MATHEMATICAL MODELLING AND NUMERICAL ANALYSIS FOR INVESTIGATING THE SELF SENSING CAPABILITIES OF A PIEZOPATCH IN A COMPOSITE LAMINATE

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Abstract

This paper presents a mathematical framework for evaluating the self-sensing capabilities of a piezoelectric patch. The system comprises of a composite material with piezopatch embedded at a given location. The piezo is energized via an input voltage which in turn induces strain in the piezo i.e. at this stage piezo acts as an induced strain actuator. However, due to its interaction with the composite material stress is generated in the entire structure. This stress further induces a voltage, thus harnessing the self-sensing capabilities of the piezo. In the presented model, the configuration of the composite can be altered as per requirement. In the findings, the focus is on the self-sensing capabilities as a function of the number of layers of the composite material and its configuration above and below the piezopatch. The technique is envisaged to be of good use indamage detection and health monitoring of composites.

Keywords : self-sensing, advanced composite material, piezoelectric actuators, piezoelectric sensors

1 Introduction

Currently composite materials are widely used in an array of applications ranging from household equipments to aviation industry. Therefore, it becomes important to analyze their reliability without compromising their performance. Smart materials such as magnetostrictive and piezoelectric have gained widespread popularity among researchers for dynamic analysis of structures for applications such as vibration control[1][6]. Few have also attempted to perform failure analysis of composite structures using magnetostrictive materials. Murthy et al[2] ,Tarun[3] and others have integrated magnetostrictive material with composite for damage detecting applications. In this paper, we approach the main objective by categorizing the problem statement into three subdivisions which are –
1. Composite Modeling
2. Piezoelectric Actuotor Modeling and
3. Piezosensing and Voltage Determination

In section 2.1, we briefly present the mathematical formulation of composite material using the Classical Laminate theory (CLT) . In the subsequent section, we deal with the constitutive equations of the piezo material and integrate them with the CLT. Finally, we present the voltage follower method to obtain measurable entity in the form of voltage for further analysis.

2 System Modelling

In this section, we will discuss the major approaches that had been undertaken for modeling the entire self-sensing actuator. We begin with discussing Composite modeling and then proceed towards piezo modeling followed by voltage follower approach. In our approach to obtain a measurable entity using the piezopatch, we first evaluate the electric charge displacement from the constituitive equations of the piezoelectric materials and laminated composite theory. Thereafter, we apply voltage follower method to obtain measurable voltages.

2.1 Composite Modeling

For analysis, composite laminate with unidirectional fiber mats were considered and the thickness of each layer was considered to be uniform. The elastic moduli of the composites were derived from the moduli of the fiber and of the matrix by the Halpin Sai[4] equations -

$$E_1 = E_f V_f + E_m V_m \quad (1)$$

$$\nu_{12} = \nu_f V_f + \nu_m V_m \quad (2)$$

$$\frac{M}{M_m} = \frac{1+\zeta\eta V_f}{1-\eta V_f} \quad (3)$$

$$\eta = \frac{(M_f/M_m)-1}{(M_f/M_m)+\zeta} \quad (4)$$

where
E elastic modulus
G shear modulus
ν Poisson ratio
M composite modulus E_2, G_{12}, or ν_{23}
M_f corresponding fiber modulus E_f, G_f, or ν_{23}
M_m correponding matrix modulus E_m, G_m, or ν_m
V_f fiber volume fraction
ζ = 2 for calculation for E_2
ζ = 1 for calculation for G_{12}.

The aforementioned formulation provides the equiavalent mechanical properties of an individual lamina. However, for strength and loading analysis, it is generally more convenient to express the combined properties of all the lamina i.e. effective laminate engineering constants. The following equations establish the relationship between loading and deformation of the laminate.

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \tag{5}$$

N_x Stress resultant along x direction per unit width (Nm^{-1})
N_y Stress resultant along y direction per unit width (Nm^{-1})
N_{xy} Shear stress resultant force per unit width (Nm^{-1})
M_x Moment resultant along x direction per unit width (N)
M_y Moment resultant along y direction per unit width (N)
M_{xy} Torsional moment resultant per unit width (N)
h_k Height of the kth layer

In the above formulation, the sub-matrices A, B and D are defined as follows -

$$[A] = \sum_{k=1}^{n} [\bar{Q}_{ij}]_k (h_k - h_{k-1}) \tag{6}$$

$$[B] = \frac{1}{2} \sum_{k=1}^{n} [\bar{Q}_{ij}]_k (h_k^2 - h_{k-1}^2) \tag{7}$$

$$[D] = \frac{1}{3} \sum_{k=1}^{n} [\bar{Q}_{ij}]_k (h_k^3 - h_{k-1}^3) \tag{8}$$

Here,

$$\bar{Q} = T^{-1} Q T \tag{9}$$

$$T = \begin{bmatrix} cos^2\theta & sin^2\theta & 2sin\theta cos\theta \\ cos^2\theta & sin^2\theta & 2sin\theta cos\theta \\ -sin\theta cos\theta & sin\theta cos\theta & cos^2\theta - sin^2\theta \end{bmatrix} \tag{10}$$

θ - Fiber inclination angle from the x-axis

Q - Stiffness Matrix of a single lamina aligned with the principal axes

2.2 Piezo Modelling

In the piezoeletric material, the dielectric charge density D and the mechanical strain ϵ are dependent on the mechanical stress σ and the field strength E. The constitutive equations of such a material are as follows -

$$D = d\sigma + \epsilon^{*\sigma} E \tag{11}$$

$$\epsilon = s^E \sigma + dE \tag{12}$$

In these equations, the matrix of piezoelectric constants d indicates the strength of the piezoelectric effect. ϵ^{*T} is the matrix of dielectric constants for constant σ and s^E the elasticity matrix for constant E On application of external electric field, the state of the composite can be represented by figure 1.With respect to the piezolayer, the composite gets divided into two sub-laminates.
The upper laminate is having p_1 number of layers and a resultant tensile stress N_1, lower laminate is having p_2 number of layers and a resultant tensile stress N_2. Since, it is a general case of loading, bending is also expected, which is represented as M.

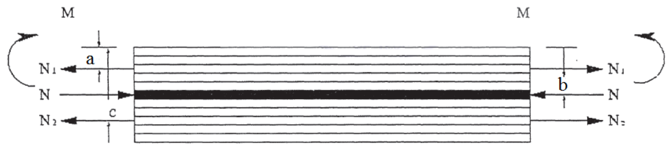

Figure 1: Laminate Structure with moment acting on it

Further, the following relations hold true -

$$\{N_1\} = [A_1]\{\epsilon_p\} \tag{13}$$

$$\{N_2\} = [A_2]\{\epsilon_p\} \tag{14}$$

Also,

$$\{N\} = \{N_1\} + \{N_1\} \tag{15}$$

Here, A_1 and A_2 are the in-place stiffness matrices of the composite layers above and below the piezo respectively. N_1 and N_2 are the force/width of the piezo and ϵ_p is the strain in the piezo patch given by -

$$\{\epsilon_p\} = \{\epsilon_0\} + z_p\{\kappa\} \tag{16}$$

$\{\epsilon_0\}$ is the mid-place strain, z_p is the distance of the piezo from the midplane and κ is the curvature of the composite. The moment balance yields -

$$\{M\} = \{N_1\}a + \{N_2\}b - \{N\}c \tag{17}$$

a,b,c are illustrated in figure 1. Also, available is the force balance equation on the piezo.

$$\{N\} = \{\sigma_p\}h_p \tag{18}$$

Where σ_p and h_p are the stress on the piezo and its height respectively

All the above equations are solved simultaneously to obtain the electric-field E

$$\{E\} = ([d][\alpha_3][d] + [\mu])^{-1}\{D\} \tag{19}$$

Where,

$$[\alpha_3] = [\bar{A}_{12}h_p - [s] + y_p h_p[\alpha_2]]^{-1} \tag{20}$$

$$[\alpha_2] = [D]^{-1}(\alpha_1 - [B]\bar{A}_{12})\{\sigma_p\}h_p \tag{21}$$

$$[\alpha_1] = A_1\bar{A}_{12}a + A_2\bar{A}_{12}c - Ib, \bar{A}_{12} = [A_1 + A_2]^{-1} \tag{22}$$

I is the identity matrix of size 3X3. Furthermore, stress on the piezomaterial can be expressed as,

$$\{\sigma_p\} = [\alpha_3][d]\{E\} \tag{23}$$

2.3 Voltage Determination

In the previous section, the charge displacement D is the the electric charge displacement across the piezoelectric material due to the applied voltage, E is the electric field generated due to the self-sensing capabilities of the piezo-patch. To obtain this representative electric field in the form a measurable entity, the circuit shown in Fig.2 is used[5]. Mathematically, this electric field and the finally obtained voltage v_p are related as follows -

$$v_p = ([d][\alpha_3][d] + [\mu])^{-1}\{D\}t_p \tag{24}$$

Where t_p is the thickness of piezopatch

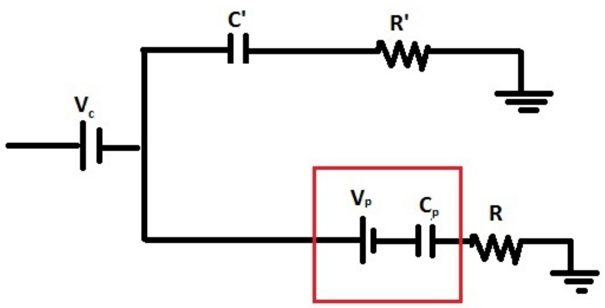

Figure 2: Self Sensing Actuator Circuit

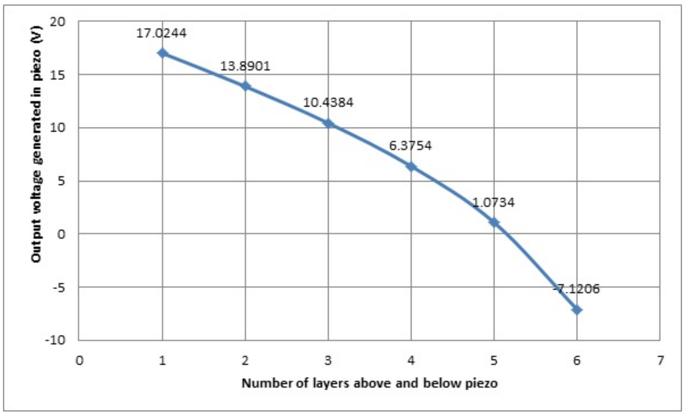

Figure 4: Output voltage generated as a function of number of layers of composite above and below the piezopatch

3 Results and Discussions

We have simulated the problem in MATLAB using the data available in Appendix. Obtained results are as follows -

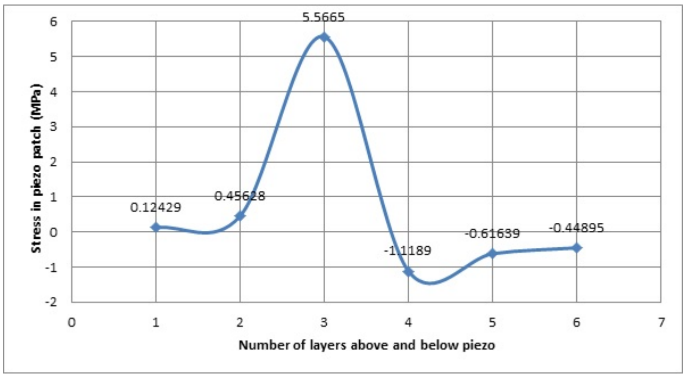

Figure 3: Stress generated in the piezo as a function of number of layers of composite above and below the piezopatch

For the cases below we have assumed the composite to be symmetric with respect to the piezopatch. In fig. 3 and 4, the configuration of the layers above and below the piezo is assumed to be same. Figure 3 represents the stress generated as a function of number of composite layers. As seen from the plot in fig 3, the stress generated in the piezo increases as the structure becomes more rigid. The stress reaches its maxima when the composite has three layers above and below piezo. After that due to the stiffness of the structure, the composite layers did not allow the expansion of piezo rather they exert a compressive force which results in an increasing compressive stress as can be observed from the trend.

The stress generated in fig. 3 is used to evaluate the voltage output in the piezo which shows the trend observed in fig. 4. The layout of fig.4 states that the output voltage generated in the piezopatch decreases as the structure becomes more and more rigid i.e. the number of layers above/below piezo increases. The voltage generated is a function of two variables, first, the input voltage supplied and secondly, due to the stress acting on the piezoelectric material both of which acts in the opposite sense. The input voltage supplied tends to induce a positive voltage in the piezo whereas the stress induced tends to compress it, thereby inducing a negative or opposite voltage. When the number of layers are less, voltage induced by the input electric field exceeds the voltage generated by the stress and thus, the overall voltage is positive. But as the structure becomes more and more rigid, voltage decreases and finally becomes negative when the piezo is placed between a composite consisting of a total of 12 layers.

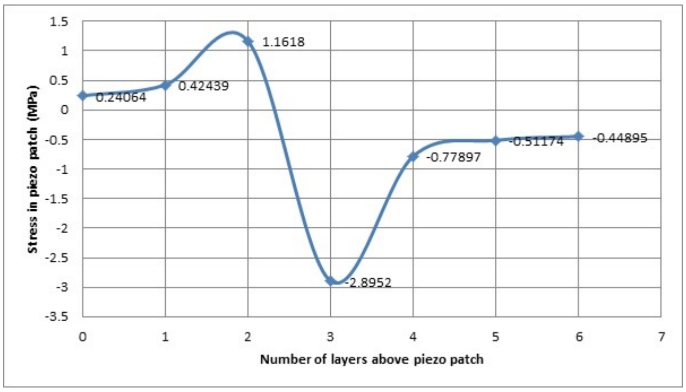

Figure 5: Voltage generated in the piezo as a function of position of piezopatch in the entire composite structure

In order to find the variation of stress and voltage generated as a function of the position of peizopatch in the composite structure as depicted in figures 5 and 6 we have considered a total of 12 layers in the composite structure for the analysis. From fig 5, it can be concluded that when piezopatch is placed at the top, it is in a free state, therefore, minimum stress is generated in it. As the structure becomes more and more symmetric with respect to the piezopatch, we can observe that stress tends to be decrease. It has minimum stress when it is fully symmetric.

In fig. 6 we can see that the output voltage generated in the piezopatch decreases as the number of layers above piezo increases. This can be attributed to the reasons stated previously. From Fig.7, it is found that for all the three cases, voltage decreases with increase in rigidity. When the number of the layers above piezo are four, voltage generated is approximately equal for all the test configurations. In general, the

Table 1: Configuration for test cases

Symmetric	0	90	0	0	90	0
Asymmetric	0	90	0	90	0	90
Antisymmetric	0	30	60	-60	-30	0
Thickness(mm)	0.8	0.8	0.8	0.8	0.8	0.8

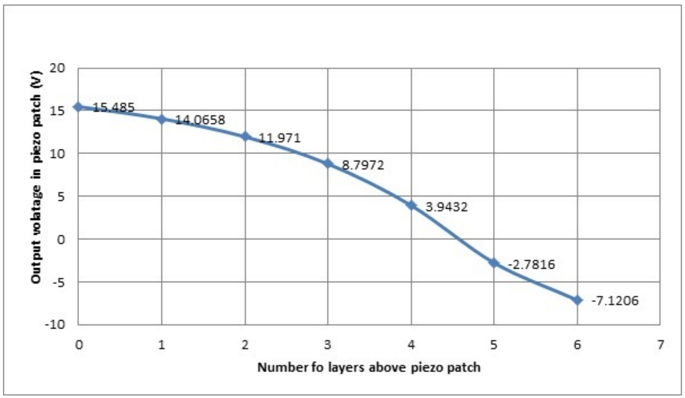

Figure 6: Voltage generated in the piezo as a function of position of piezopatch in the entire composite structure

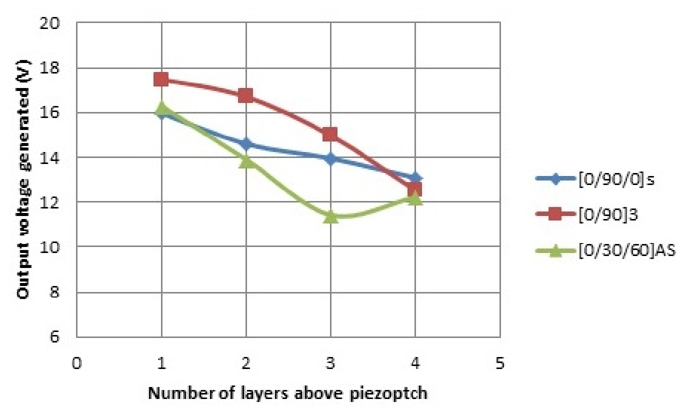

Figure 7: Comparison of voltage generated in symmetric, asymmetric and anti-symmetric configurations

voltage output for the asymmetric case is higher as compared to the other two.

4 Conclusion

In this paper, the self-sensing capabilities of the piezo electric material was realized by incorporating it with a multi-layered laminated composite structure. The main focus of this research was laid on developing a relationship between the configuration/position of piezopatch in the composite structure and electromechanical properties including stress and electric voltage. It was found that when piezo was held in a symmetric configuration, the stress reaches a maxima and then it starts decreasing as the structure becomes more and more rigid. Interestingly, the output voltage, which happens to be a function of input electric field and induced stress, showed a monotonous decrement with increasing stiffness of the composite structure. Finally, while trying to position the piezopatch at different locations we concluded that the stress showed similar trend as explained above with an increase in rigidity. Moreover, a plodding decrement was there in the output voltage generated. Further publications will deal with the application of the procedure presented here for damage detection purposes.

5 References

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6 Appendix

Table 2: Configuration of Layers above and below the Piezo

thickness(mm)	0.5	0.5	0.5	0.5	0.5	0.5
orientation	45	45	45	45	45	45

Table 3: Properties of Fiber

E1(Gpa)	60
E2(Gpa)	20
G2(Gpa)	30
gamma12	0.3
gamma21	0.4
Vf	0.6

Table 4: Properties of Matrix

E1(Gpa)	2
E2(Gpa)	2
G2(Gpa)	1
gamma	0.4

Table 5: Mechanical Properties of Piezomaterial

E1(Gpa)	67
E2(Gpa)	67
G2(Gpa)	24.8
gamma12	0.28
gamma21	0.28

Table 6: More Piezoelectric Properties

thickeness of piezo patch(mm)	0.3
Length of Piezo patch(mm)	10
width of Piezo patch(mm)	2
Voltage Applied in V (z axis)	20
d31	22