# Rough Terrain Footstep Plan for a Biped Robot

## Gaurav Gupta

BT-MT Dual Degree Student Thesis Supervisor - Dr. Ashish Dutta Department of Mechanical Engineering, IIT Kanpur, India



#### Introduction

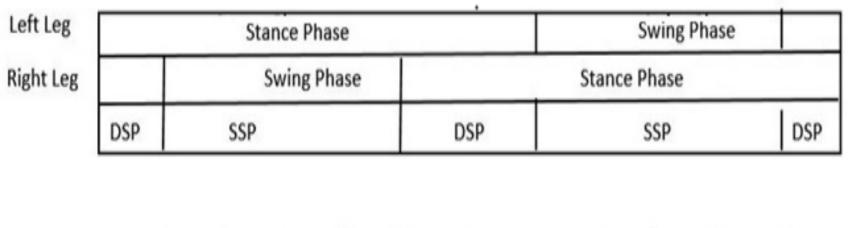
Bipeds have significant advantages over wheeled robots in terms of the variety of terrain they can traverse. It becomes important to develop navigation plans for such robots in a general 3 dimensional environment. To accomplish such a task, following steps are taken -

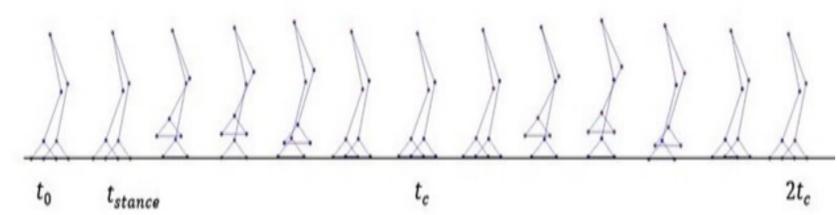
- Discussion on phases of walking.
- Kinematic solutions for the 12 DOF biped.
- Pattern generation for a single step.
- Dynamics using Euler-Lagrange Method.
- Zero Moment Point.
- Real time evaluation of optimal trajectory.
- Potential Field Method for step planning.

#### **Phases of Walking**

Gait: is the pattern of movement of the limbs of animals, humans or robots during locomotion A step of bipedal gait has two phases -

- Single Support Phase (SSP): Swing leg moves forward and stance leg remains on ground
- Double Support Phase (DSP): Both legs are in contact with the ground while trunk moves forward.





**Figure 1:** Phase of Walking

### **Kinematics**

The biped can be treated as a serial manipulator. For performing kinematics, DH parameters are written starting from the base frame towards the end effector. These parameters are mentioned in Table 1 and corresponding frames are depicted in Fig.2. For the inverse kinematics, positions of a reference points on the stance and swing foot, in addition to the position of mid-hip are needed. Also required are the orientations of the two feet in the form of Euler angles.

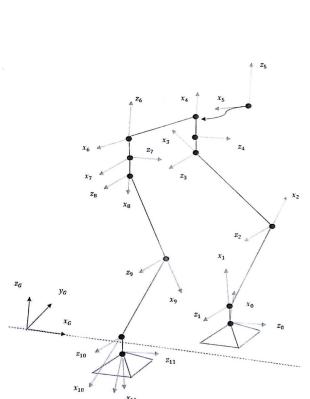


Figure 2: Biped Schematic

Z <sub>6</sub>	$x_1$ $a_2$ $a_3$ $a_4$ $a_5$ $a_6$ $a_{10}$
$x_{6}$ $x_{7}$ $x_{8}$ $x_{8}$ $x_{1}$ $x_{2}$ $x_{2}$ $x_{3}$ $x_{4}$ $x_{5}$ $x_{1}$ $x_{2}$	$x_{10}$ $a_{11}$ $a_{11}$ $a_{12}$ $a_{13}$ $a_{14}$ $a_{15}$ $a$
$x_0$	$\theta_1$ $x_0$ $\theta_{12}$ $\theta_{12}$

 $\alpha = atan2(y_{11} + y_0, x_{11} + x_0)$ 

Frame	$a_{i-1}$	$\alpha_{i-1}$	$d_i$	$ heta_i$
0	0	0	0	$\theta_1$
1	0	$-\pi/2$	0	$\theta_2$
2	$l_1$	0	0	$\theta_3$
3	$l_2$	0	0	$\theta_4$
4	0	$\pi/2$	0	$\pi/2+\theta_5$
5	0	$\pi/2$	0	$\theta_6$
6	$l_3$	0	0	$\theta_7$
7	0	0	0	$\pi/2+\theta_8$
8	0	$-\pi/2$	0	$\theta_9$
9	$l_2$	$\pi/2$	0	$\theta_{10}$
10	11	0	0	$\theta_{11}$
11	0	$-\pi/2$	0	$\theta_{12}$

**Table 1:** DH parameters

х	, O <sub>4</sub> , O <sub>5</sub> , X <sub>2</sub>	a <sub>g</sub>
	x <sub>1</sub> · o <sub>2</sub>	010 x9
	x4 05	$x_{10}$ $\theta_{11}$ $\theta_{12}$ $\theta_{13}$ $\theta_{14}$
	0 <sub>1</sub> x <sub>0</sub>	θ <sub>6</sub> x <sub>5</sub>
	Figure 3: J	Toint Angles

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(811 80)	
$X_{h_l} = (-l_3 sin\alpha + x_{mh}, l_3 cos\alpha + y_{mh}, z_{mh})^T$	
$X_{h_r} = (l_3 sin\alpha + x_{mh}, -l_3 cos\alpha + y_{mh}, z_{mh})^T$	
$X_{h_l}^b = {}^{org} T_b^{-1} X_{h_l}$	
$\theta_1 = atan2(X_{h_l}^b(2), X_{h_l}^b(1))$	
$\theta_3 = \pi - \cos^{-1}\left(\frac{l_2^2 - d^2 + l_1^2}{2l_2l_1}\right), d =  X_{a_l} - X_{a_l} $	h
$\theta_2 = atan2(-X_{k_l}^b(3), X_{k_l}^b(1))$	
$\beta = atan2(X_{h_r}(2) - X_{h_l}(2), X_{h_r}(1) - X_{h_l}(2))$	
$T_h = [R^{\beta}, X_{h_l}; 0, 0, 0, 1]$	
$T_l = T_b^2 T_l$	
$\theta_5 = sin^{-1}(T_l(3,3));$	
$\theta_6 = cos^{-1}(T_l(3,1)/cos(\theta_5));$	

 $\theta_4 = \cos^{-1}(T_l(1,3)/\cos(\theta_5));$ 

#### **Pattern Generation**

 $(n_i + 6)$  order polynomial is used for trajectory generation when n intermediate positions are used for the point i on the biped. This is done to account for zero, first and second order conditions at the 2 end-points. All trajectories are generated in a local coordinate frame situated at the base of the stance foot. Intermediate points are expressed in terms of fractions in the range [0,1].

$$t_m = 0.5t_f, t_{d1} = 0.2, t_a = (t_m + t_{d1})/2, t_{d2} = 0.8t_f, t_b = (t_m + t_{d2})/2$$

	$t_{d1}$	$t_a$	$t_m$	$t_b$	$t_{d2}$
$x_{a_{sw}}$	-	-	$x_{0_f} + f_1(x_{f_f} - x_{0_f})$	-	-
$y_{a_{sw}}$	-	-	$y_{0_f} + 0.5(y_{f_f} - y_{0_f})$	-	-
$z_{a_{sw}}$	-	-	$z_{0_f} + f_2(z_{f_f} - z_{0_f})$	-	-
$x_{mh}$	-	$x_{mh_0} + f_3(x_{mh_f} - x_{mh_0})$	-	$x_{mh_0} + f_4(x_{mh_f} - x_{mh_0})$	-
$y_{mh}$	$y_{mh_0} + f_7(y_{mh_f} - y_{mh_0})$	-	$y_{mh_0} + f_6(y_{mh_f} - y_{mh_0})$	-	$y_{mh_{t_{d1}}}$
$z_{mh}$	-	-	$max(z_{mh_0}, z_{mh_f}) + f_5(\Delta z_{max})$	-	-

**Table 2:** Intermediate Points for Pattern Generation

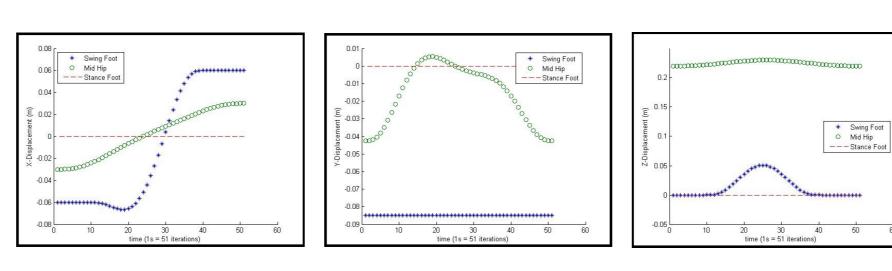


Figure 6: Z-axis Pattern Figure 5: Y-axis Pattern **Figure 4:** X-axis Pattern

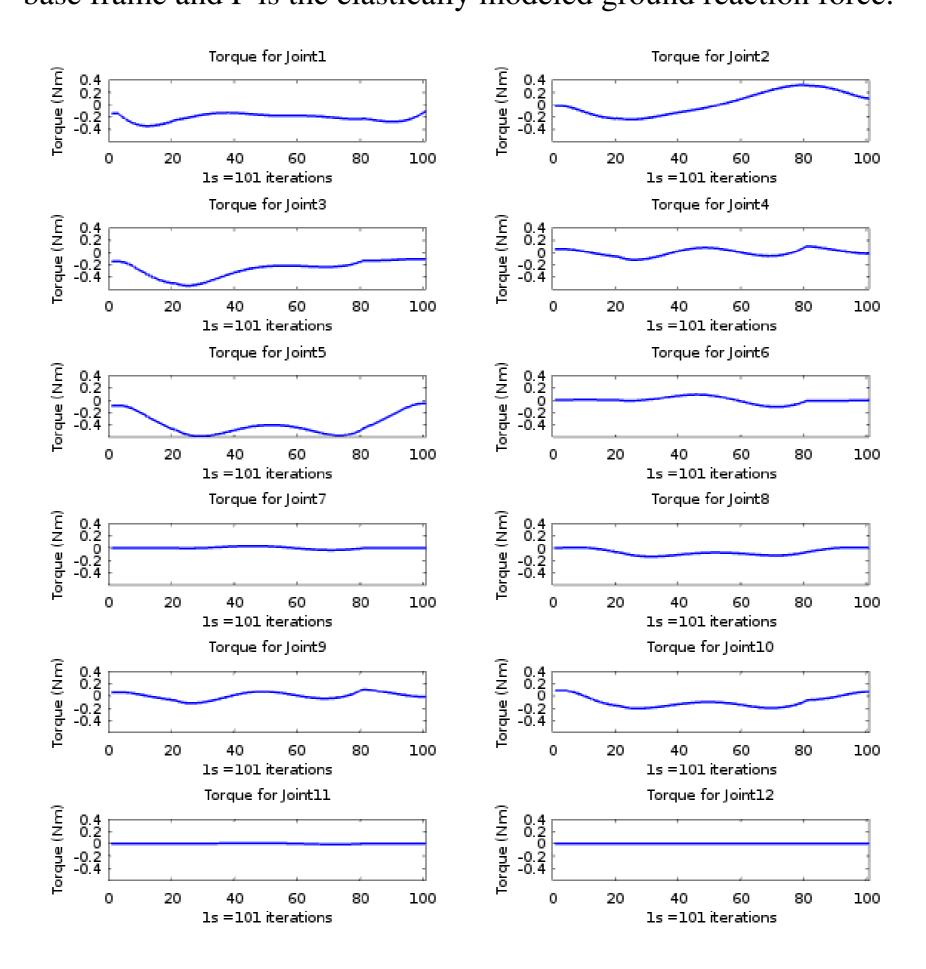
#### **Dynamics**

Joint Torque are calculated using Euler-Lagrange formulation.

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = \tau_i$$

$$\tau = D \left( \ddot{\theta} \right) + H \left( \theta, \dot{\theta} \right) + C \left( \theta \right) - J^T F$$

Here, J is the Jacobian relating the end effector (swing foot) to the base frame and F is the elastically modeled ground reaction force.



**Figure 7:** Joint Torques for a Single Step

#### **Zero Moment Point**

The ZMP is the point on the ground where the tipping moment acting on the biped, due to gravity and inertia forces, equals zero, the tipping moment being defined as the component of the moment that is tangential to the supporting surface. The resultant of the gravity plus inertia forces (superscript gi) may be expressed as,

$$R^{gi} = mg - ma_G$$

and the moment about any point Q as

$$M_Q^{gi} = QG \times mg - QG \times ma_G - \dot{H}_G$$

The ZMP D is given by the expression -

$$QD = \frac{n \times M_Q^{gi}}{R^{gi}.n}$$

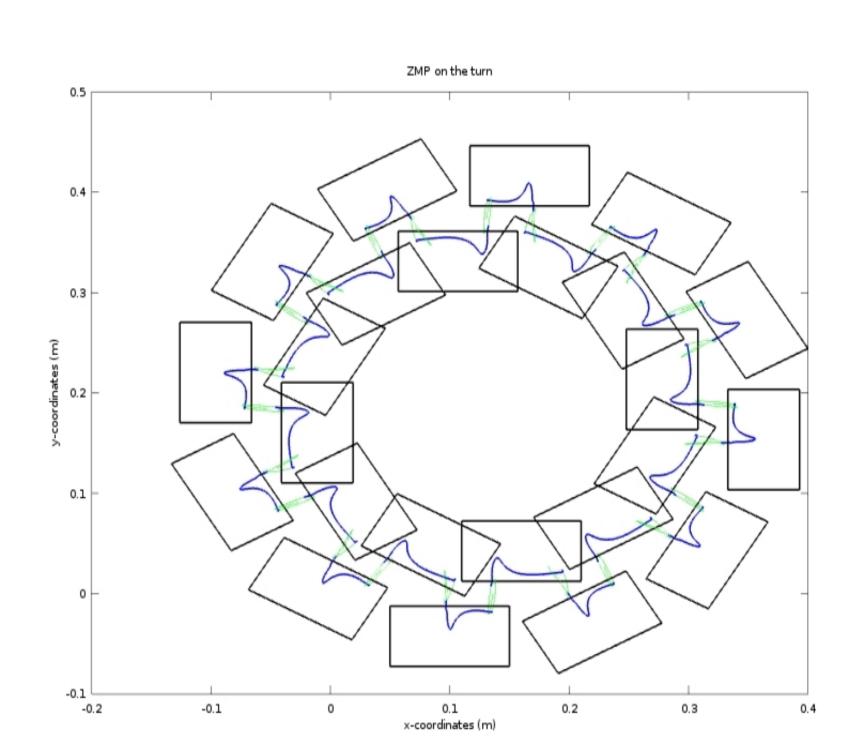


Figure 8: ZMP Trajectory on the turn

#### **Optimal Trajectory**

The 'optimality' of a biped trajectory can be defined in terms of motor torques, movement of joints or power consumption. Currently, the positive work done by the motors is taken to be the objective function.

$$W = \int_{t_0}^{t_f} |\tau d\theta|$$

The choice of intermediate points in the pattern generator determines the energy consumption of the step. Genetic Algorithm is used to obtain an optimum solution.

#### **Forthcoming Research**

#### Machine Learning for Pattern Generation

As the optimization of the biped dynamics is computationally expensive, it becomes infeasible to generate optimal walking patterns in real time. In order to circumvent this problem, it is proposed that the optimal trajectories be pre-computed for a wide array of initial and final states and suitable regression technique be used to generate motion in real time.

#### **Development of Step Plan**

Once step cost can be evaluated in real time, a variety of motion planning algorithms can be deployed. Potential field method is envisaged to be a good alternative.

#### **Conclusions**

Work in its current form is capable of developing energy optimal trajectories for given initial and final states. Post completion, it is expected that optimal step planning will be accomplished in a general workspace.

#### References

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