2x}{dt^2} = \left(F_{D_x} + F_{M_x}\right)......(44)$$

$$m\frac{d^2y}{dt^2} = \left(F_{D_y} + F_{M_y}\right)......(45)$$

$$m\frac{d^2z}{dt^2} = \left(F_{D_z} + F_{M_z} + F_g\right)......(46)$$

Effect of Wind:

Above formulations have been carried out assuming still medium. However, as the wind blows, the relative velocity of the ball with respect to the fluid medium i.e. air changes. The range over which the flight of the ball lasts is typically no more than 30 meters. In such a scenario, wind velocity can be assumed constant. Therefore, $\vec{v} \to \vec{v} + \vec{v}_w$. Here $\vec{v}_w$ is the wind velocity.

DISCUSSION:

The governing equations (Eqns. 44-46) were solved using *ode45* in MATLAB.

By appropriately choosing the initial values, some of the most popular goals in the history of the game were simulated. Two of such goals are illustrated below –

a. **Roberto Carlos (1997):**

The ball was placed at around 35m from the goal, the player was left footed. The ball first went wide and then *magically* turned towards goal leaving the goalkeeper awestruck.

The input parameters chosen were –

$\theta_1 = -110°$

$\phi_1 = -12.5°$

$F = 1,500N$

$\theta_2 = 59.5°$

$\phi_2 = 11°$

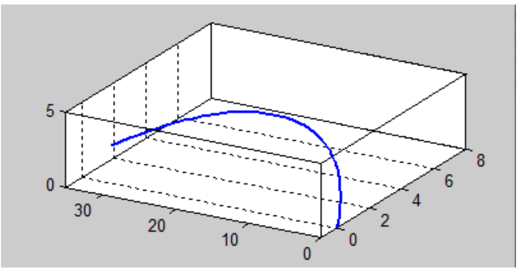

Fig.4 Matlab simulation of the free kick by Roberto Carlos in 1997

b. **David Beckham(2001)**

The ball was placed at a distance of about 30m from the goal. The player was right footed. The ball ended up in the left top corner of the goal before the goal keeper could understand the movement.

The input parameters chosen were –

$$\theta_1 = -90°$$
$$\phi_1 = -14°$$
$$F = 1,200N$$
$$\theta_2 = 89°$$
$$\phi_2 = 14.5°$$

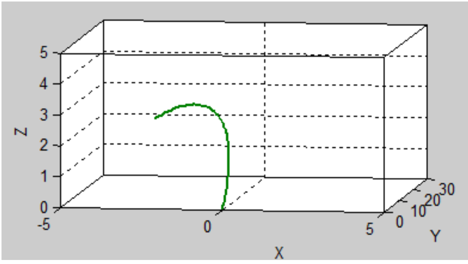

Fig.5 Matlab simulation of the free kick by David Beckham in 1997.

CONCLUSIONS:

A single model was revealed which incorporates the governing laws of the motion of a spinning ball. The model so developed accurately predicted the motion of the soccer ball. This made it easy to test the impact of various parameters such as friction coefficient, ambient conditions as well as the role of the individual player on the trajectory of the soccer ball.

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