



DeepLearning.AI

Math for Machine Learning

Linear algebra - Week 3

Vectors

Matrices

Dot product

Matrix multiplication

Linear transformations



DeepLearning.AI

Vectors and Linear Transformations

Machine Learning motivation

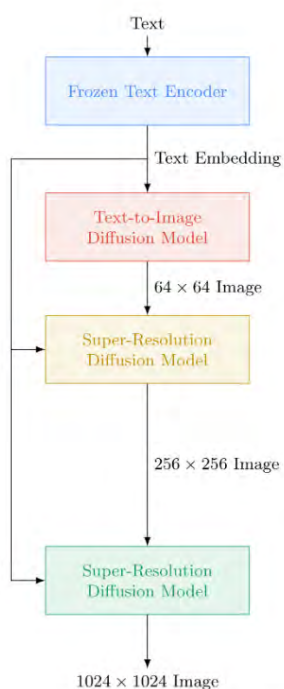
Neural Networks - AI generated images



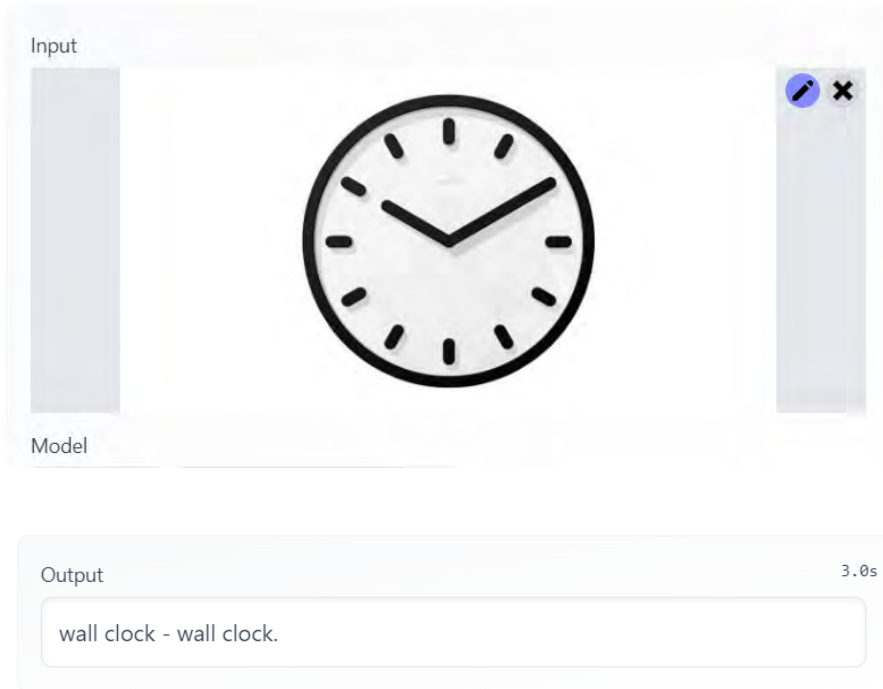
AI-generated human faces.

- Generative learning: Generating realistic looking images.

Text-to-image and image-to-text generation



"A Golden Retriever dog wearing a blue checkered beret and red dotted turtleneck."



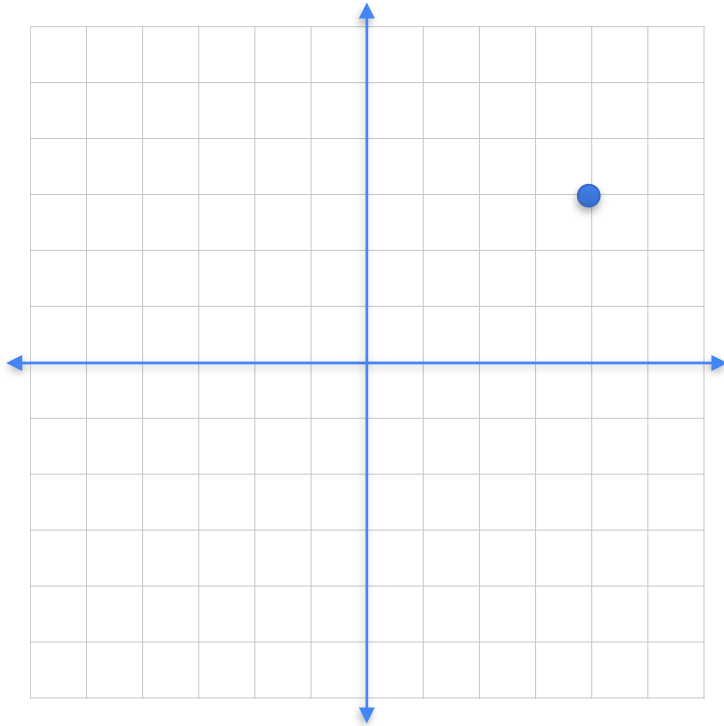


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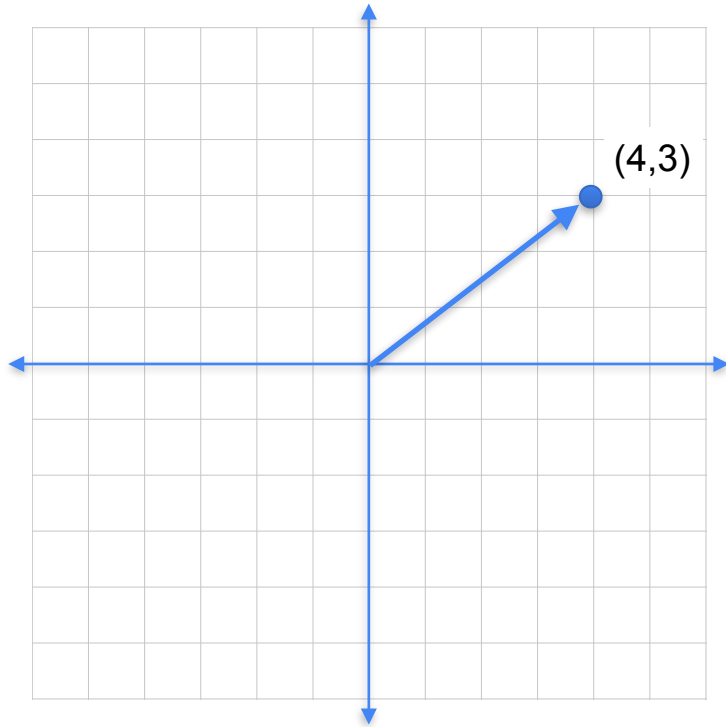
Vectors and Linear Transformations

Vectors and their properties

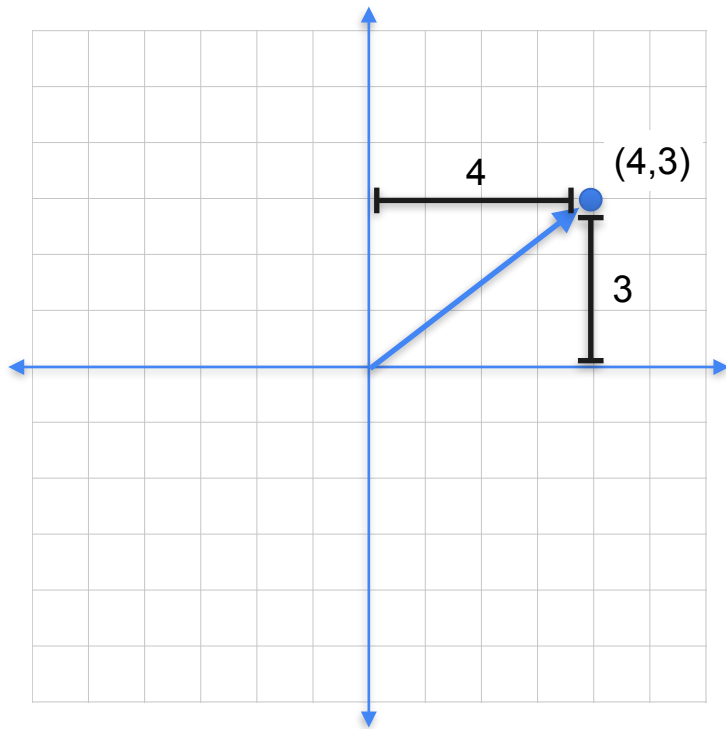
Vectors



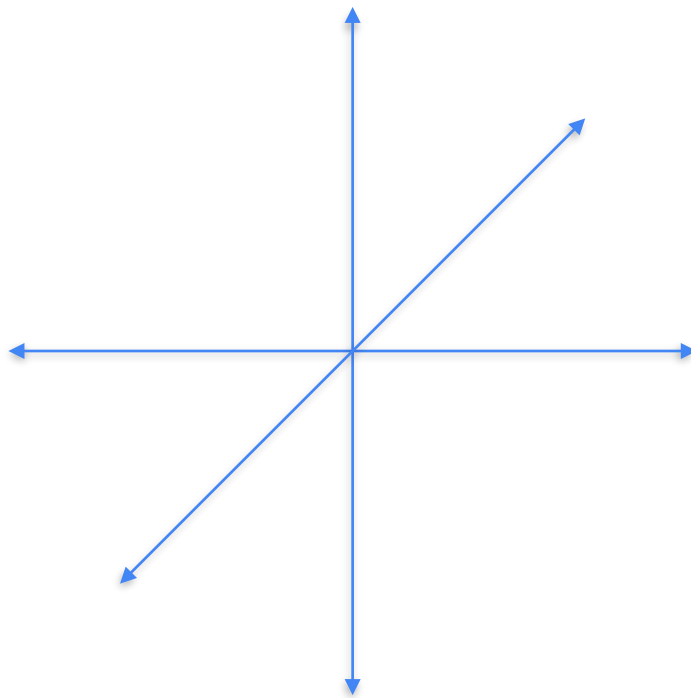
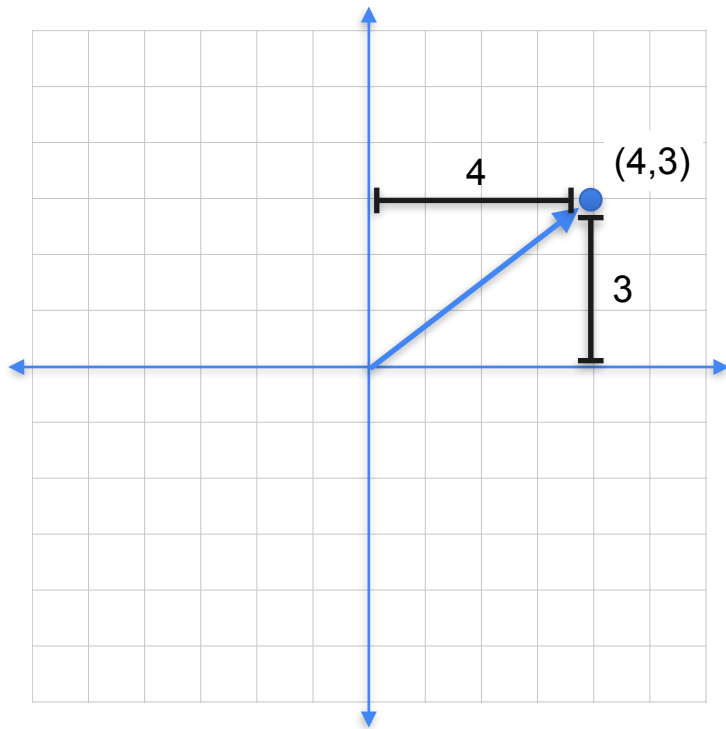
Vectors



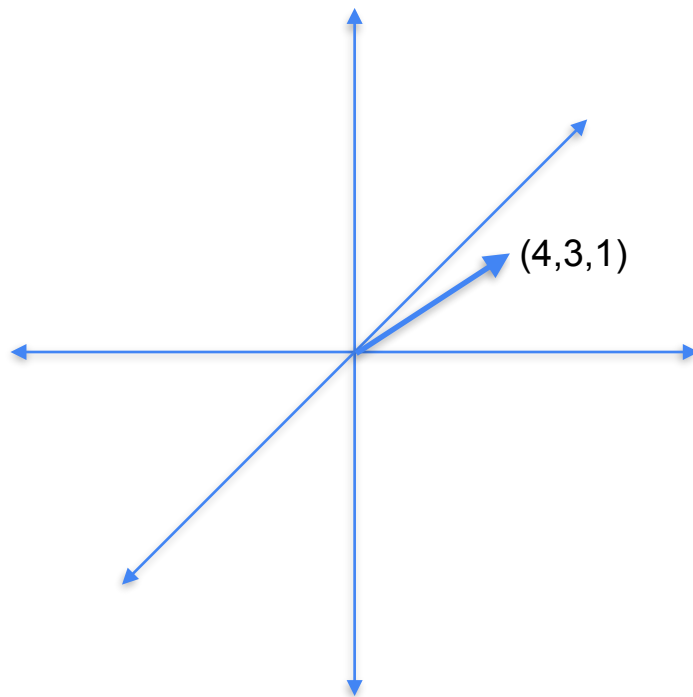
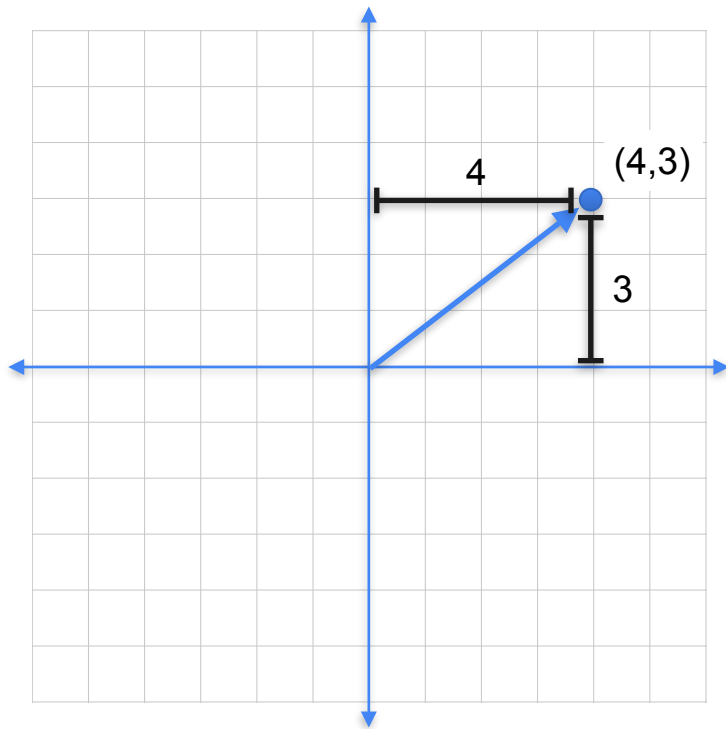
Vectors



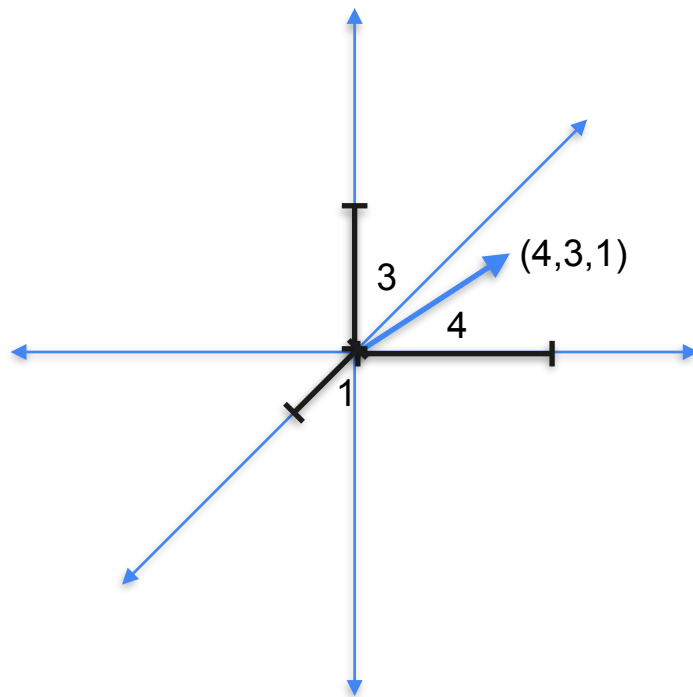
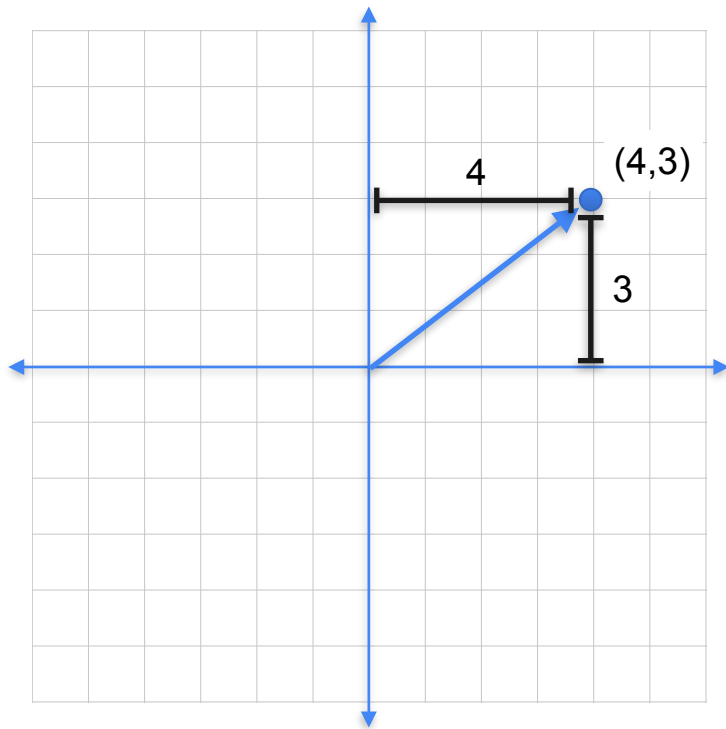
Vectors



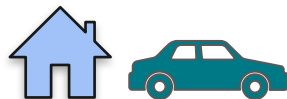
Vectors



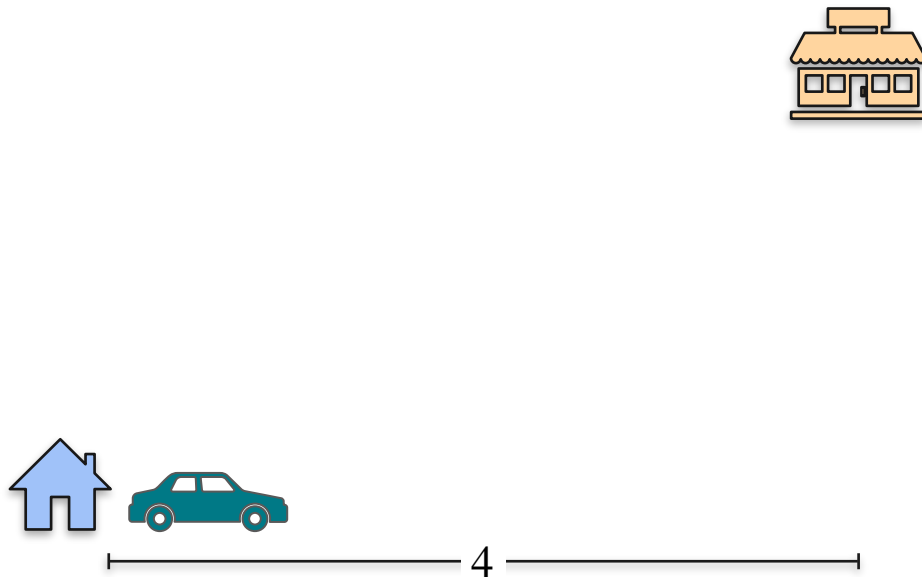
Vectors



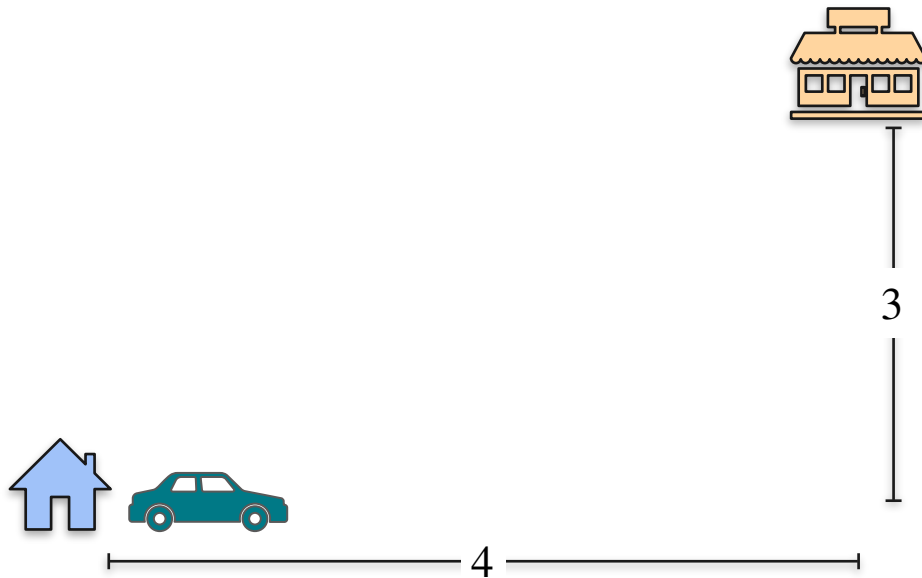
How to get from point A to point B?



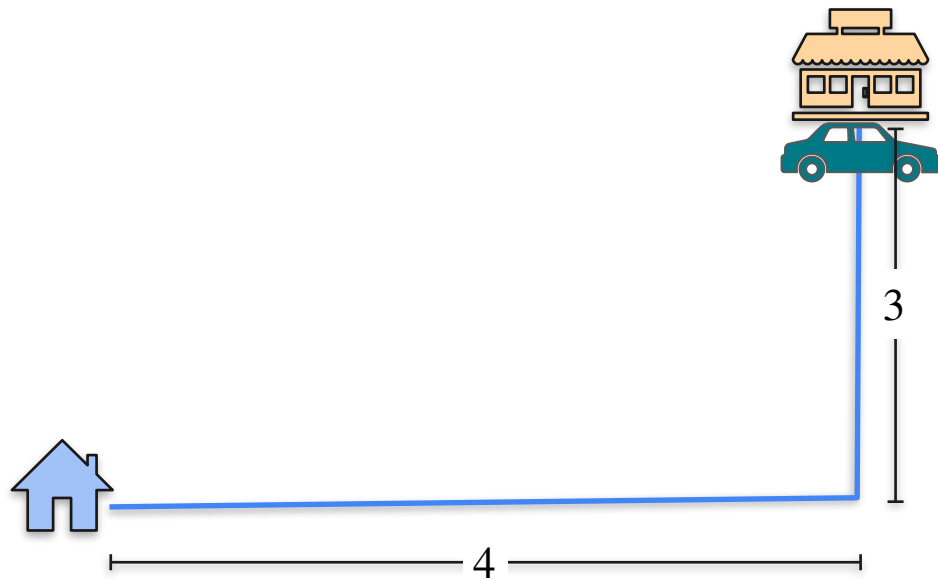
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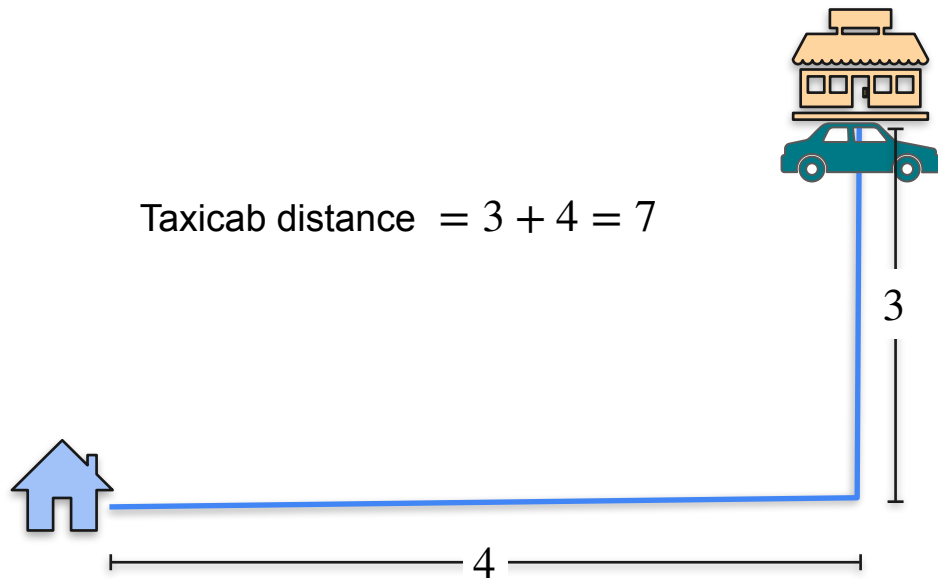
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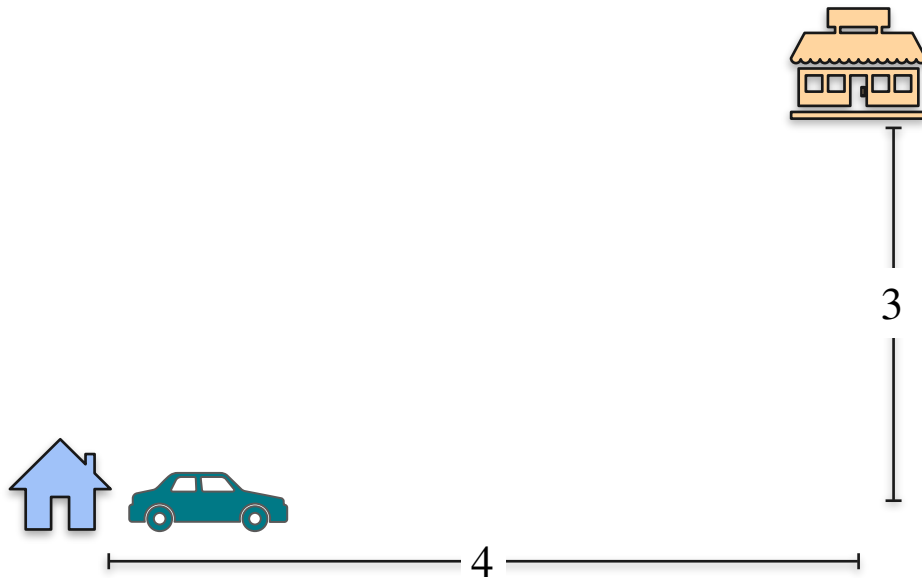
How to get from point A to point B?



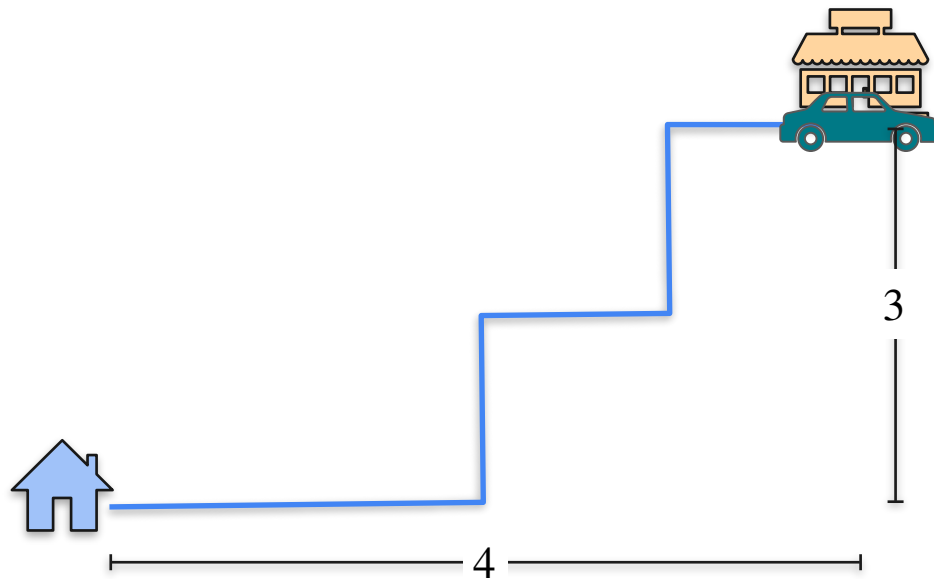
How to get from point A to point B?



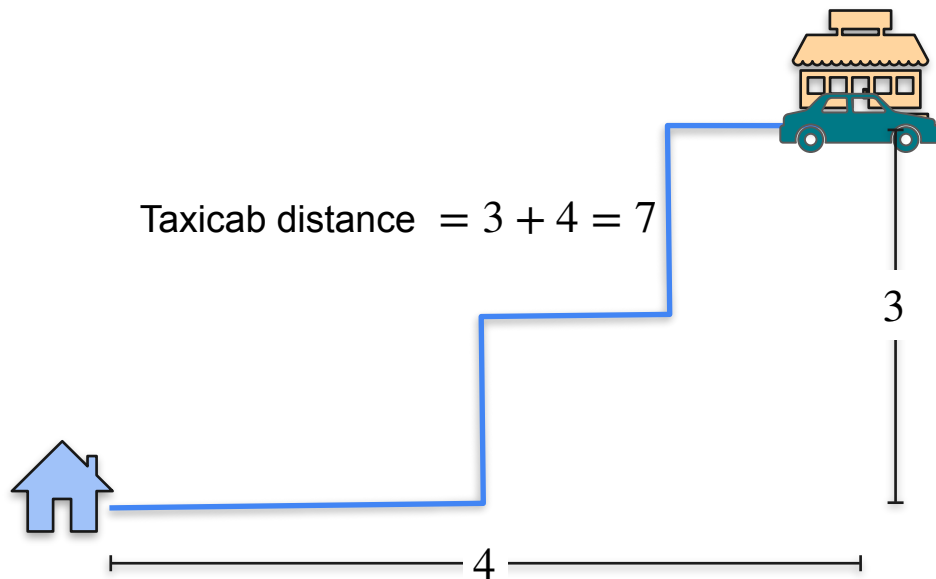
How to get from point A to point B?



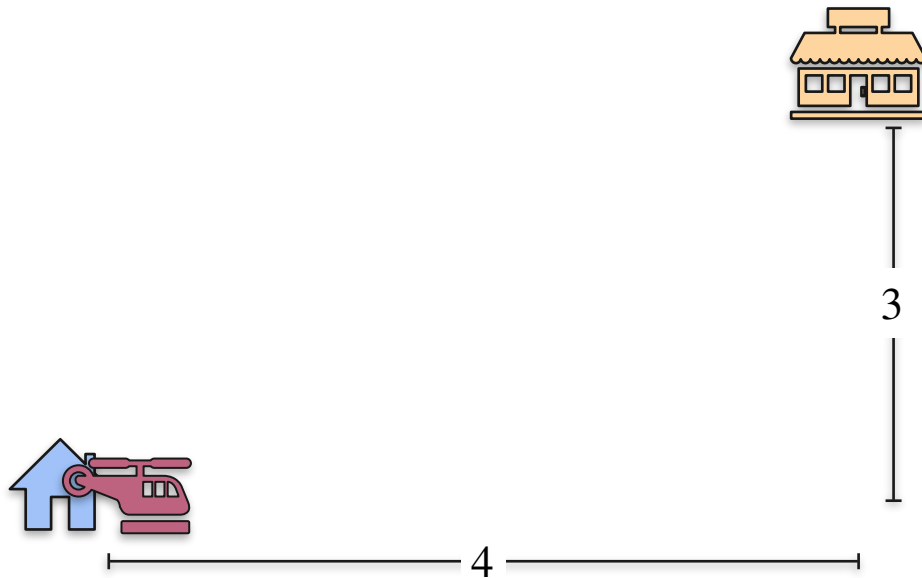
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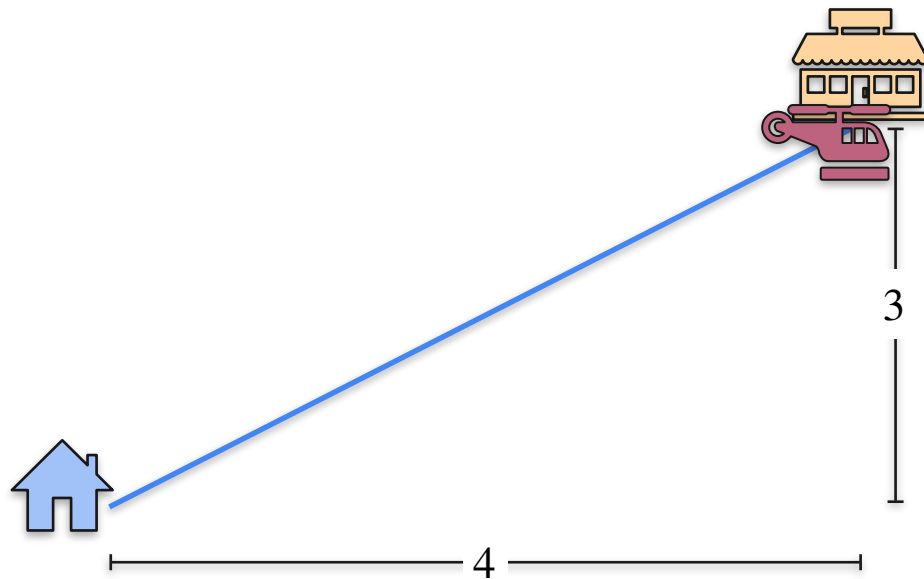
How to get from point A to point B?



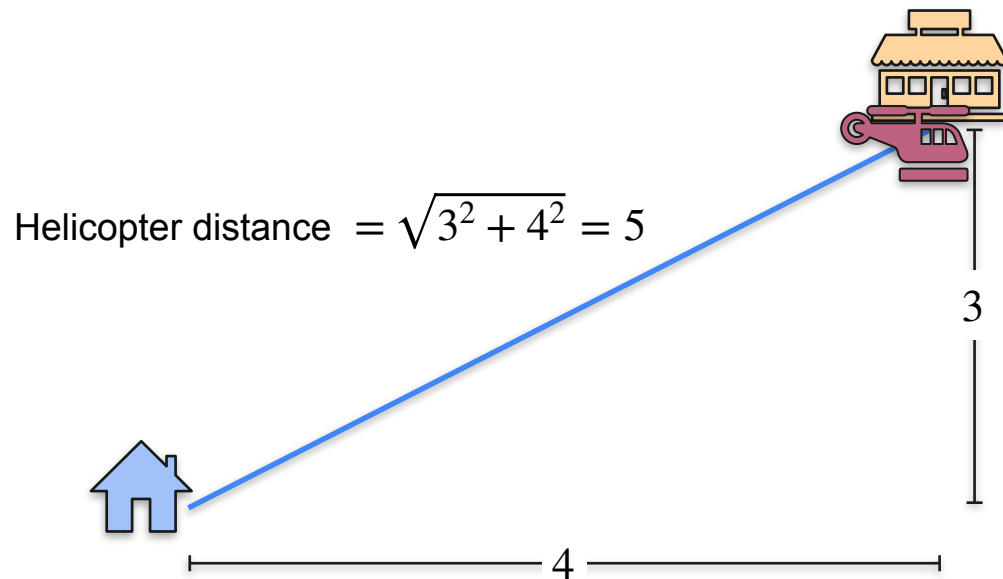
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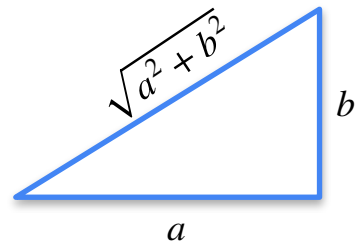
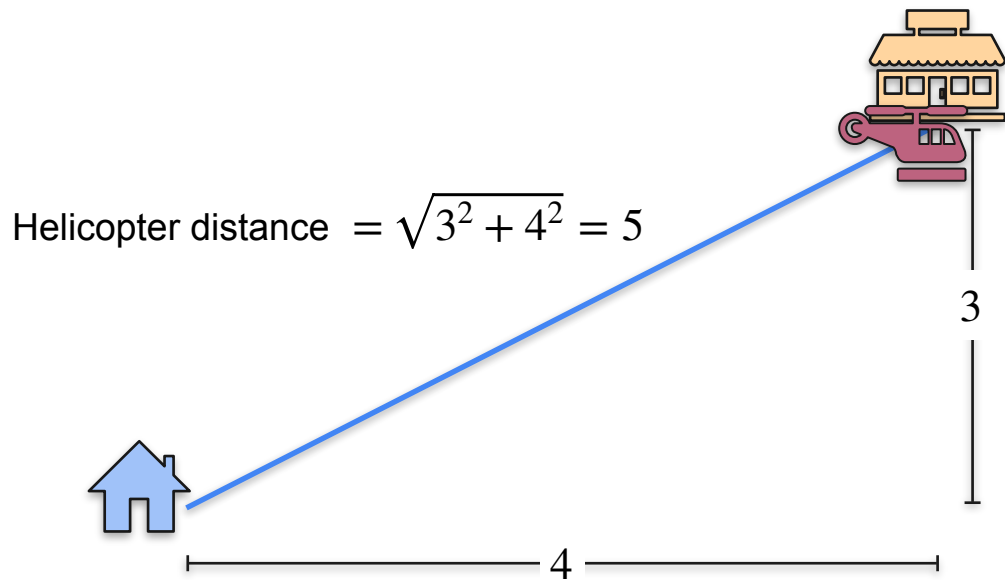
How to get from point A to point B?



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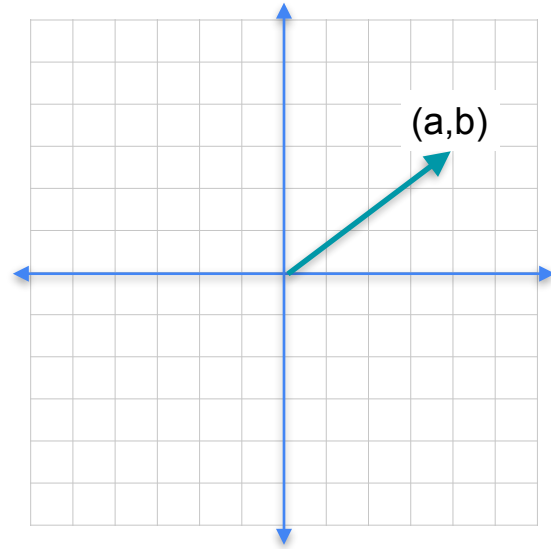


How to get from point A to point B?

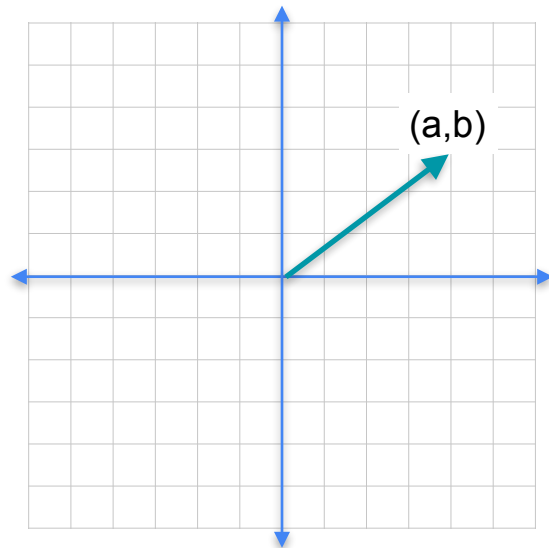


Pythagorean Theorem

Norms

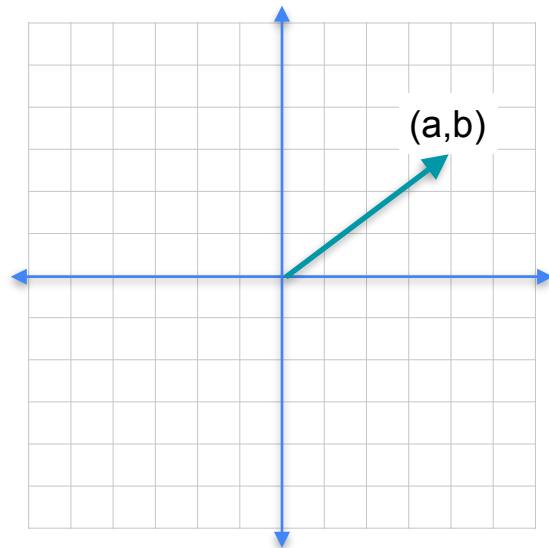


Norms



$$\text{L1-norm} = |(a,b)|_1 = |a| + |b|$$

Norms

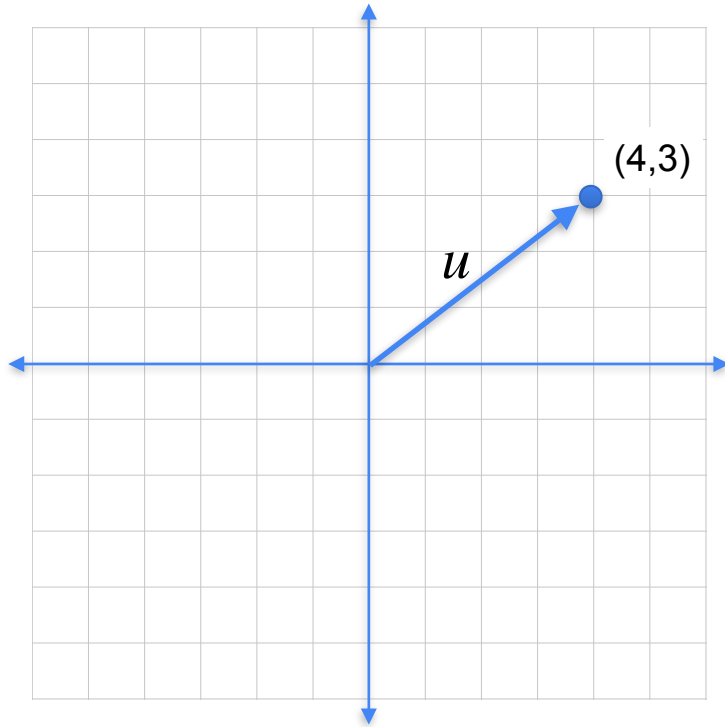


$$\text{L1-norm} = |(a,b)|_1 = |a| + |b|$$

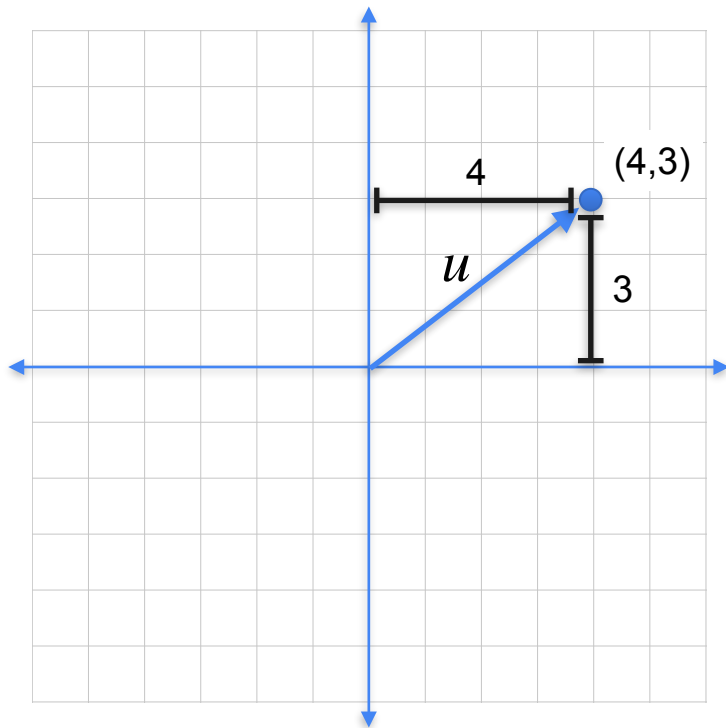


$$\text{L2-norm} = |(a,b)|_2 = \sqrt{a^2 + b^2}$$

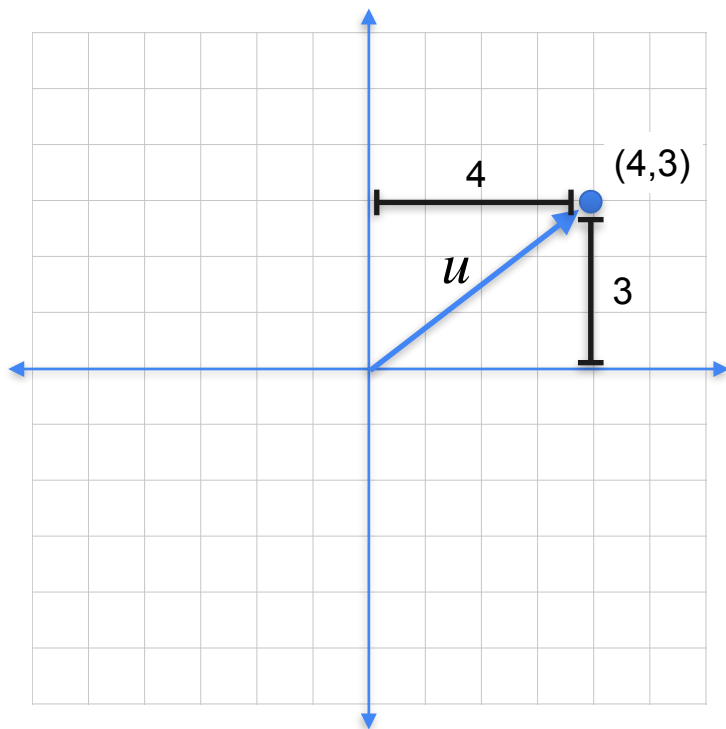
Norm of a vector



Norm of a vector

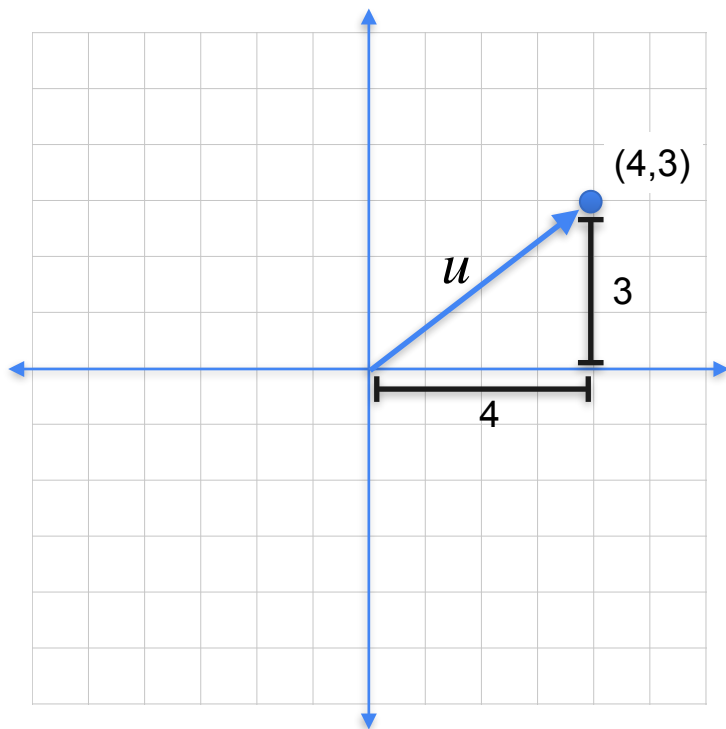


Norm of a vector

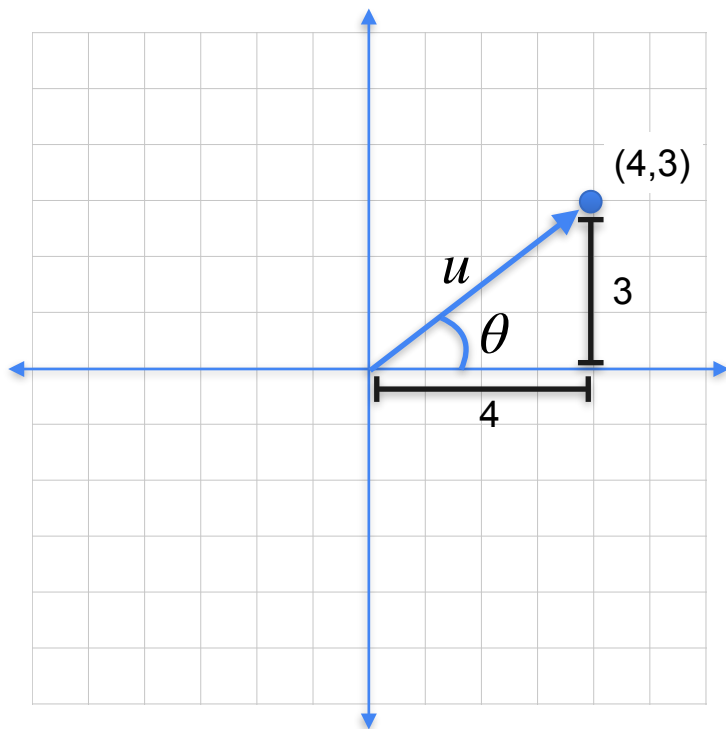


$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

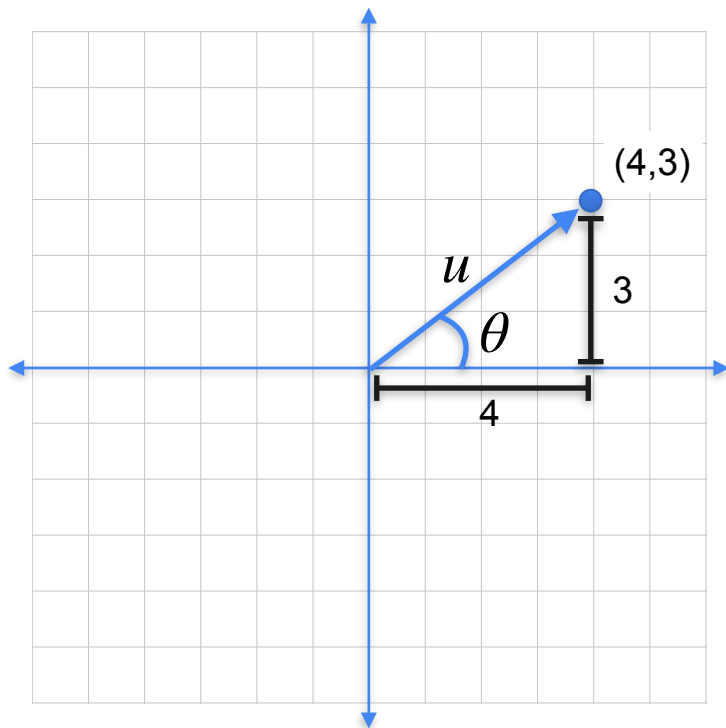
Direction of a vector



Direction of a vector

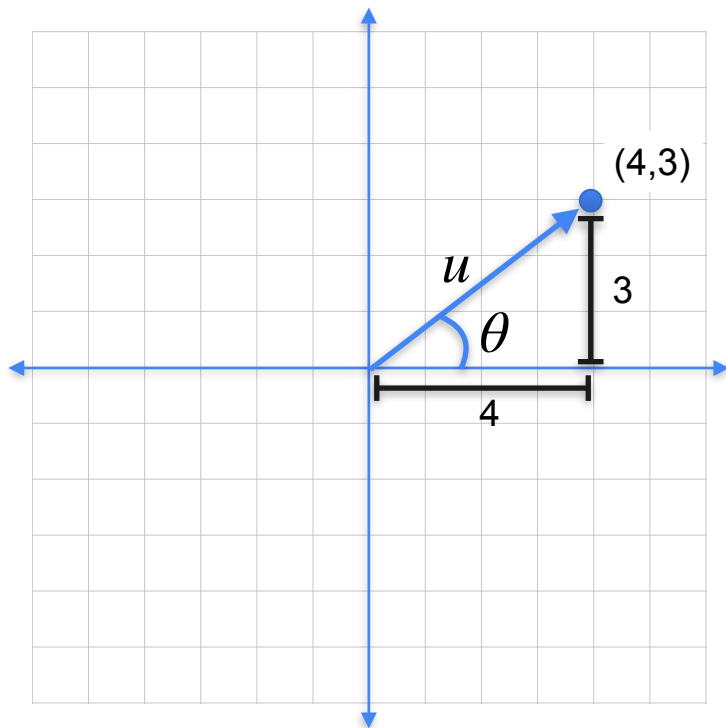


Direction of a vector



$$\tan(\theta) = \frac{3}{4}$$

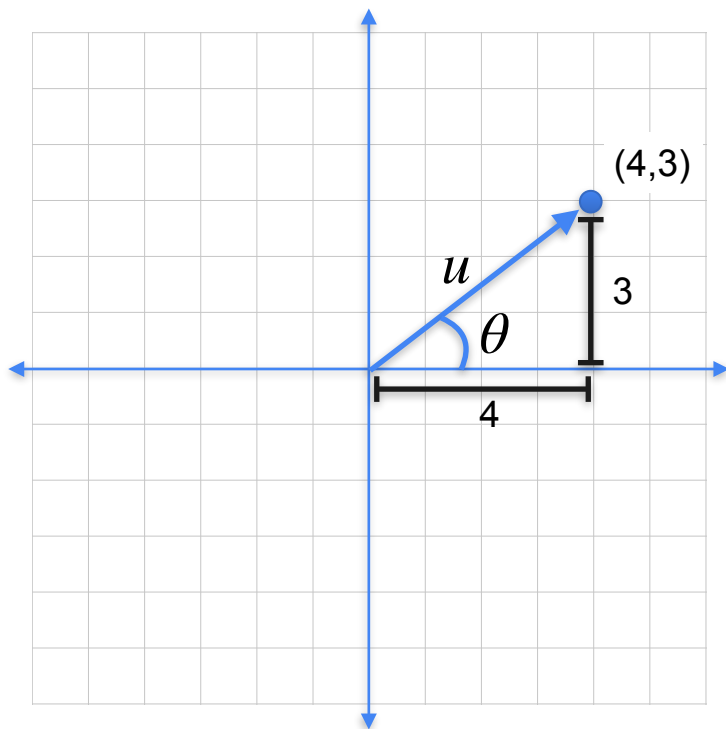
Direction of a vector



$$\tan(\theta) = \frac{3}{4}$$

$$\theta = \arctan(3/4) = 0.64$$

Direction of a vector



$$\tan(\theta) = \frac{3}{4}$$

$$\theta = \arctan(3/4) = 0.64 = 36.87^\circ$$

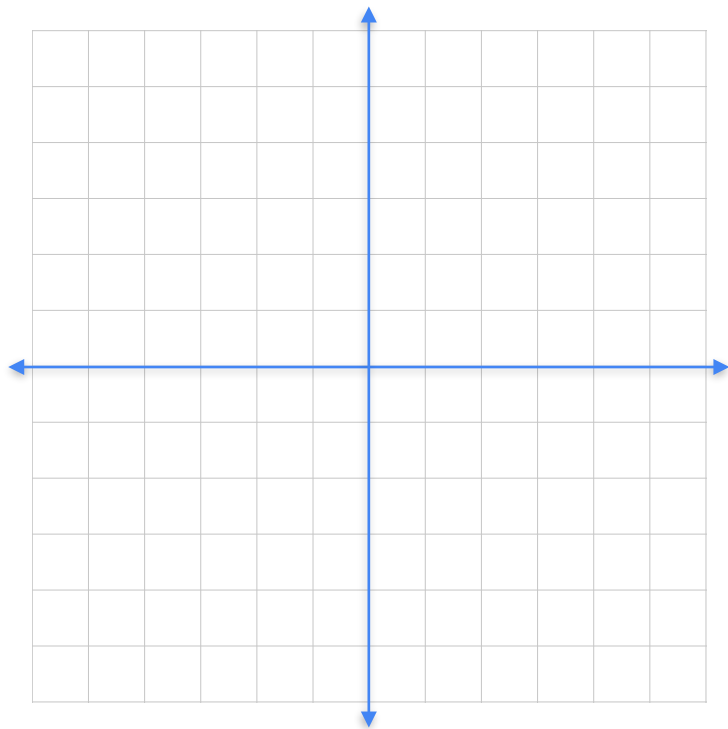


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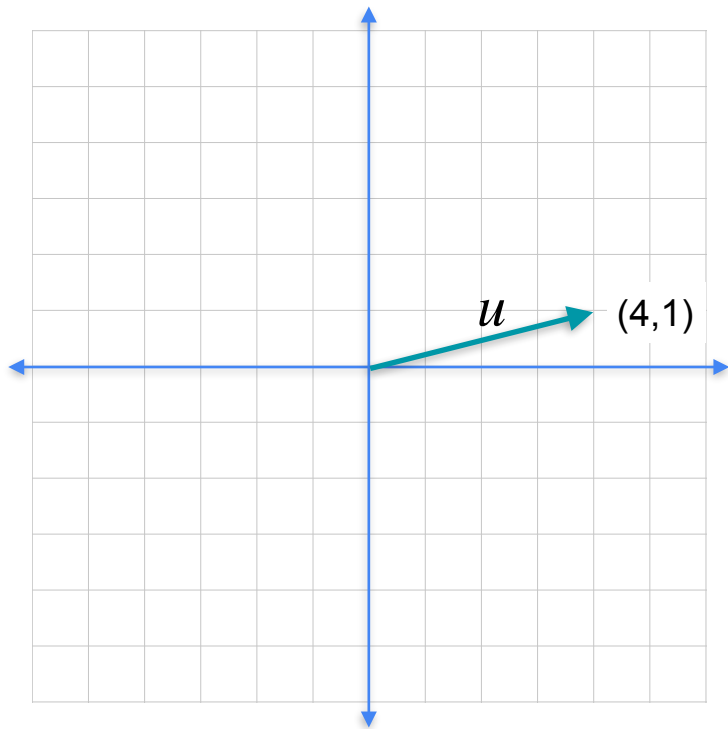
Vectors and Linear Transformations

Sum and difference of vectors

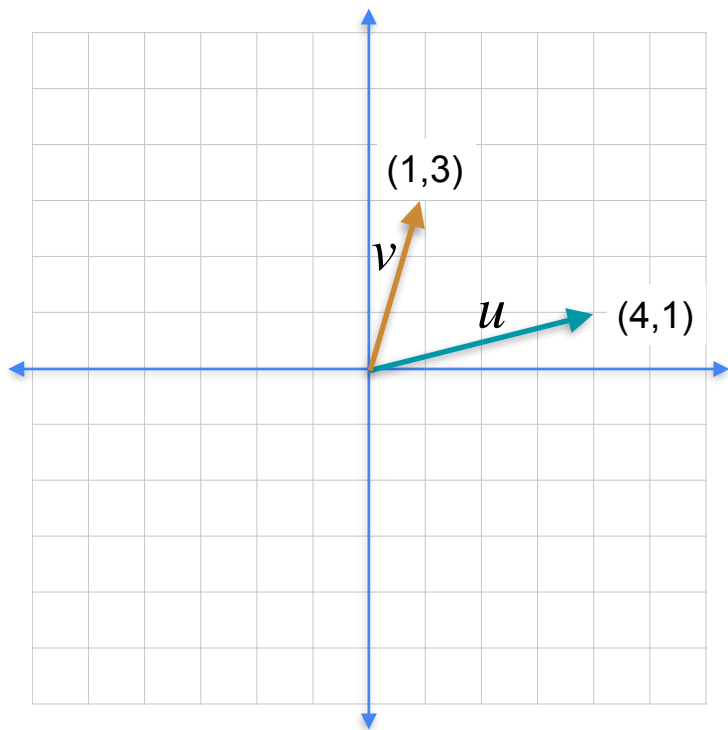
Sum of vectors



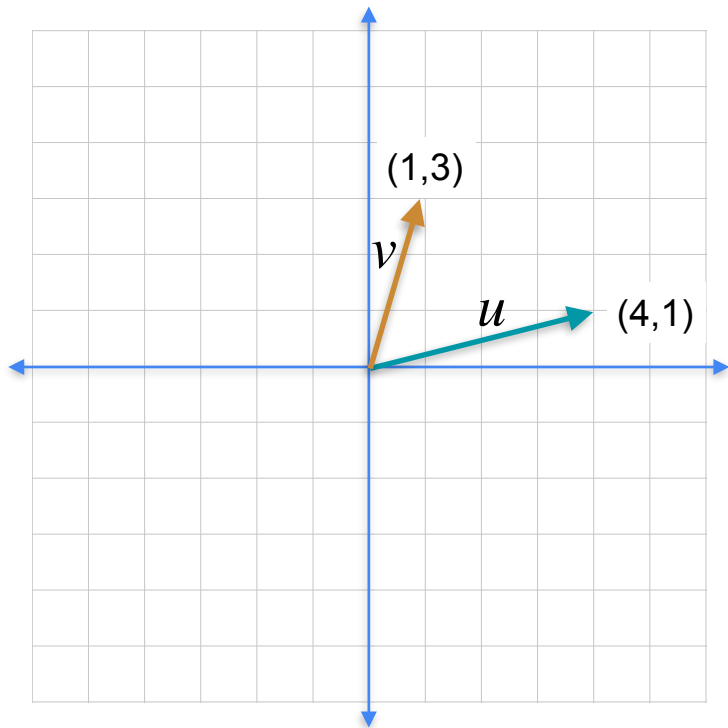
Sum of vectors



Sum of vectors

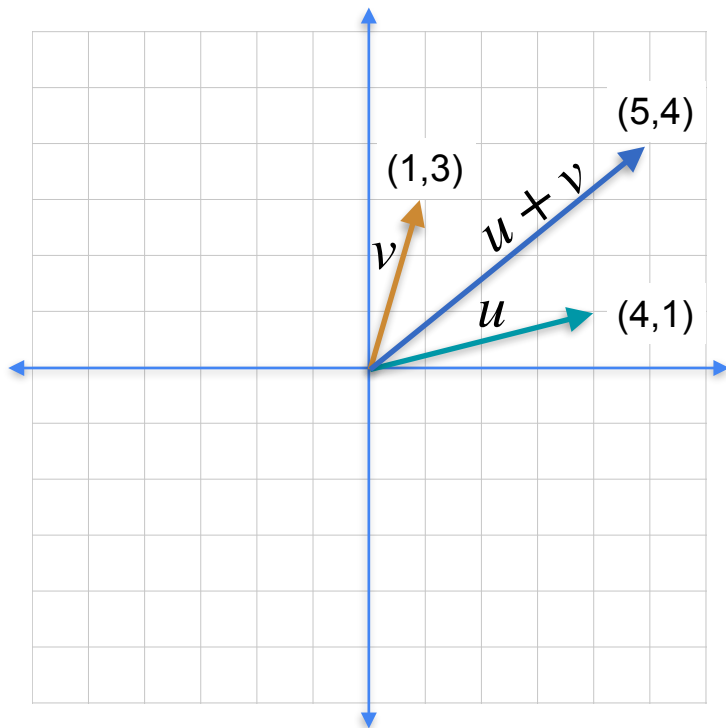


Sum of vectors



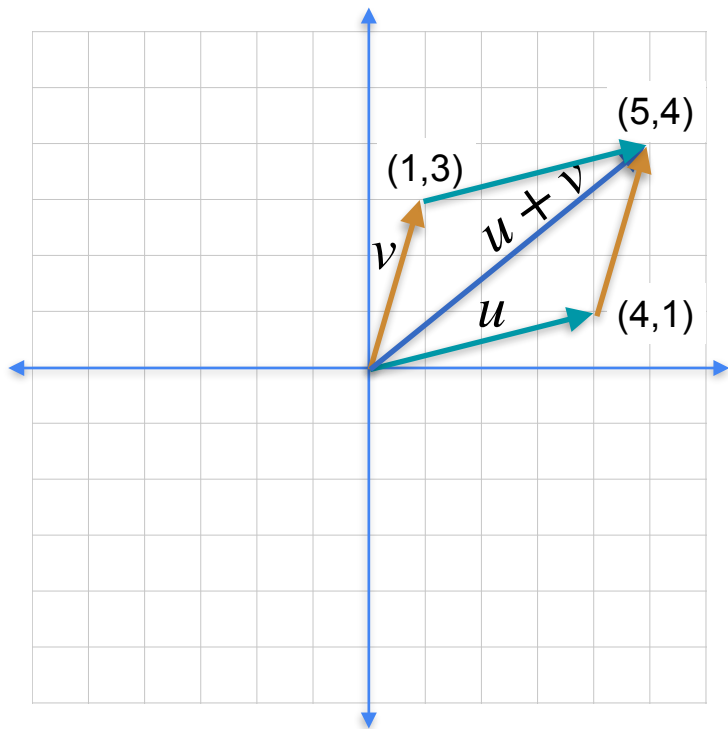
$$u + v = (4 + 1, 1 + 3) = (5, 4)$$

Sum of vectors



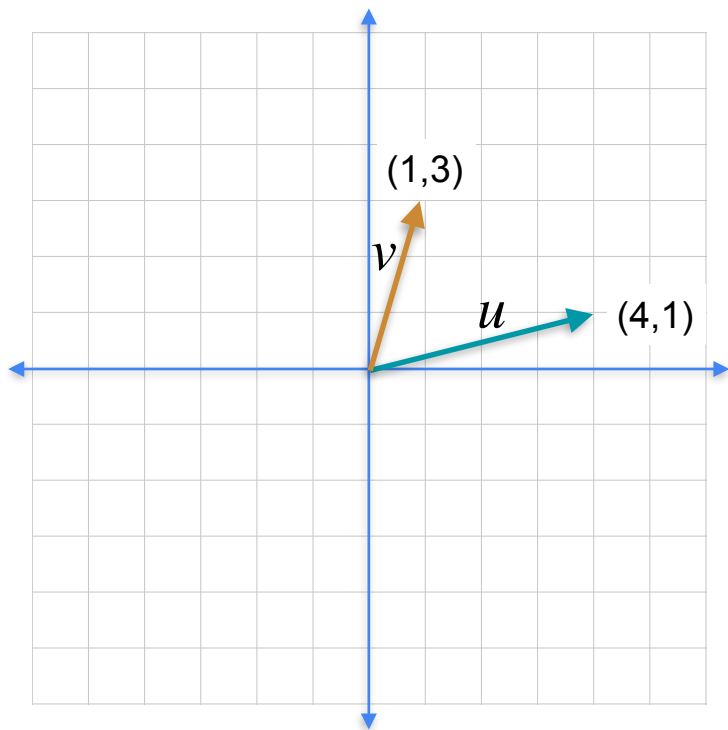
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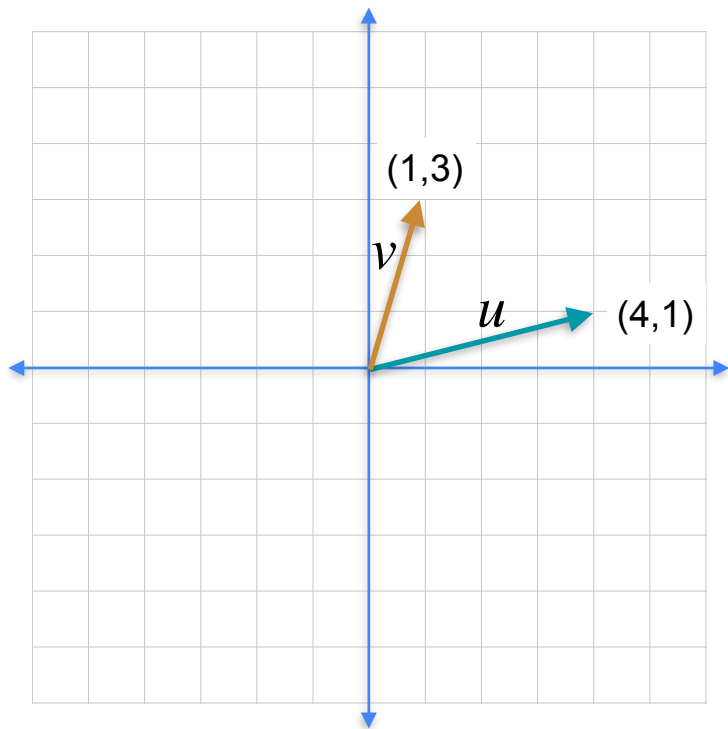


$$u + v = (4 + 1, 1 + 3) = (5, 4)$$

Difference of vectors

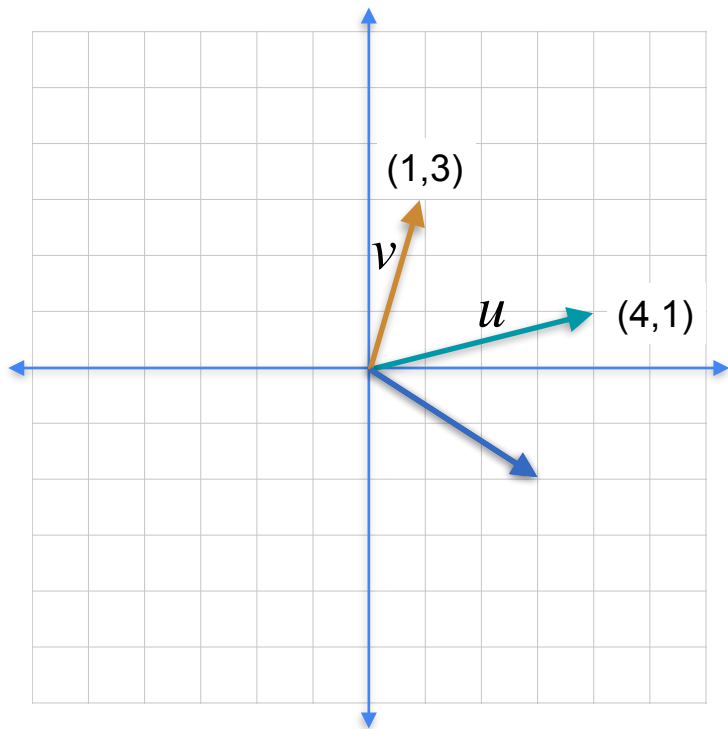


Difference of vectors



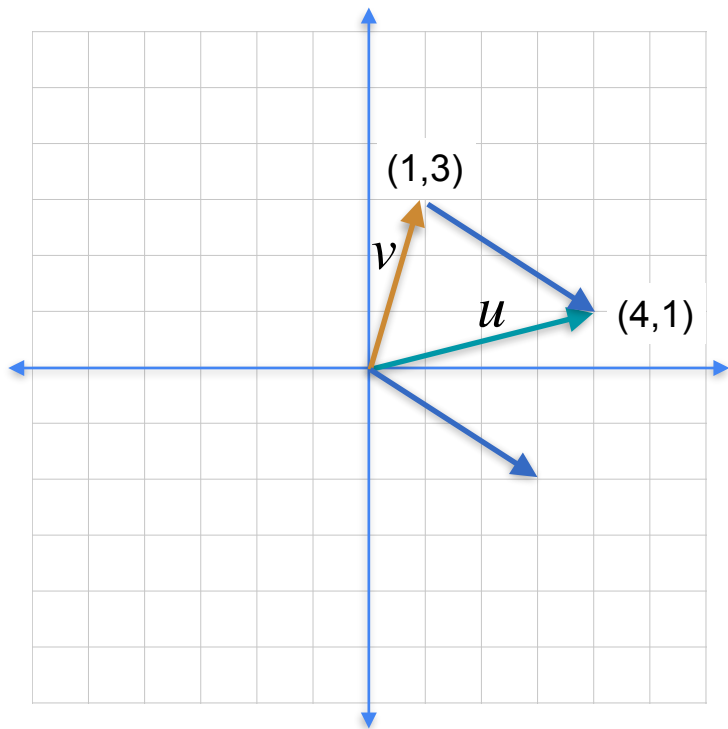
$$u - v = (4 - 1, 1 - 3) = (3, -2)$$

Difference of vectors



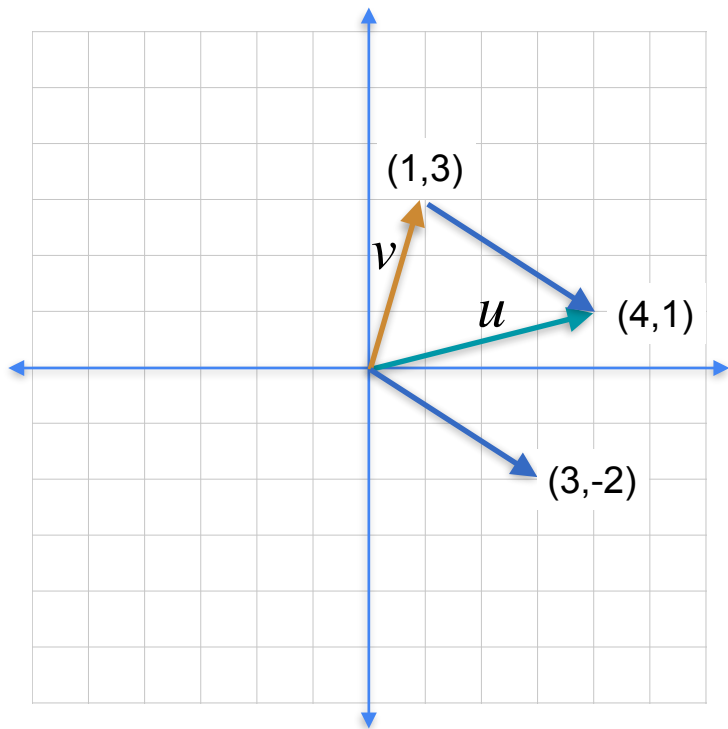
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Difference of vectors



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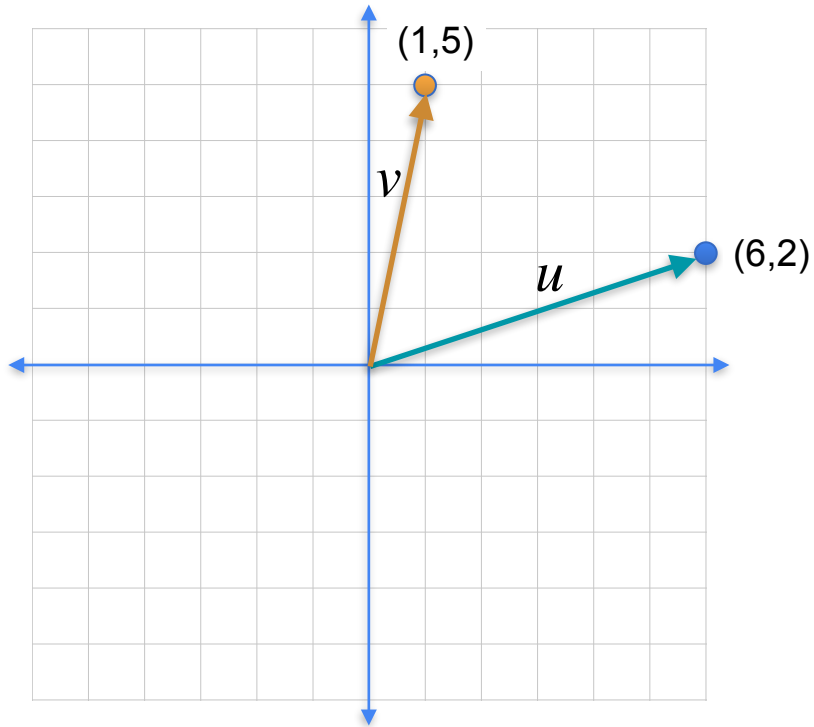


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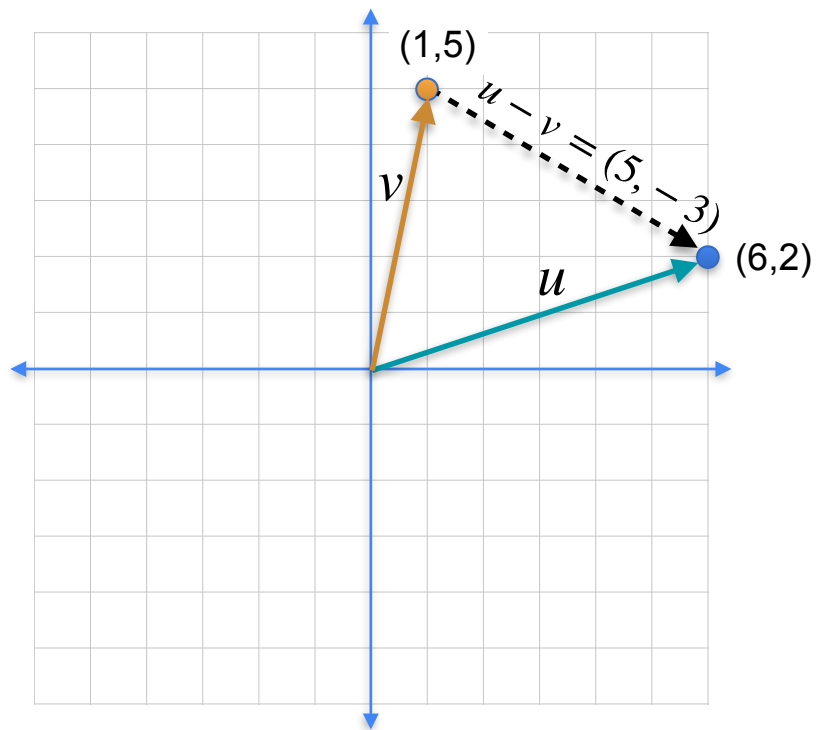
Vectors and Linear Transformations

Distance between vectors

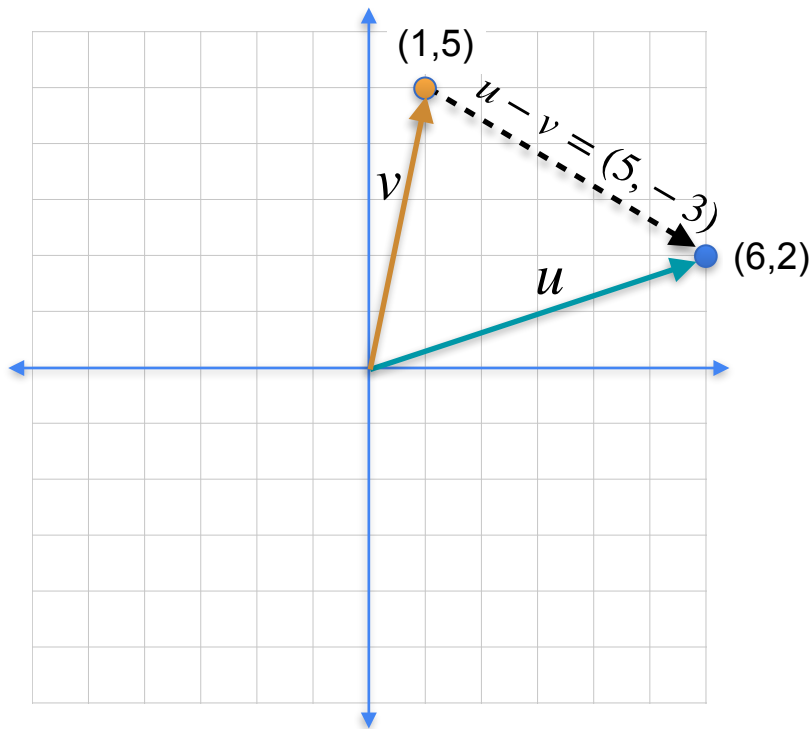
Distances



Distances



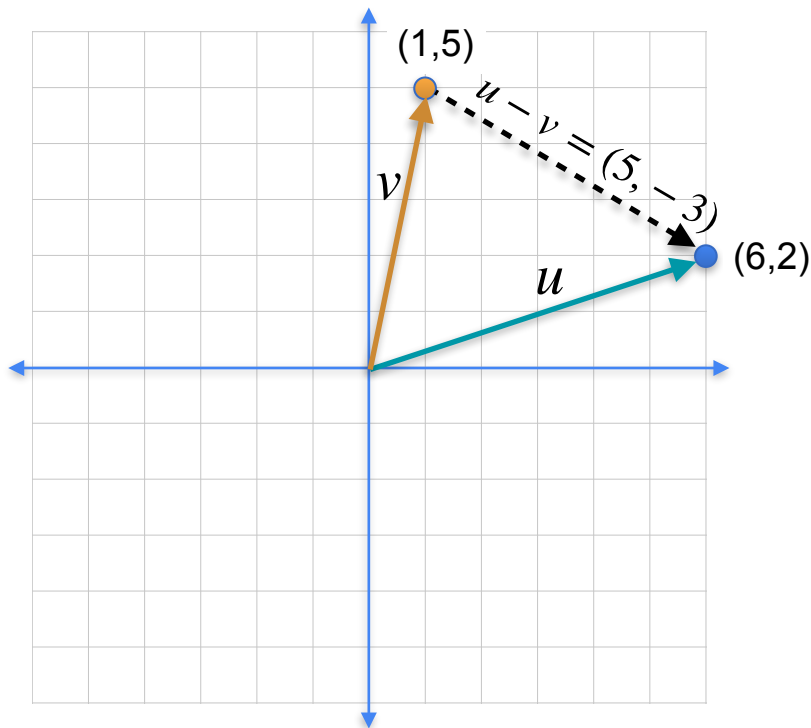
Distances



L1-distance

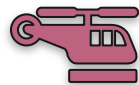
$$|u - v|_1 = |5| + |-3| = 8$$

Distances



L1-distance

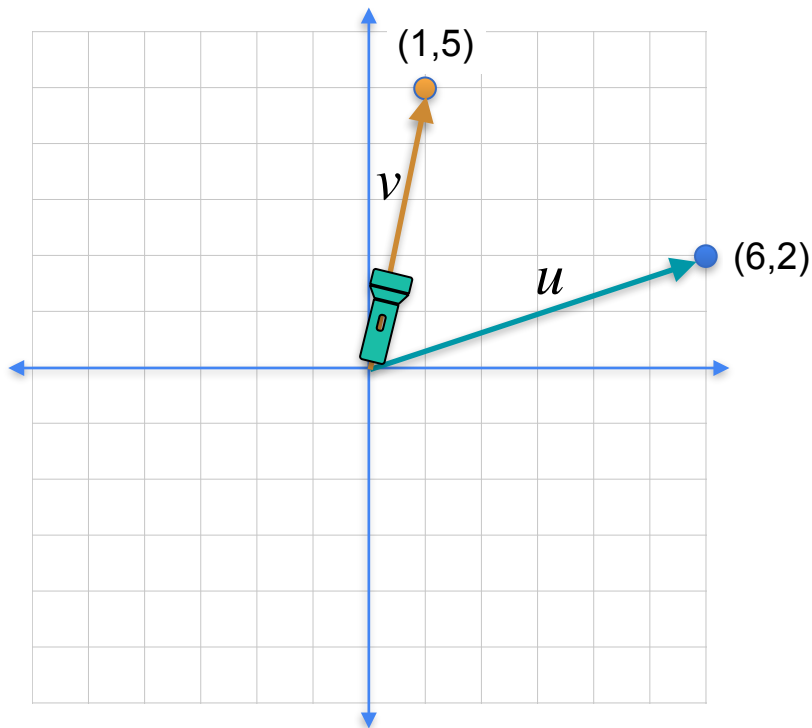
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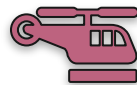
L2-distance

$$|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$$

Distances

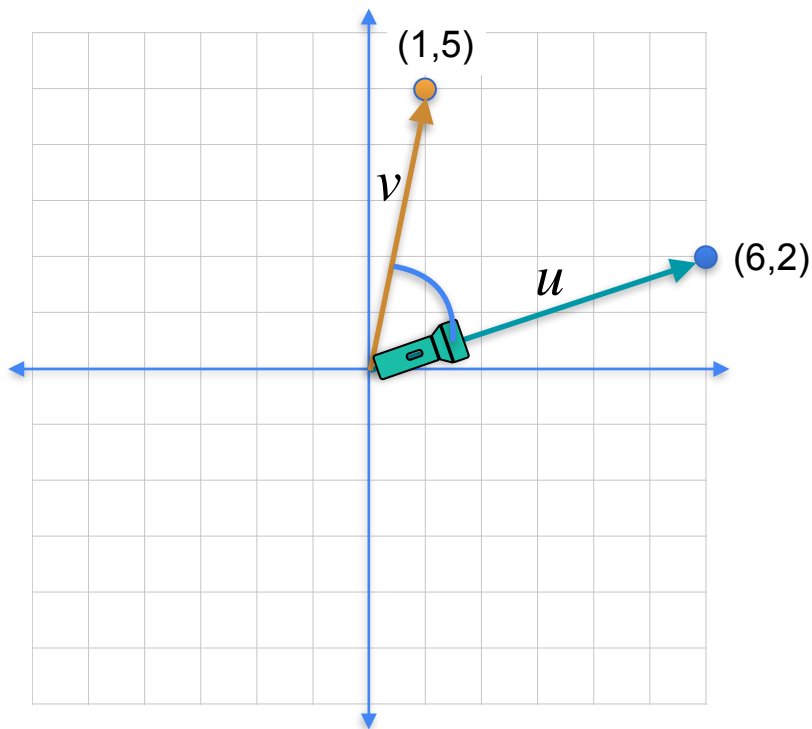


L1-distance $|u - v|_1 = |5| + |-3| = 8$



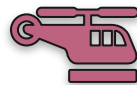
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Distances



L1-distance

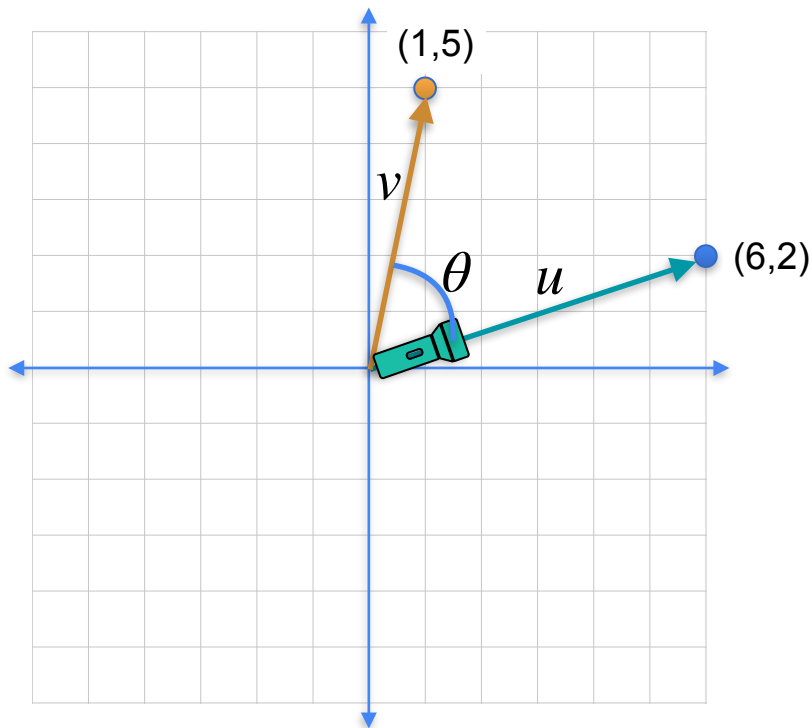
$$|u - v|_1 = |5| + |-3| = 8$$



L2-distance

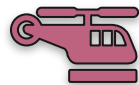
$$|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$$

Distances



L1-distance

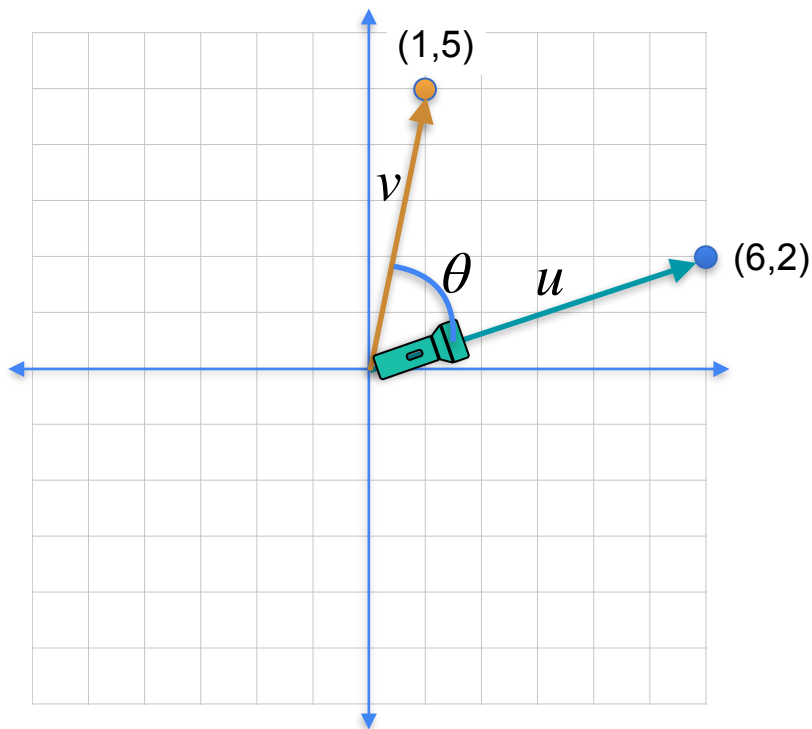
$$|u - v|_1 = |5| + |-3| = 8$$





L2-distance

$$|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$$

Distances



 $|u - v|_1 = |5| + |-3| = 8$
L1-distance

 $|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$
L2-distance

 $\cos(\theta)$
Cosine distance

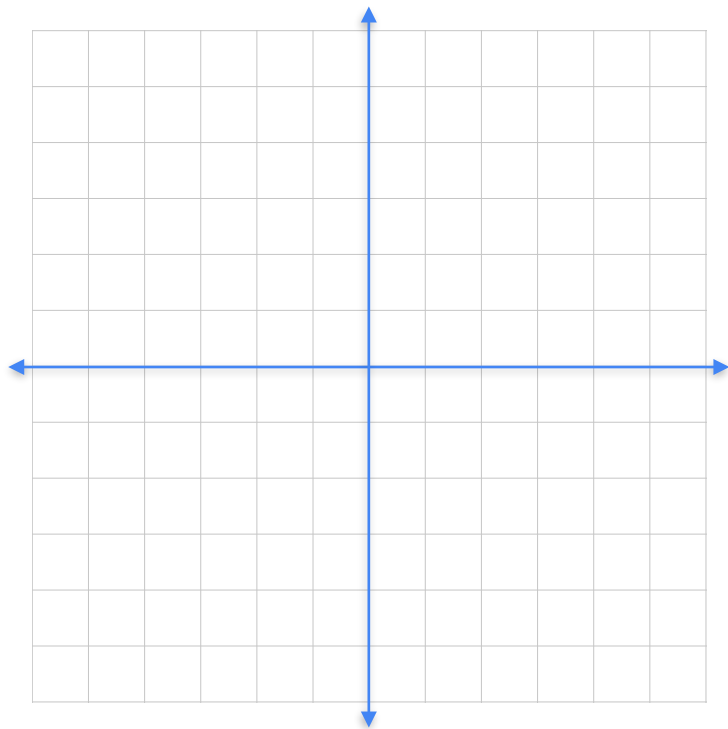


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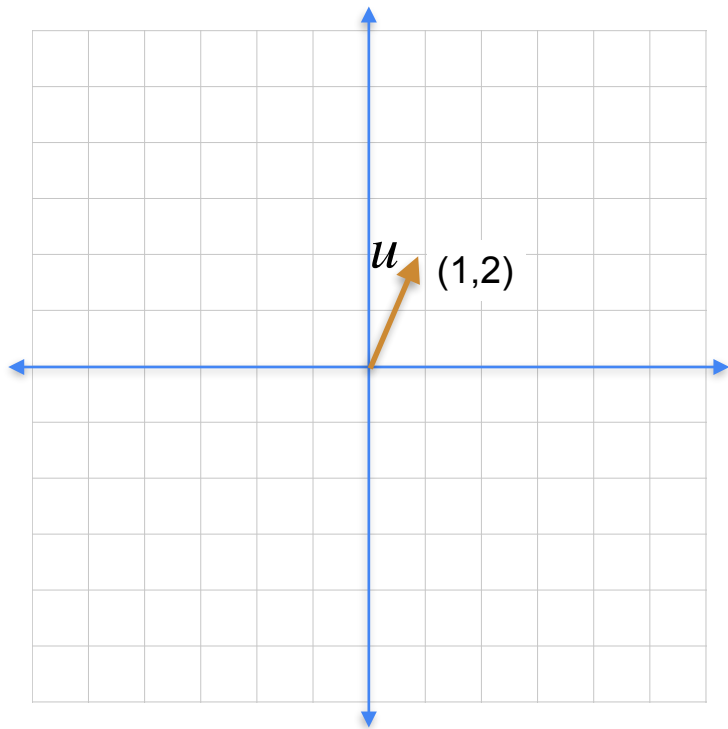
Vectors and Linear Transformations

Multiplying a vector by a scalar

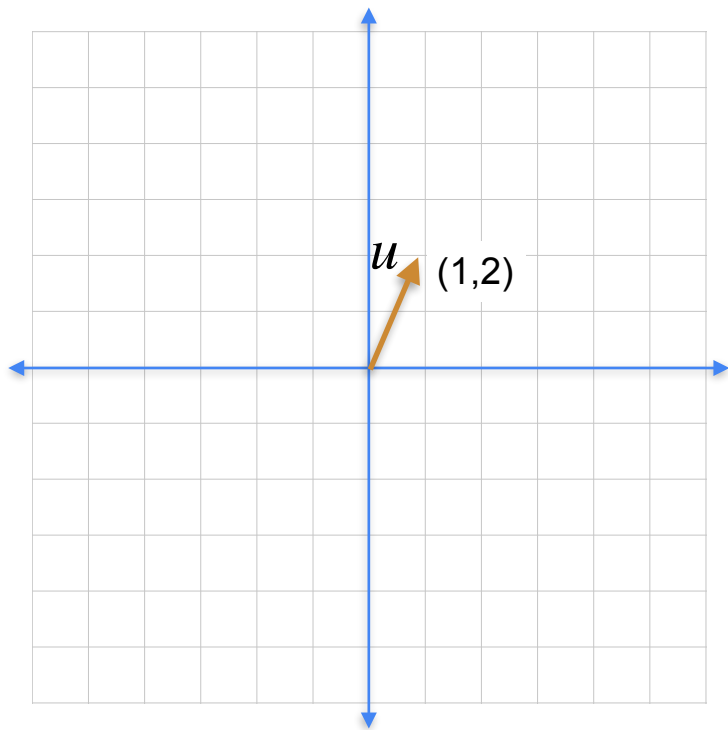
Multiplying a vector by a scalar



Multiplying a vector by a scalar

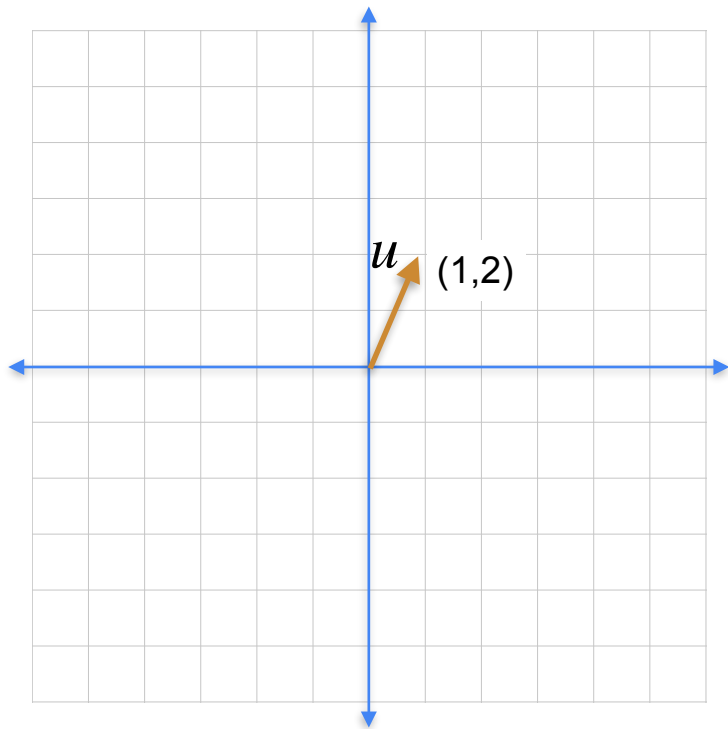


Multiplying a vector by a scalar



$$u = (1,2)$$

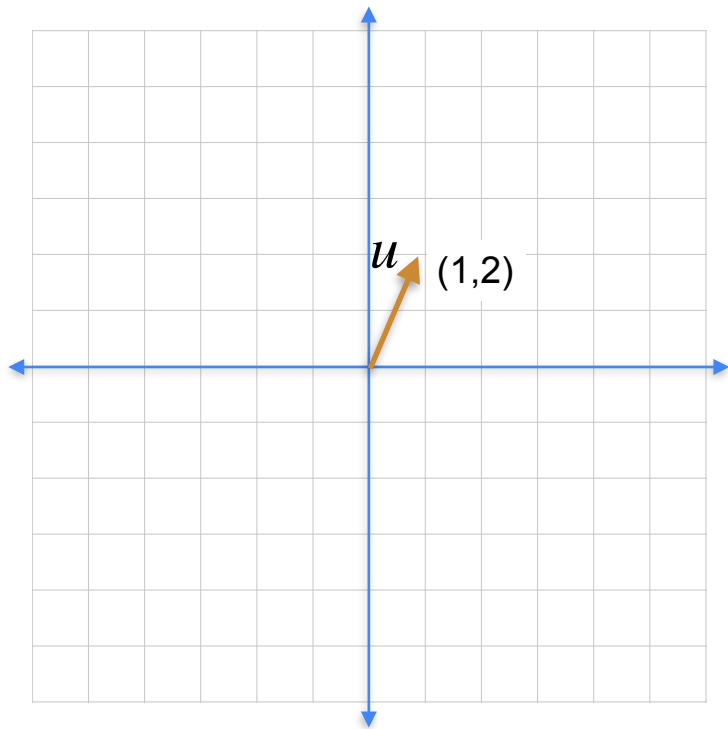
Multiplying a vector by a scalar



$$u = (1,2)$$

$$\lambda = 3$$

Multiplying a vector by a scalar

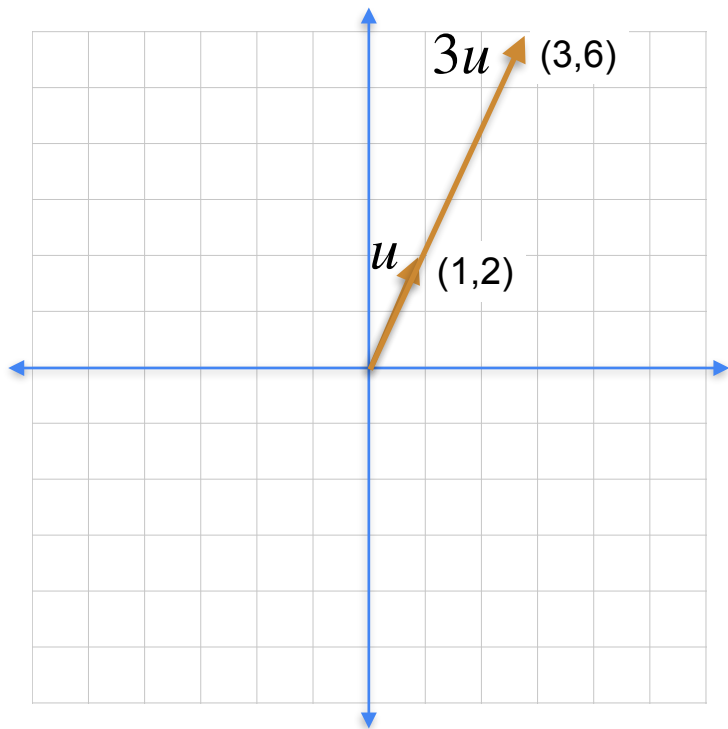


$$u = (1,2)$$

$$\lambda = 3$$

$$\lambda u = (3,6)$$

Multiplying a vector by a scalar

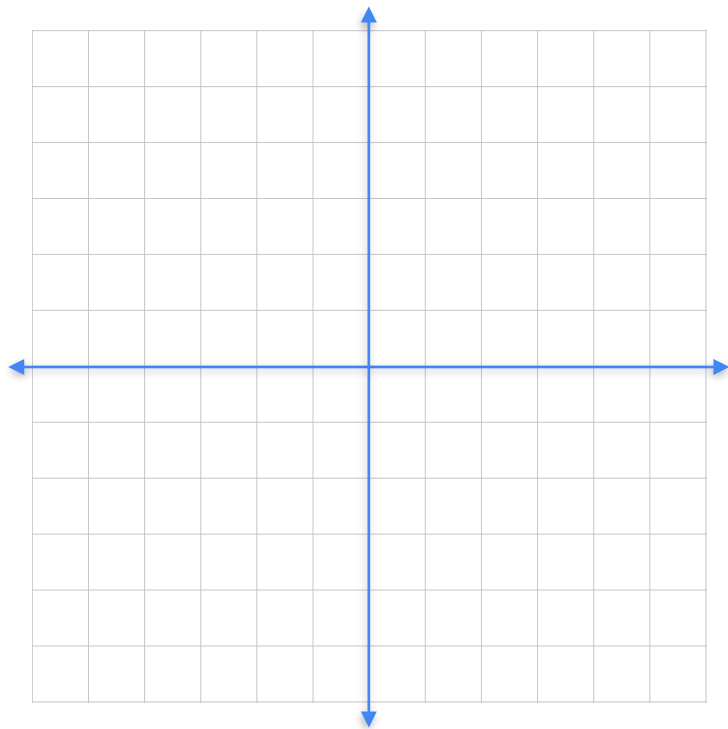


$$u = (1,2)$$

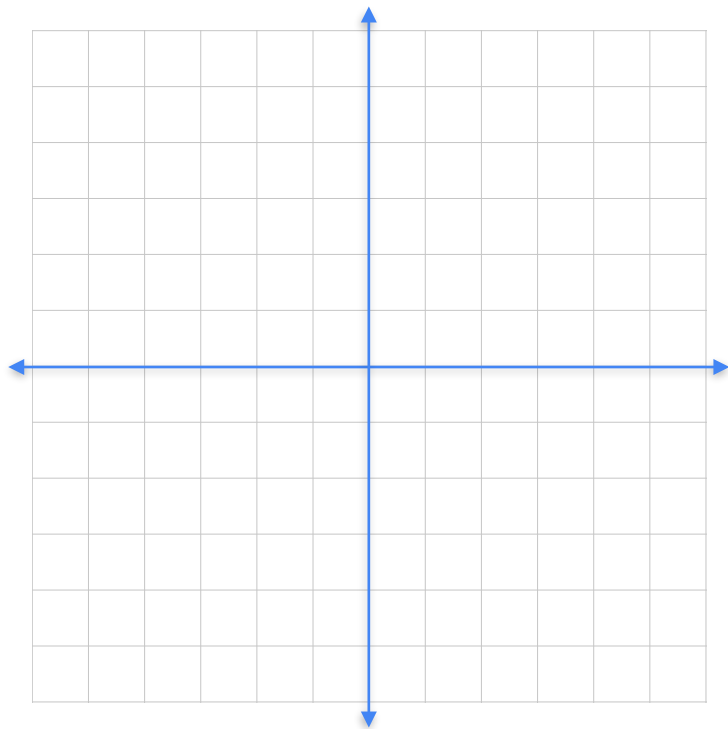
$$\lambda = 3$$

$$\lambda u = (3,6)$$

If the scalar is negative

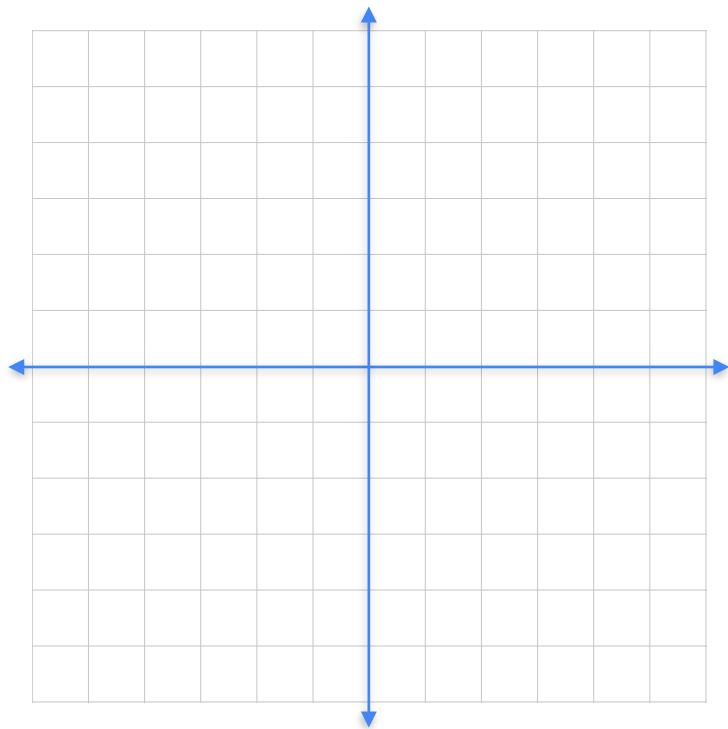


If the scalar is negative



$$u = (1,2)$$

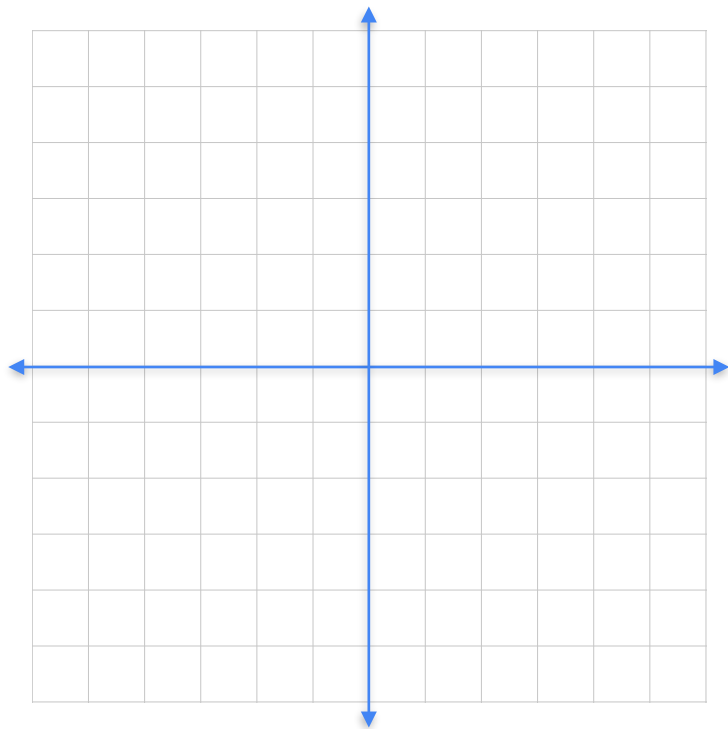
If the scalar is negative



$$u = (1, 2)$$

$$\lambda = -2$$

If the scalar is negative

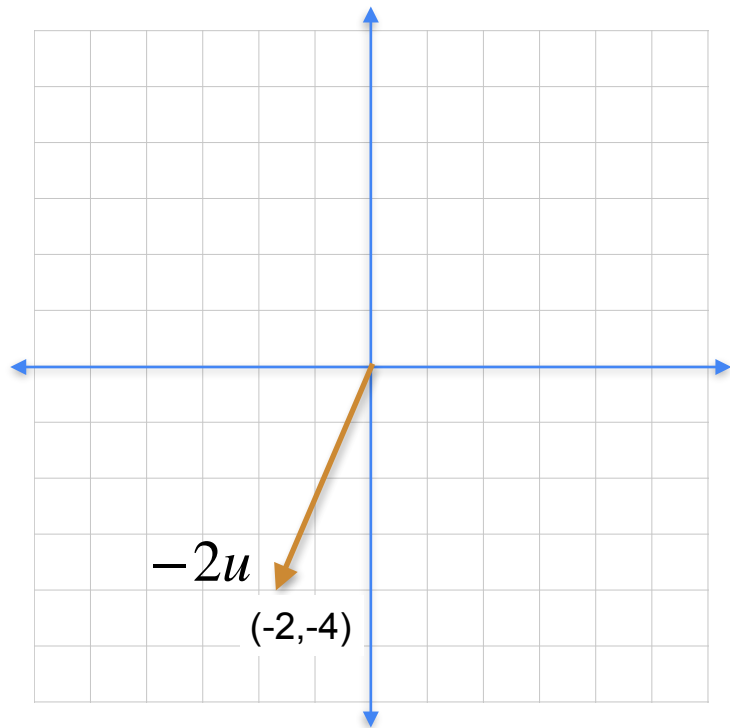


$$u = (1, 2)$$

$$\lambda = -2$$

$$\lambda u = (-2, -4)$$

If the scalar is negative

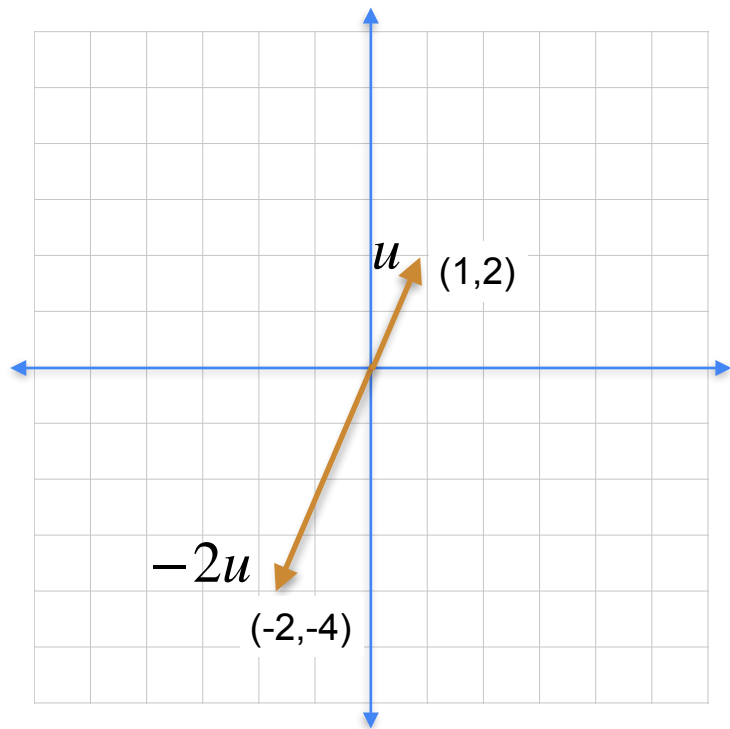


$$u = (1, 2)$$

$$\lambda = -2$$

$$\lambda u = (-2, -4)$$

If the scalar is negative



$$u = (1,2)$$

$$\lambda = -2$$

$$\lambda u = (-2, -4)$$



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Vectors and Linear Transformations

The dot product

A shortcut for linear operations

A shortcut for linear operations

Quantities

2 apples

4 bananas

1 cherry

A shortcut for linear operations

Quantities

2 apples
4 bananas
1 cherry

Prices

apples: \$3
bananas: \$5
cherries: \$2

A shortcut for linear operations

Quantities

2 apples
4 bananas
1 cherry

Prices

apples: \$3
bananas: \$5
cherries: \$2

Total price




A shortcut for linear operations

Quantities

2 apples

4 bananas

1 cherry

	2
	4
	1

Prices

apples: \$3

bananas: \$5

cherries: \$2

Total price




A shortcut for linear operations

Quantities

2 apples

4 bananas

1 cherry

	2
	4
	1

Prices

apples: \$3

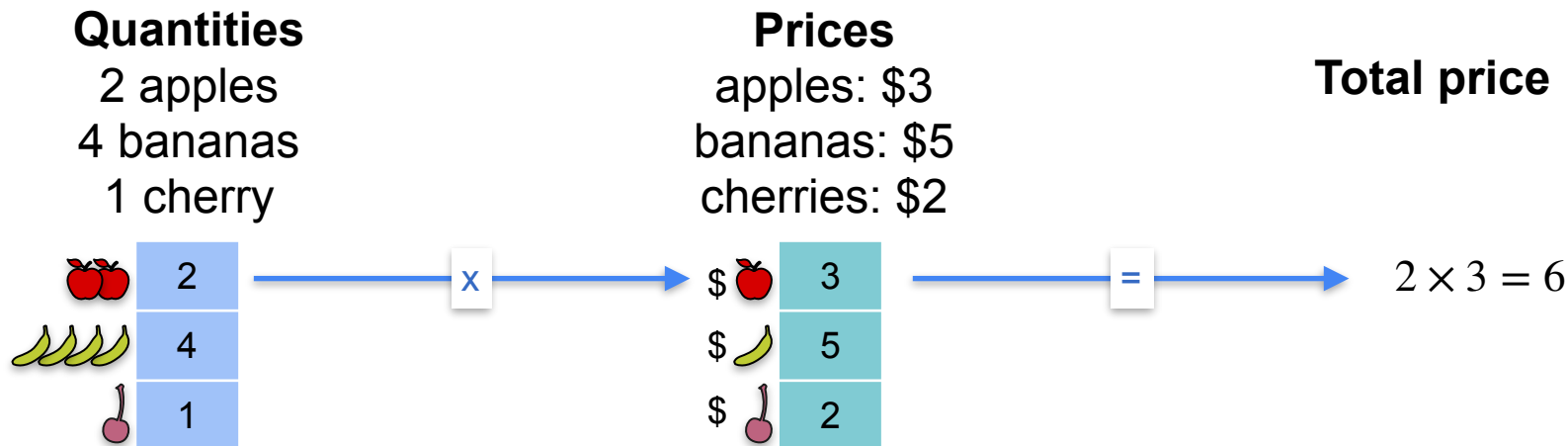
bananas: \$5

cherries: \$2

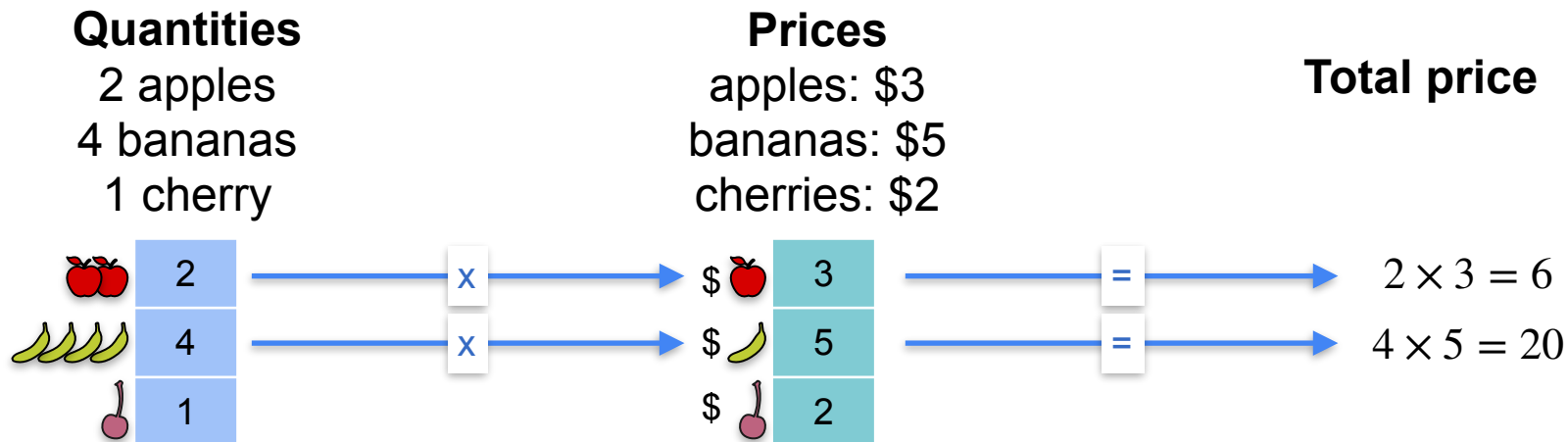
\$ 	3
\$ 	5
\$ 	2

Total price

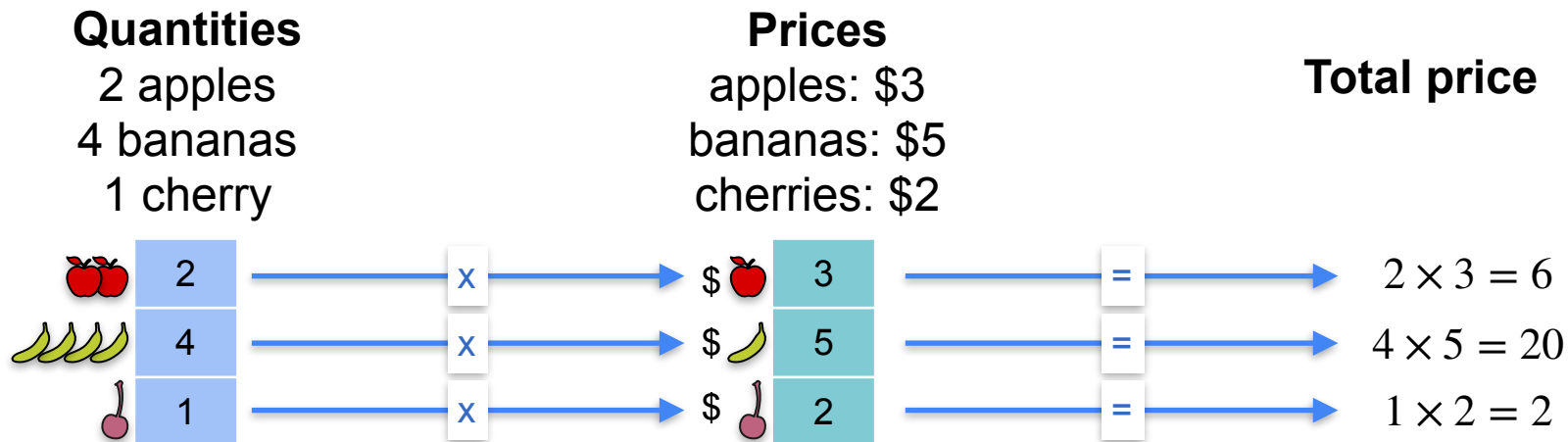
A shortcut for linear operations



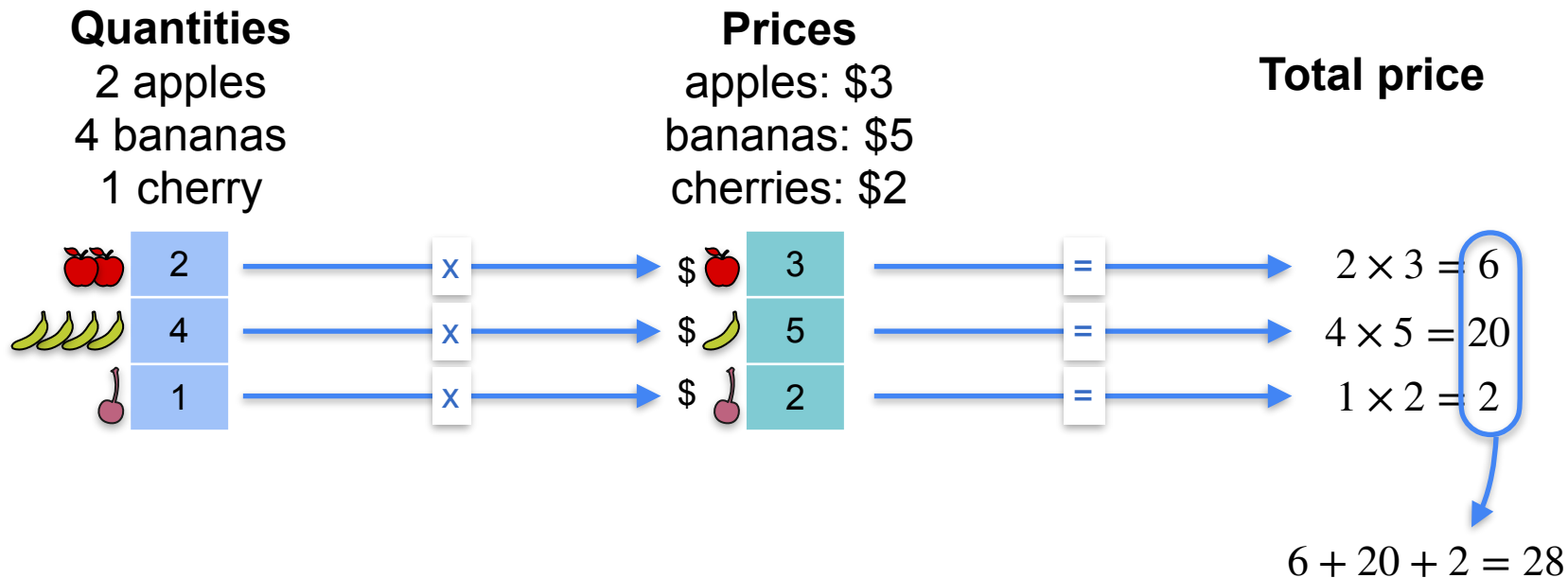
A shortcut for linear operations



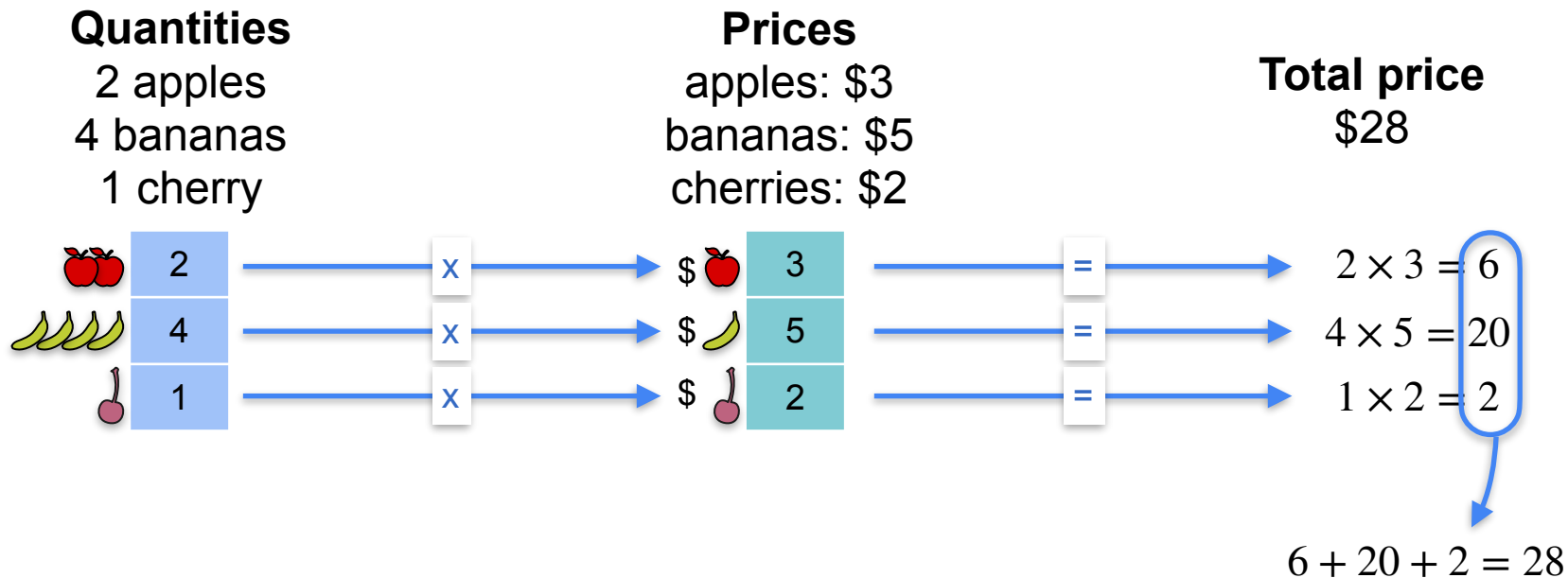
A shortcut for linear operations



A shortcut for linear operations






A shortcut for linear operations






The dot product

The diagram illustrates the dot product of two vectors using fruit prices. The first vector (blue boxes) represents quantities: 2 apples, 4 bananas, and 1 cherry. The second vector (teal boxes) represents prices: \$3 for an apple, \$5 for a banana, and \$2 for a cherry. The dot product is calculated as follows:




	2
	4
	1

·




\$ 	3
\$ 	5
\$ 	2

= \$28

The dot product

	2
	4
	1

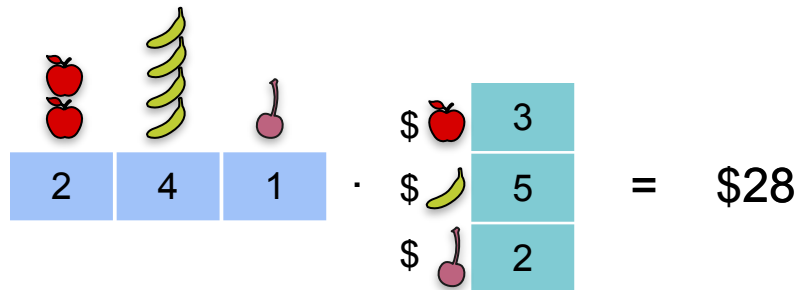
 \cdot

\$ 	3
\$ 	5
\$ 	2

 $= \$28$

$$2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = 28$$

The dot product



The diagram illustrates a dot product calculation for fruit prices. On the left, a vector of quantities is shown in blue boxes: 2 (with 2 apples above), 4 (with 4 bananas above), and 1 (with 1 cherry above). This is followed by a dot operator. To the right is a vector of prices in teal boxes: 3 (with \$ and 1 apple to the left), 5 (with \$ and 1 banana to the left), and 2 (with \$ and 1 cherry to the left). An equals sign follows, leading to the result \$28.

2	4	1	·	\$ 🍎	3	= \$28
				\$ 🍌	5	
				\$ 🍒	2	

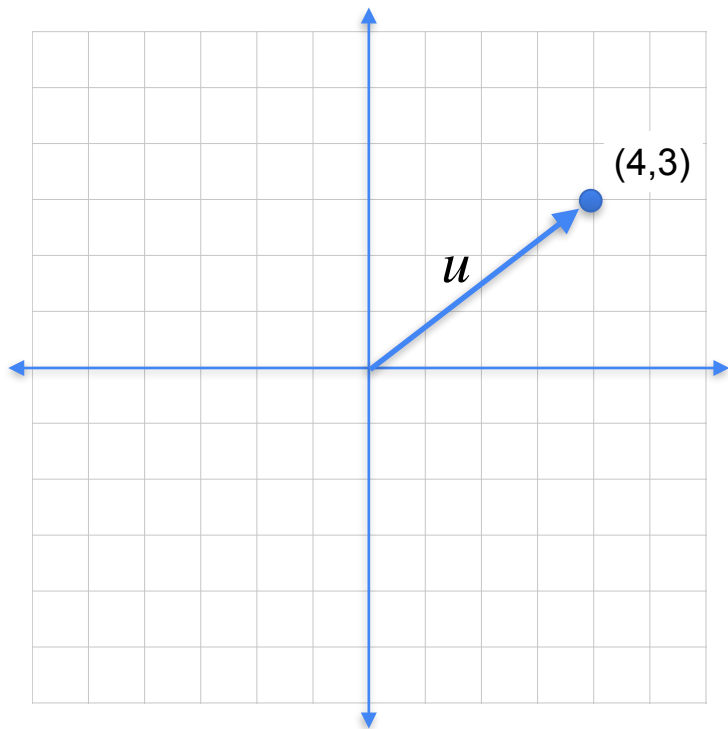
$$2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = 28$$

The dot product

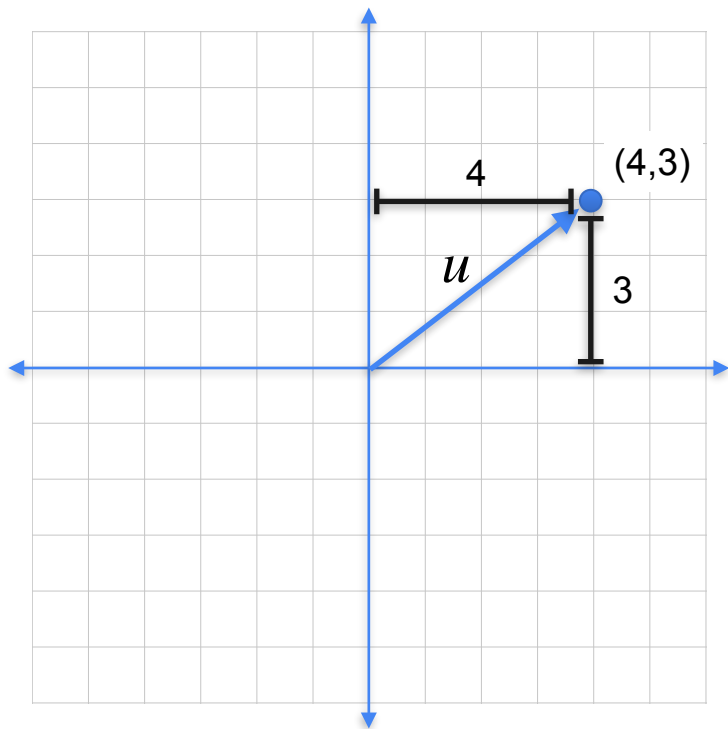
$$\begin{bmatrix} 2 & 4 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} = 28$$

$$2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = 28$$

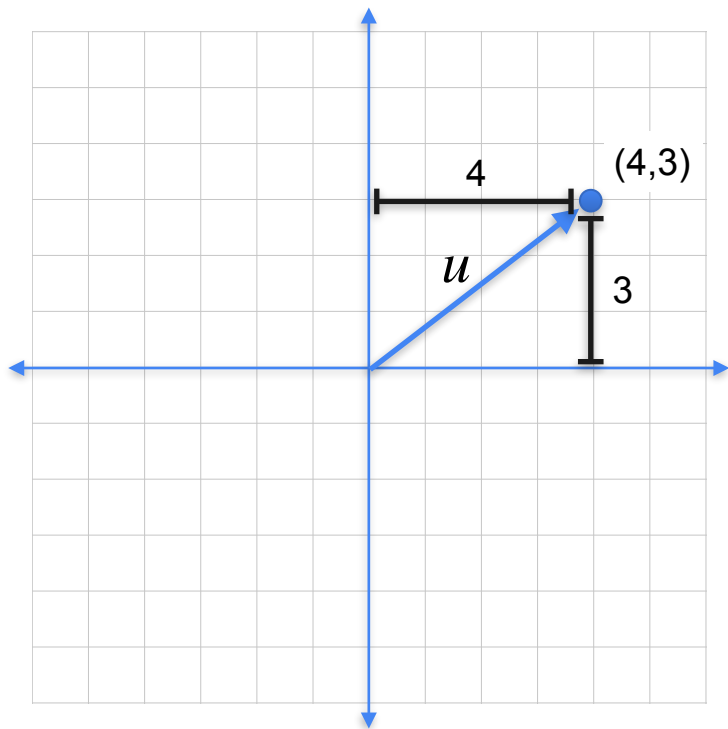
Norm of a vector using dot product



Norm of a vector using dot product

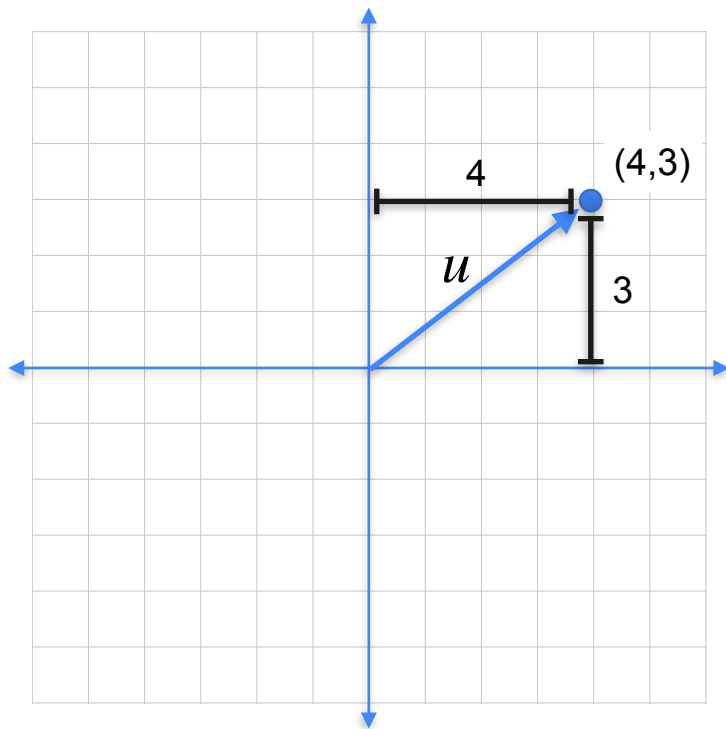


Norm of a vector using dot product



$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

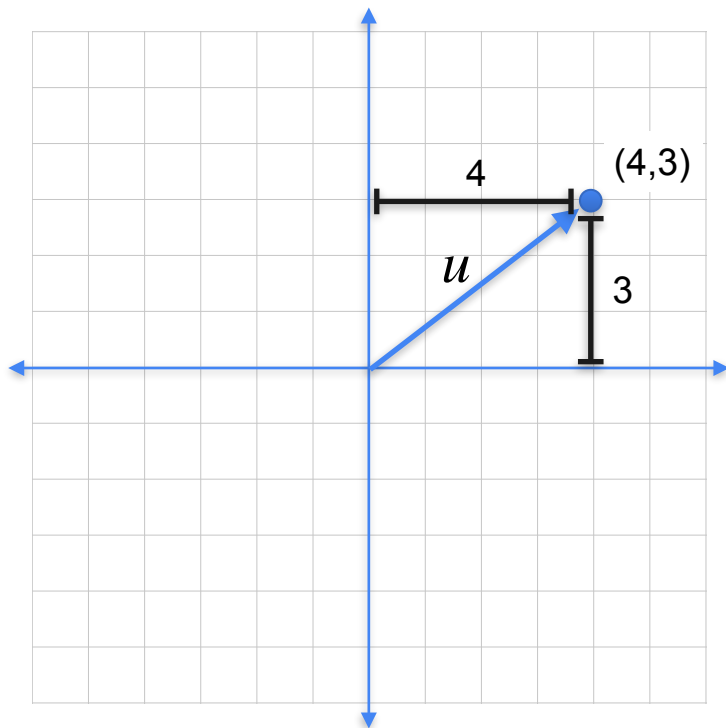
Norm of a vector using dot product



$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

$$\begin{bmatrix} 4 & 3 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 3 \end{bmatrix} = 25$$

Norm of a vector using dot product

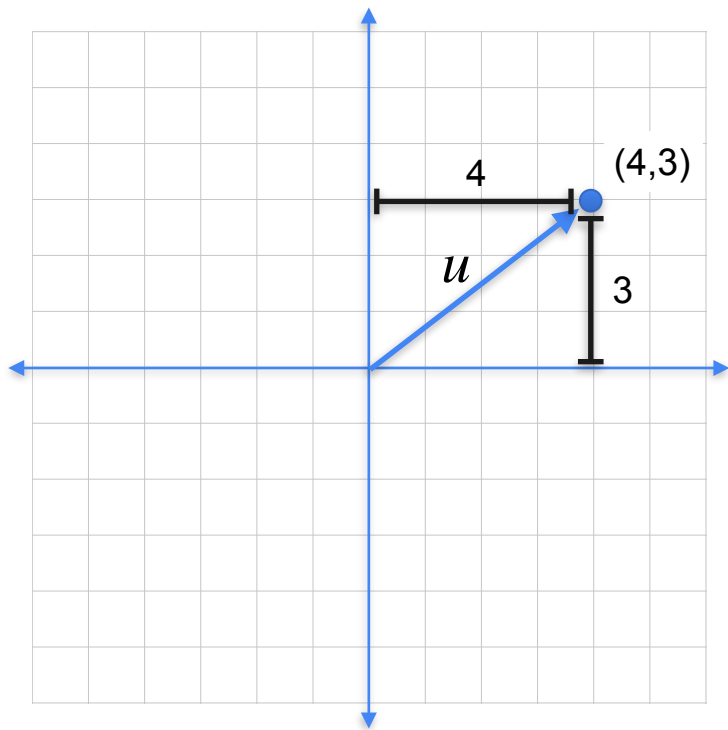


$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

$$\begin{bmatrix} 4 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = 25$$

$$L2 - norm = \sqrt{\text{dot product}(u, u)}$$

Norm of a vector using dot product



$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

$$\begin{bmatrix} 4 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = 25$$

$$L2 - norm = \sqrt{\text{dot product}(u, u)}$$

$$|u|_2 = \sqrt{\langle u, u \rangle}$$

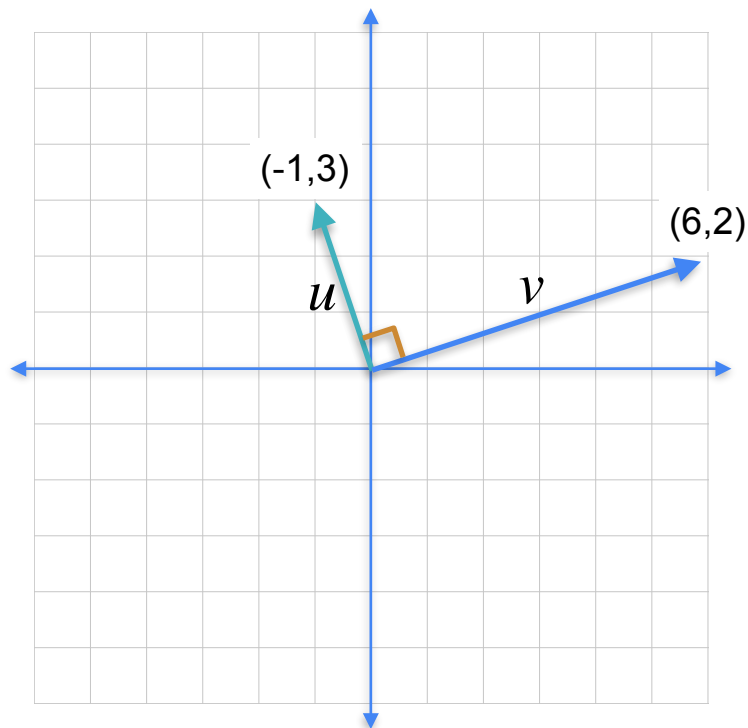


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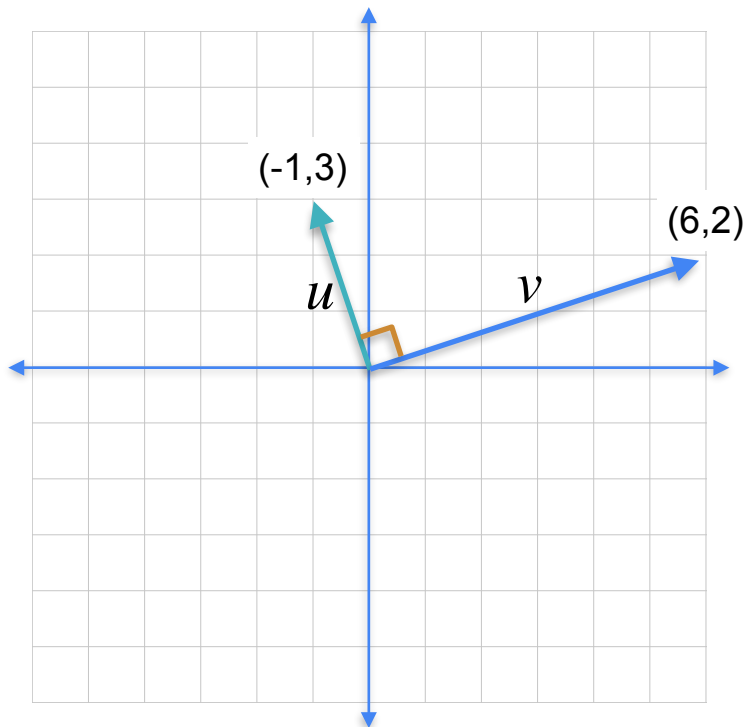
Vectors and Linear Transformations

Geometric dot product

Orthogonal vectors have dot product 0



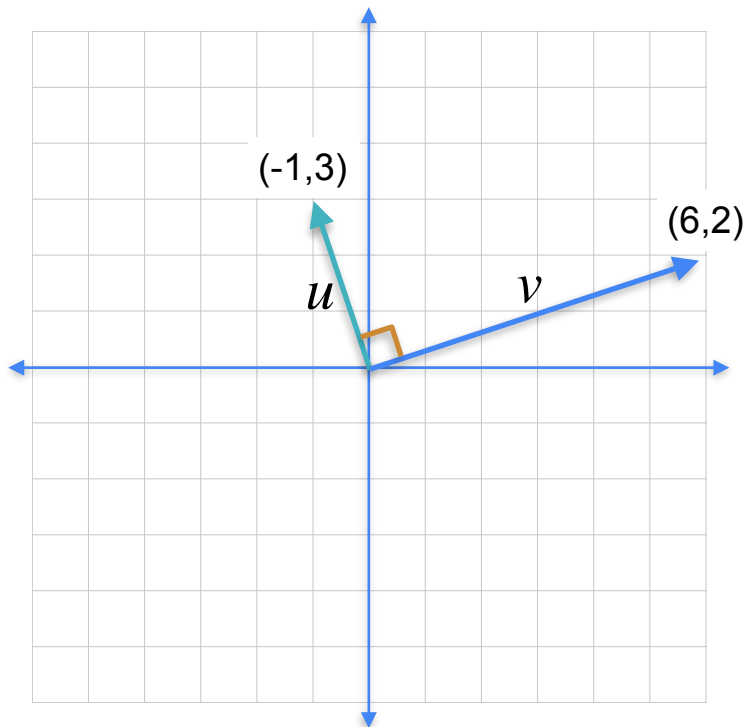
Orthogonal vectors have dot product 0



6

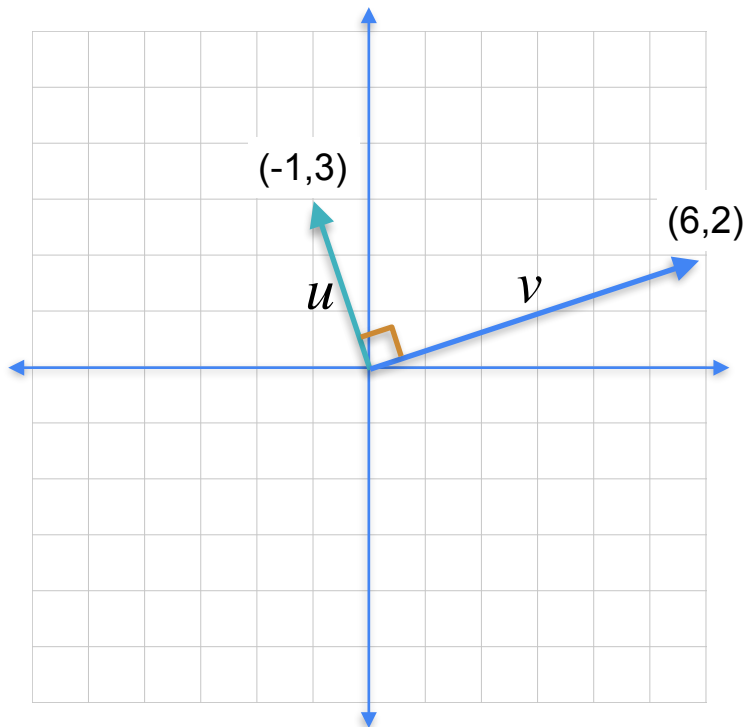
2

Orthogonal vectors have dot product 0



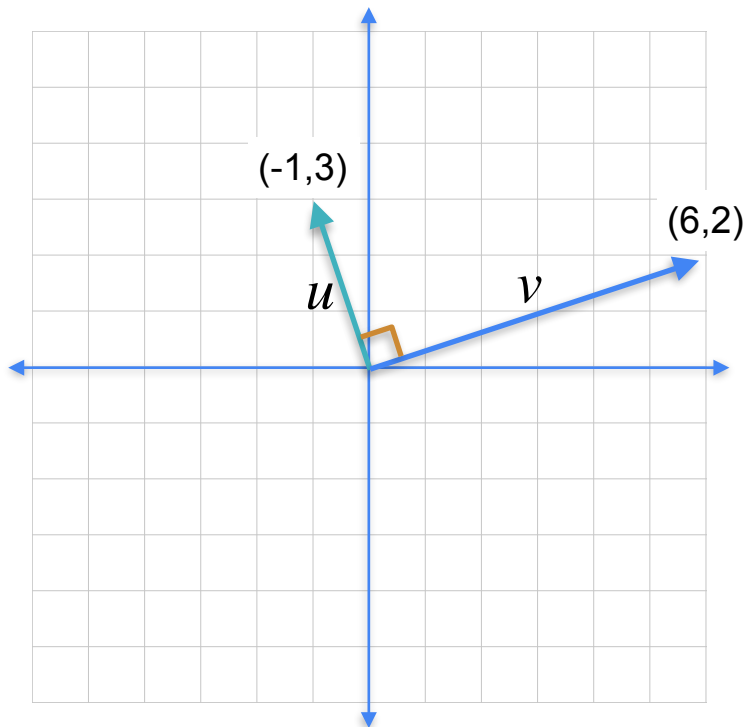
6	2	-1
		3

Orthogonal vectors have dot product 0



$$\begin{bmatrix} 6 & 2 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

Orthogonal vectors have dot product 0

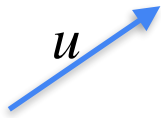


$$\begin{bmatrix} 6 & 2 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

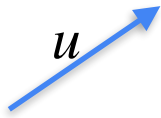
$$\langle u, v \rangle = 0$$

The dot product

The dot product

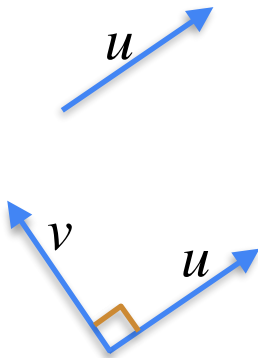


The dot product



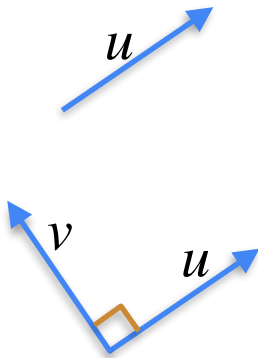
$$\langle u, u \rangle = |u|^2$$

The dot product



$$\langle u, u \rangle = |u|^2$$

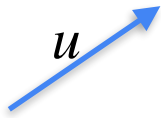
The dot product



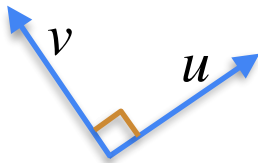
$$\langle u, u \rangle = |u|^2$$

$$\langle u, v \rangle = 0$$

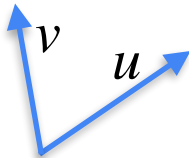
The dot product



$$\langle u, u \rangle = |u|^2$$



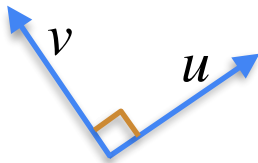
$$\langle u, v \rangle = 0$$



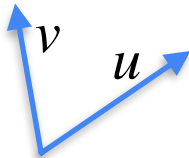
The dot product



$$\langle u, u \rangle = |u|^2$$



$$\langle u, v \rangle = 0$$



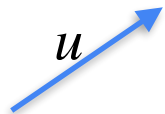
$$\langle u, v \rangle = ?$$

The dot product

The dot product



The dot product



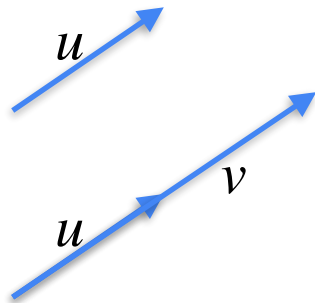
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

The dot product



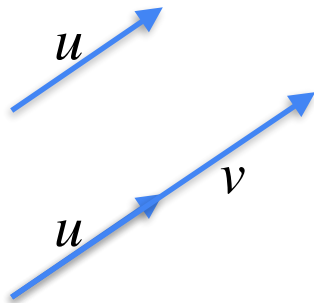
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

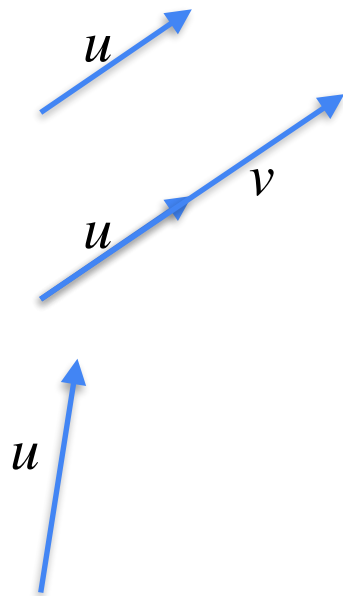
The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$

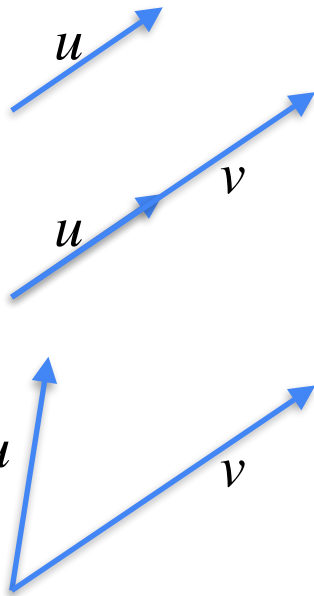
The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$

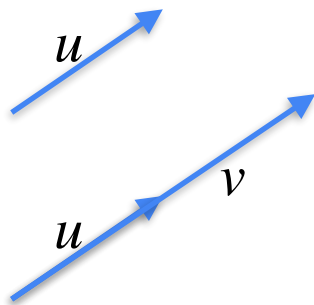
The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

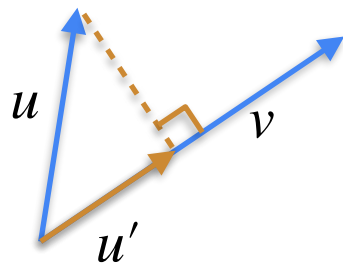
$$\langle u, v \rangle = |u| \cdot |v|$$

The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

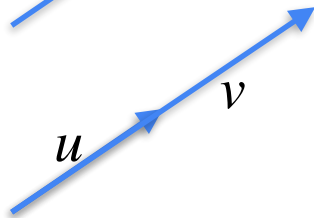
$$\langle u, v \rangle = |u| \cdot |v|$$



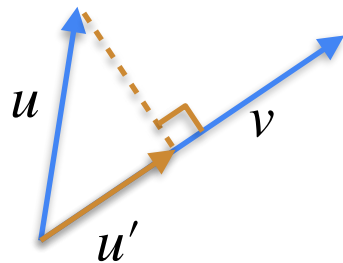
The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

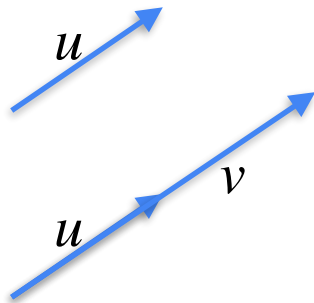


$$\langle u, v \rangle = |u| \cdot |v|$$



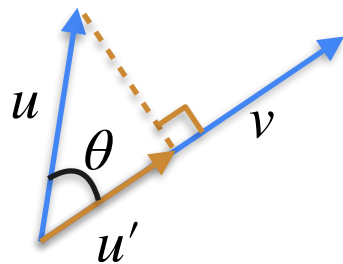
$$\langle u, v \rangle = |u'| \cdot |v|$$

The dot product



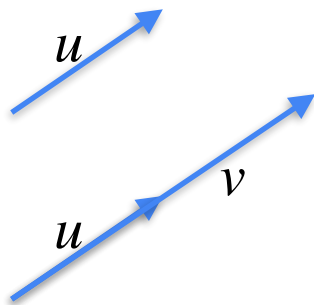
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$



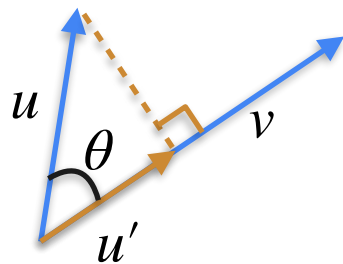
$$\langle u, v \rangle = |u'| \cdot |v|$$

The dot product



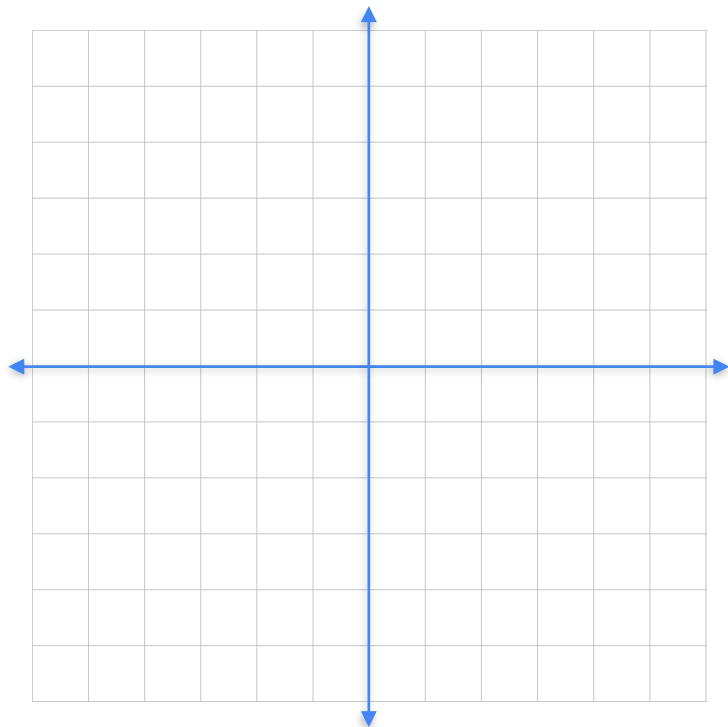
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$

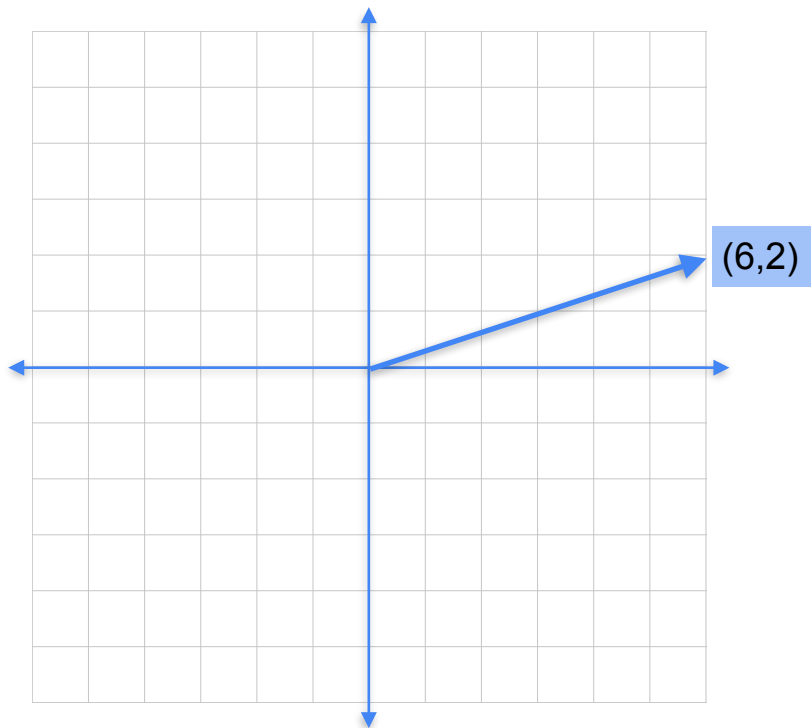


$$\begin{aligned}\langle u, v \rangle &= |u'| \cdot |v| \\ &= |u| |v| \cos(\theta)\end{aligned}$$

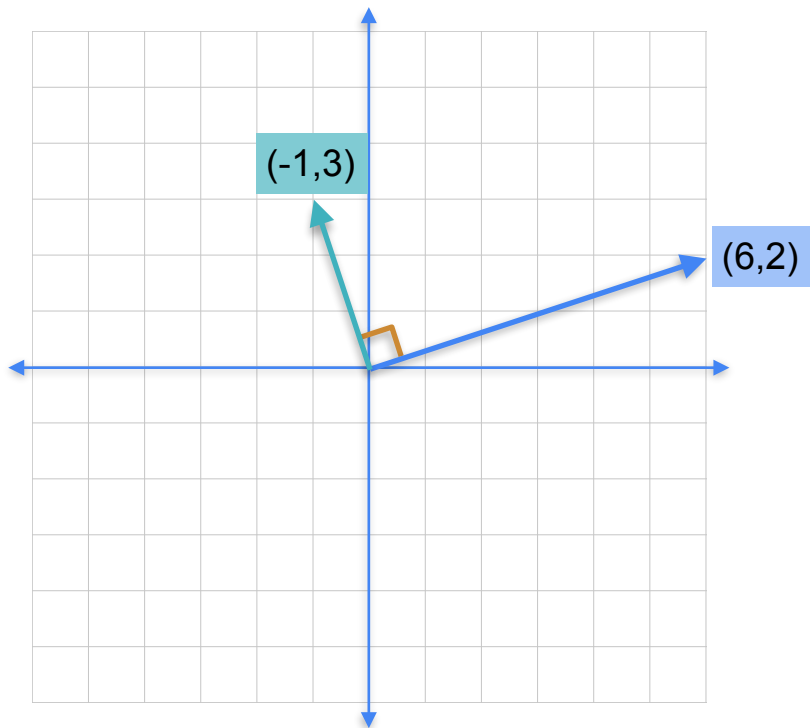
Geometric dot product



Geometric dot product

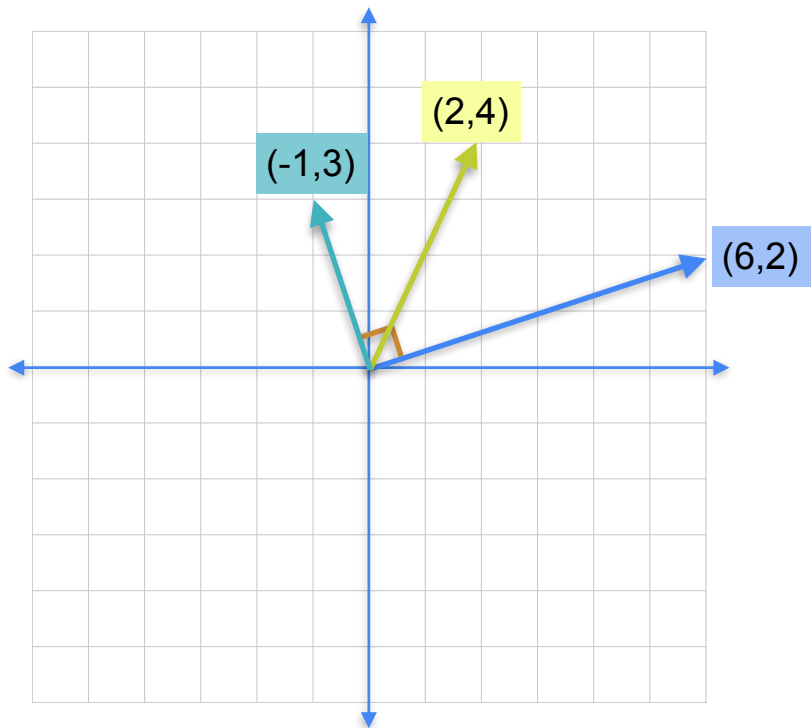


Geometric dot product



$$\begin{bmatrix} 6 & 2 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

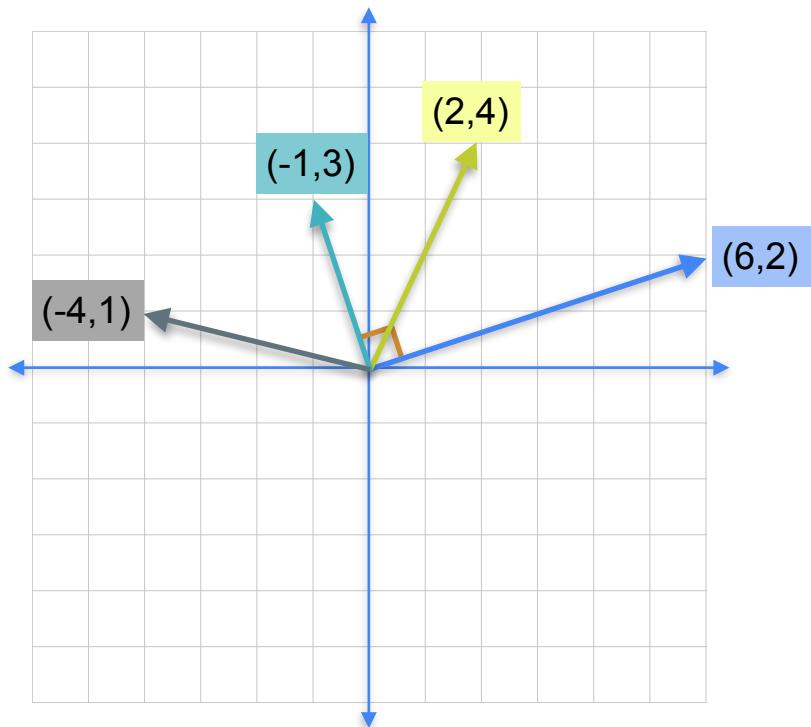
Geometric dot product



$$\begin{bmatrix} 6 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 4 \end{bmatrix} = 20$$

$$\begin{bmatrix} 6 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

Geometric dot product

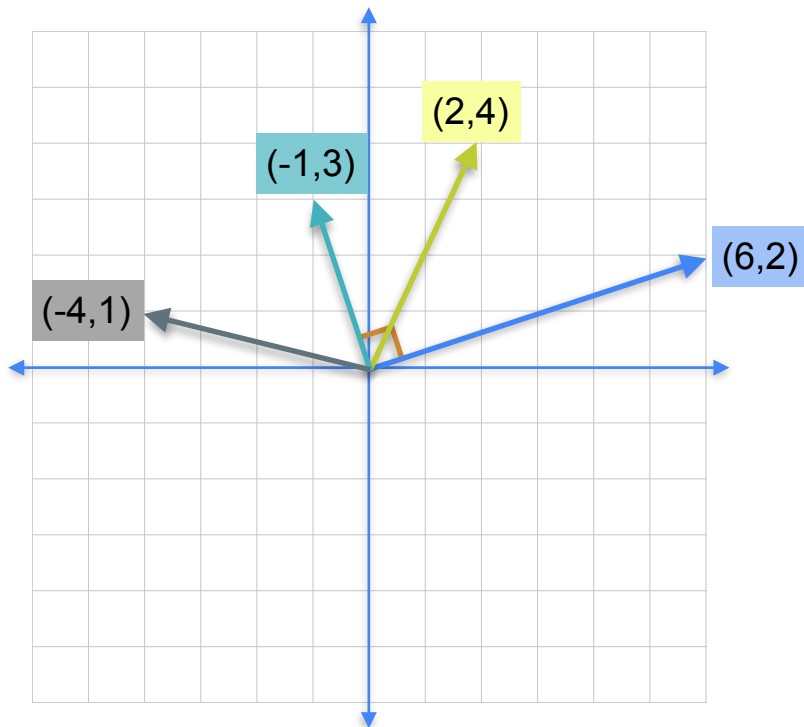


$$\begin{bmatrix} 6 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 4 \end{bmatrix} = 20$$

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \cdot \begin{bmatrix} -4 \\ 1 \end{bmatrix} = -22$$

Geometric dot product

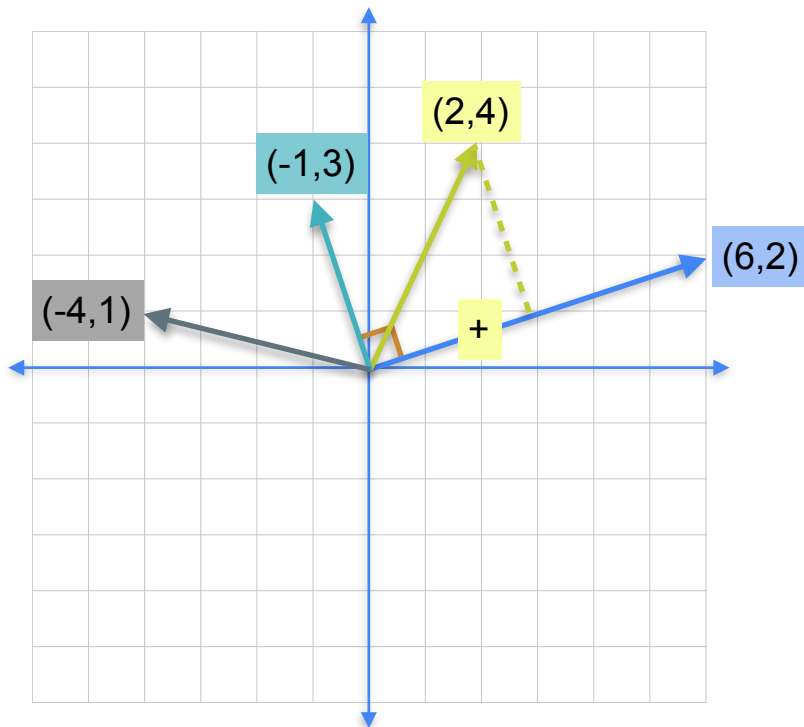


$$\begin{bmatrix} 6 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 4 \end{bmatrix} = 20 \quad \text{Positive}$$

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \cdot \begin{bmatrix} -4 \\ 1 \end{bmatrix} = -22$$

Geometric dot product

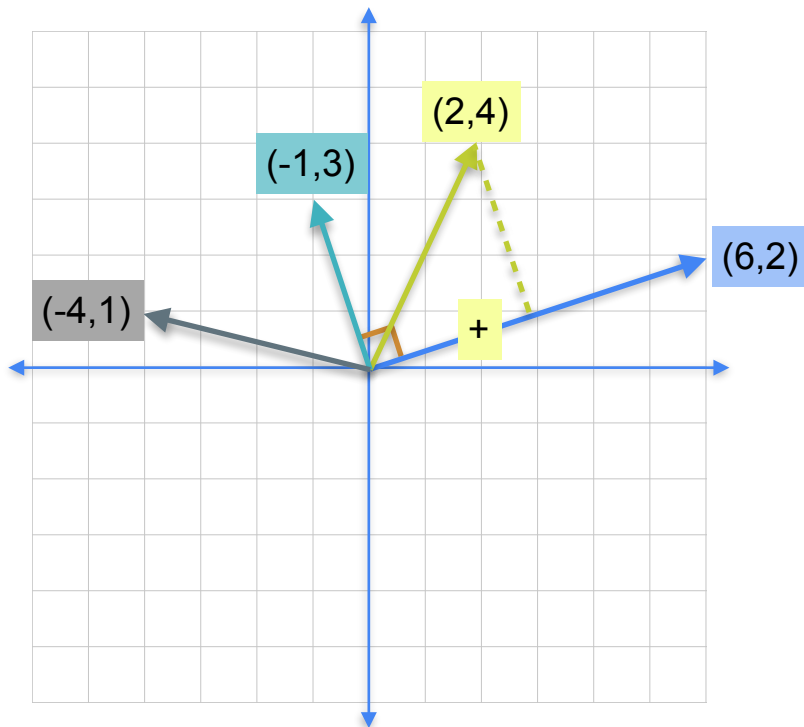


$$\begin{matrix} 6 & 2 \end{matrix} \begin{matrix} 2 \\ 4 \end{matrix} = 20 \quad \text{Positive}$$

$$\begin{matrix} 6 & 2 \end{matrix} \begin{matrix} -1 \\ 3 \end{matrix} = 0$$

$$\begin{matrix} 6 & 2 \end{matrix} \begin{matrix} -4 \\ 1 \end{matrix} = -22$$

Geometric dot product

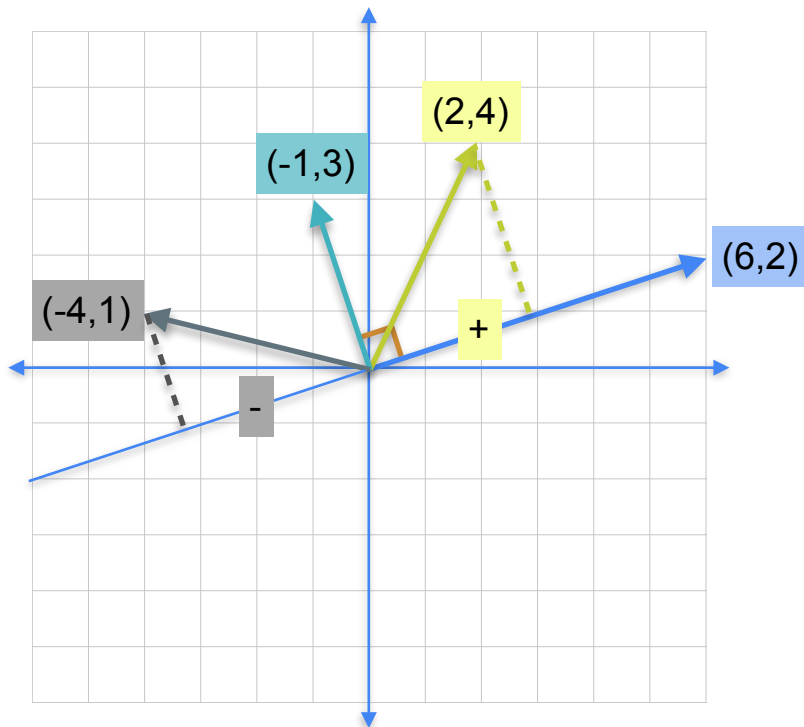


$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix} = 20 \quad \text{Positive}$$

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -4 \\ 1 \end{bmatrix} = -22 \quad \text{Negative}$$

Geometric dot product

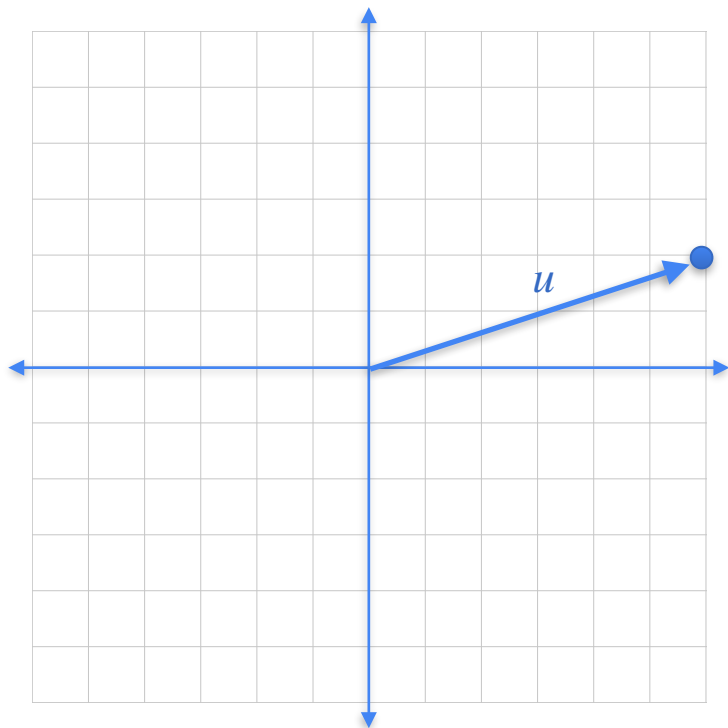


$$\begin{bmatrix} 6 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 4 \end{bmatrix} = 20 \quad \text{Positive}$$

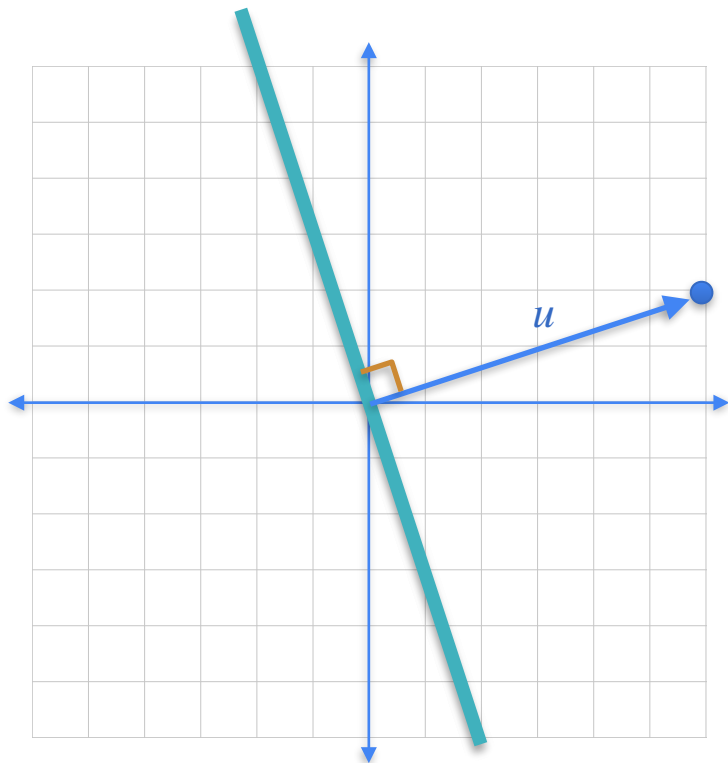
$$\begin{bmatrix} 6 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 6 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} -4 \\ 1 \end{bmatrix} = -22 \quad \text{Negative}$$

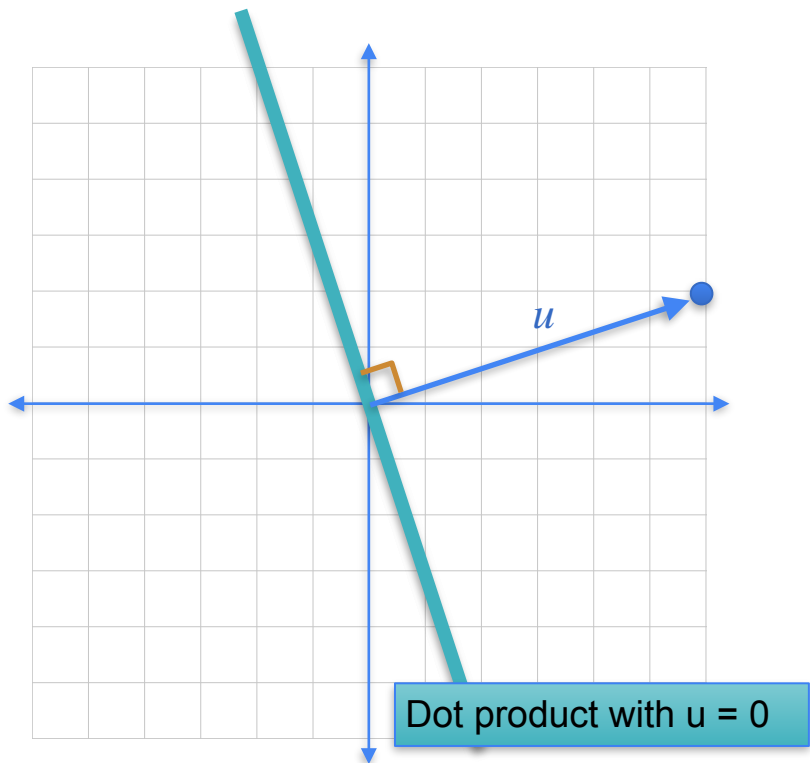
Geometric dot product



Geometric dot product

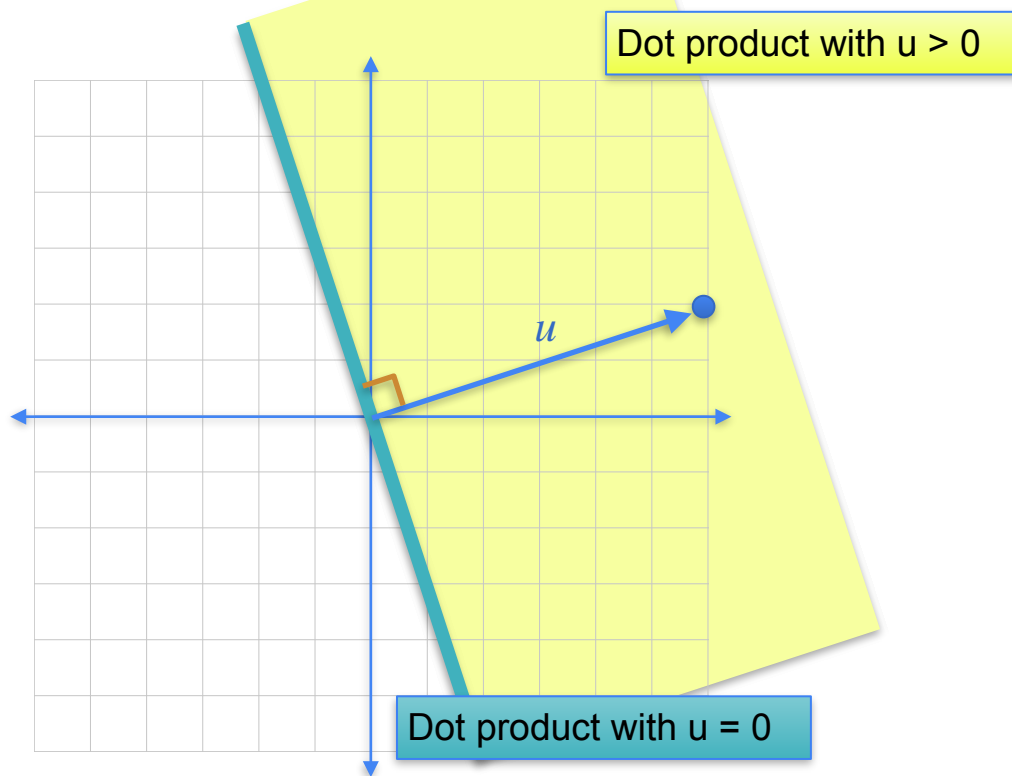


Geometric dot product



$$\langle u, v \rangle = 0$$

Geometric dot product



$$\langle u, v \rangle > 0$$

$$\langle u, v \rangle = 0$$

Geometric dot product

Dot product with $u > 0$

$$\langle u, v \rangle > 0$$

$$\langle u, v \rangle = 0$$

$$\langle u, v \rangle < 0$$

Dot product with $u < 0$

Dot product with $u = 0$



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Vectors and Linear Transformations

**Multiplying a matrix by a
vector**

Equations as dot product

$$2a + 4b + c = 28$$

The diagram illustrates the equation $2a + 4b + c = 28$ using fruit icons and boxes. On the left, there are three blue boxes containing the numbers 2, 4, and 1. Above the box with 2 are two red apples, above the box with 4 are four yellow bananas, and above the box with 1 is one red cherry. To the right of these boxes is a dot operator. Further right is a vertical stack of three light blue boxes labeled 'a', 'b', and 'c'. To the left of these boxes are the dollar signs and fruit icons: '\$' and an apple for 'a', '\$' and a banana for 'b', and '\$' and a cherry for 'c'. To the right of this stack is an equals sign, followed by a dollar sign and an orange box containing the number 28.

Equations as dot product

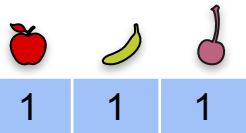
$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Equations as dot product

$$a + b + c = 10$$

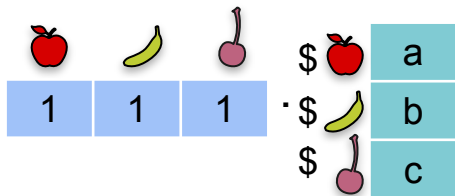


$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Equations as dot product

$$a + b + c = 10$$



$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Equations as dot product

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

The diagram illustrates the dot product for the equation $a + b + c = 10$. It shows a row vector $[1, 1, 1]$ (represented by blue boxes) multiplied by a column vector $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ (represented by teal boxes). The column vector is preceded by a dollar sign (\$) and each element is accompanied by a fruit icon: an apple for a , a banana for b , and a cherry for c . The result of the dot product is shown as $= \$ 10$ (represented by an orange box).

Equations as dot product

$$a + b + c = 10$$

Diagram illustrating the dot product for the equation $a + b + c = 10$. The row vector $[1, 1, 1]$ is multiplied by the column vector $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ to equal 10. The vector elements are represented by fruit icons: an apple for 'a', a banana for 'b', and a cherry for 'c'.

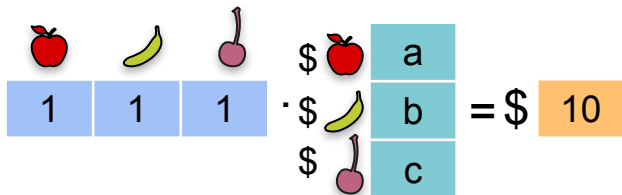
$$a + 2b + c = 15$$

Diagram illustrating the dot product for the equation $a + 2b + c = 15$. The row vector $[1, 2, 1]$ is multiplied by the column vector $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ to equal 15. The vector elements are represented by fruit icons: an apple for 'a', a banana for 'b', and a cherry for 'c'.

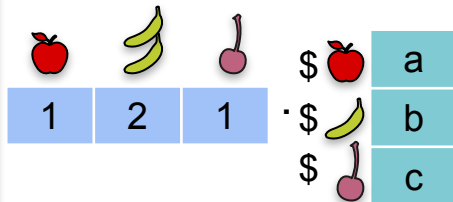
$$a + b + 2c = 12$$

Equations as dot product

$$a + b + c = 10$$



$$a + 2b + c = 15$$



$$a + b + 2c = 12$$

Equations as dot product

$$a + b + c = 10$$

Diagram illustrating the dot product for the equation $a + b + c = 10$. The vector $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is multiplied by the vector $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ to yield the scalar value 10.

$$a + 2b + c = 15$$

Diagram illustrating the dot product for the equation $a + 2b + c = 15$. The vector $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ is multiplied by the vector $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ to yield the scalar value 15.

$$a + b + 2c = 12$$

Equations as dot product

$$a + b + c = 10$$

$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \$ \text{apple} & a \\ \$ \text{banana} & b \\ \$ \text{cherry} & c \end{bmatrix} = \$ 10$

$$a + 2b + c = 15$$

$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} \$ \text{apple} & a \\ \$ \text{banana} & b \\ \$ \text{cherry} & c \end{bmatrix} = \$ 15$

$$a + b + 2c = 12$$

$\begin{bmatrix} 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} \$ \text{apple} & a \\ \$ \text{banana} & b \\ \$ \text{cherry} & c \end{bmatrix}$

Equations as dot product

$$a + b + c = 10$$

Diagram illustrating the equation $a + b + c = 10$ as a dot product. The coefficients are 1, 1, and 1, and the variables are a , b , and c . The result is 10.

$$a + 2b + c = 15$$

Diagram illustrating the equation $a + 2b + c = 15$ as a dot product. The coefficients are 1, 2, and 1, and the variables are a , b , and c . The result is 15.

$$a + b + 2c = 12$$

Diagram illustrating the equation $a + b + 2c = 12$ as a dot product. The coefficients are 1, 1, and 2, and the variables are a , b , and c . The result is 12.

Equations as dot product

$$a + b + c = 10$$

$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \$ \text{apple} & a \\ \$ \text{banana} & b \\ \$ \text{cherry} & c \end{bmatrix} = \$ 10$

$$a + 2b + c = 15$$

$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} \$ \text{apple} & a \\ \$ \text{banana} & b \\ \$ \text{cherry} & c \end{bmatrix} = \$ 15$

$$a + b + 2c = 12$$

$\begin{bmatrix} 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} \$ \text{apple} & a \\ \$ \text{banana} & b \\ \$ \text{cherry} & c \end{bmatrix} = \$ 12$

Equations as dot product

$$a + b + c = 10$$

$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \$ \text{apple} \\ \$ \text{banana} \\ \$ \text{cherry} \end{bmatrix} = \$ 10$

$$a + 2b + c = 15$$

$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} \$ \text{apple} \\ \$ \text{banana} \\ \$ \text{cherry} \end{bmatrix} = \$ 15$

$$a + b + 2c = 12$$

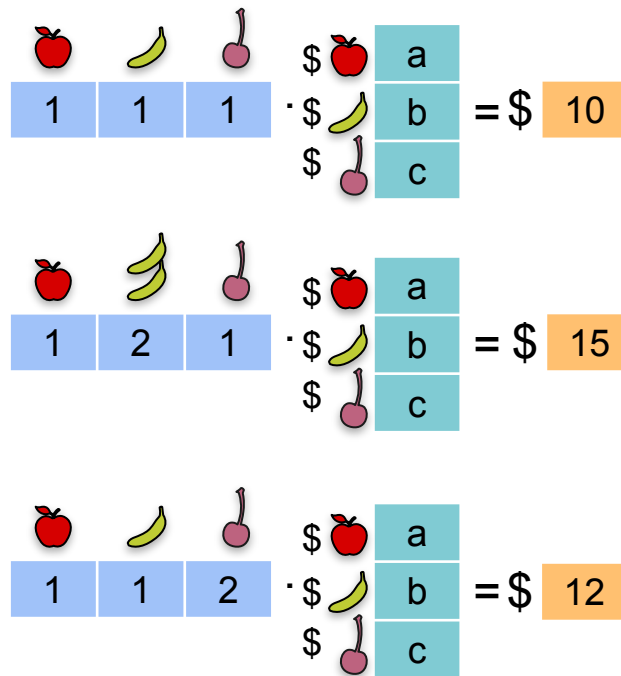
$\begin{bmatrix} 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} \$ \text{apple} \\ \$ \text{banana} \\ \$ \text{cherry} \end{bmatrix} = \$ 12$

Equations as dot product

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

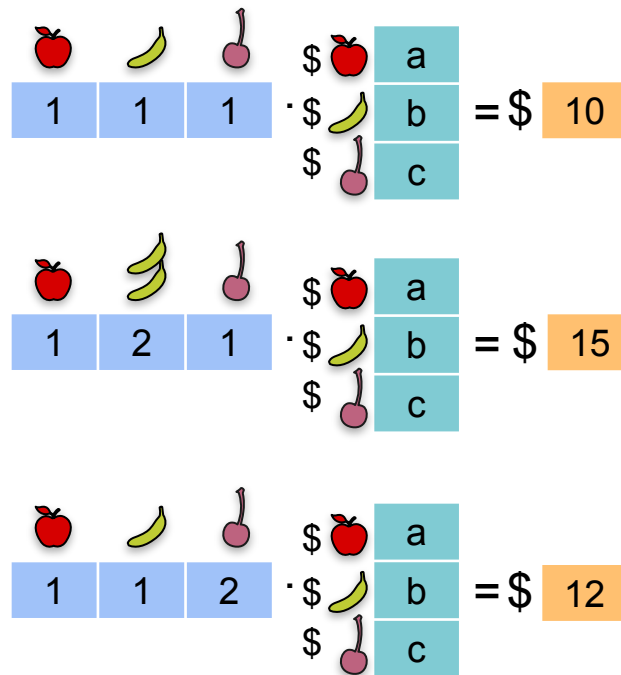


Equations as dot product

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$



Equations as dot product







System of equations

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Matrix product

					
1	1	1	\$ 	a	10
1	2	1	· \$ 	b	15
1	1	2	\$ 	c	12

Equations as dot product

System of equations

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Matrix product

1	1	1	a	=	10
1	2	1	b	=	15
1	1	2	c	=	12





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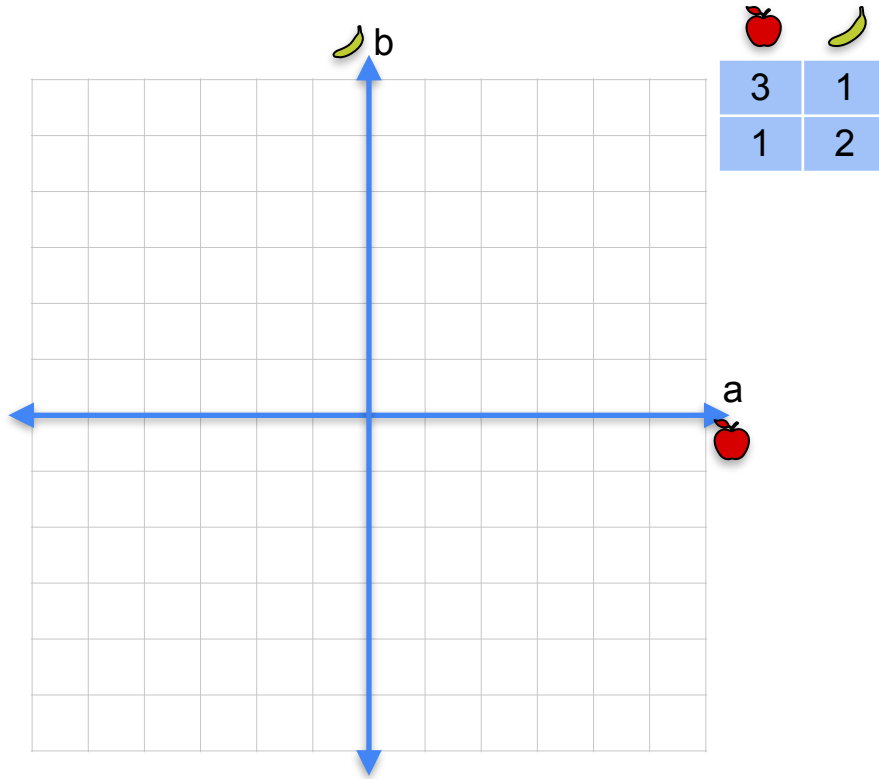
Vectors and Linear Transformations

**Matrices as linear
transformations**

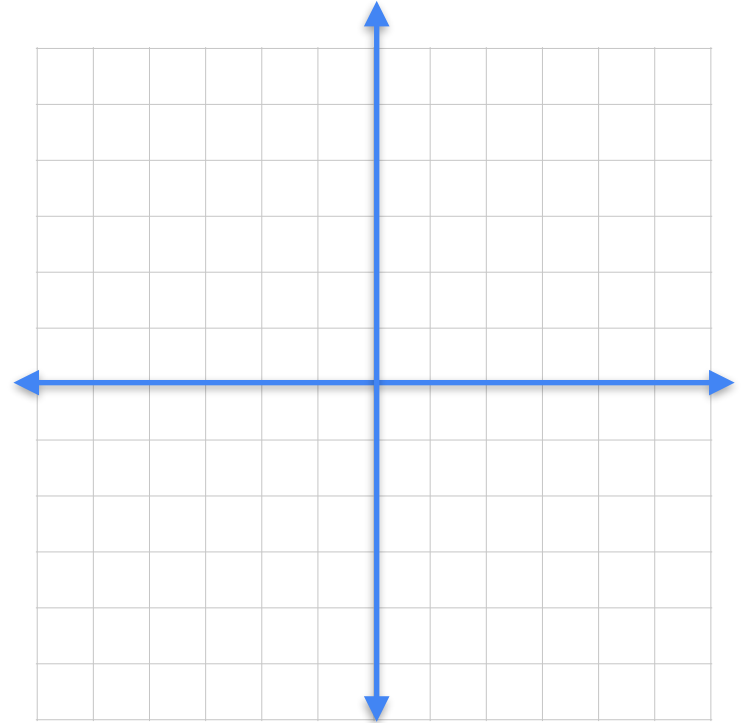
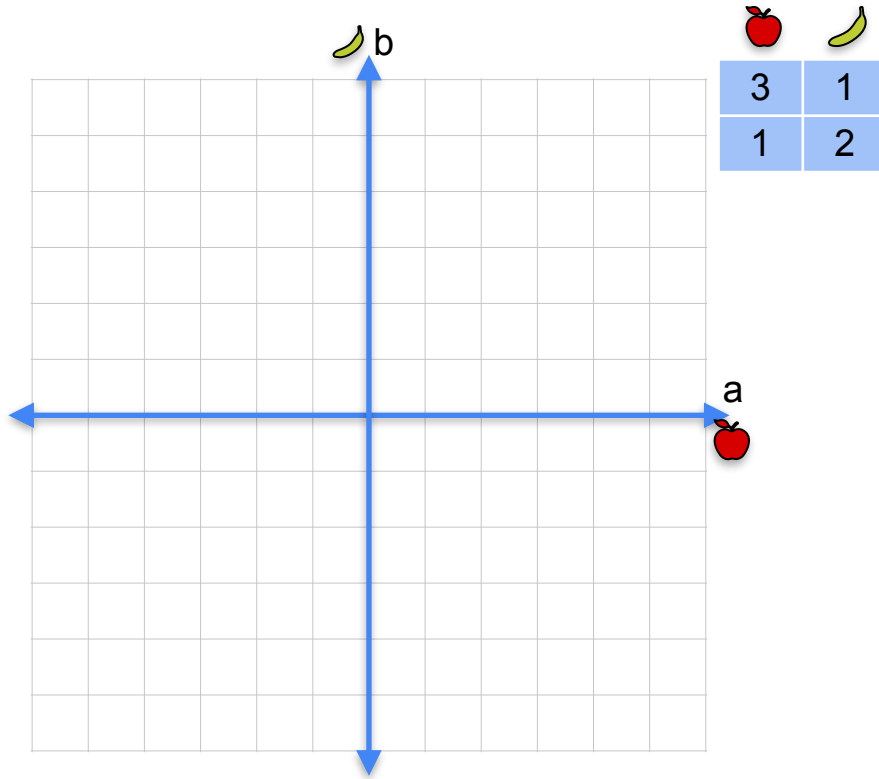
Matrices as linear transformations

	
3	1
1	2

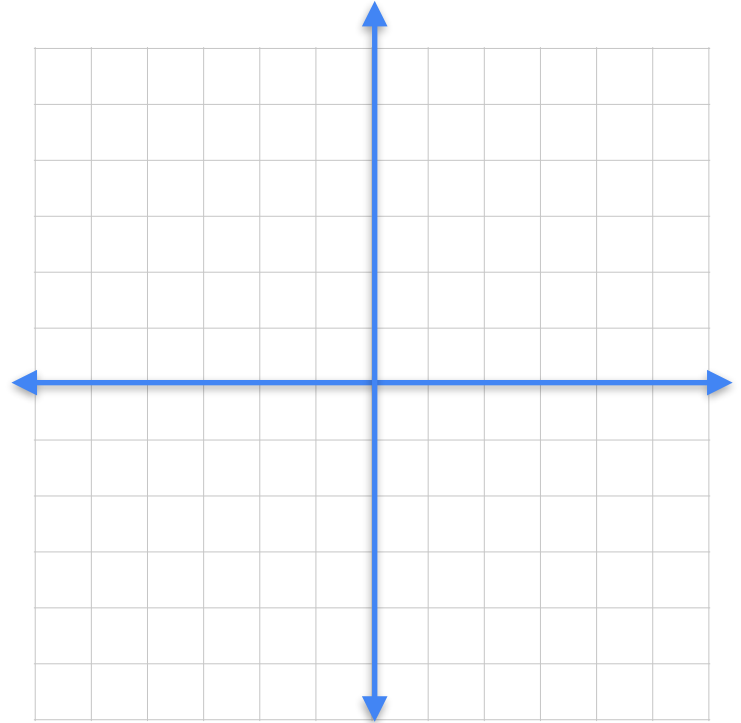
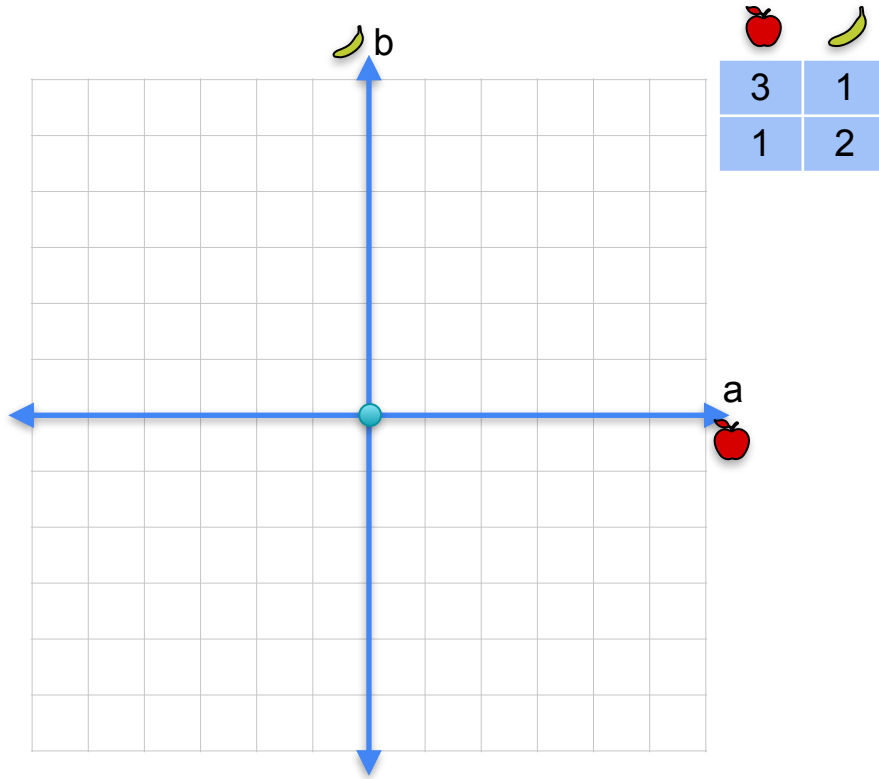
Matrices as linear transformations



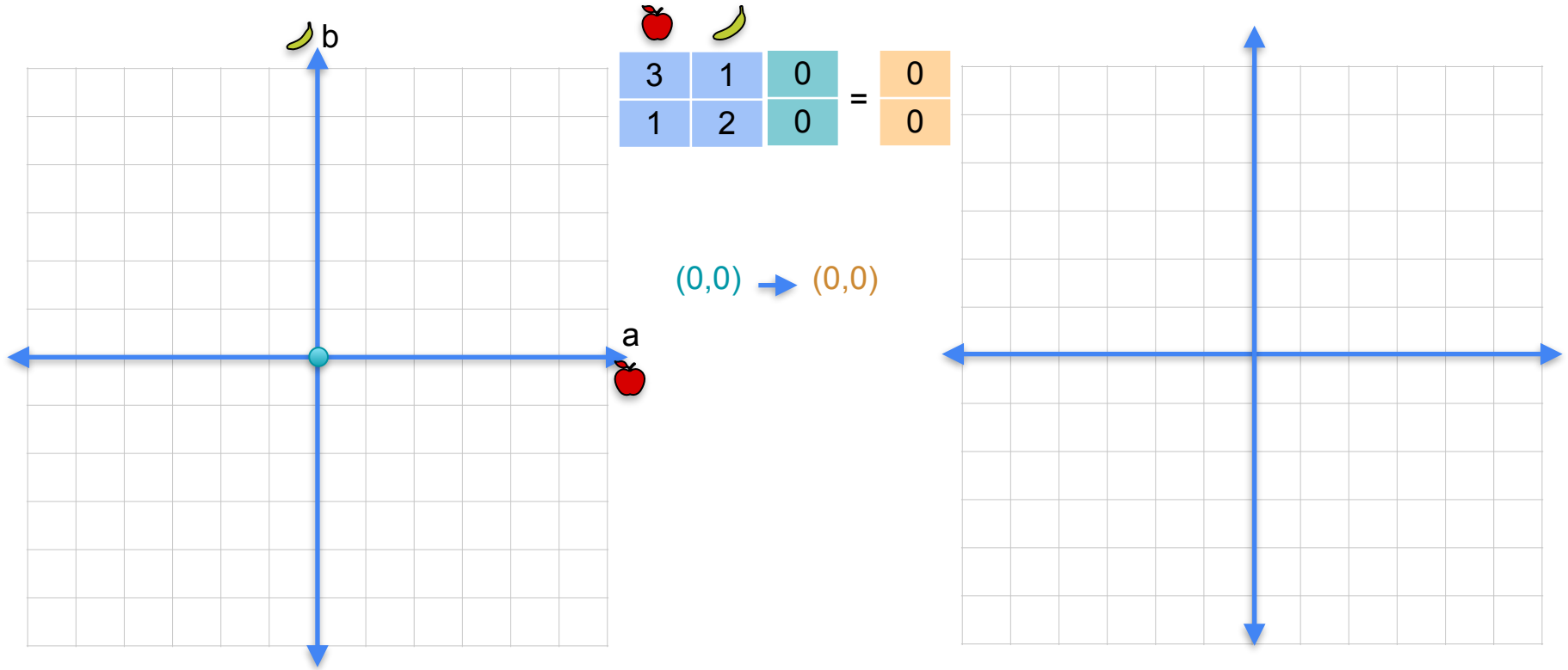
Matrices as linear transformations



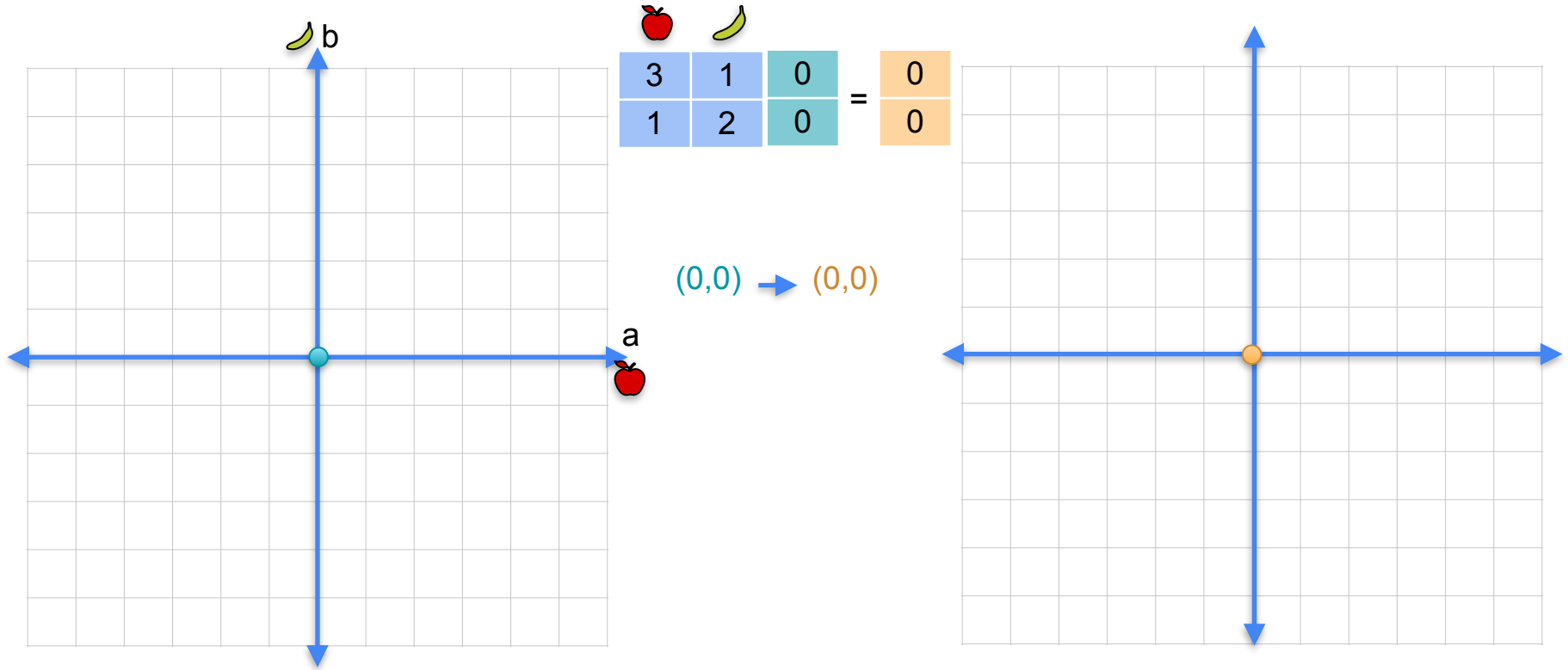
Matrices as linear transformations



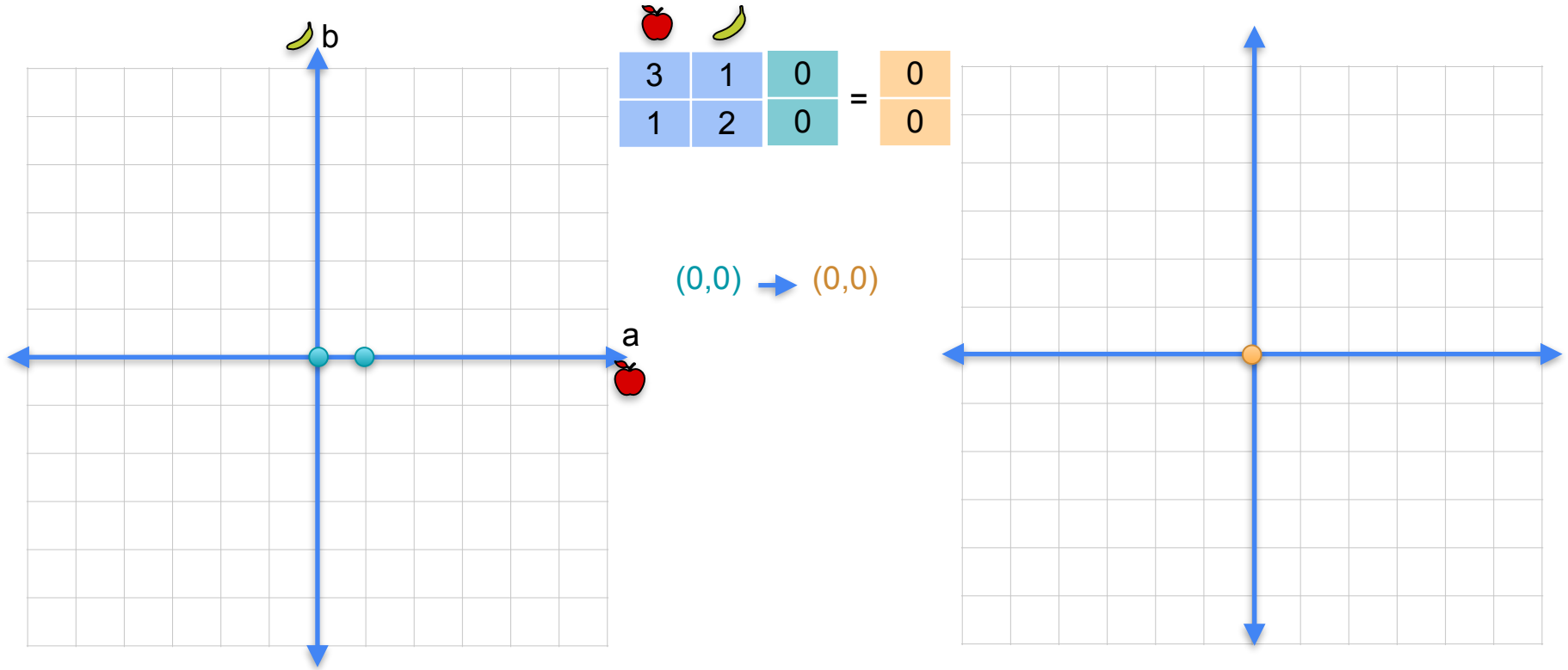
Matrices as linear transformations



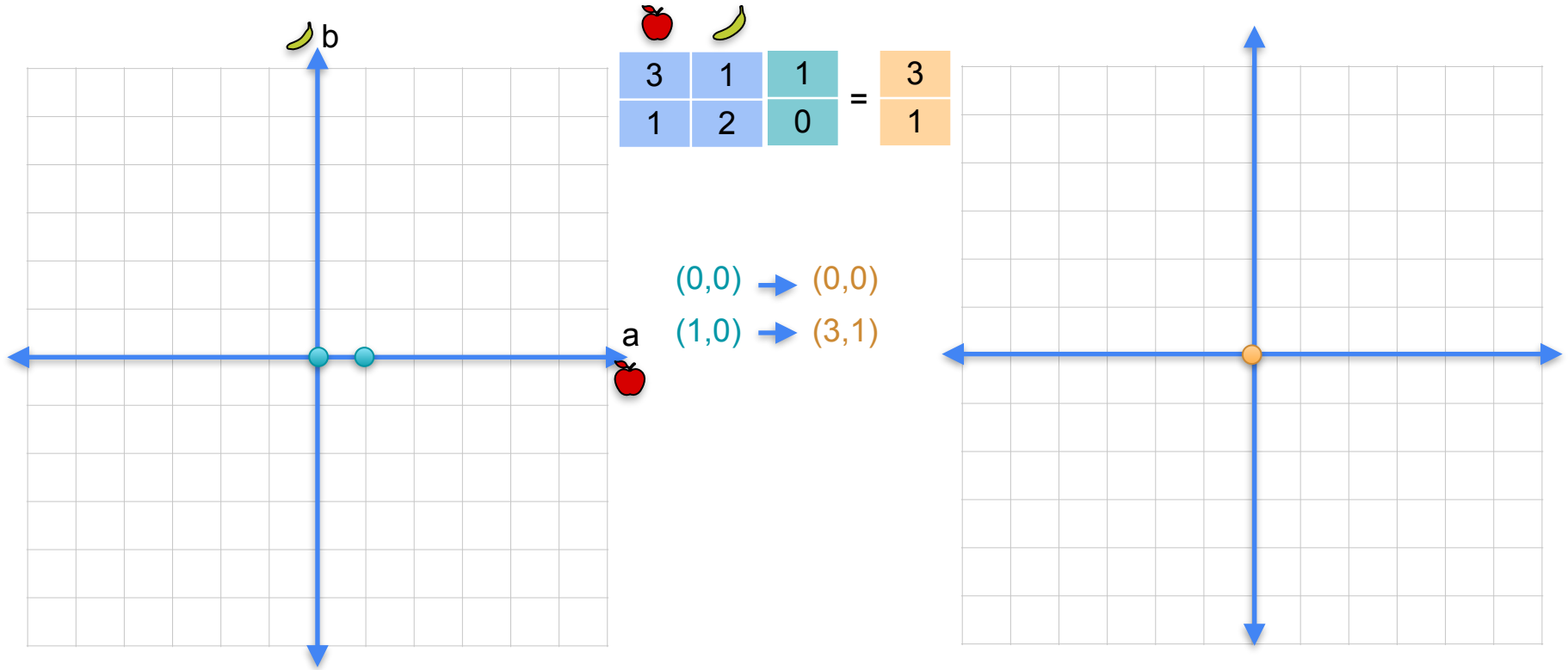
Matrices as linear transformations



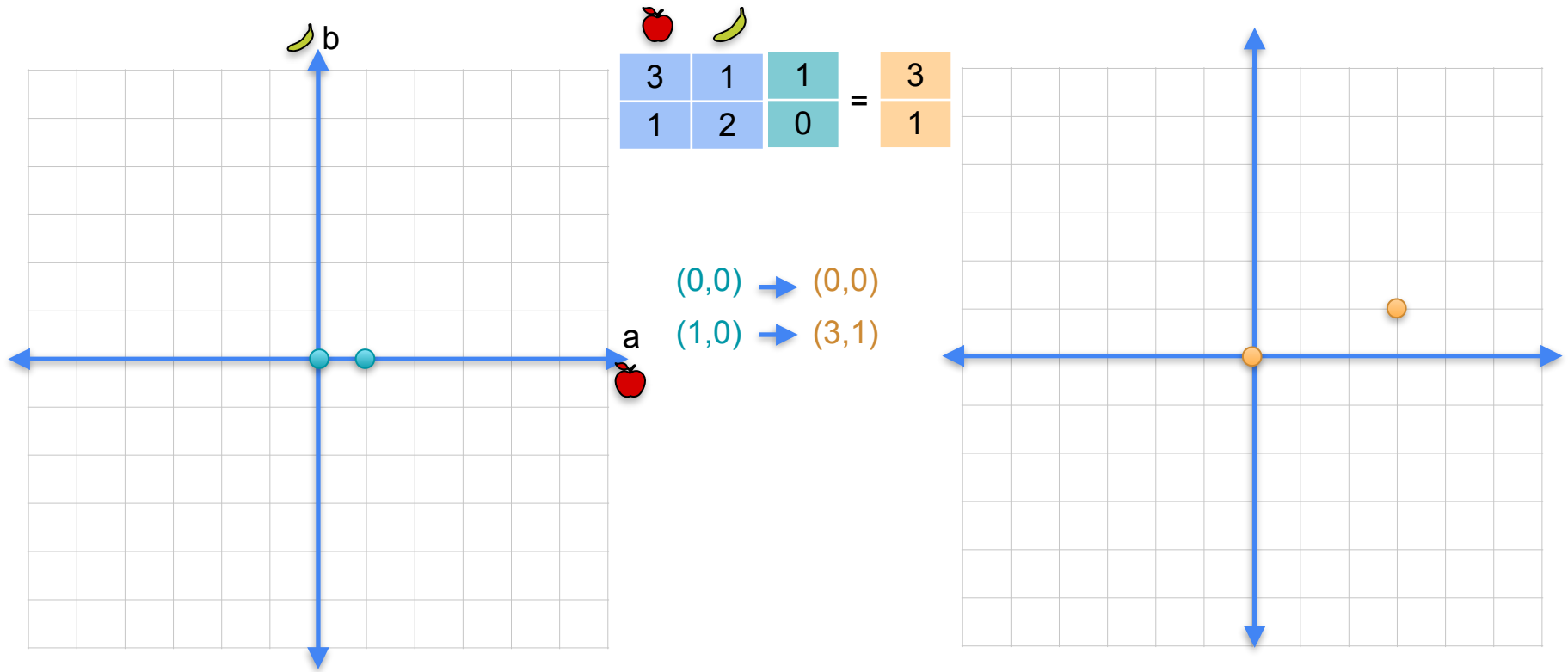
Matrices as linear transformations



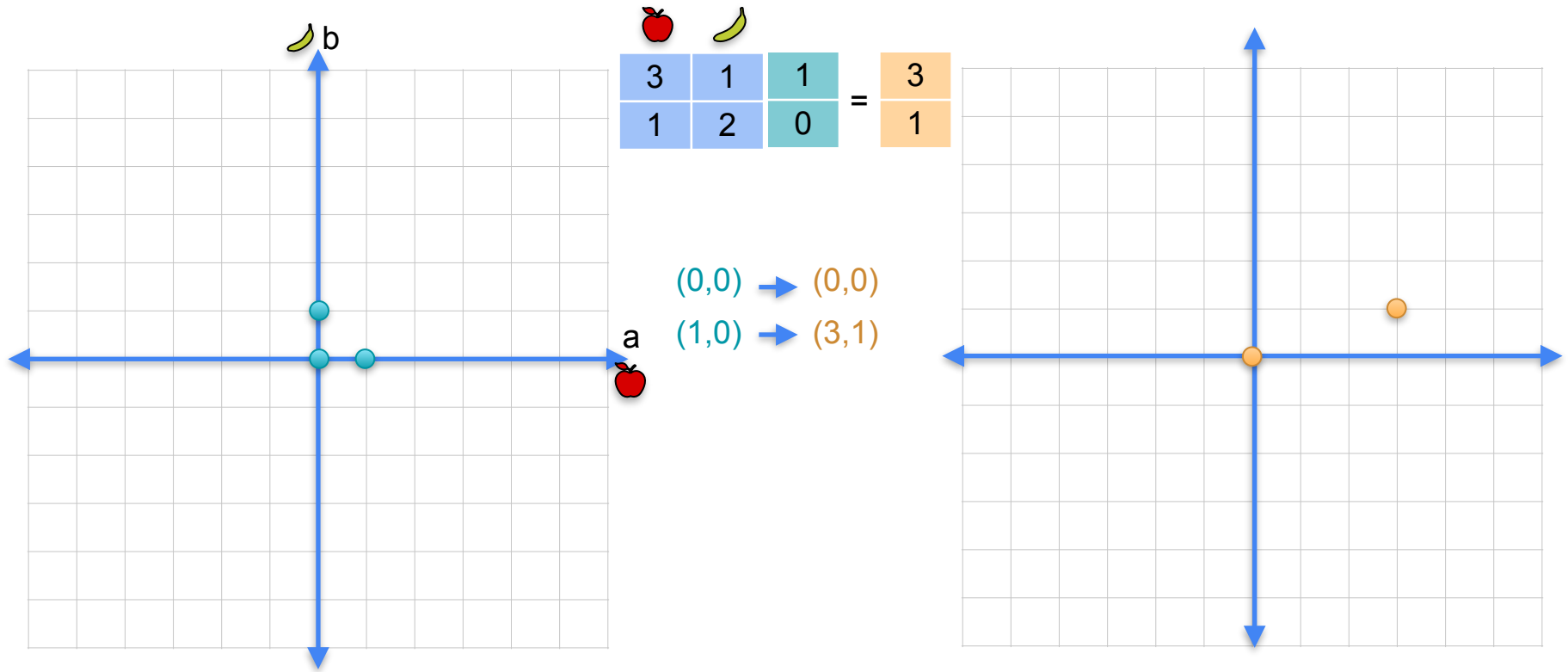
Matrices as linear transformations



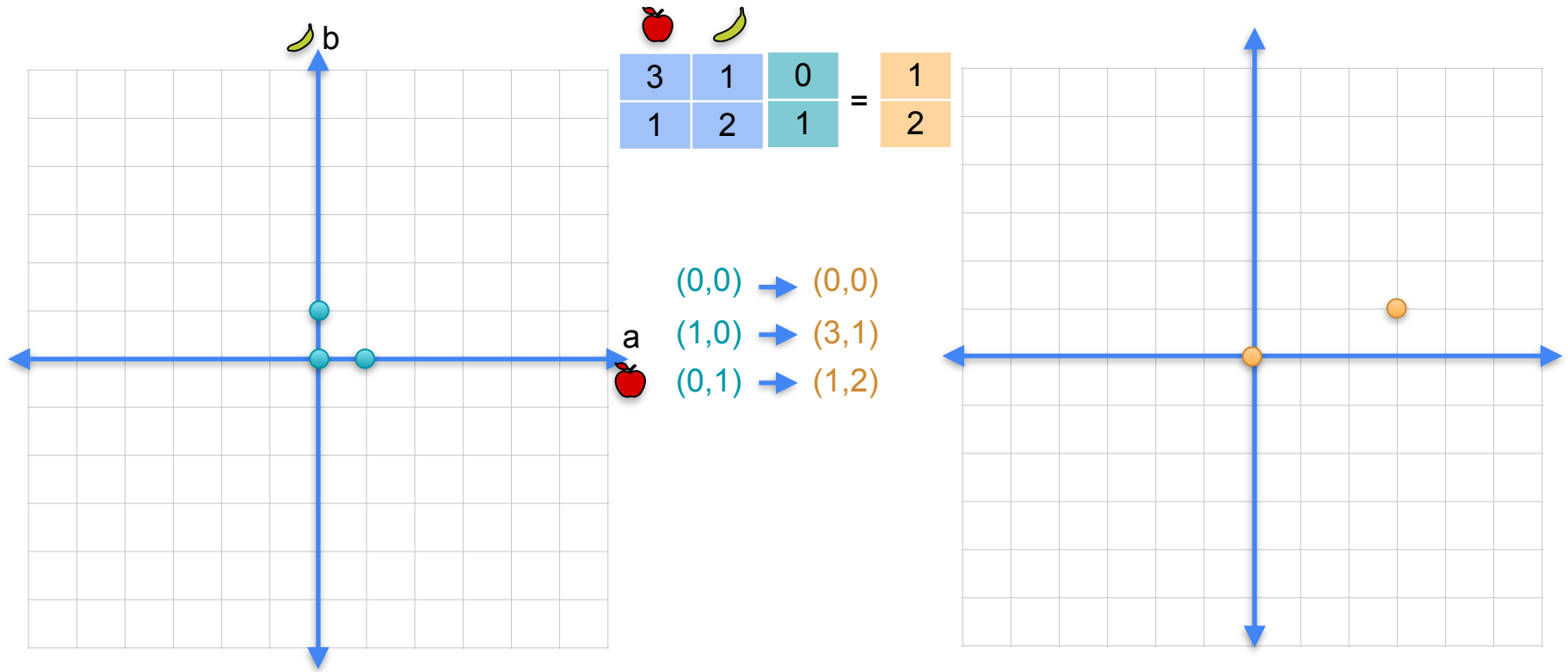
Matrices as linear transformations



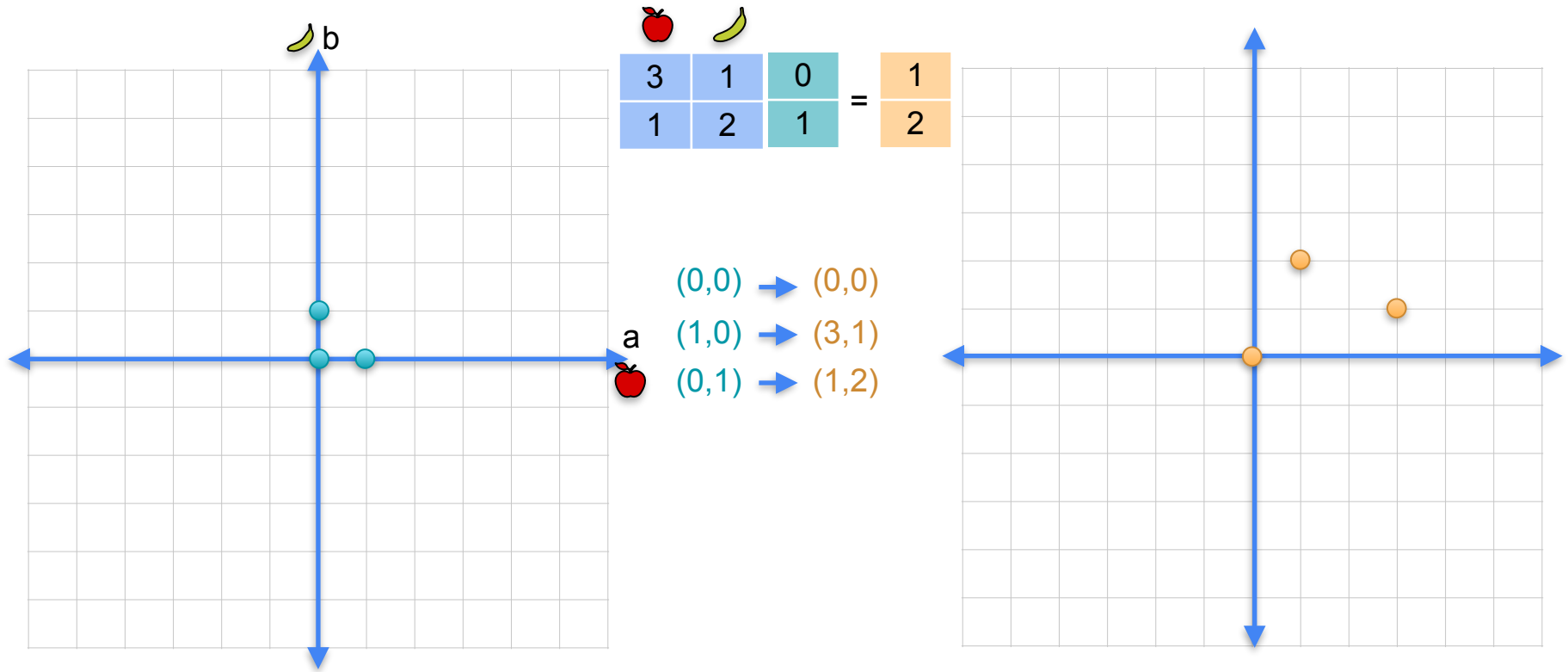
Matrices as linear transformations



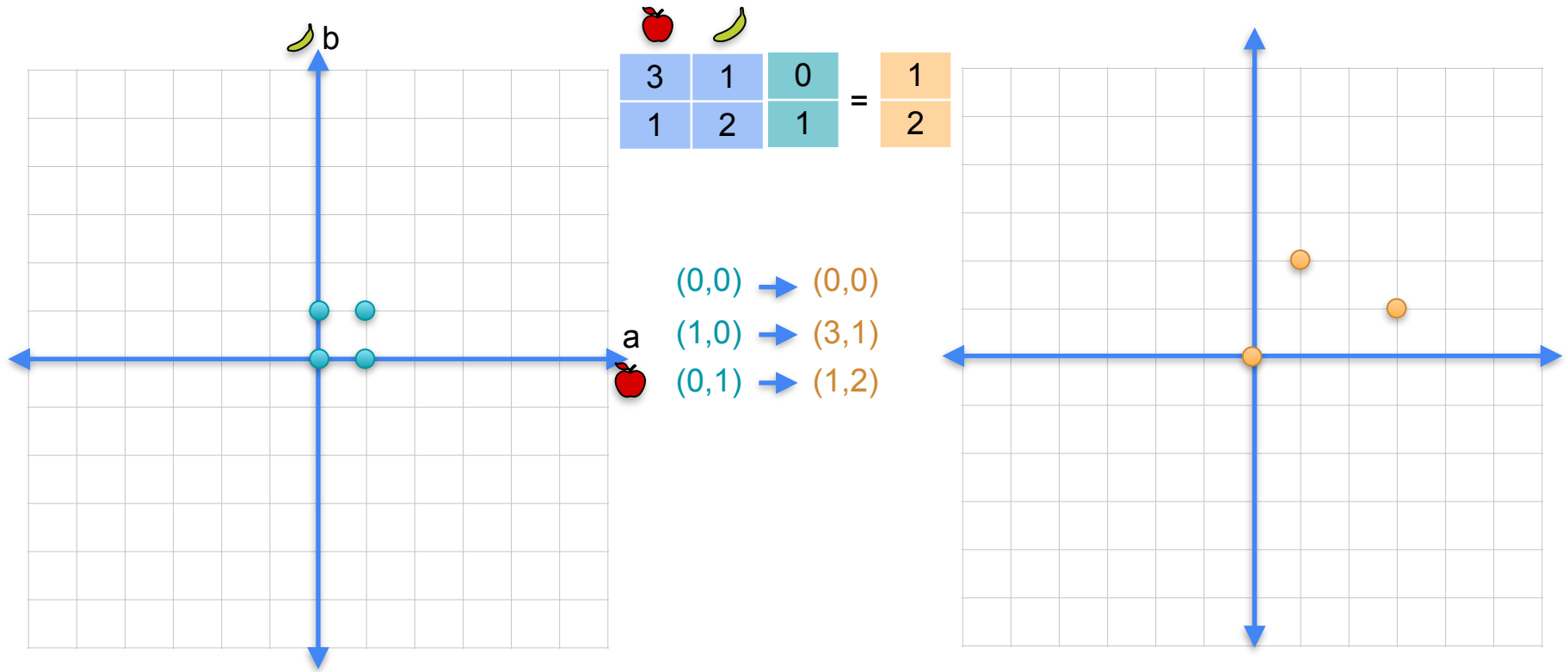
Matrices as linear transformations



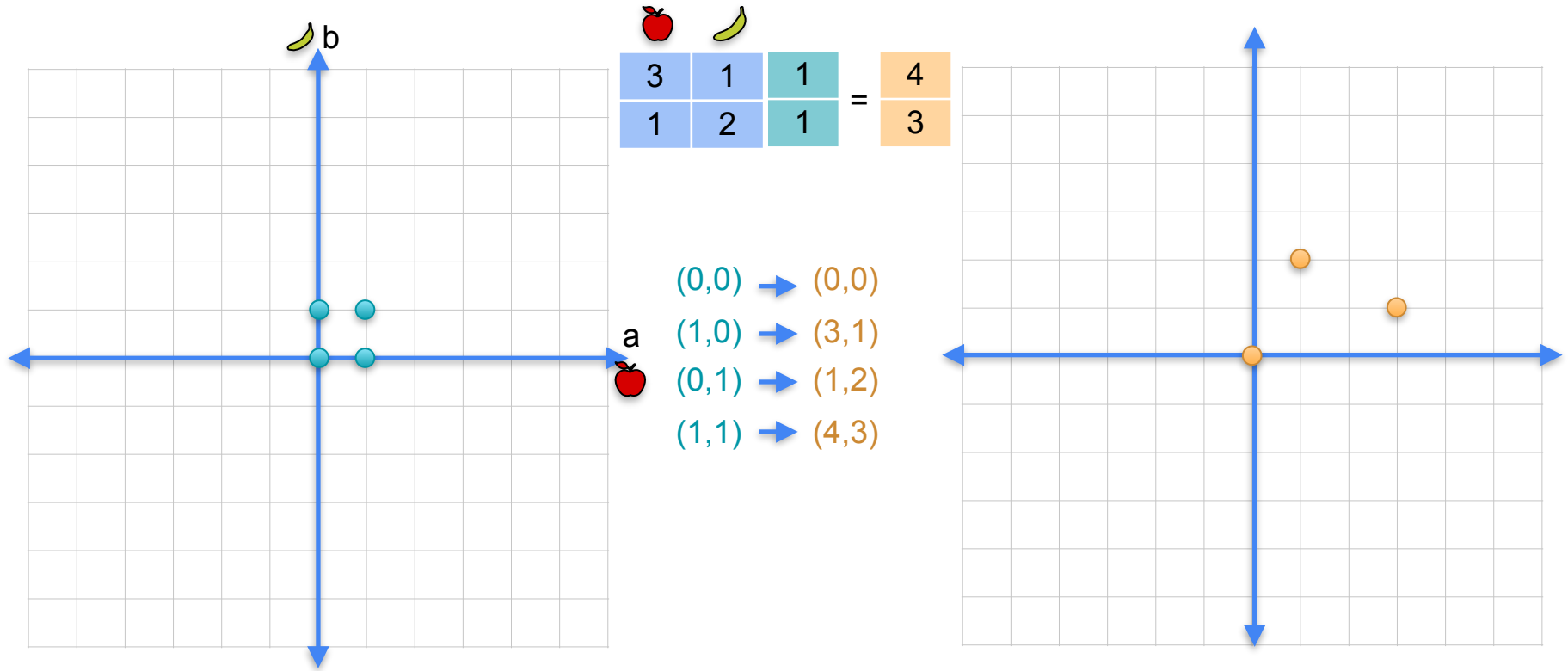
Matrices as linear transformations



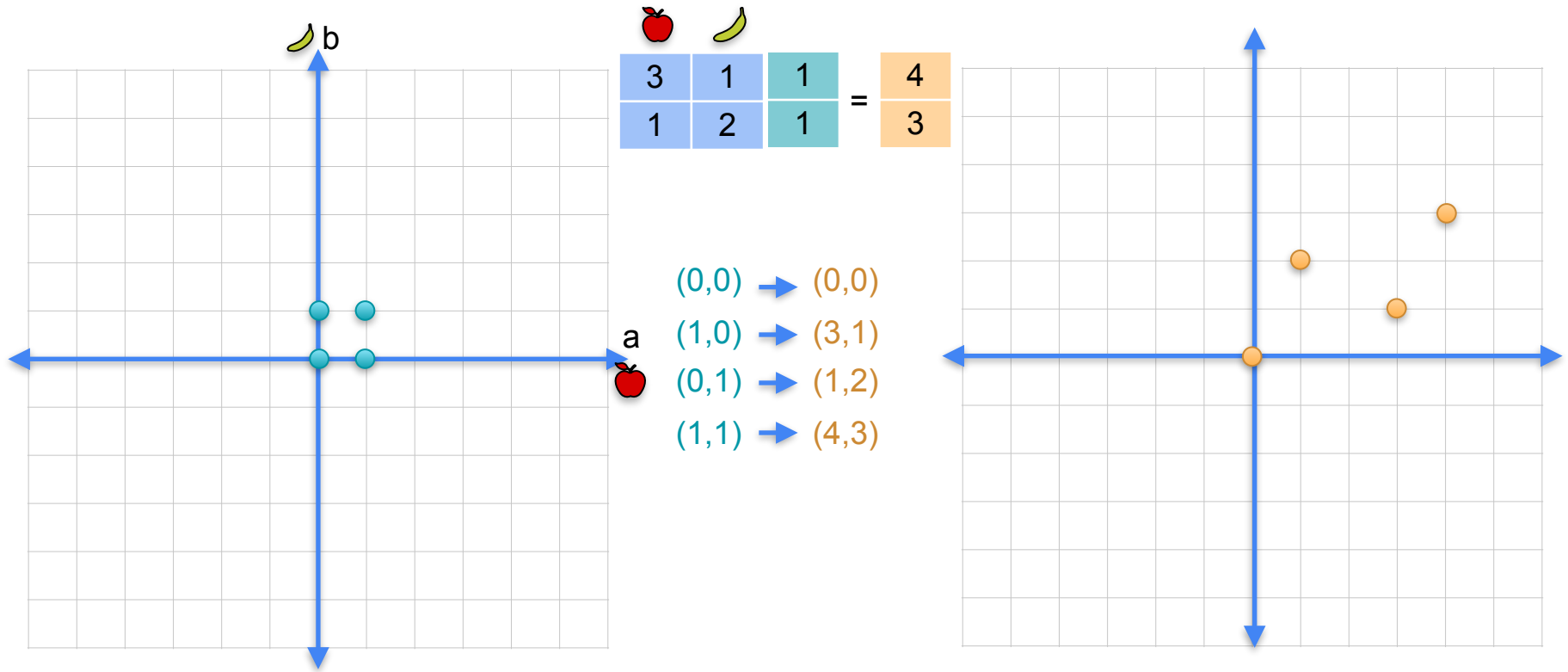
Matrices as linear transformations



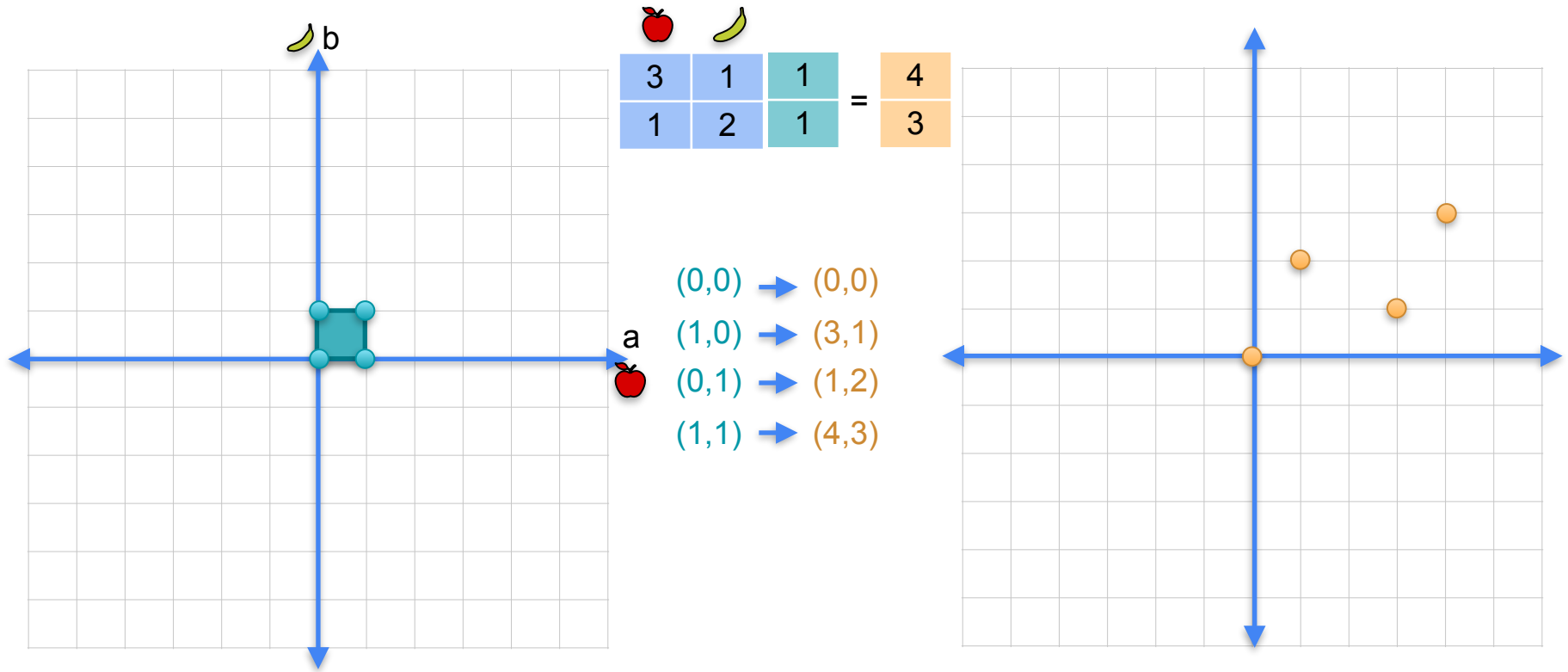
Matrices as linear transformations



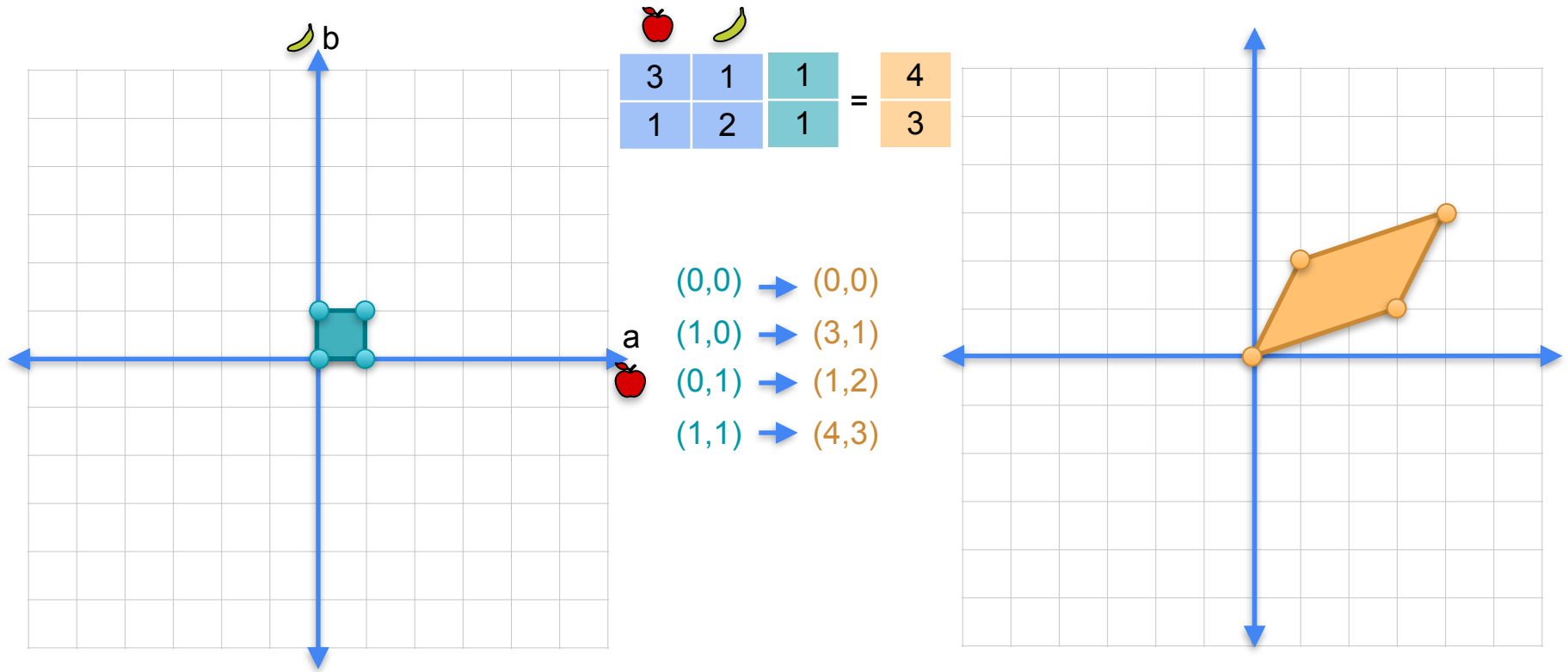
Matrices as linear transformations



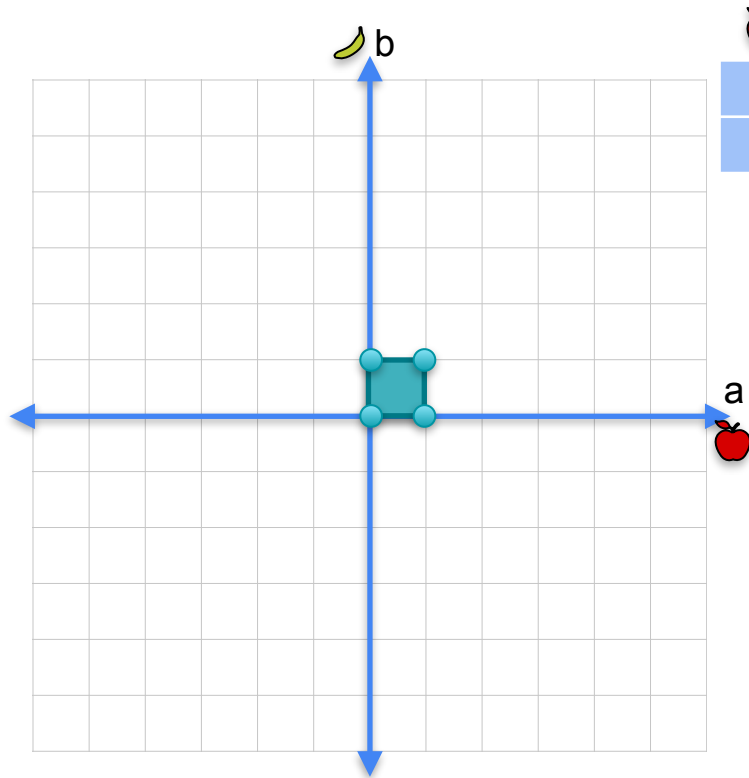
Matrices as linear transformations



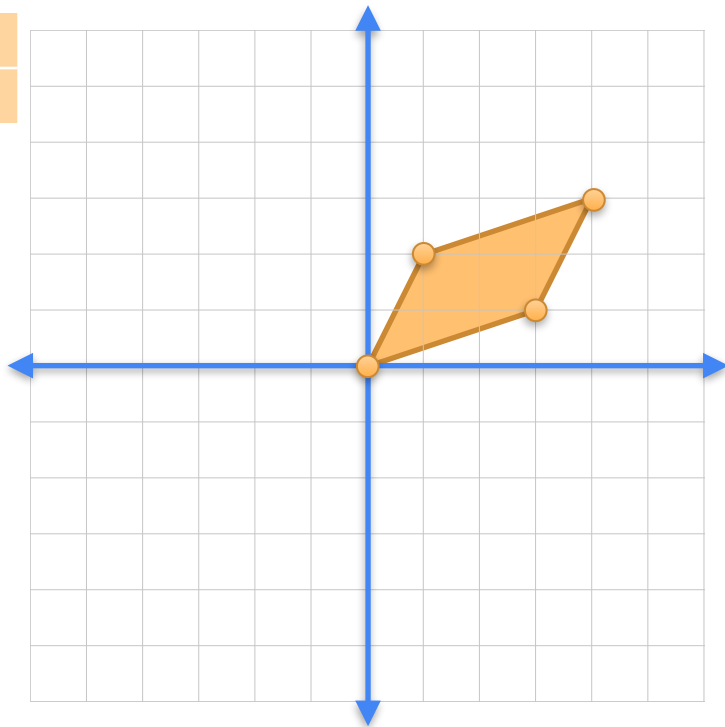
Matrices as linear transformations



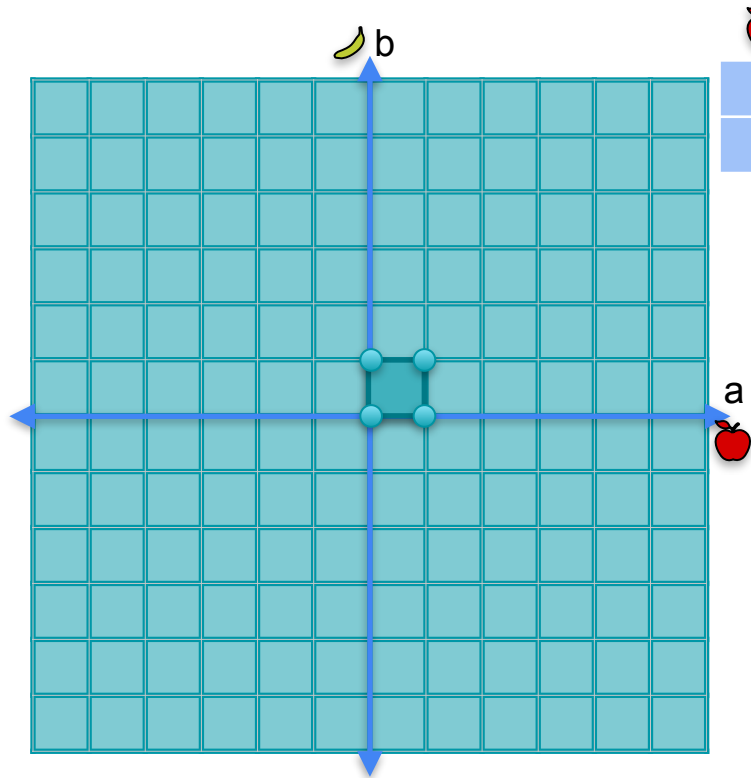
Matrices as linear transformations



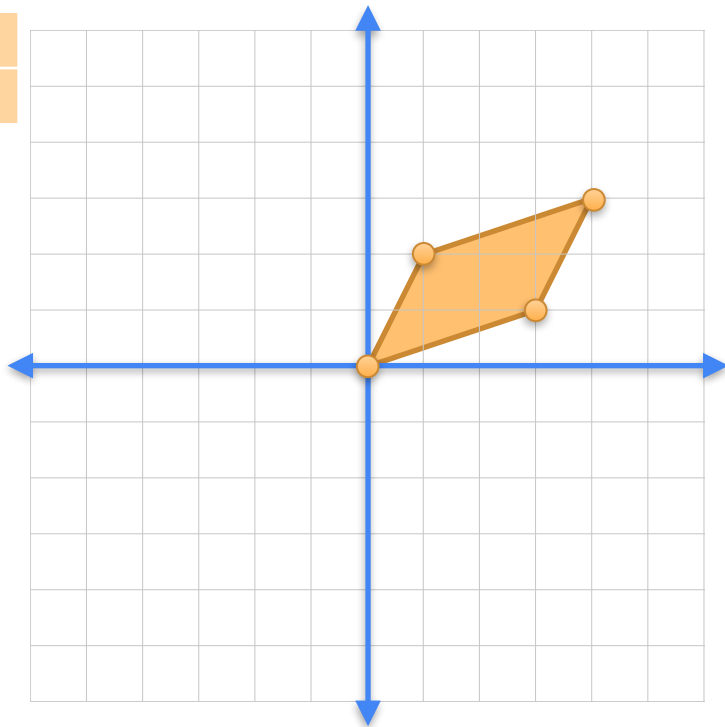
$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} & \begin{bmatrix} -2 \\ 3 \end{bmatrix} \end{matrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$



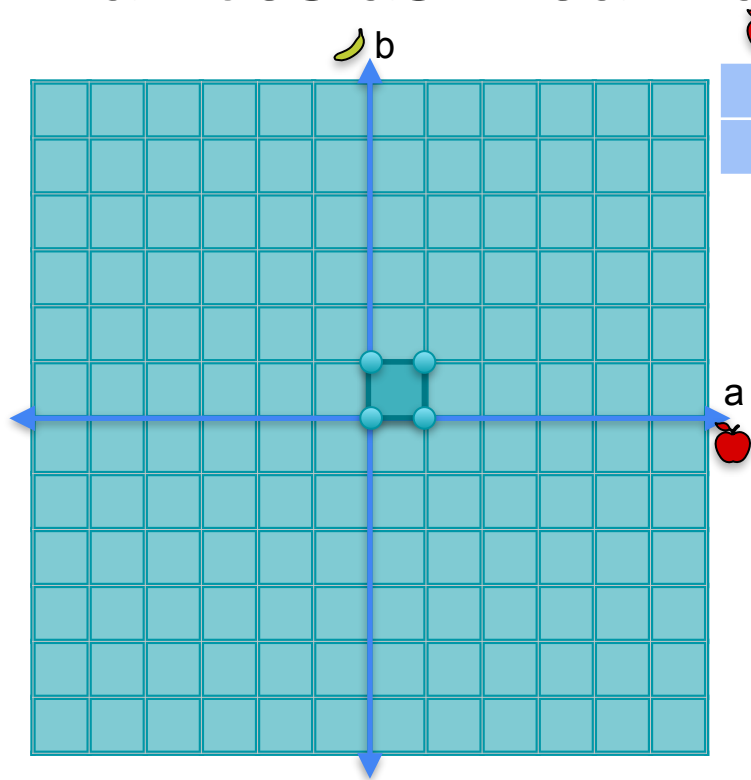
Matrices as linear transformations



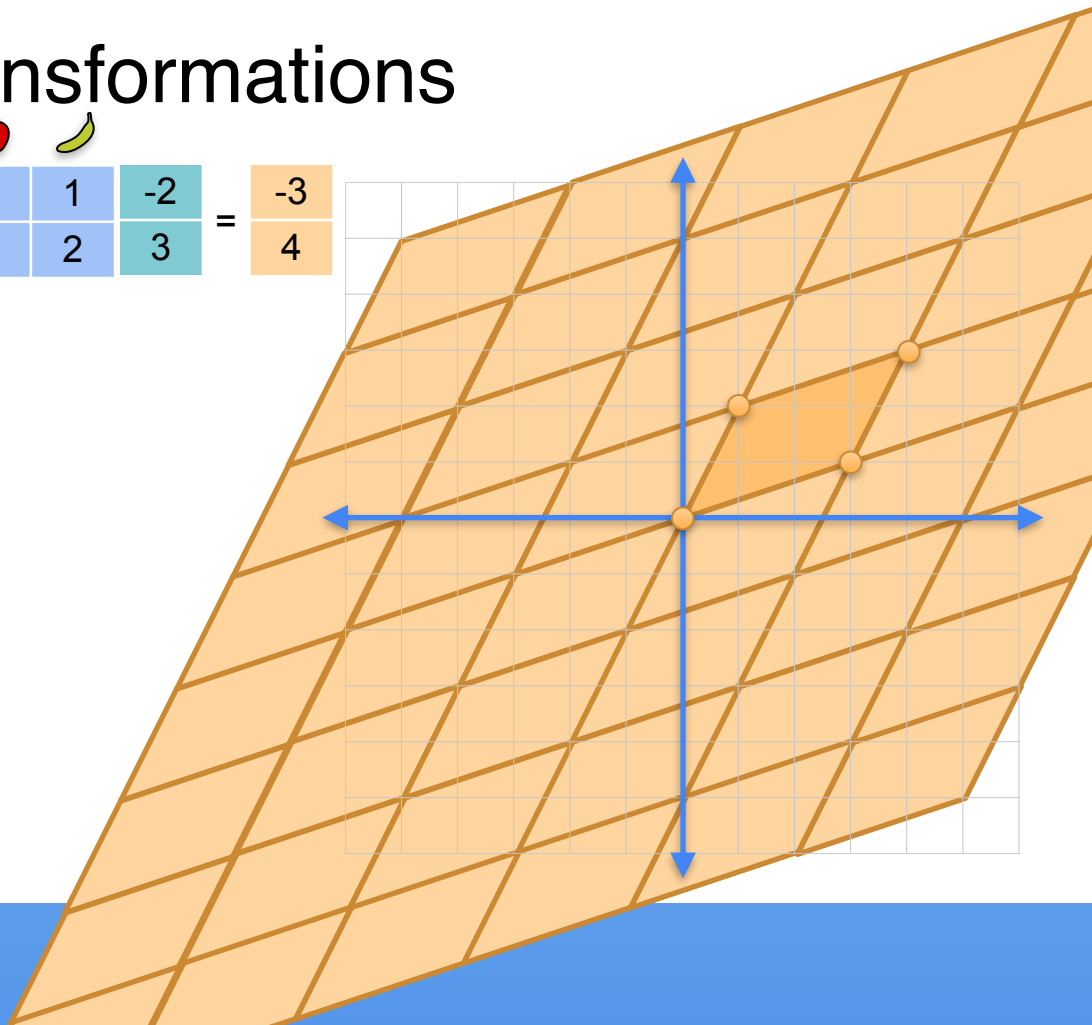
$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} & \begin{bmatrix} -2 \\ 3 \end{bmatrix} \end{matrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$



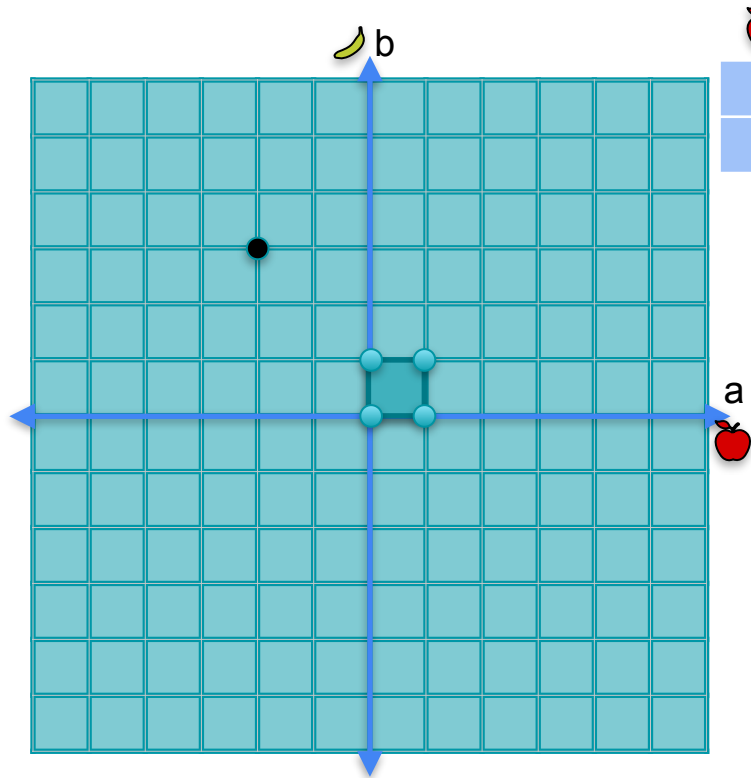
Matrices as linear transformations



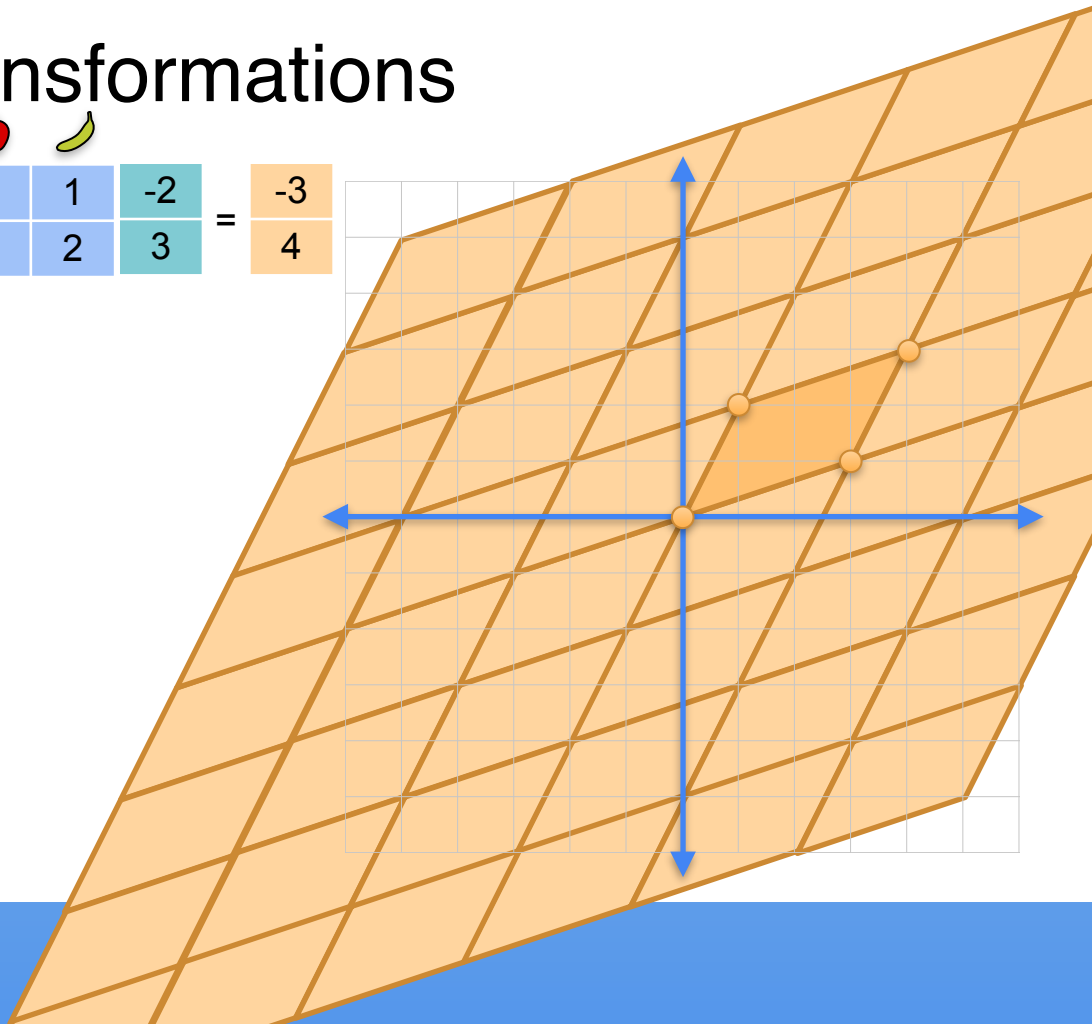
$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} & \begin{bmatrix} -2 \\ 3 \end{bmatrix} \end{matrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$



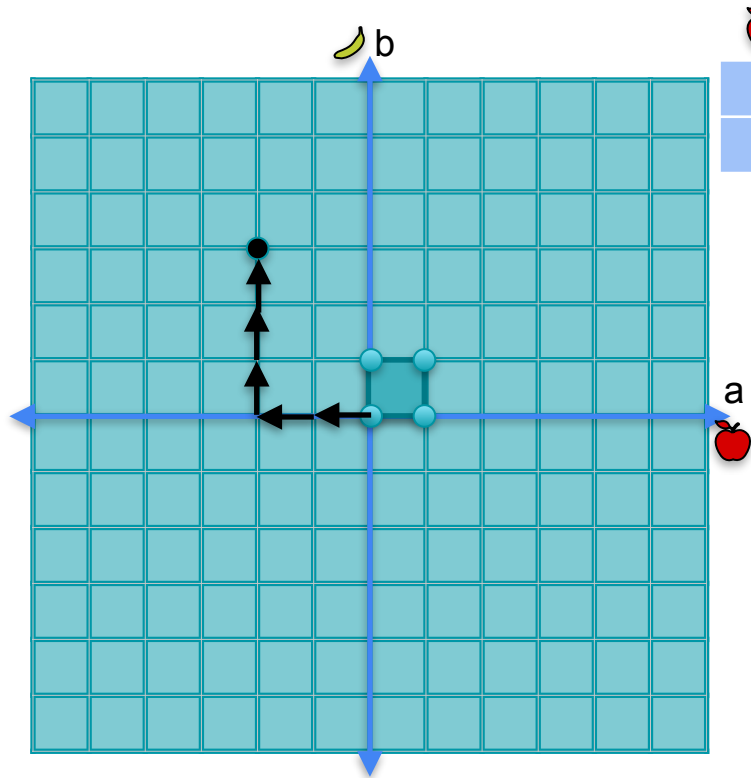
Matrices as linear transformations



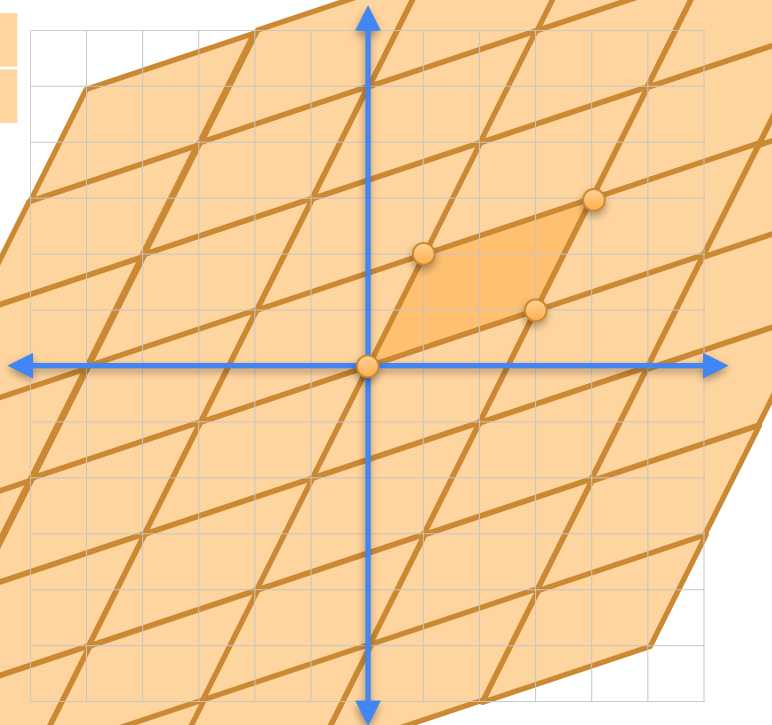
$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} & \begin{bmatrix} -2 \\ 3 \end{bmatrix} \end{matrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$



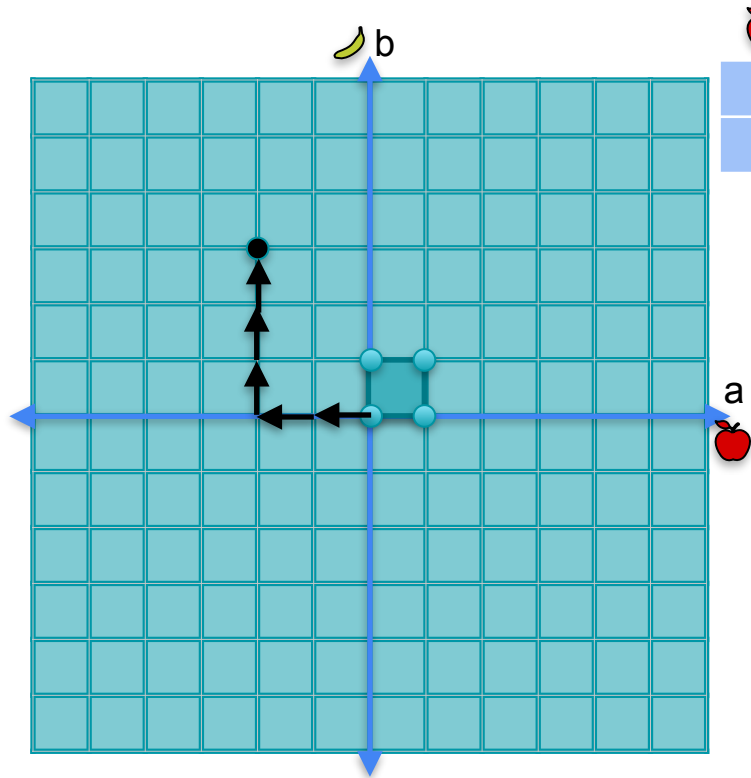
Matrices as linear transformations



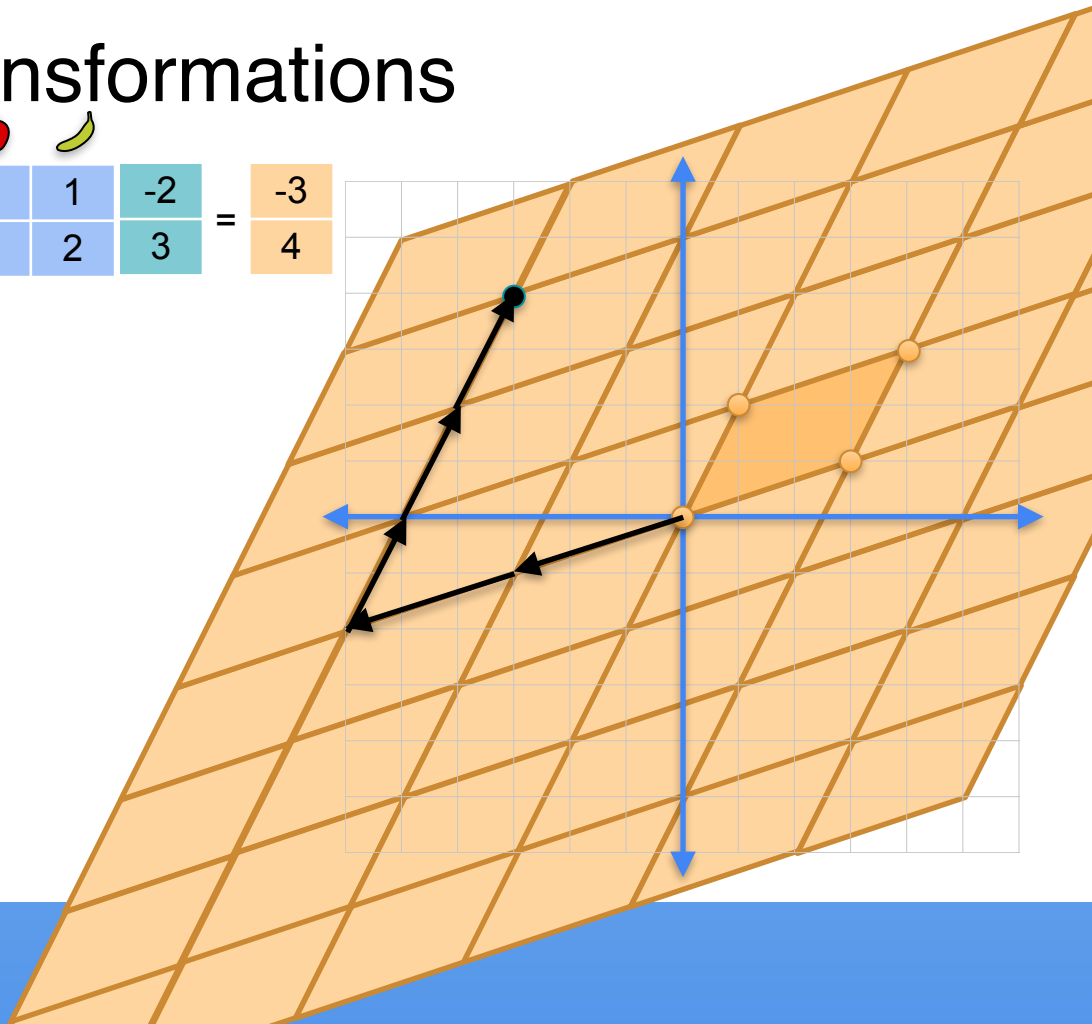
$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} & \begin{bmatrix} -2 \\ 3 \end{bmatrix} \end{matrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$



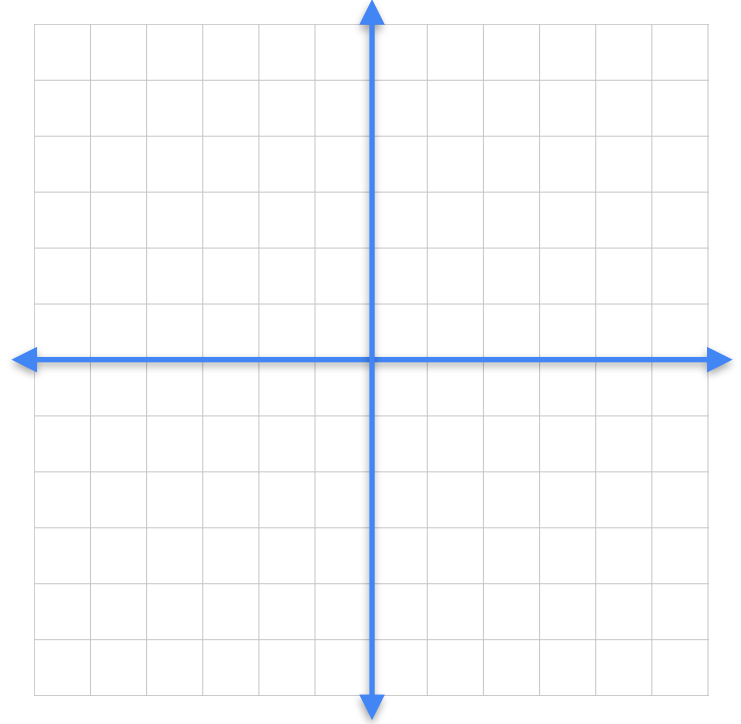
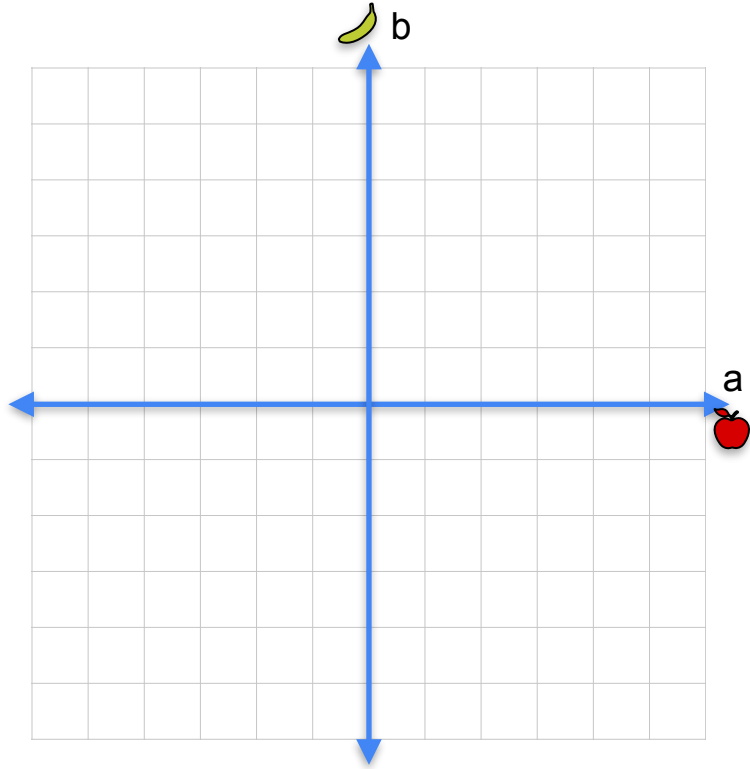
Matrices as linear transformations



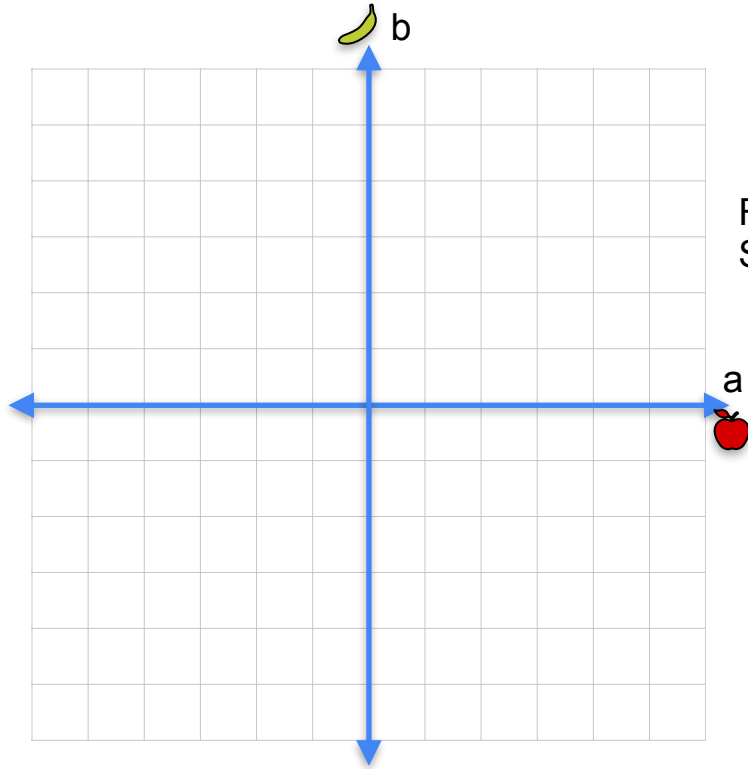
$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} & \begin{bmatrix} -2 \\ 3 \end{bmatrix} \end{matrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$



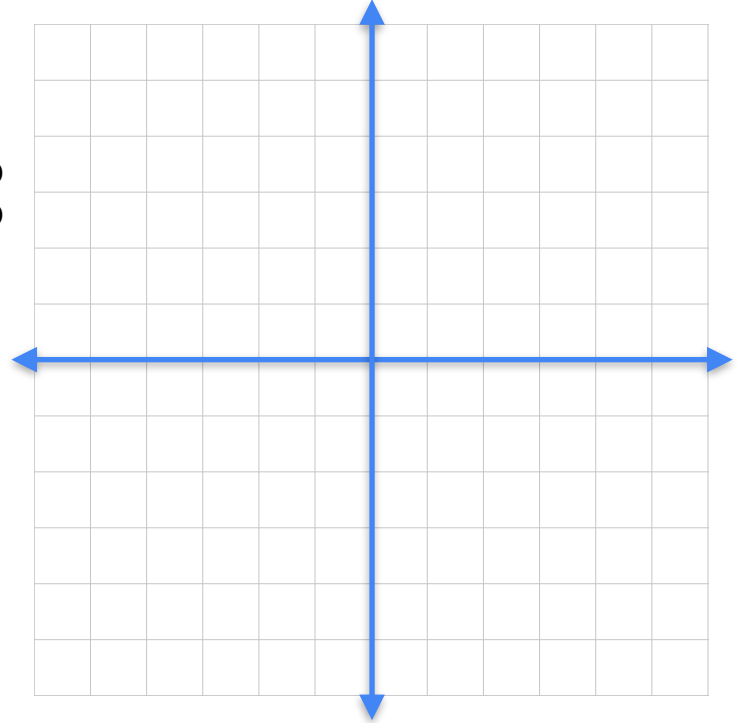
Systems of equations as linear transformations



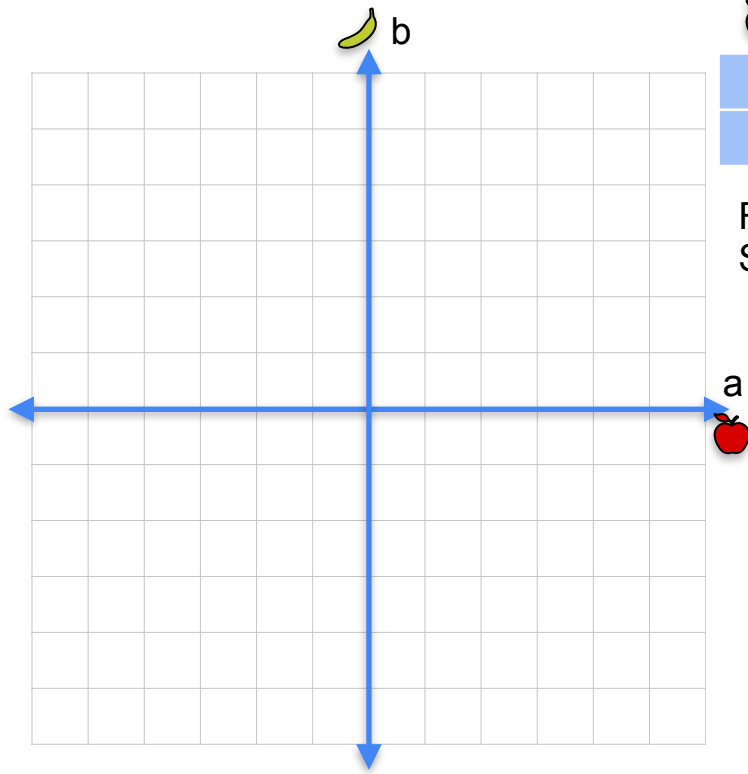
Systems of equations as linear transformations



First day: $3a + b$
Second day: $a + 2b$



Systems of equations as linear transformations

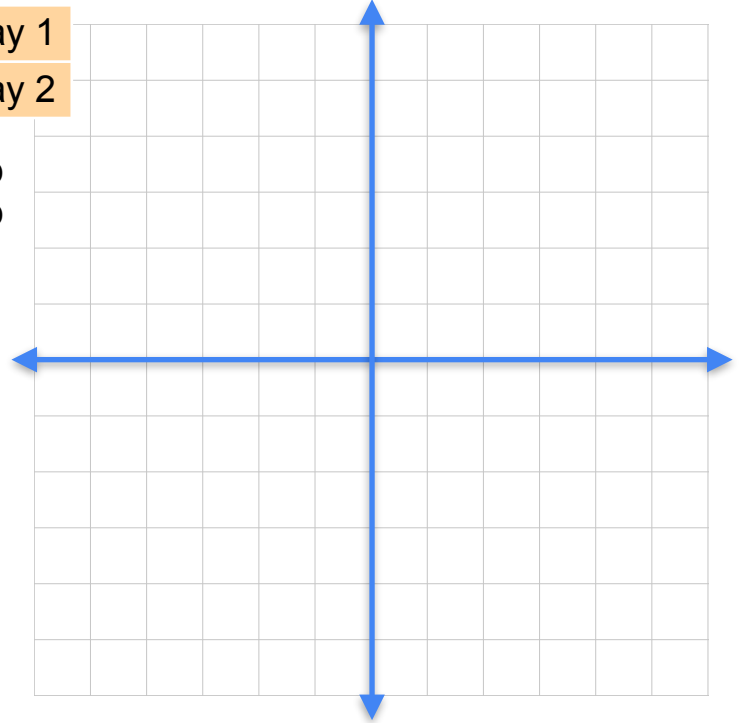


3	1	a
1	2	b

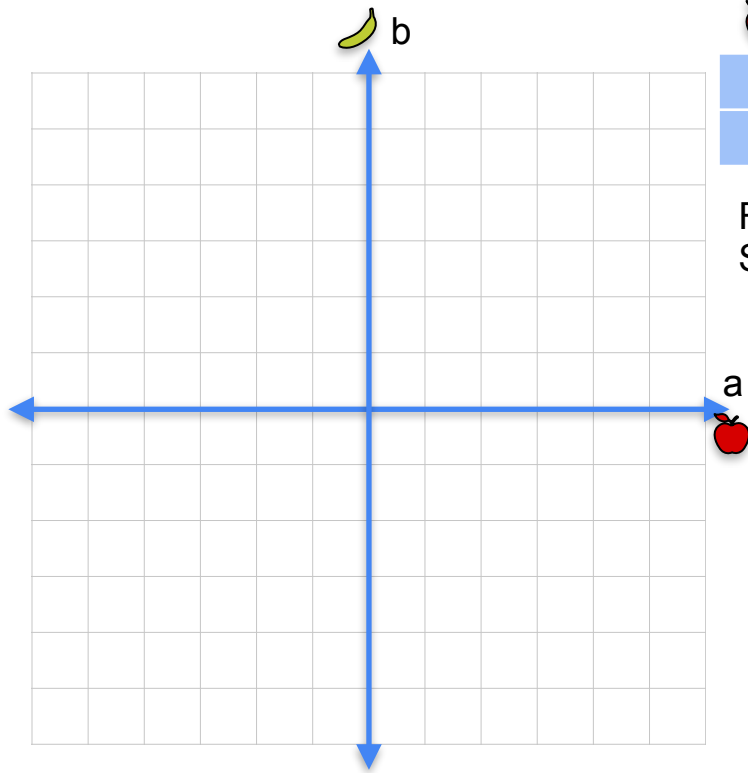
 =

Day 1
Day 2

First day: $3a + b$
Second day: $a + 2b$

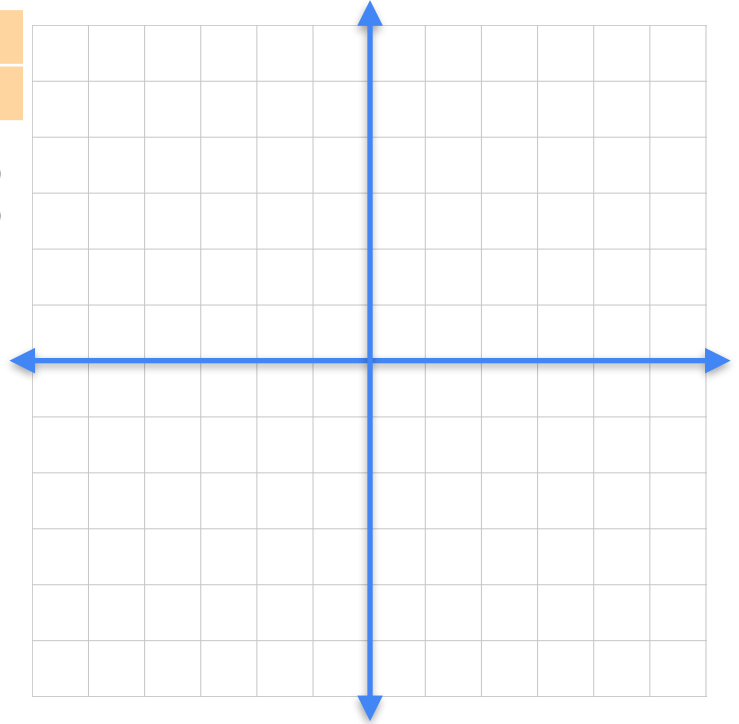


Systems of equations as linear transformations



3	1	1	=	4
1	2	1	=	3

First day: $3a + b$
Second day: $a + 2b$



Systems of equations as linear transformations

 b

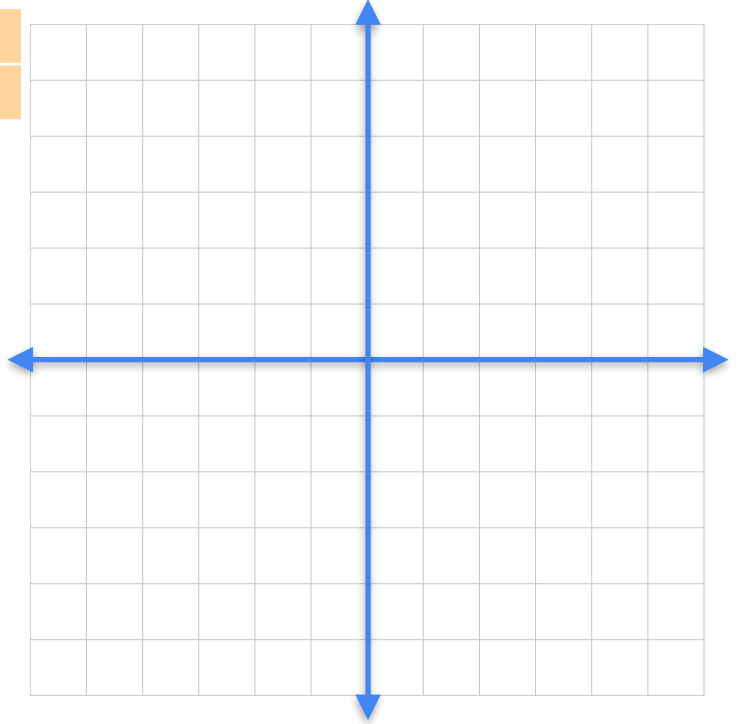
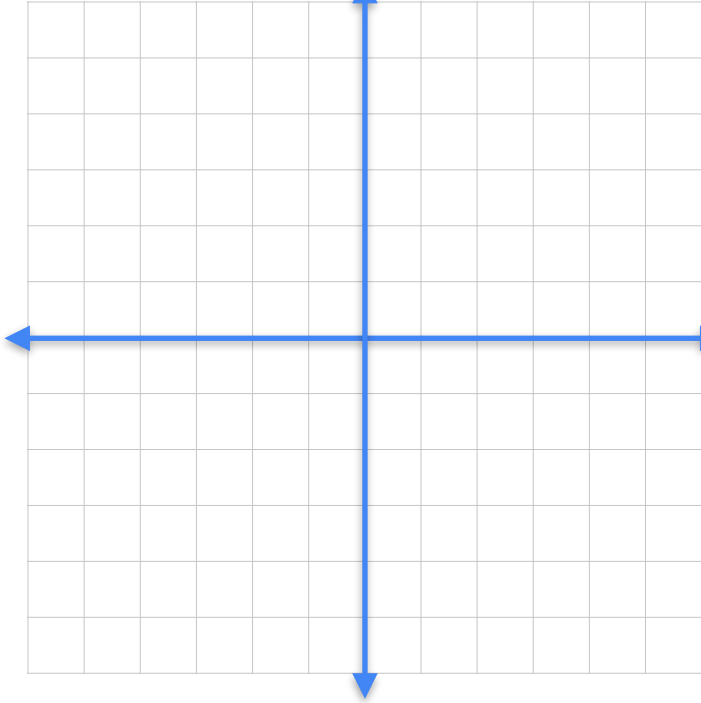


3	1	1	=	4
1	2	1	=	3

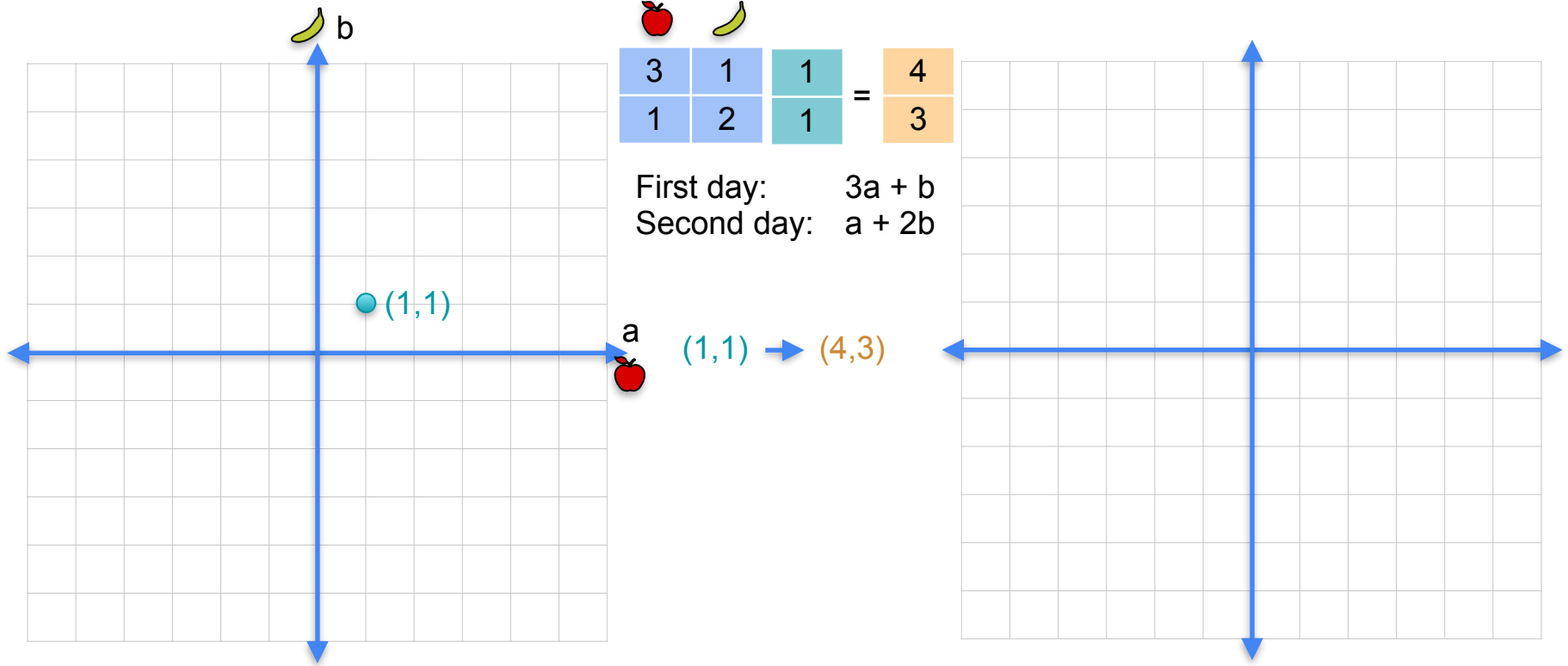
First day: $3a + b$
Second day: $a + 2b$

a

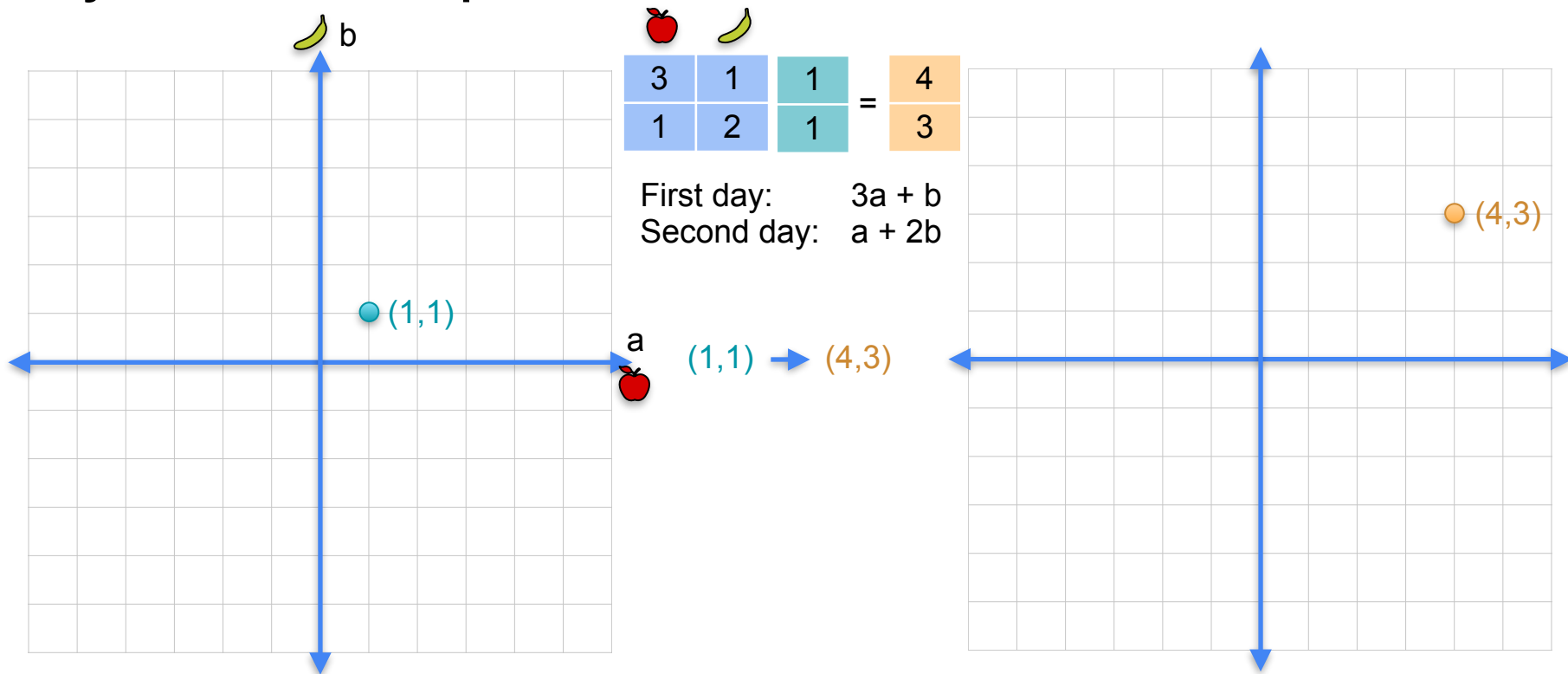
$(1,1) \rightarrow (4,3)$



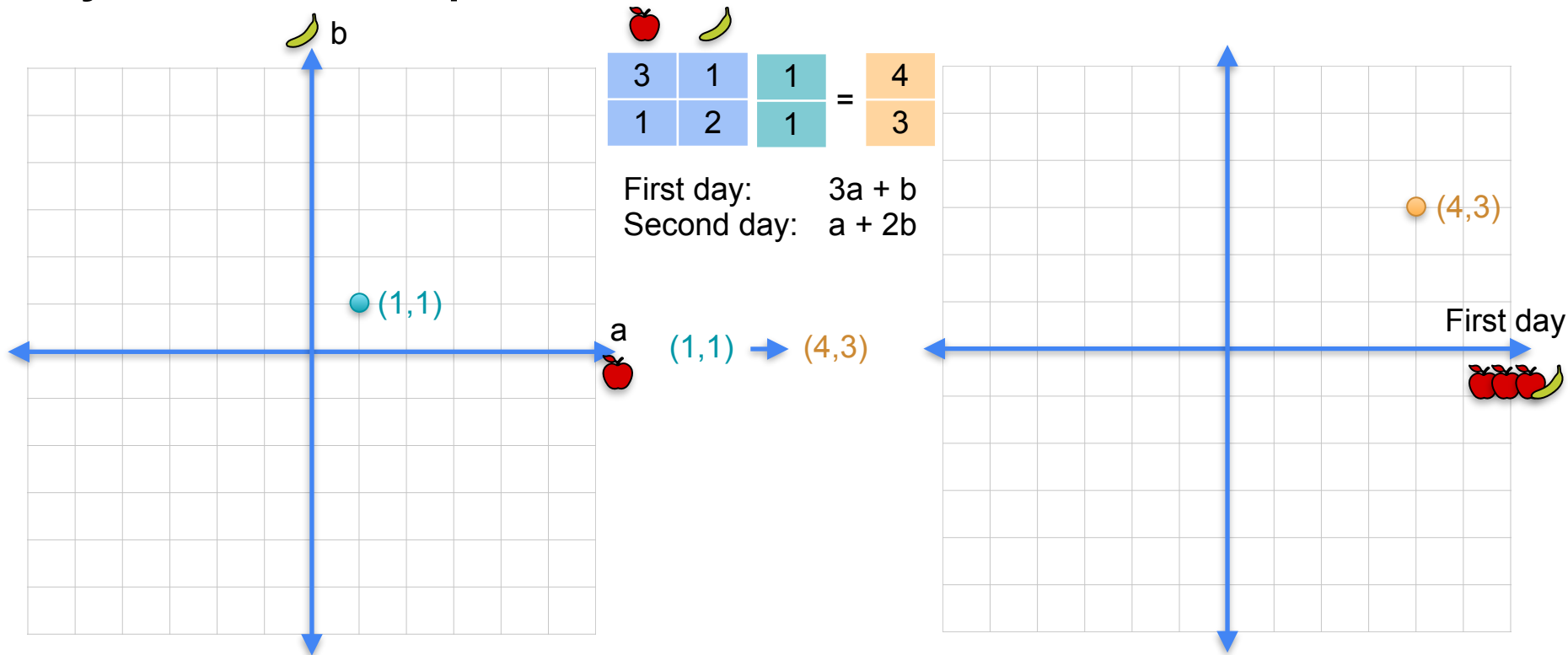
Systems of equations as linear transformations



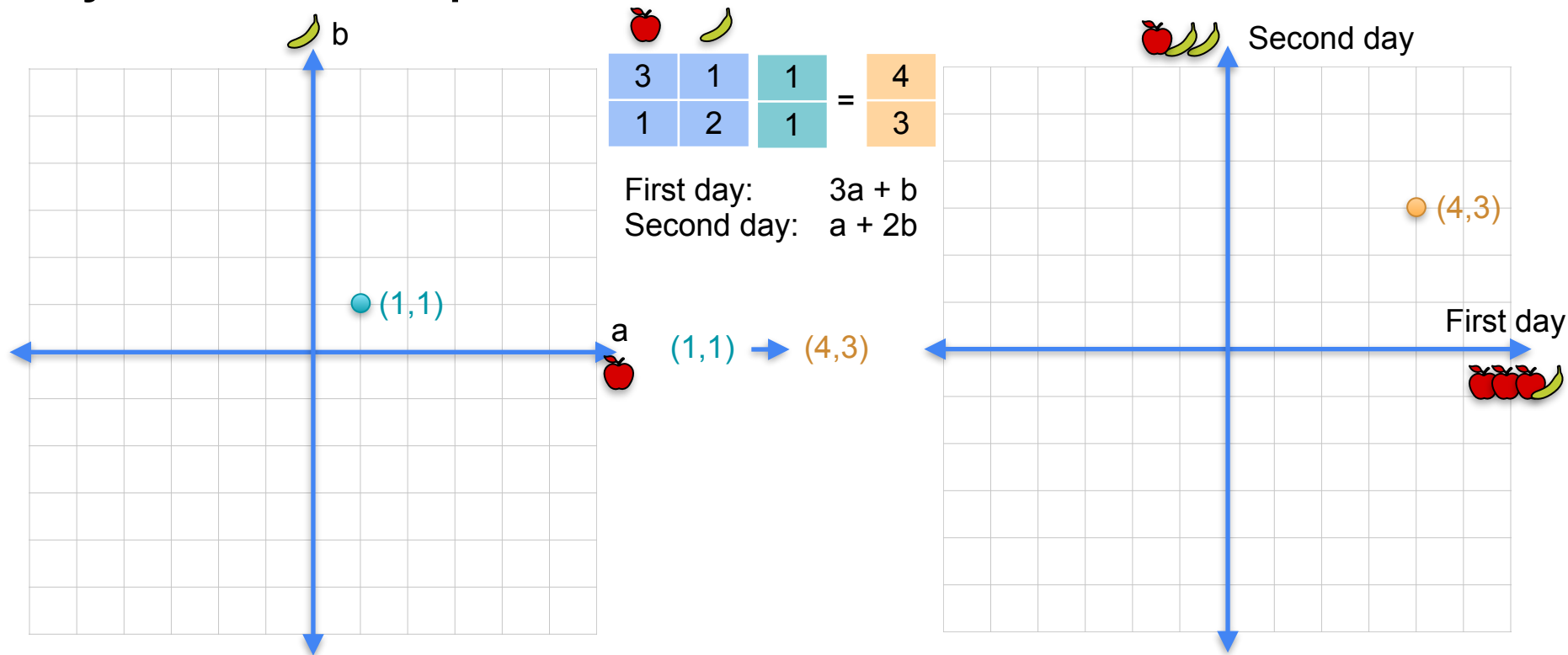
Systems of equations as linear transformations



Systems of equations as linear transformations



Systems of equations as linear transformations



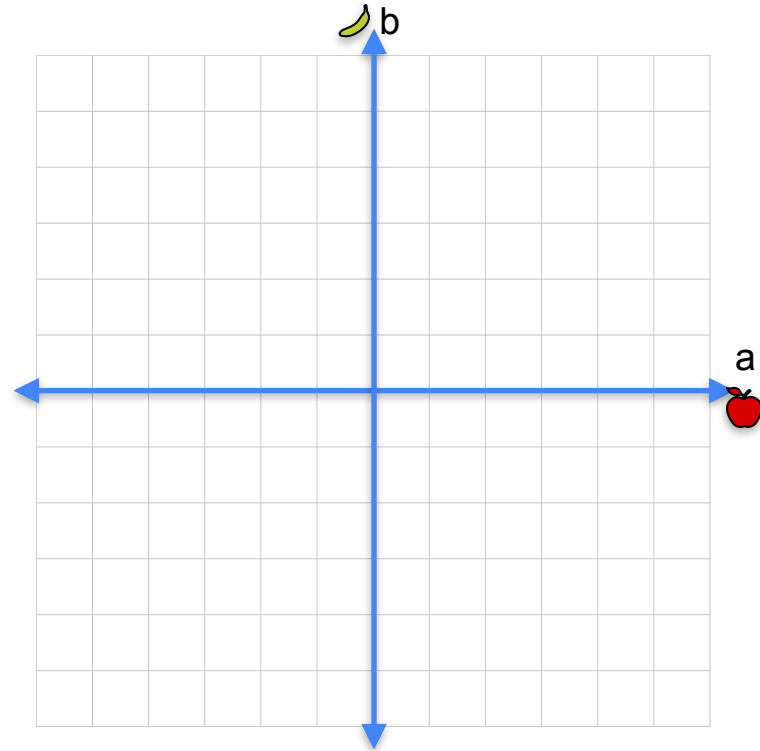
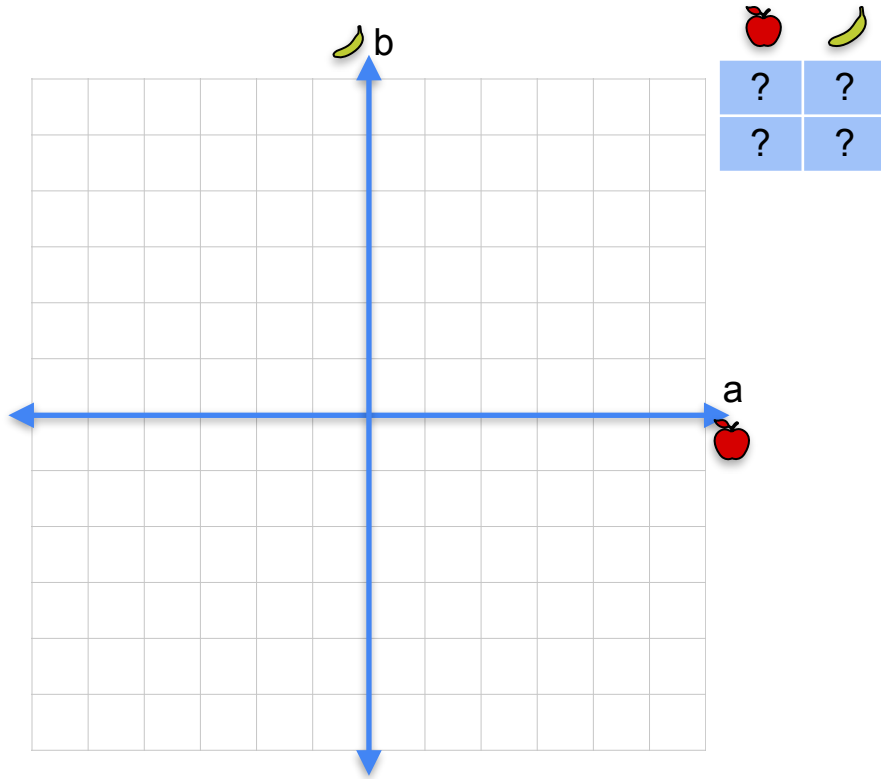


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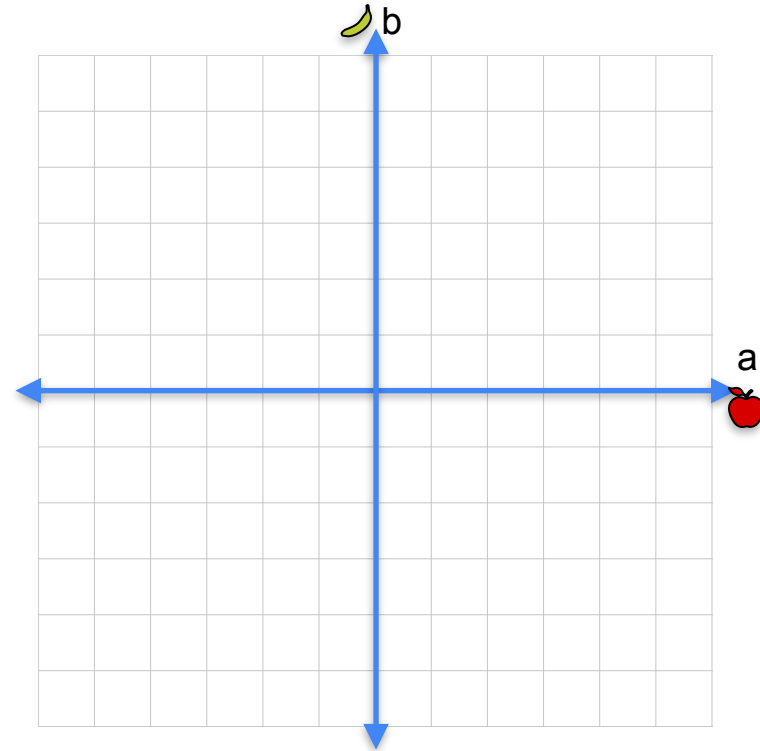
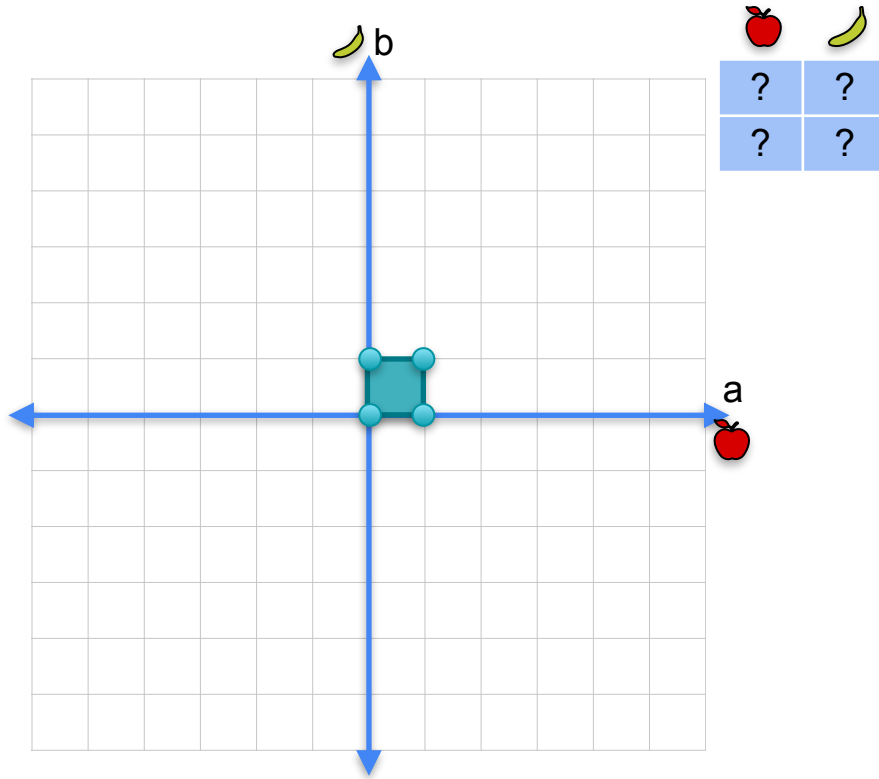
Vectors and Linear Transformations

**Linear transformations as
matrices**

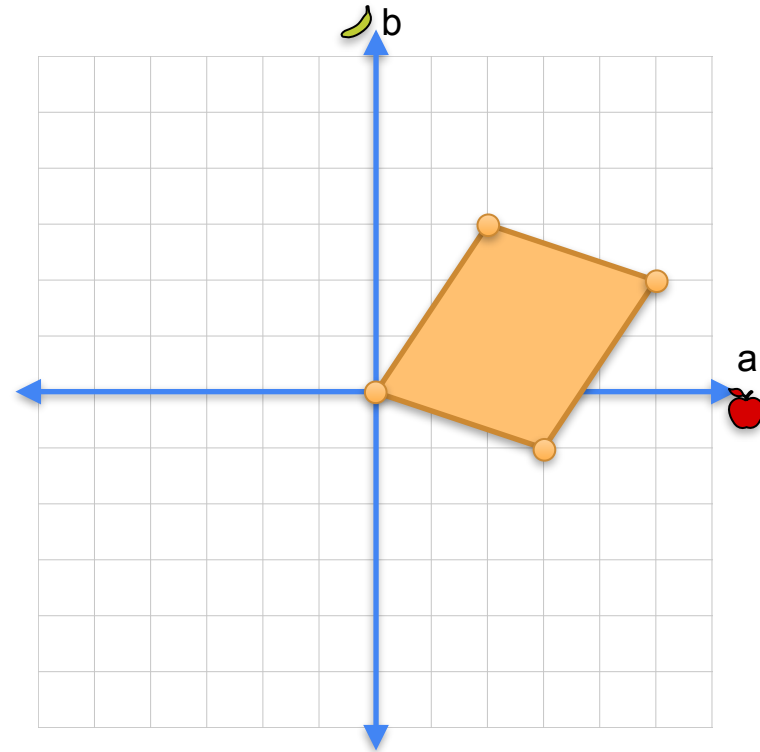
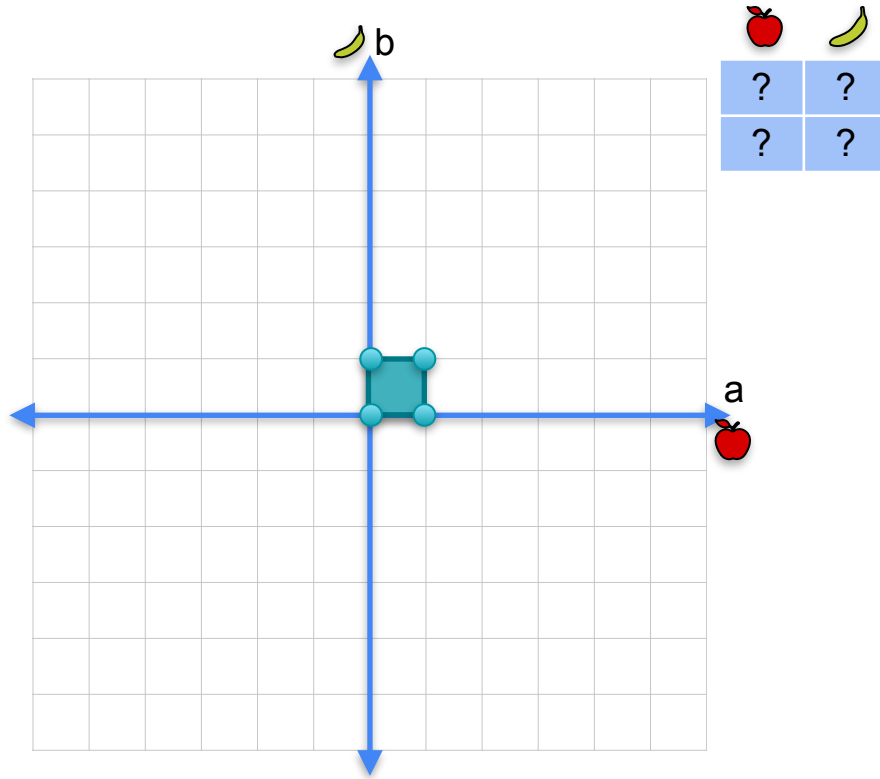
Linear transformations as matrices



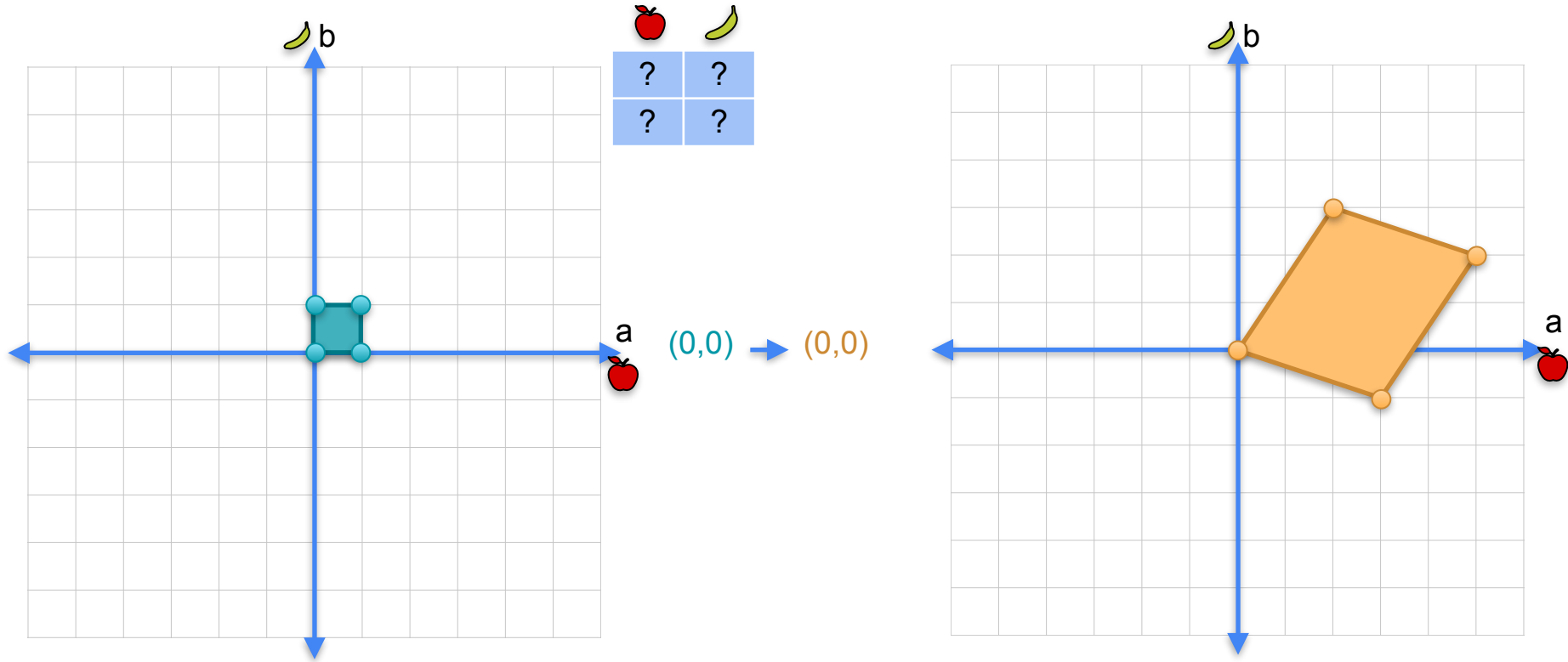
Linear transformations as matrices



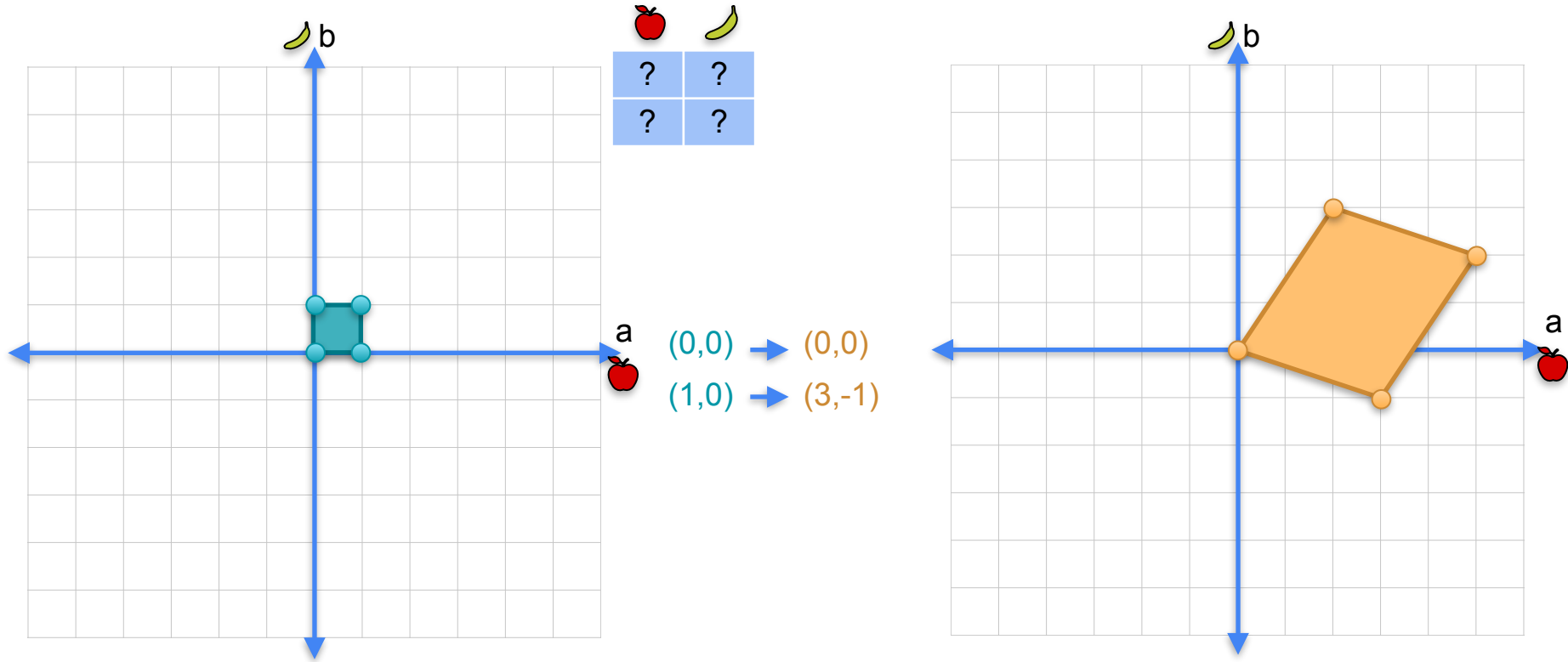
Linear transformations as matrices



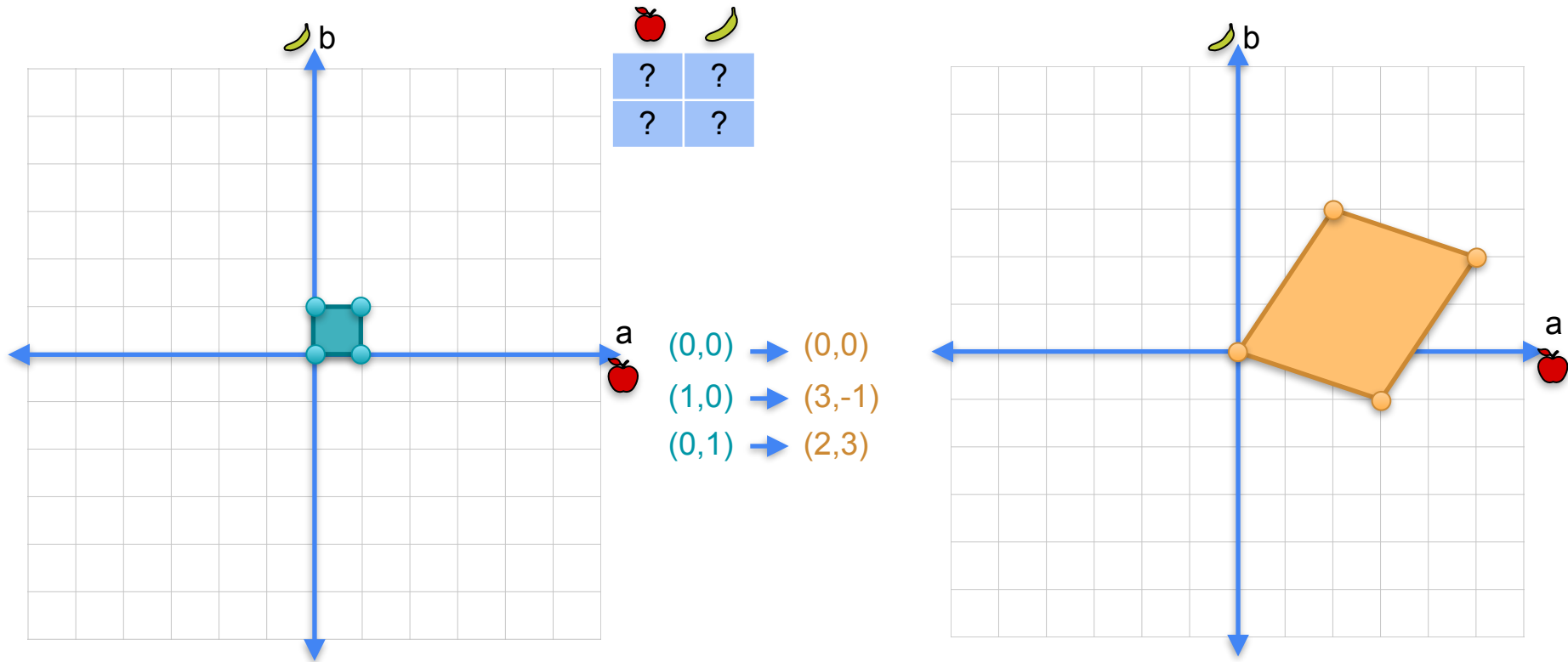
Linear transformations as matrices



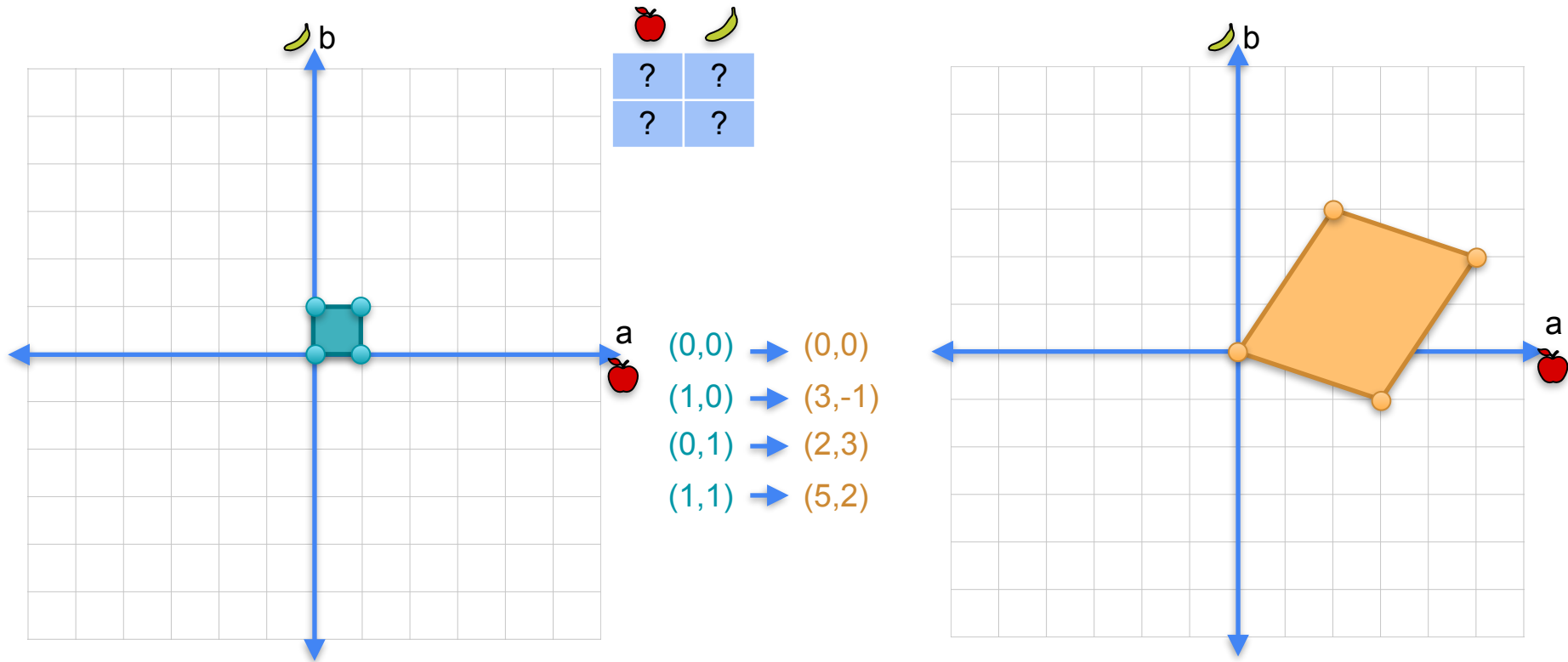
Linear transformations as matrices



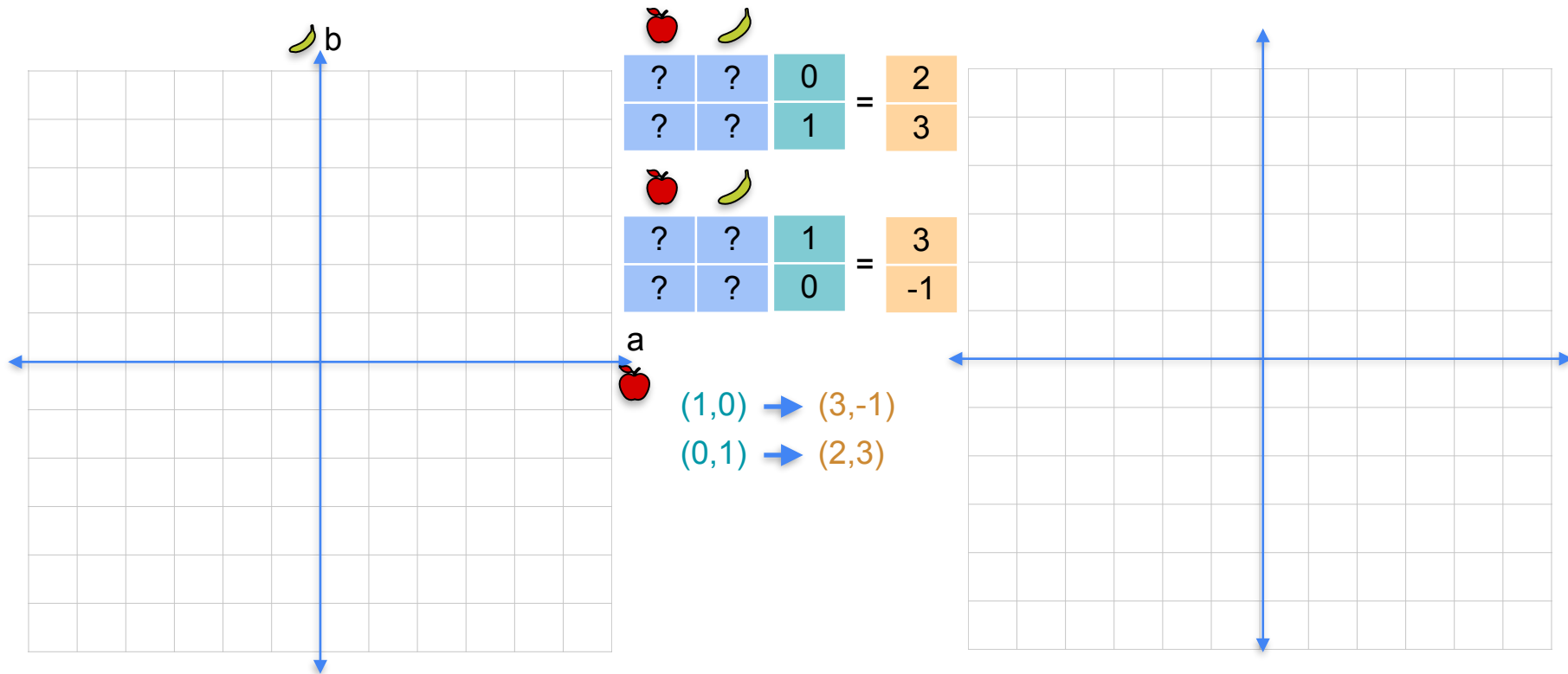
Linear transformations as matrices



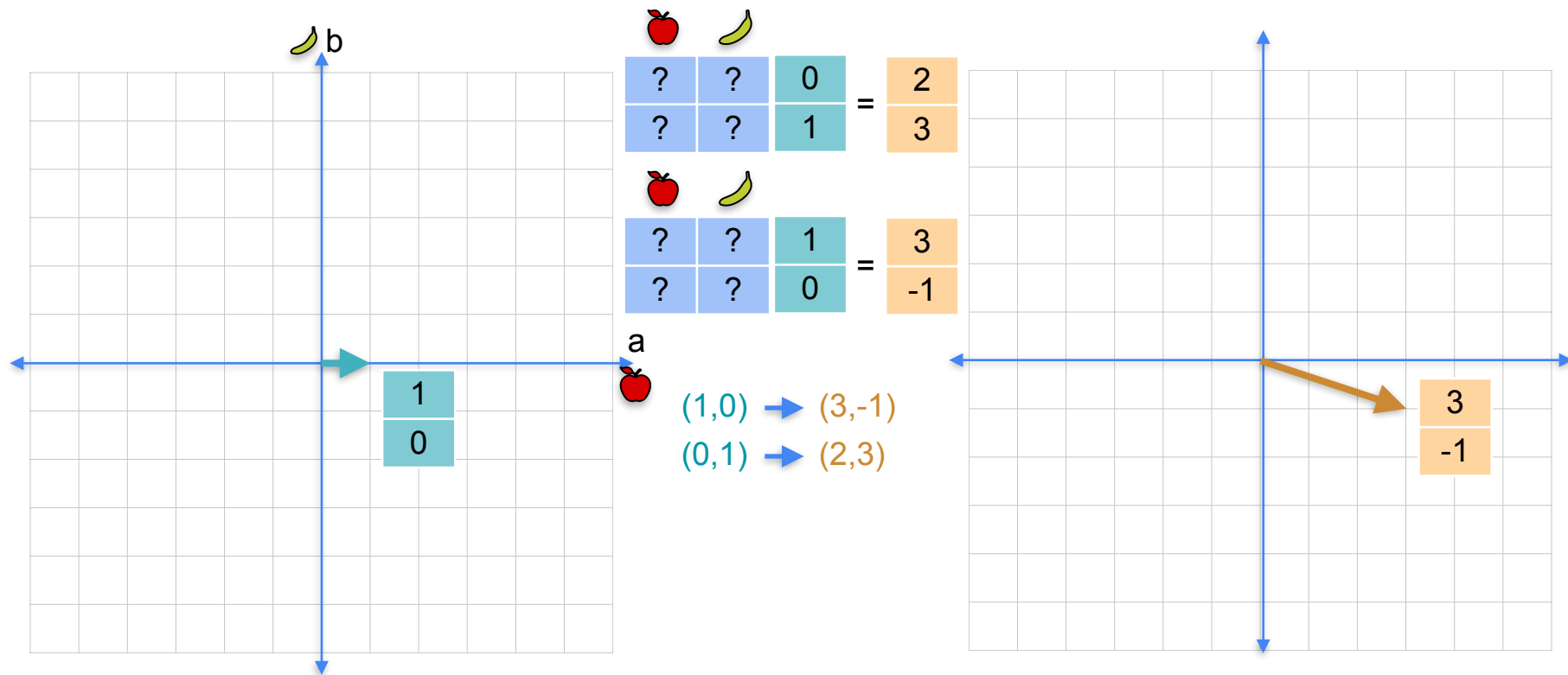
Linear transformations as matrices



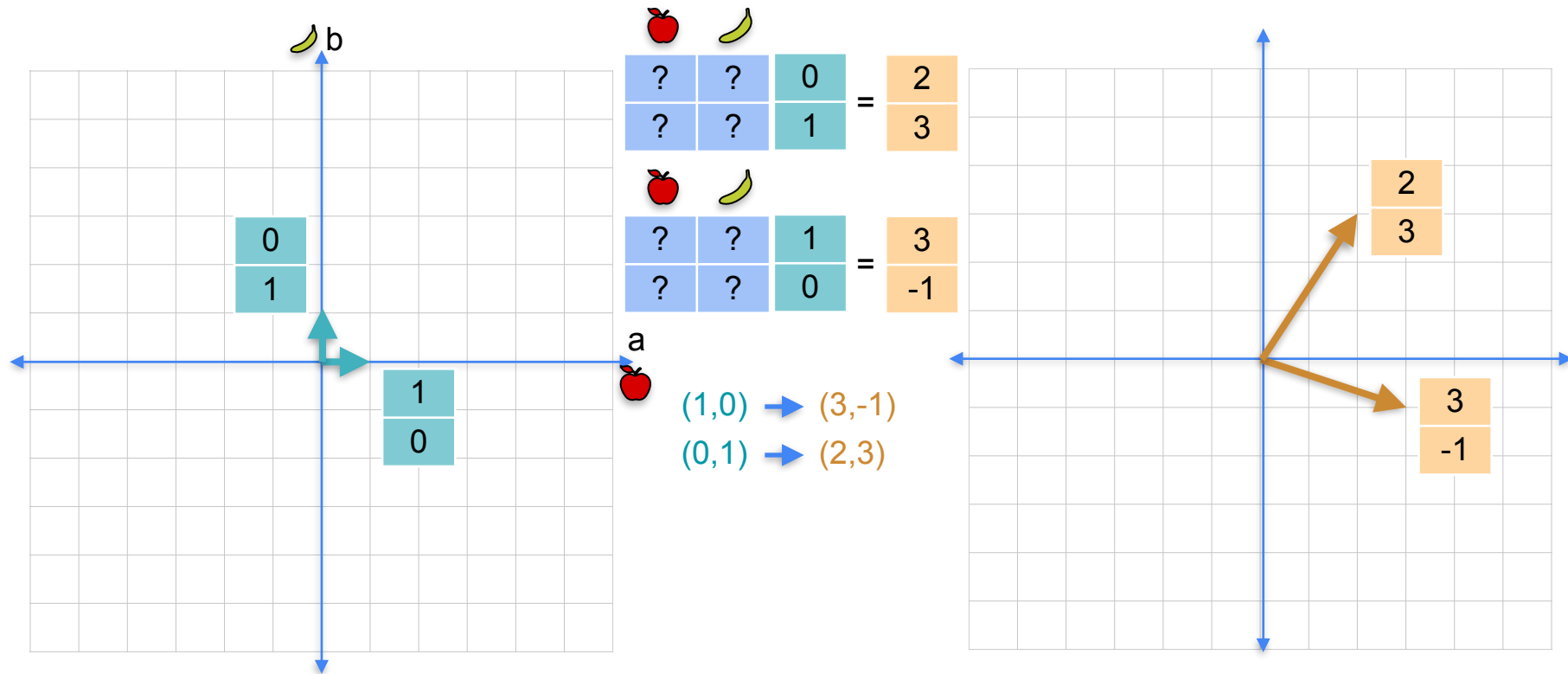
Linear transformations as matrices



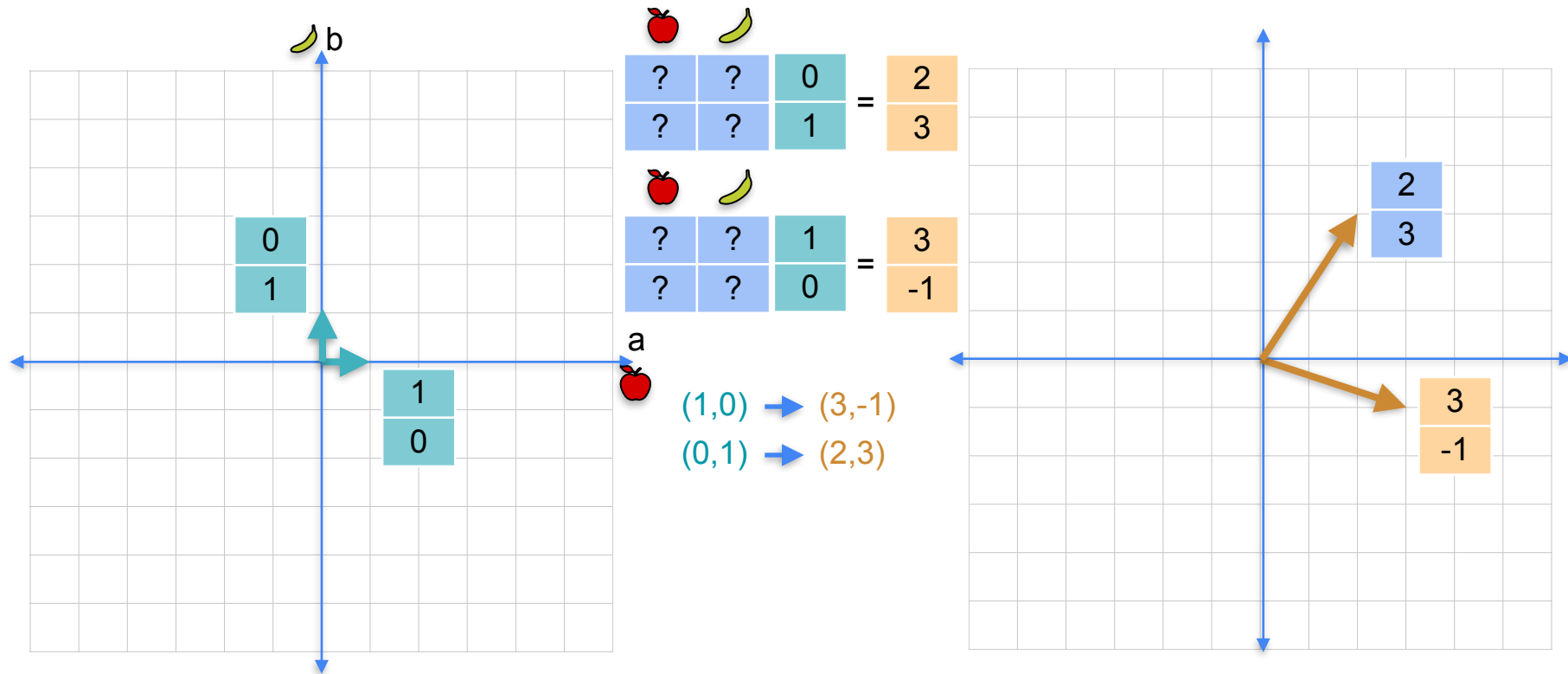
Linear transformations as matrices



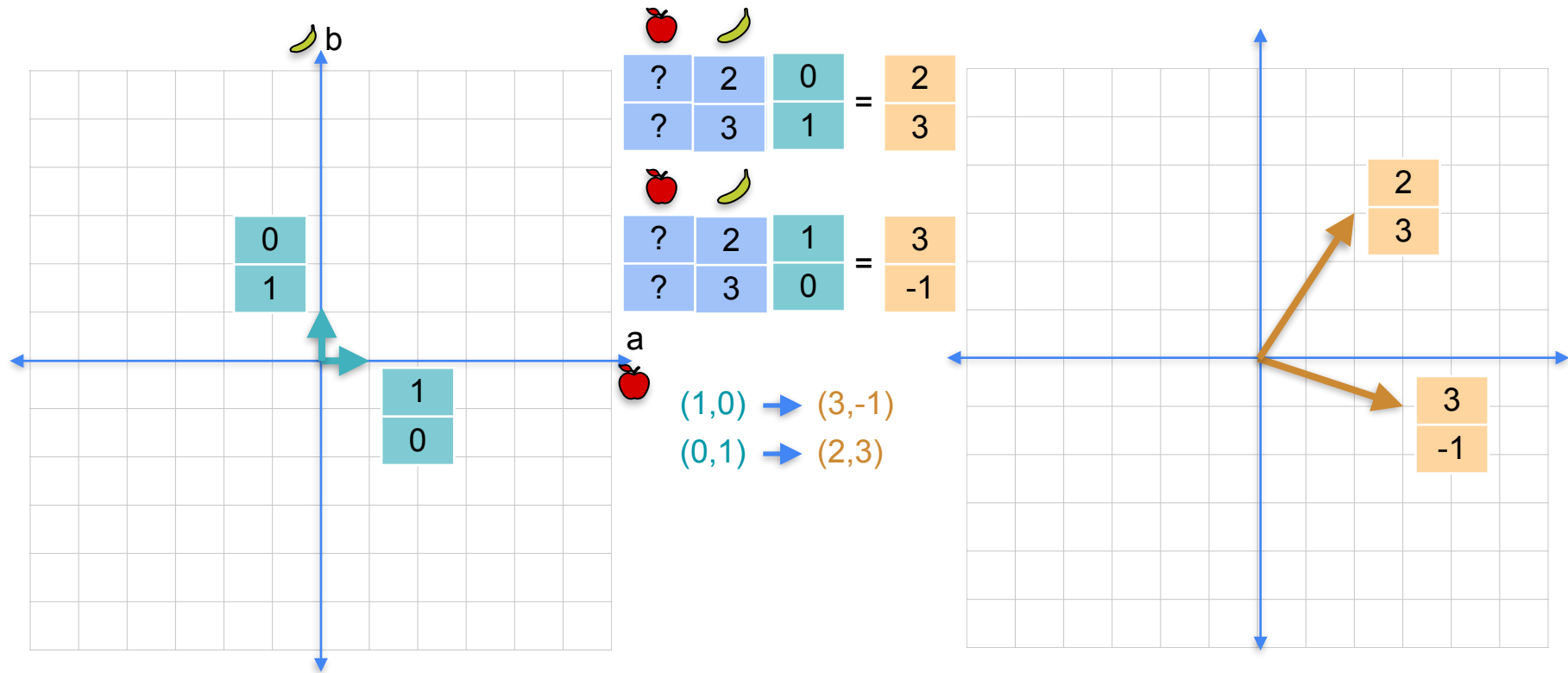
Linear transformations as matrices



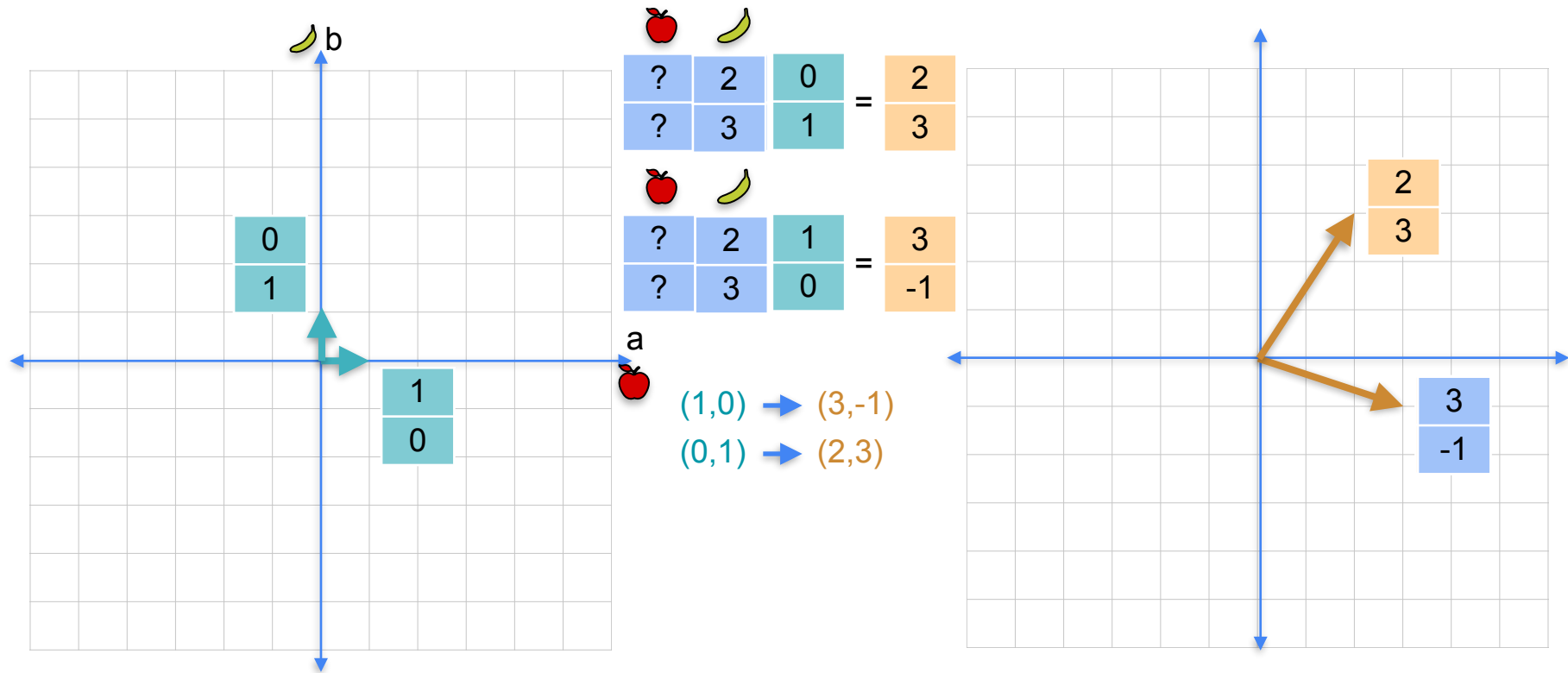
Linear transformations as matrices



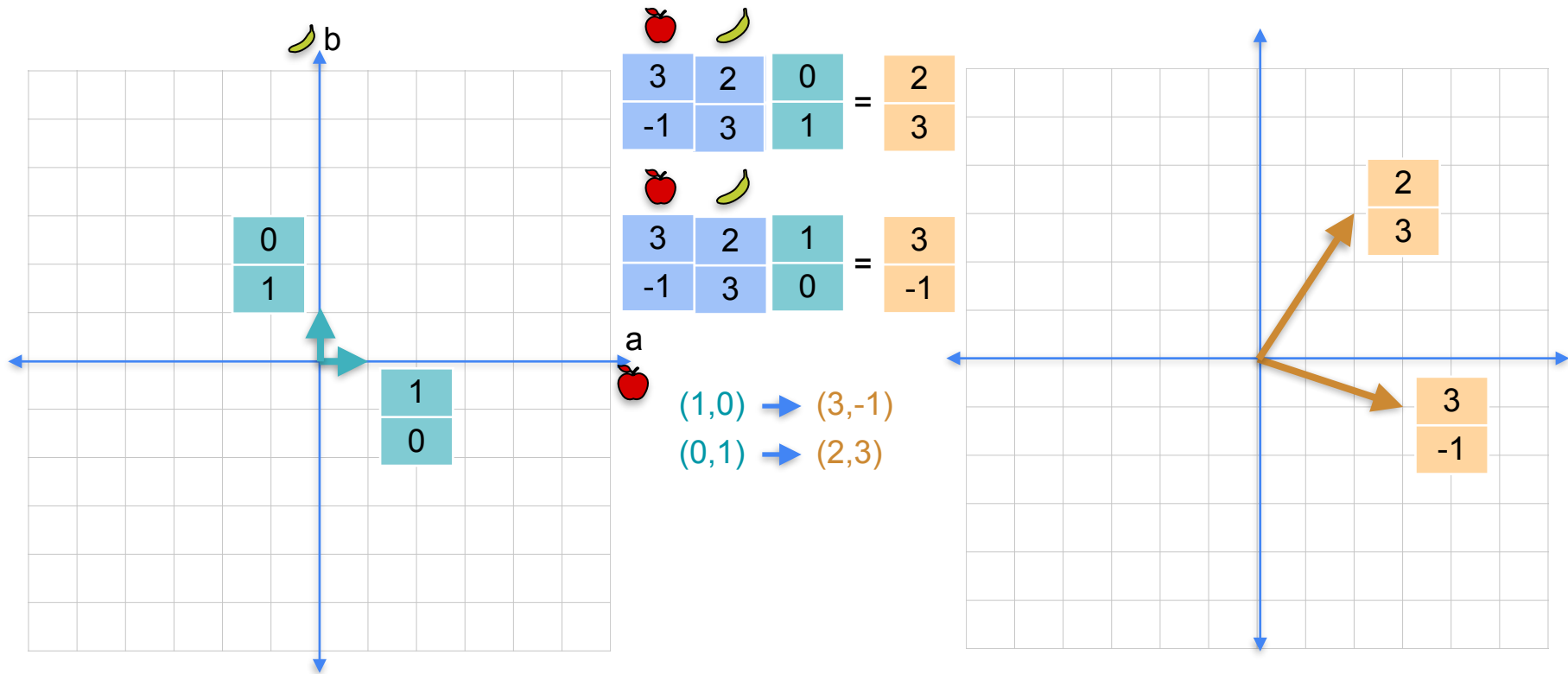
Linear transformations as matrices



Linear transformations as matrices



Linear transformations as matrices



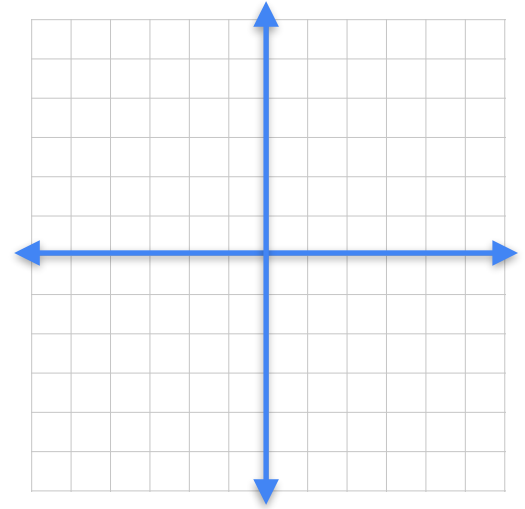
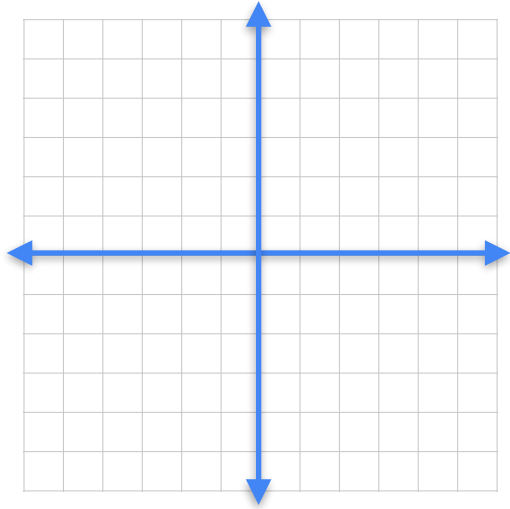


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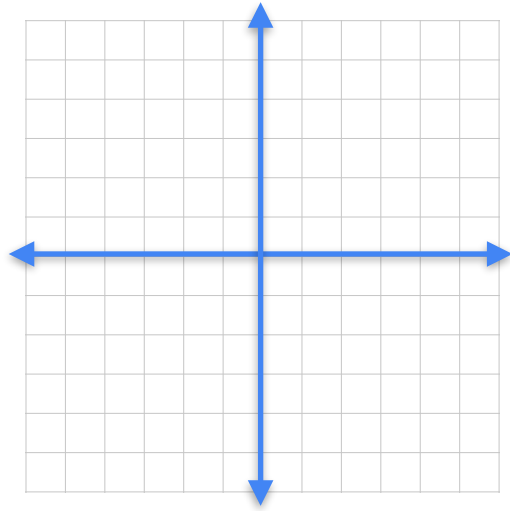
Vectors and Linear Transformations

Matrix multiplication

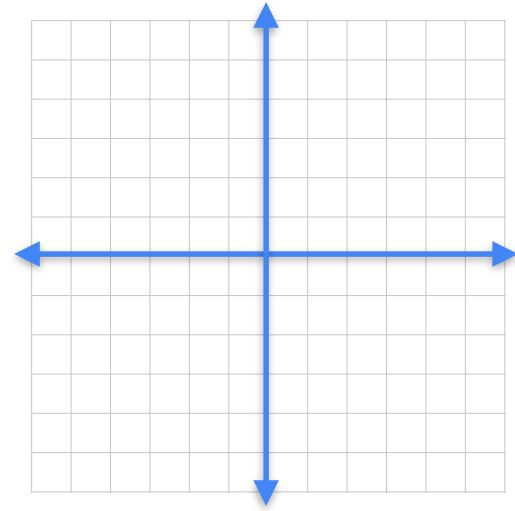
Combining linear transformations



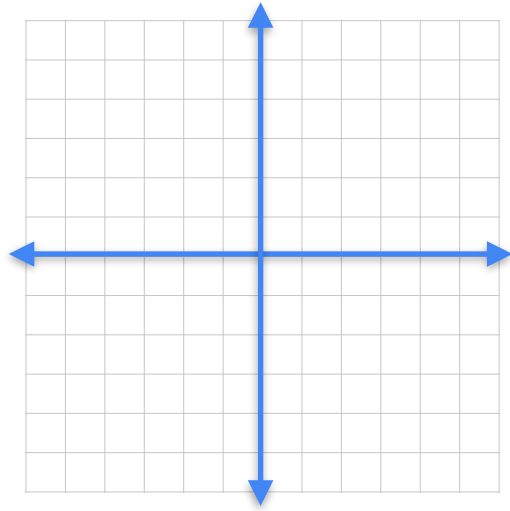
Combining linear transformations



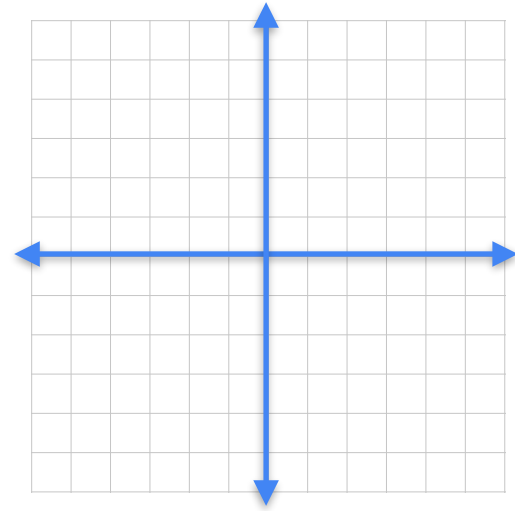
3	1
1	2



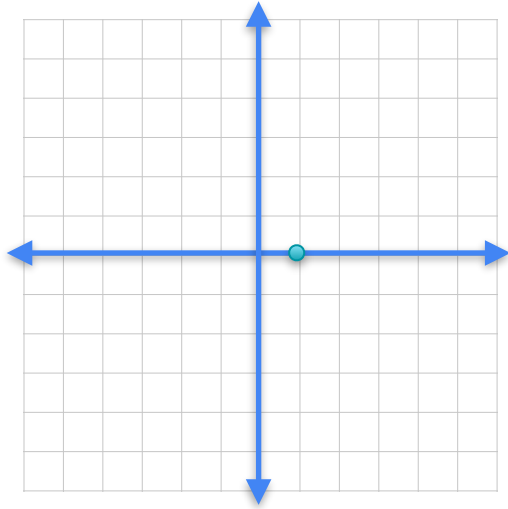
Combining linear transformations



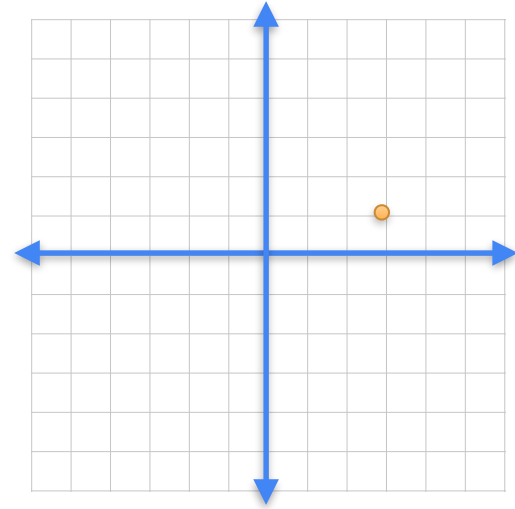
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$



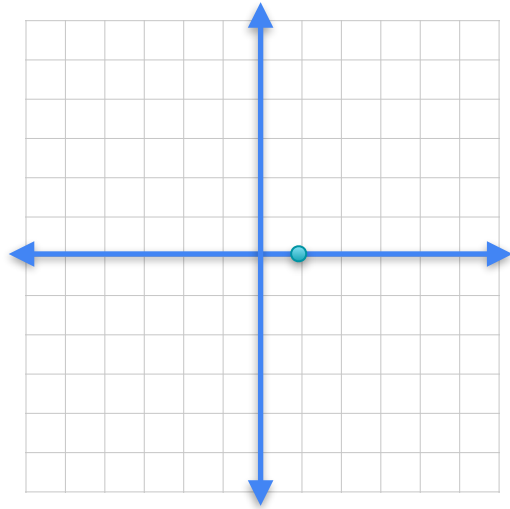
Combining linear transformations



$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

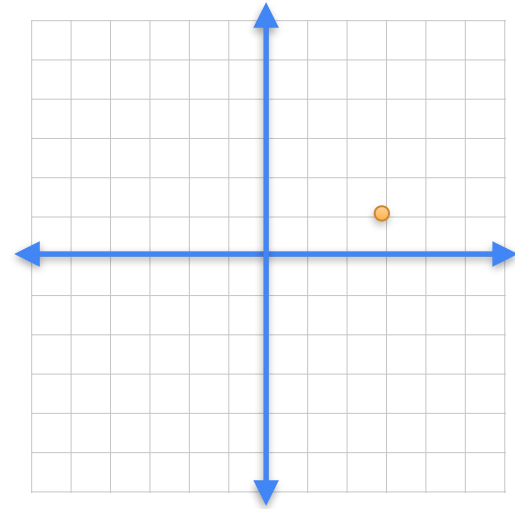


Combining linear transformations

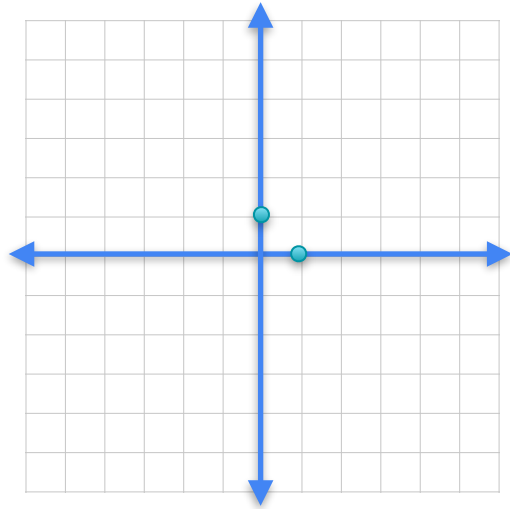


$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

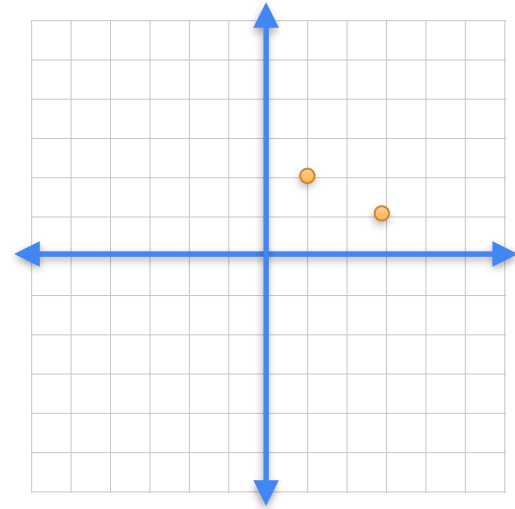


Combining linear transformations

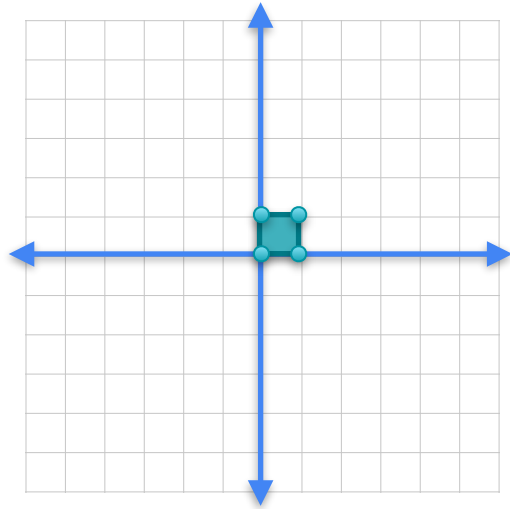


$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

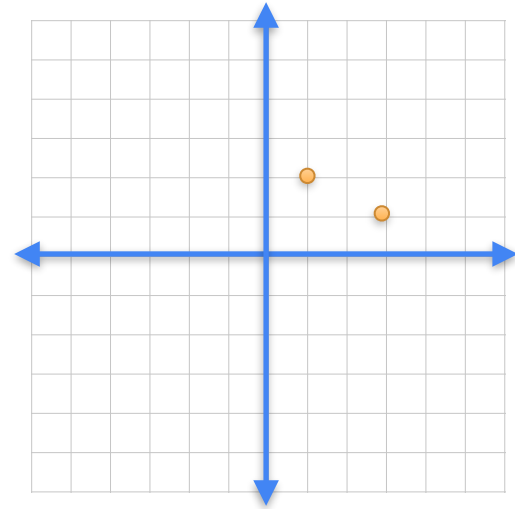


Combining linear transformations

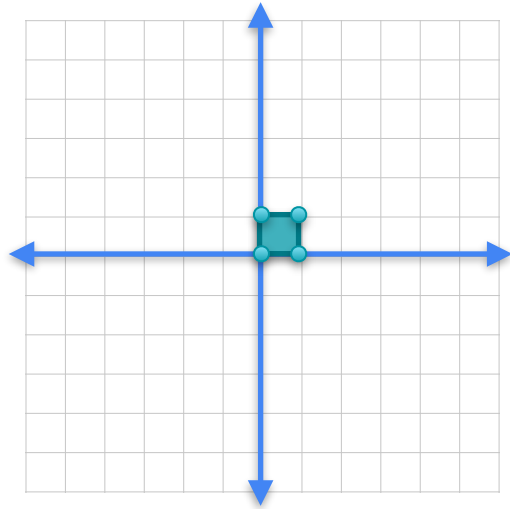


$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

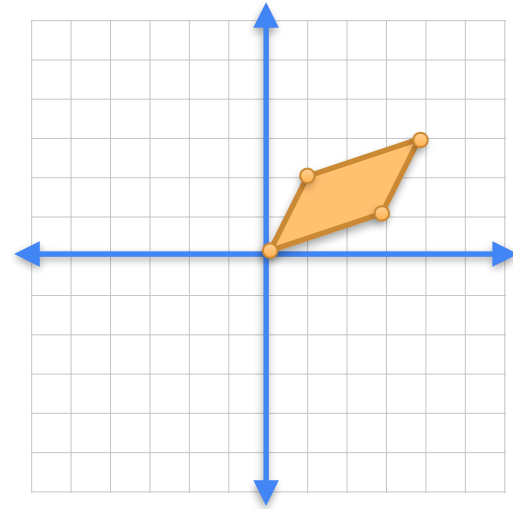


Combining linear transformations

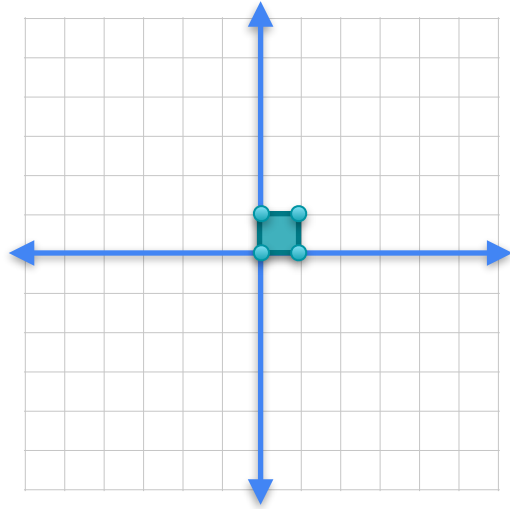


$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

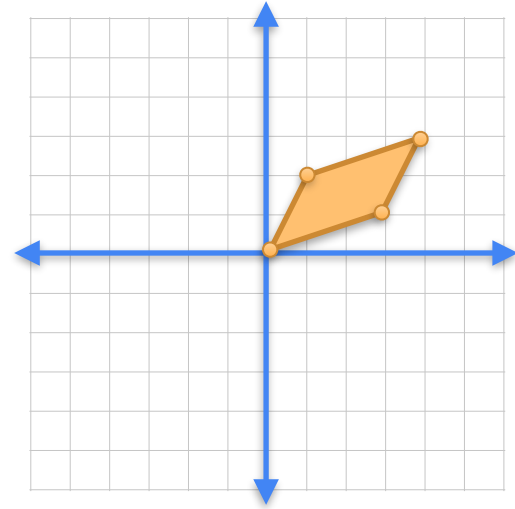


Combining linear transformations

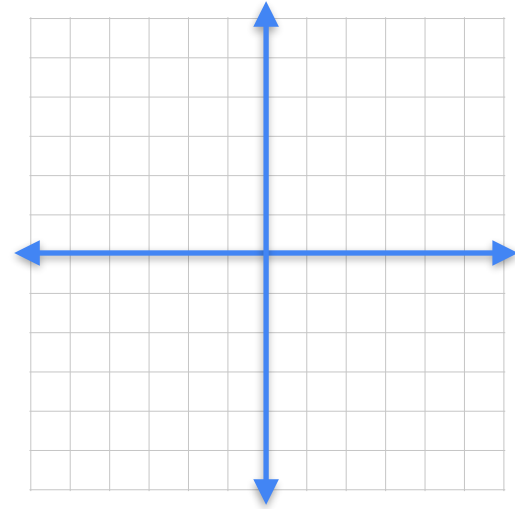
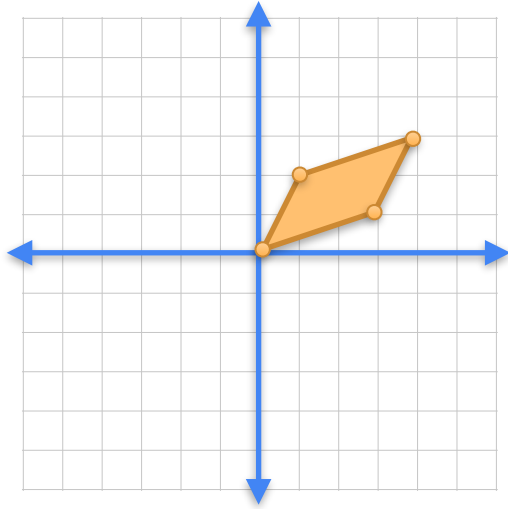


$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

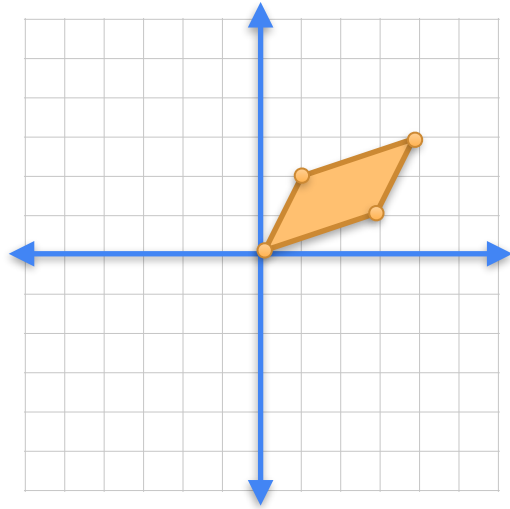
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$



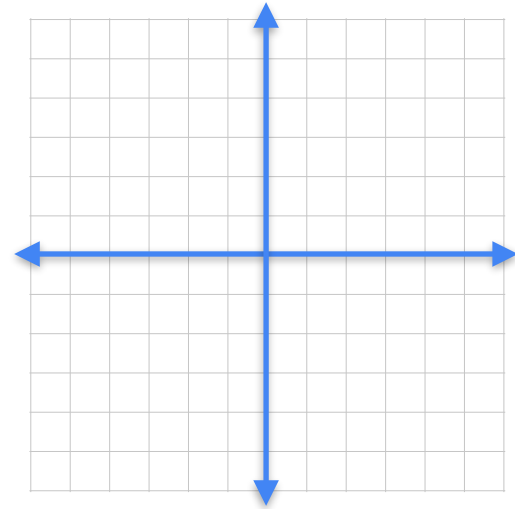
Combining linear transformations



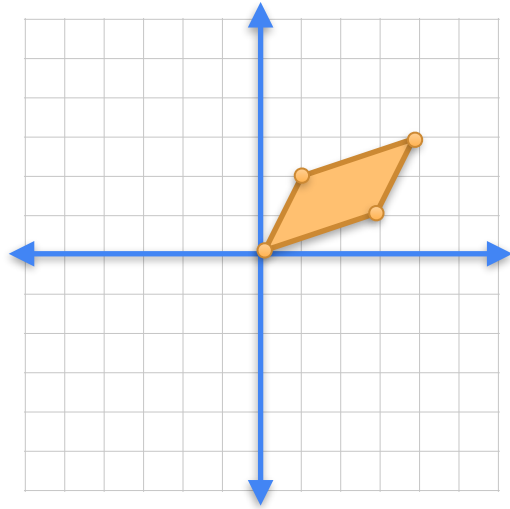
Combining linear transformations



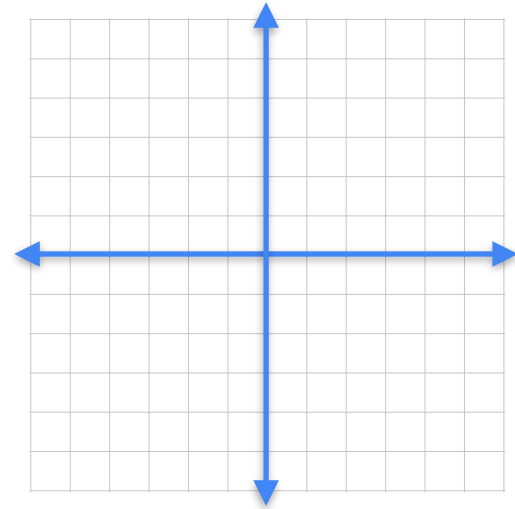
2	-1
0	2



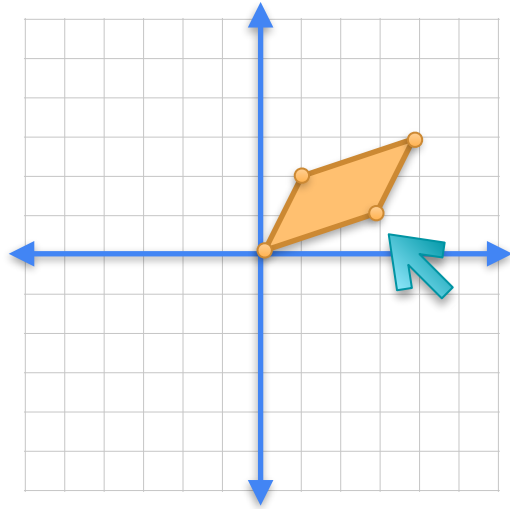
Combining linear transformations



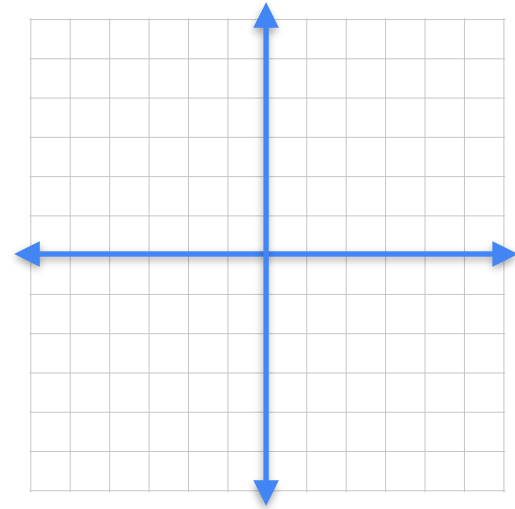
$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$



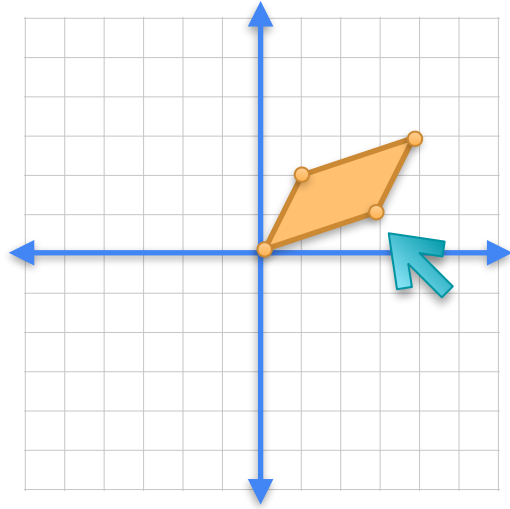
Combining linear transformations



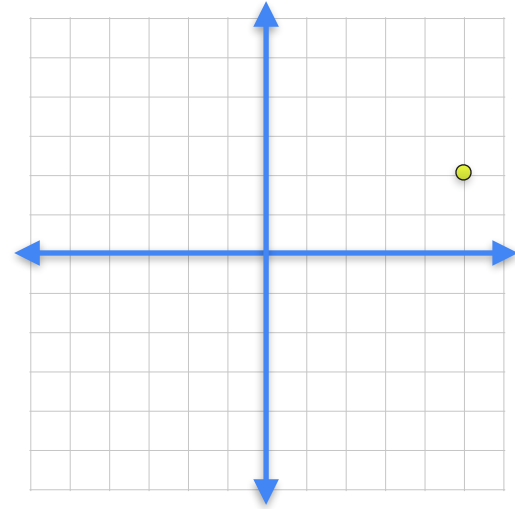
$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$



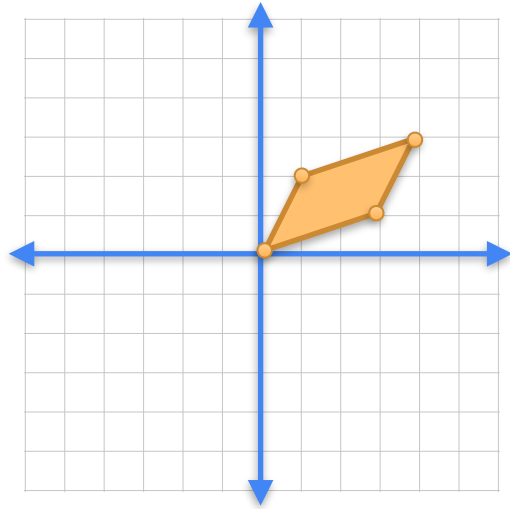
Combining linear transformations



$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

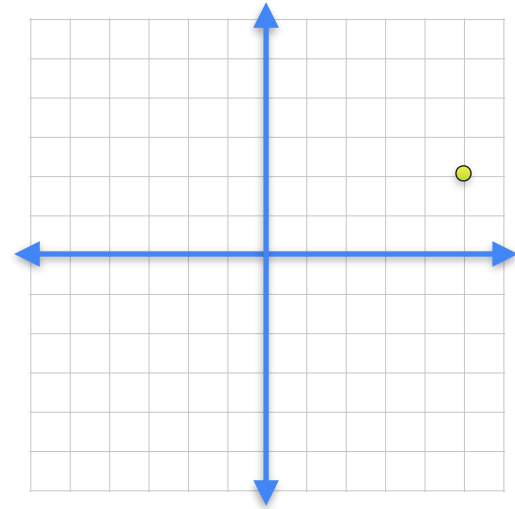


Combining linear transformations

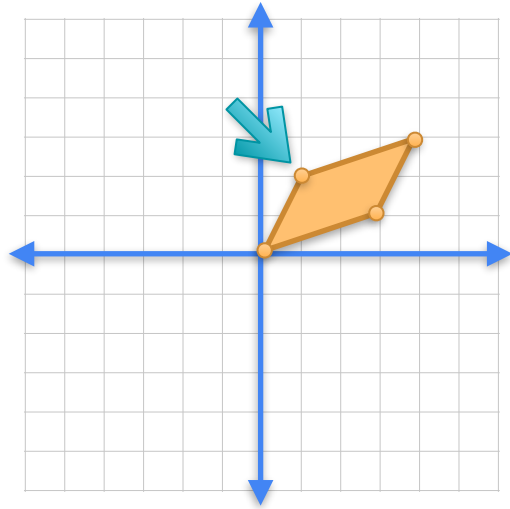


$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

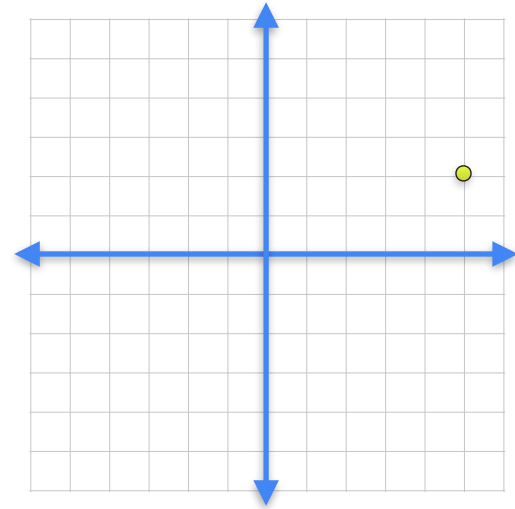


Combining linear transformations

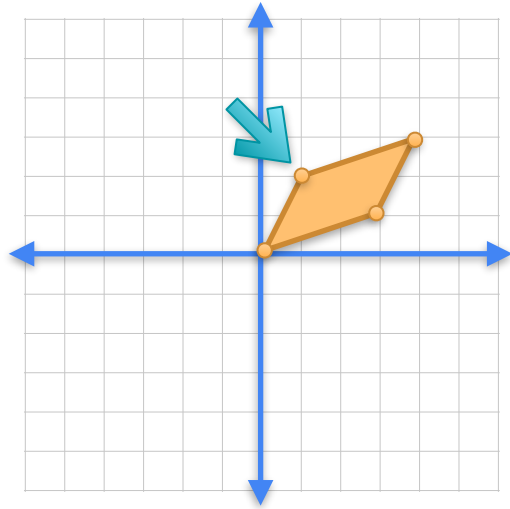


$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

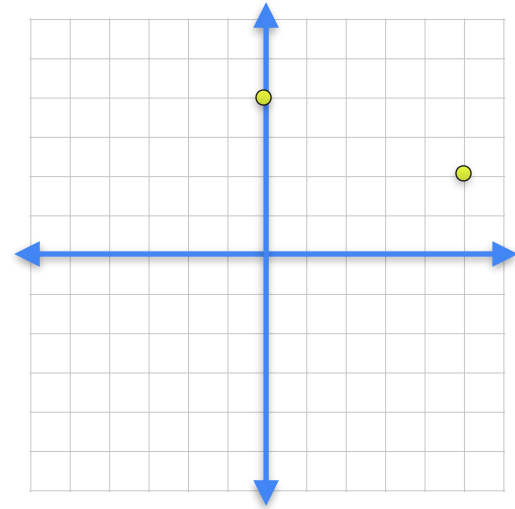


Combining linear transformations

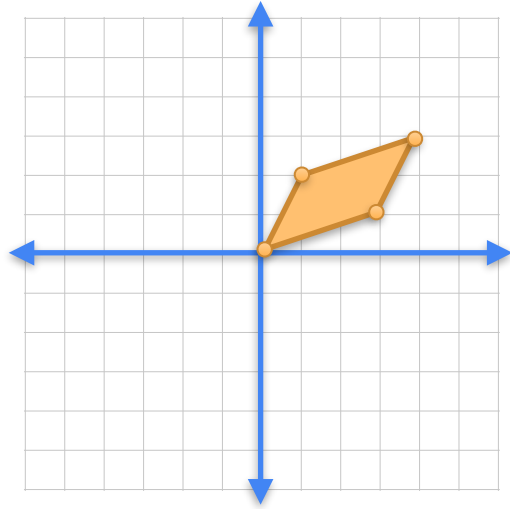


$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

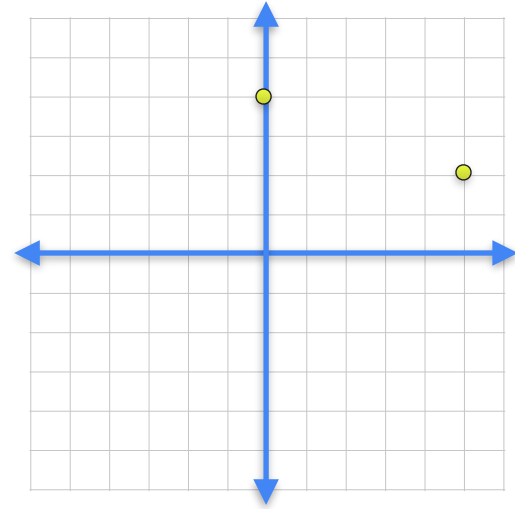


Combining linear transformations

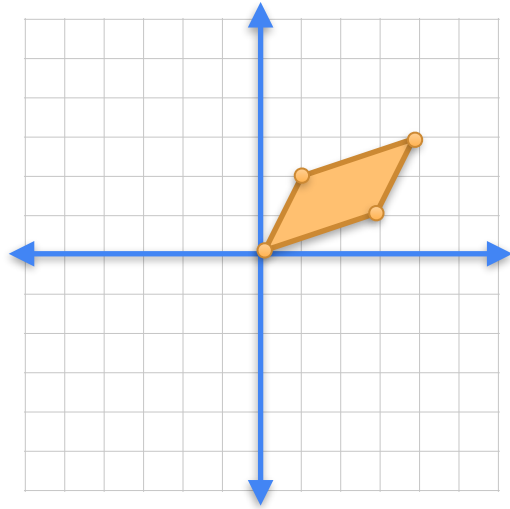


$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

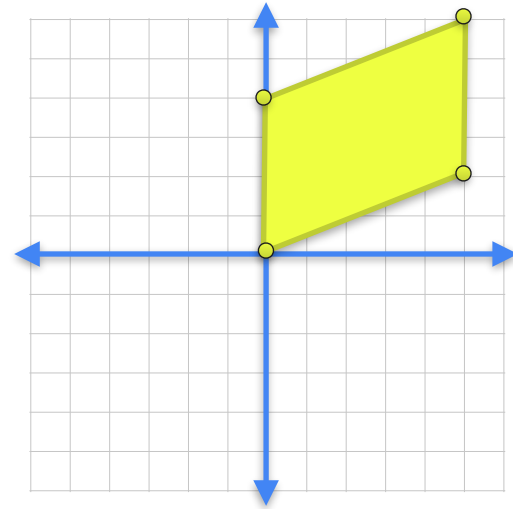


Combining linear transformations

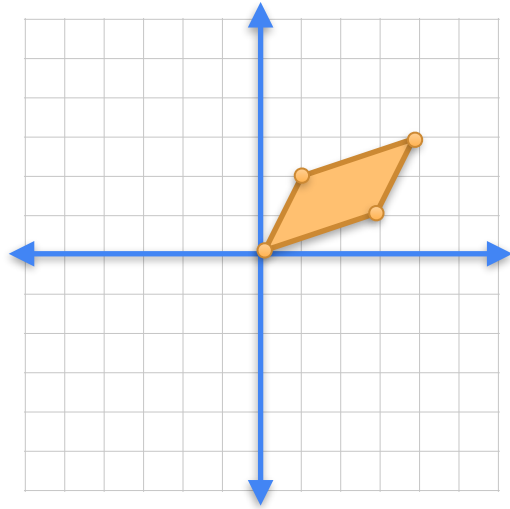


$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

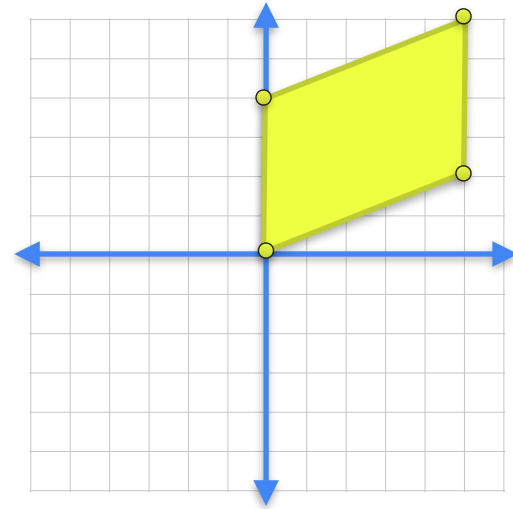


Combining linear transformations

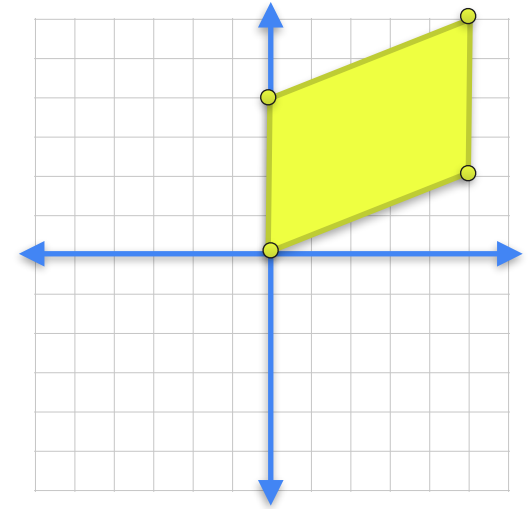
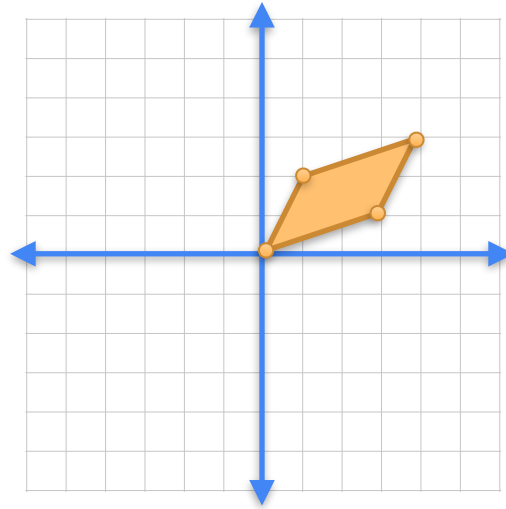
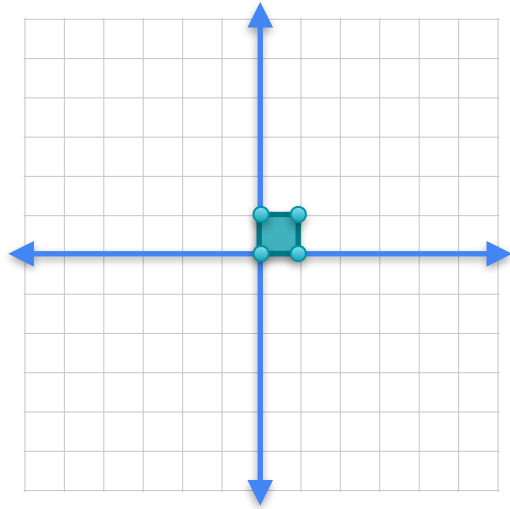


$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

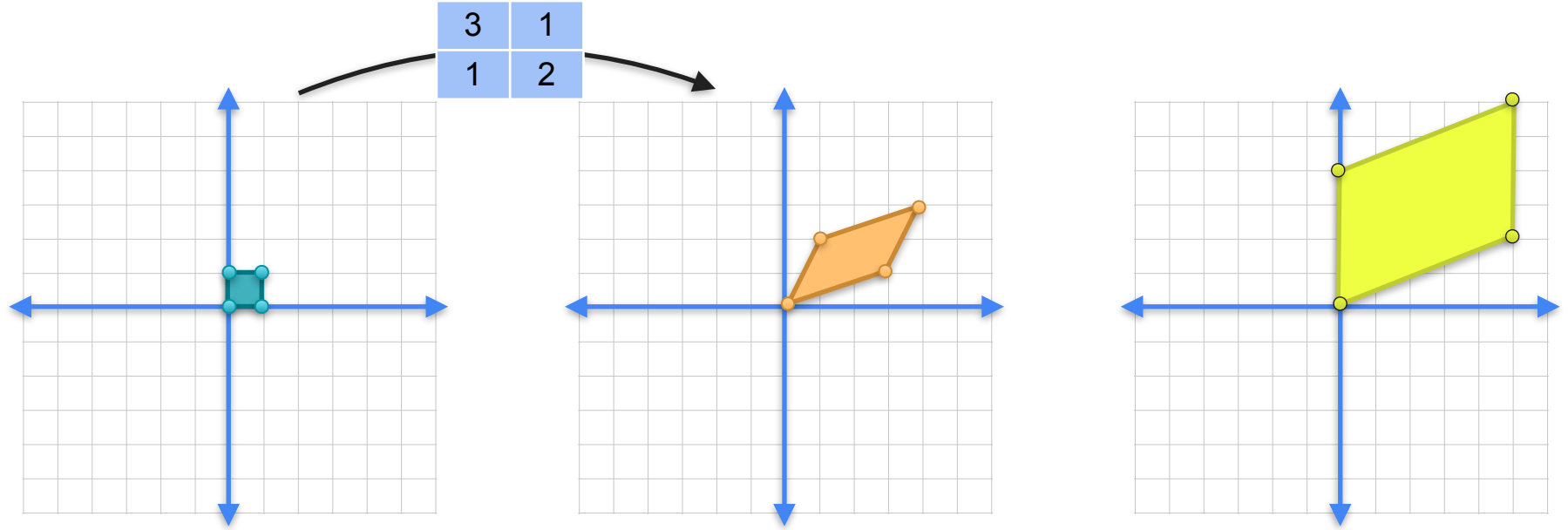
$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$



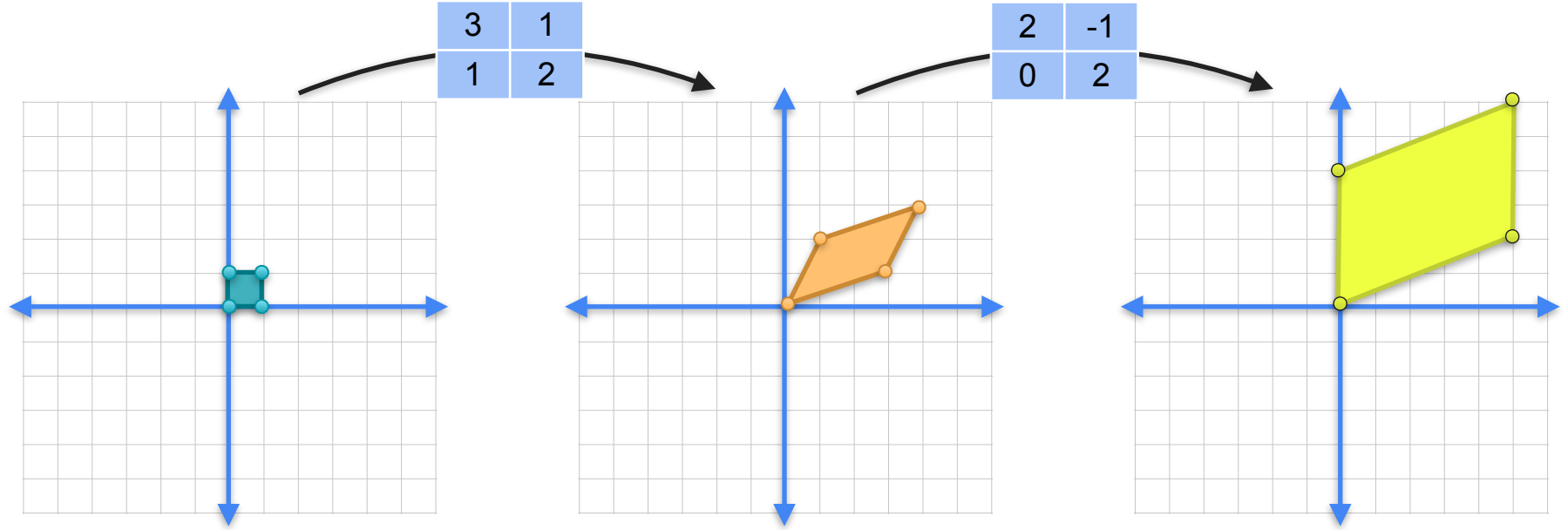
Combining linear transformations



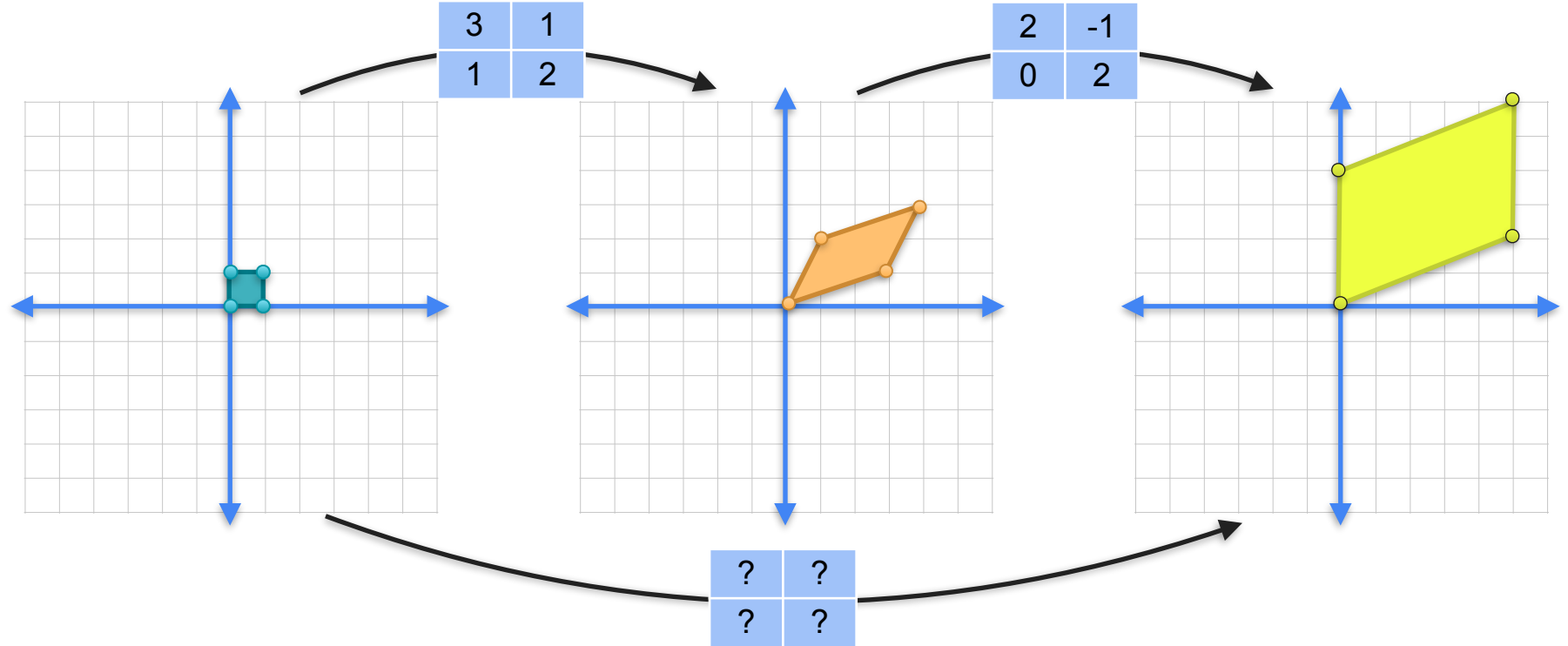
Combining linear transformations



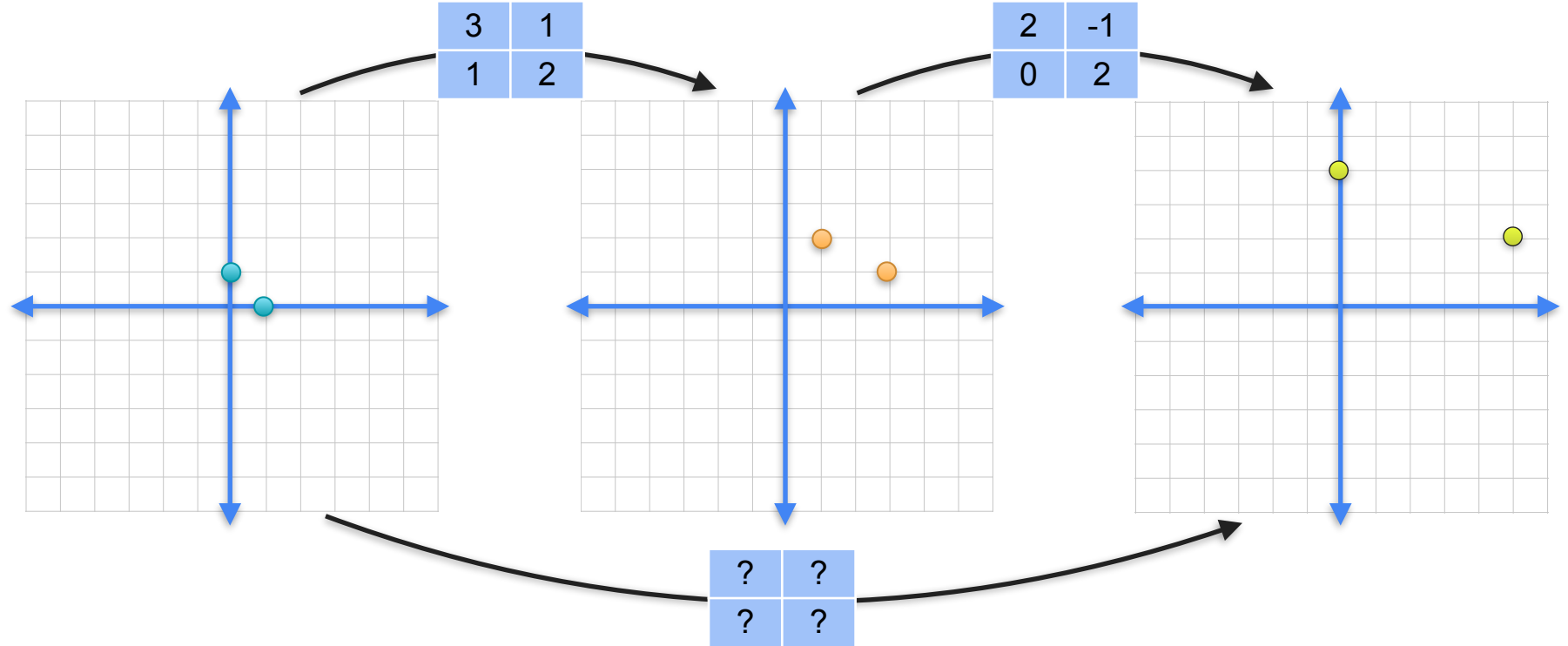
Combining linear transformations



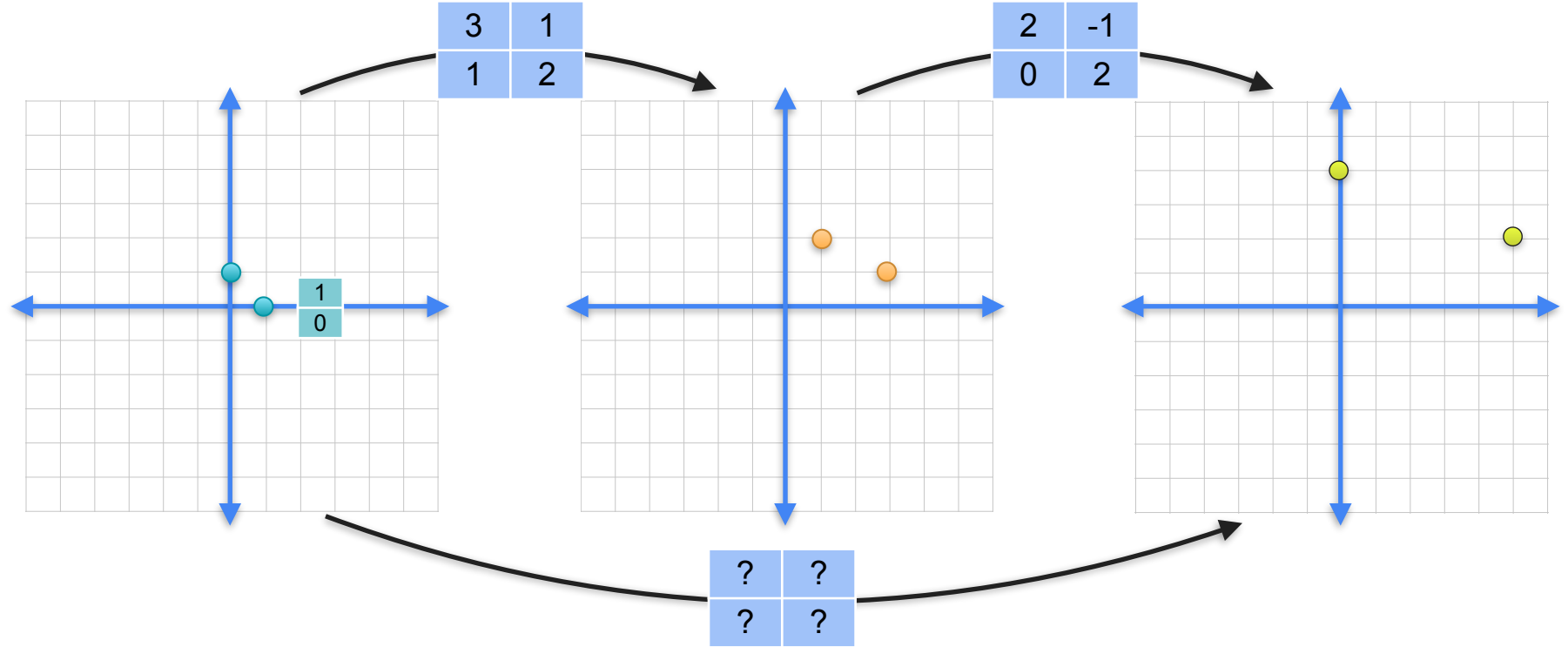
Combining linear transformations



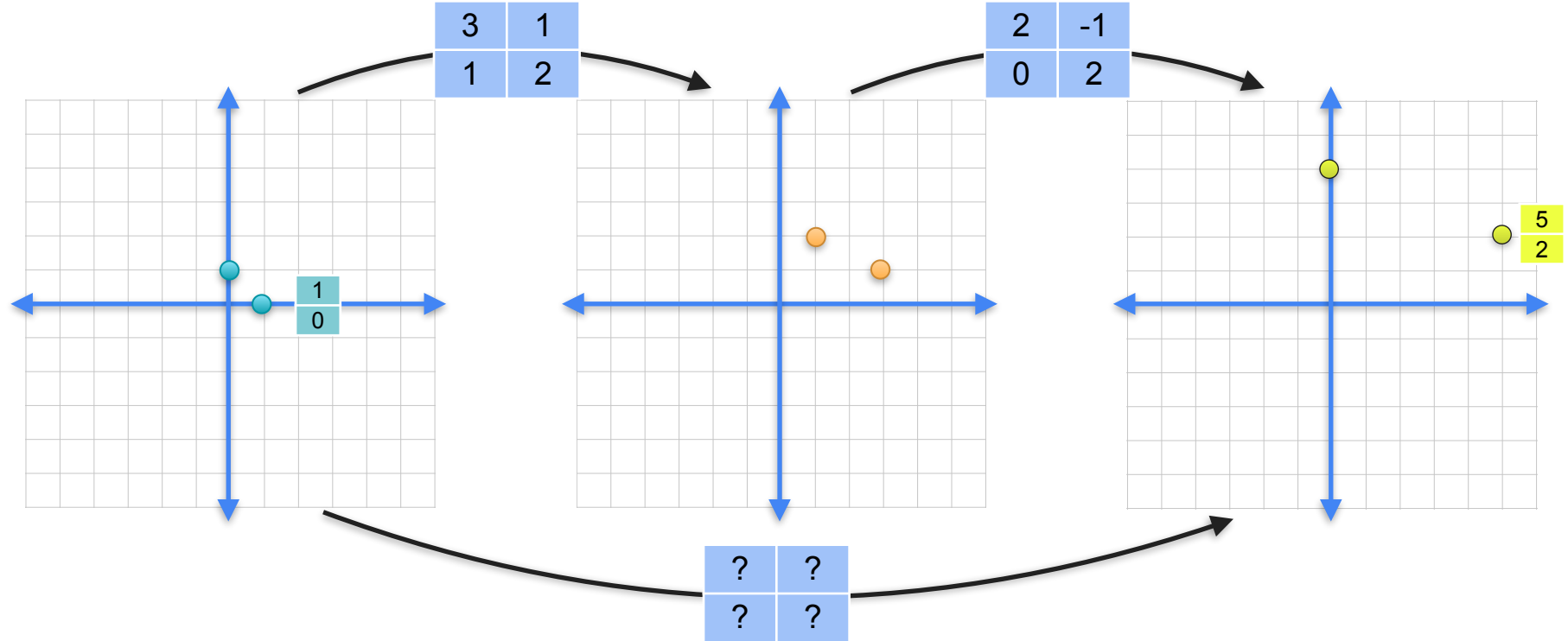
Combining linear transformations



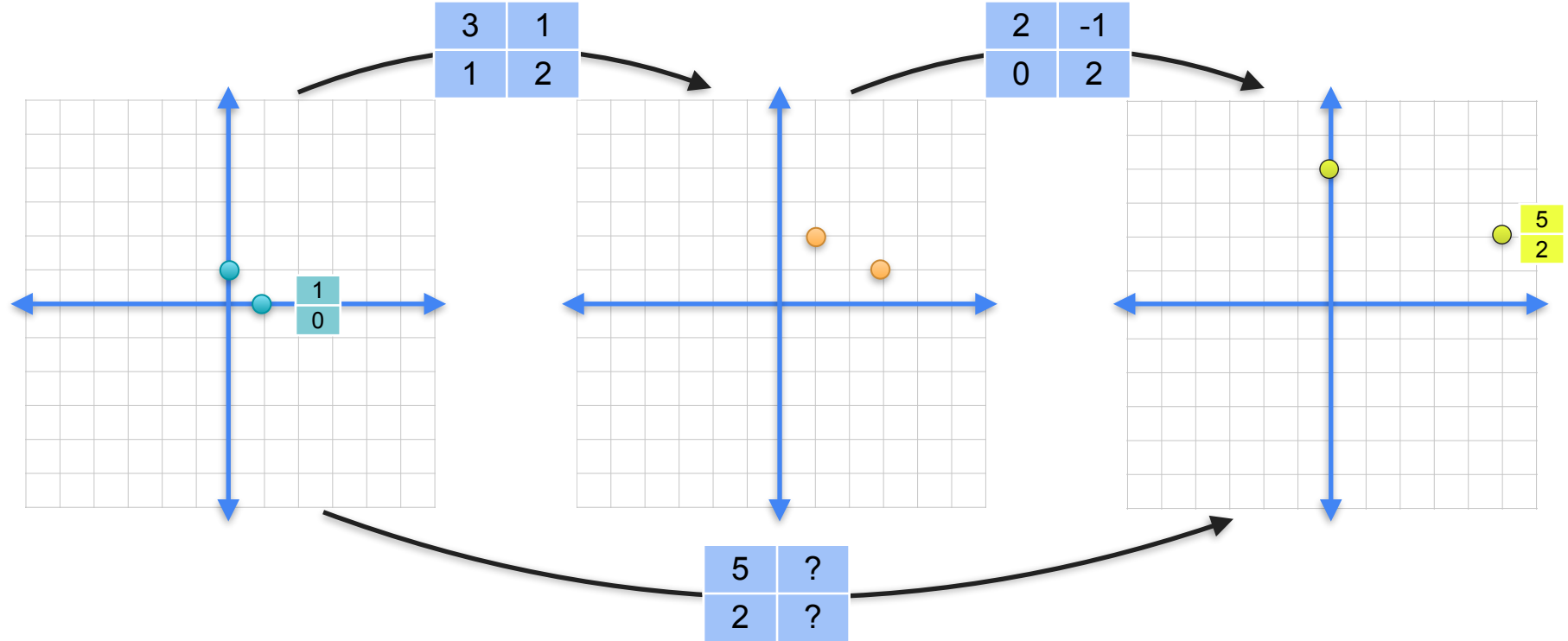
Combining linear transformations



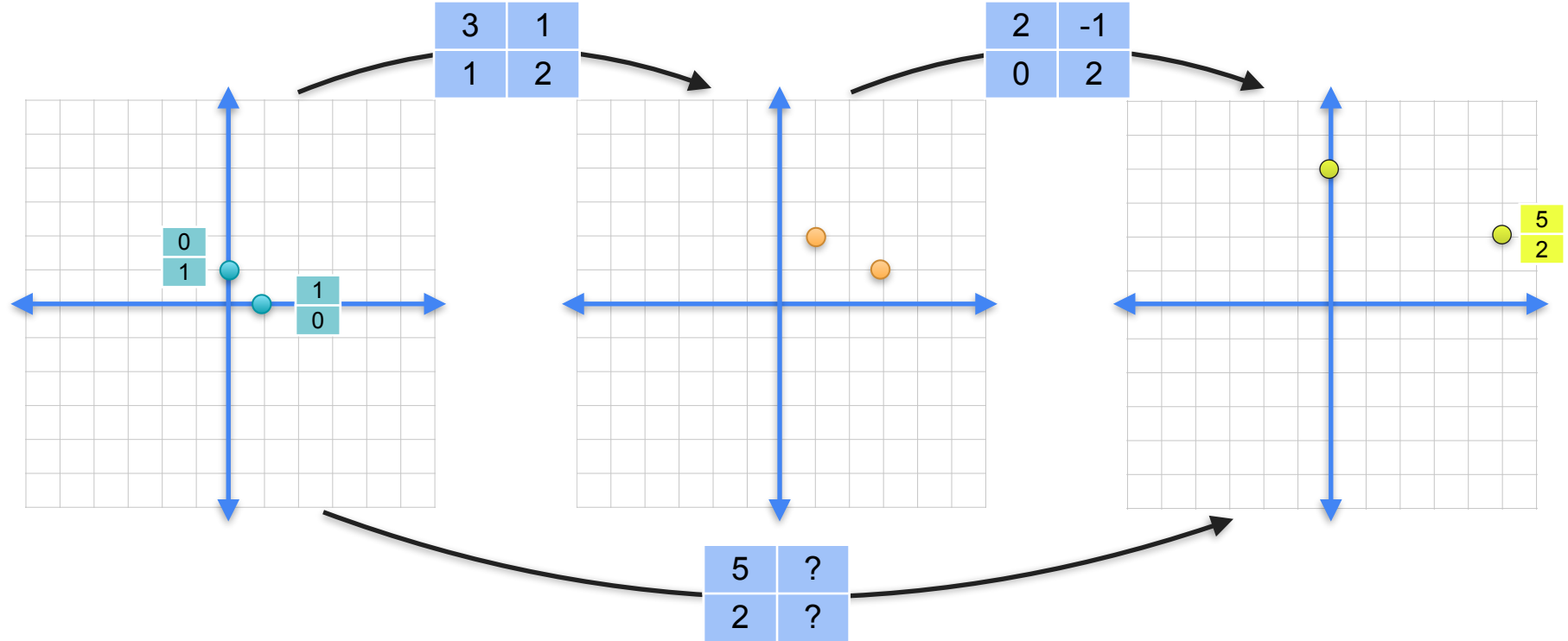
Combining linear transformations



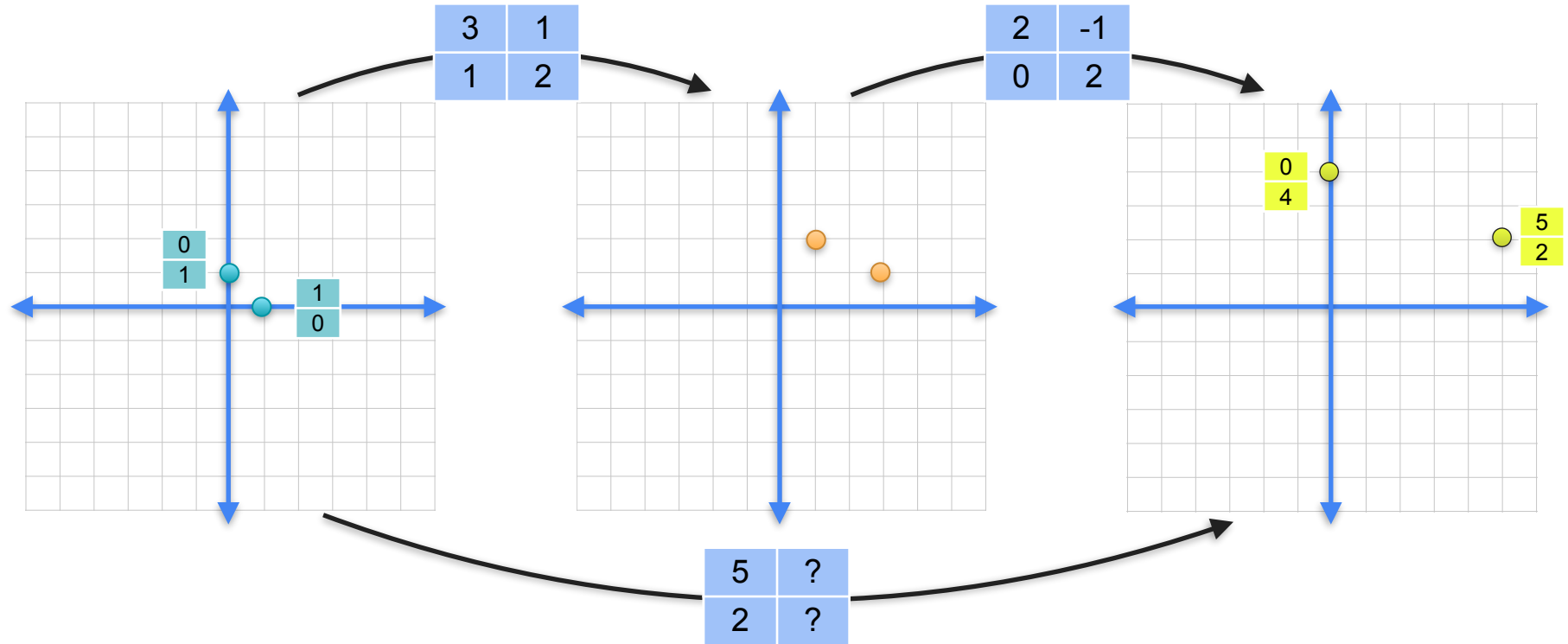
Combining linear transformations



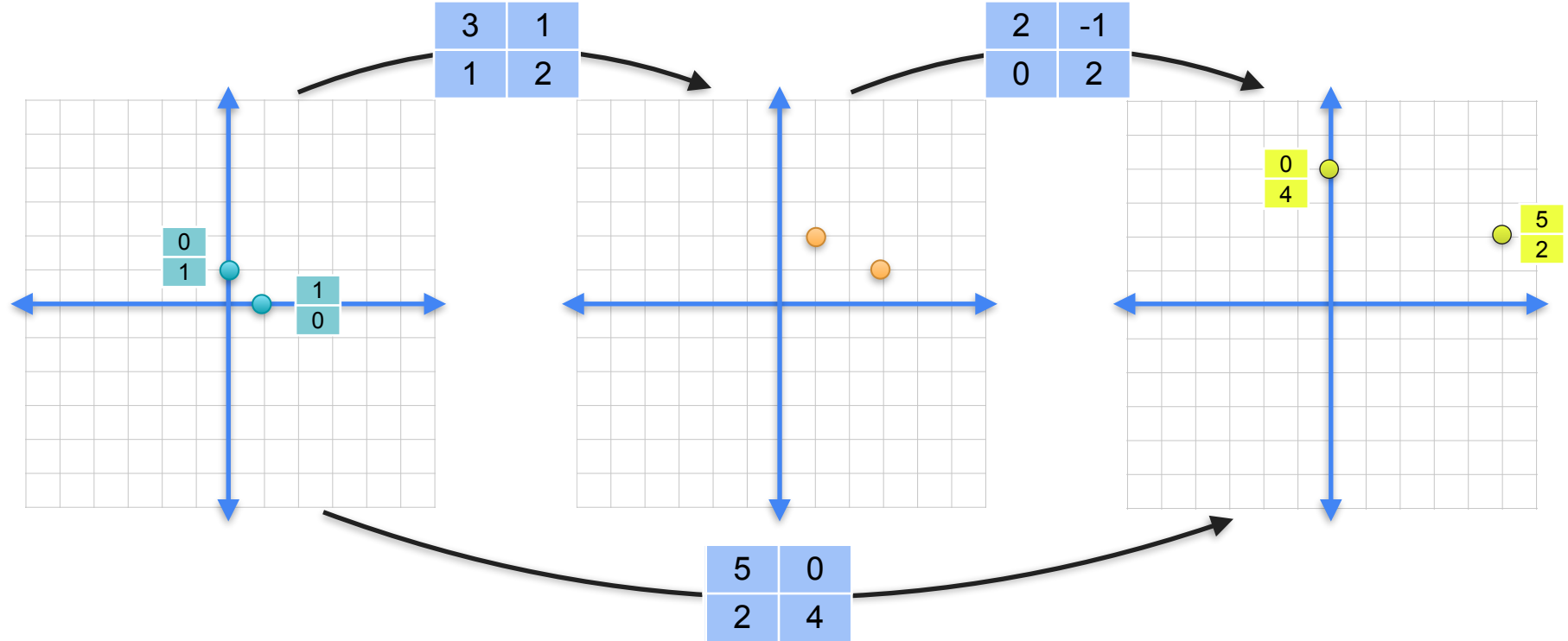
Combining linear transformations



Combining linear transformations



Combining linear transformations



Combining linear transformations

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 2 & 4 \end{bmatrix}$$

Combining linear transformations

First

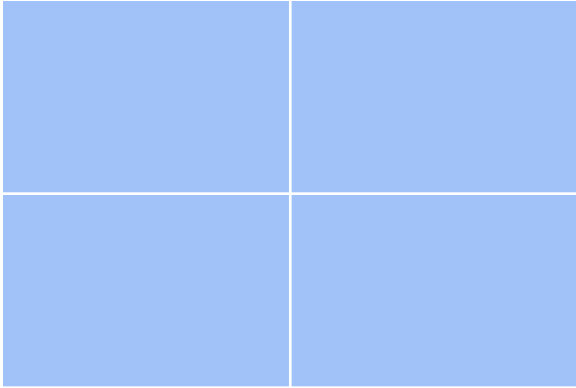
↓

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 2 & 4 \end{bmatrix}$$

Combining linear transformations

$$\begin{array}{c} \text{Second} \\ \downarrow \\ \begin{array}{|c|c|} \hline 2 & -1 \\ \hline 0 & 2 \\ \hline \end{array} \end{array} \cdot \begin{array}{c} \text{First} \\ \downarrow \\ \begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array} \end{array} = \begin{array}{|c|c|} \hline 5 & 0 \\ \hline 2 & 4 \\ \hline \end{array}$$

Multiplying matrices

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 5 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix}$$


Multiplying matrices

The diagram illustrates the process of multiplying two 2x2 matrices. The first matrix (teal) is $\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}$ and the second matrix (orange) is $\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$. The result is a 2x2 matrix (light blue) where each element is the dot product of a row from the first matrix and a column from the second matrix.

Row 1 of the result: $2 \cdot 3 + (-1) \cdot 1 = 6 - 1 = 5$ and $2 \cdot 1 + (-1) \cdot 2 = 2 - 2 = 0$

Row 2 of the result: $0 \cdot 3 + 2 \cdot 1 = 0 + 2 = 2$ and $0 \cdot 1 + 2 \cdot 2 = 0 + 4 = 4$

The resulting matrix is $\begin{bmatrix} 5 & 0 \\ 2 & 4 \end{bmatrix}$.

Multiplying matrices

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & \begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} \\ \begin{bmatrix} 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} & \begin{bmatrix} 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} \end{bmatrix}$$

Multiplying matrices

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ \begin{bmatrix} 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} & \begin{bmatrix} 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} \end{bmatrix}$$

Multiplying matrices

The diagram illustrates the multiplication of two 2x2 matrices. The first matrix (teal) has values $\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}$. The second matrix (orange) has values $\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$. The result is a 2x2 matrix (blue) with values $\begin{bmatrix} 5 & 0 \\ 2 & \begin{bmatrix} 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} \end{bmatrix}$. The bottom-right element of the result matrix is shown as the product of the row vector $\begin{bmatrix} 0 & 2 \end{bmatrix}$ and the column vector $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 2 & \begin{bmatrix} 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} \end{bmatrix}$$

Multiplying matrices

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 2 & 4 \end{bmatrix}$$



DeepLearning.AI

Vectors and Linear Transformations

The identity matrix

The identity matrix

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

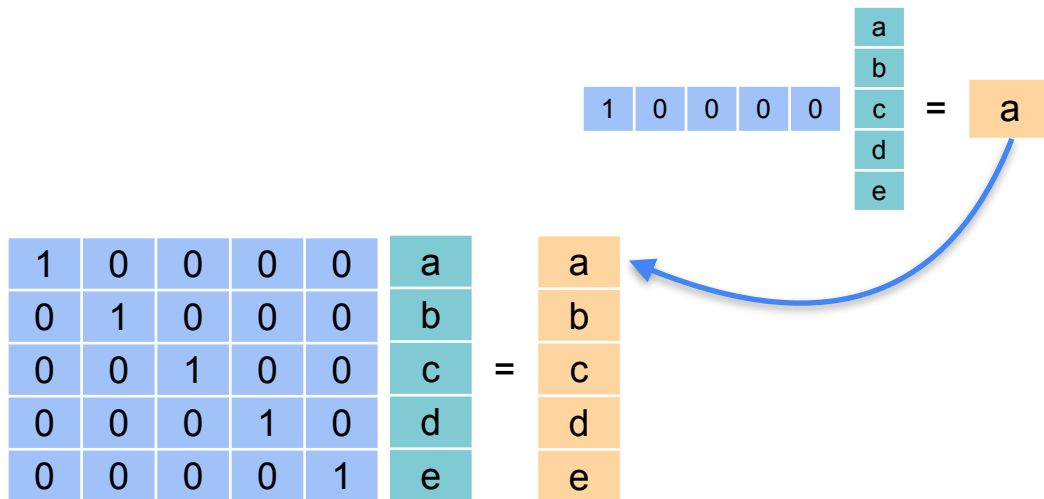
The identity matrix

1	0	0	0	0	a
0	1	0	0	0	b
0	0	1	0	0	c
0	0	0	1	0	d
0	0	0	0	1	e

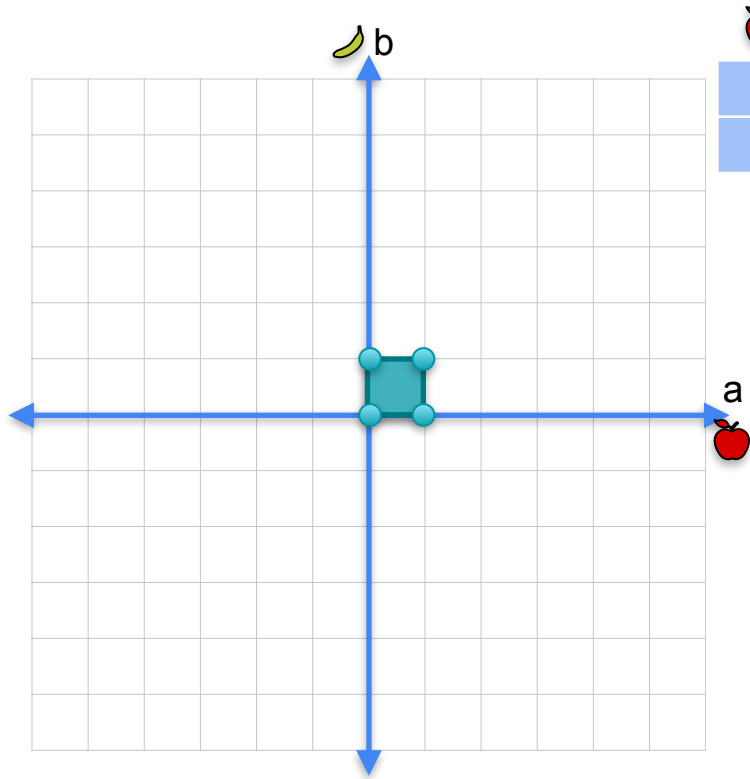
The identity matrix

1	0	0	0	0	a	=	a
0	1	0	0	0	b		b
0	0	1	0	0	c		c
0	0	0	1	0	d		d
0	0	0	0	1	e		e

The identity matrix

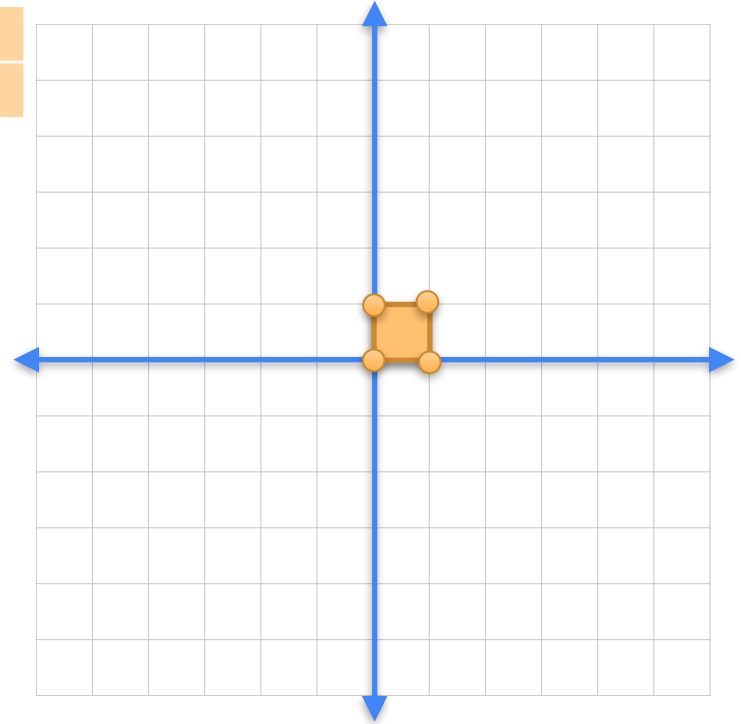


The identity matrix

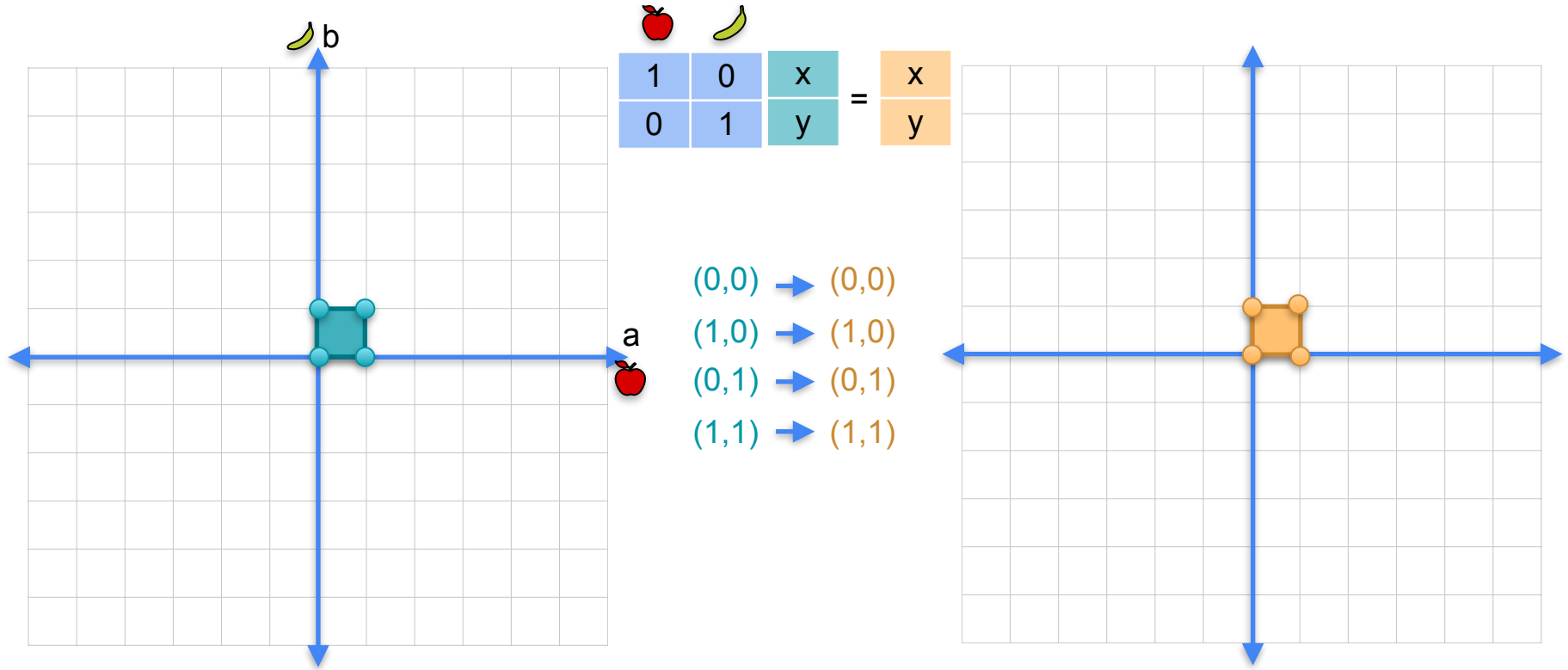


$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

The matrix is shown with a red apple icon above the first column and a yellow banana icon above the second column. The result vector is shown in an orange box.



The identity matrix





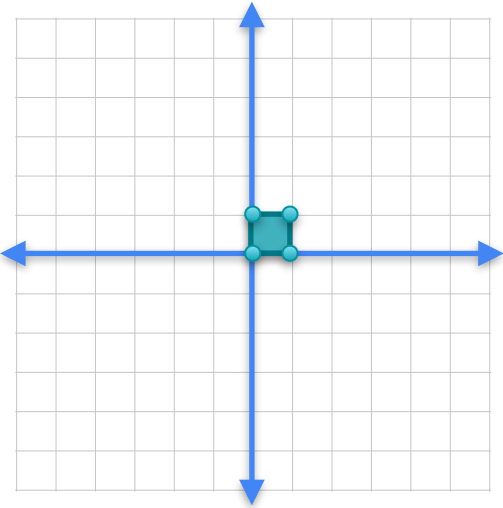
DeepLearning.AI

Vectors and Linear Transformations

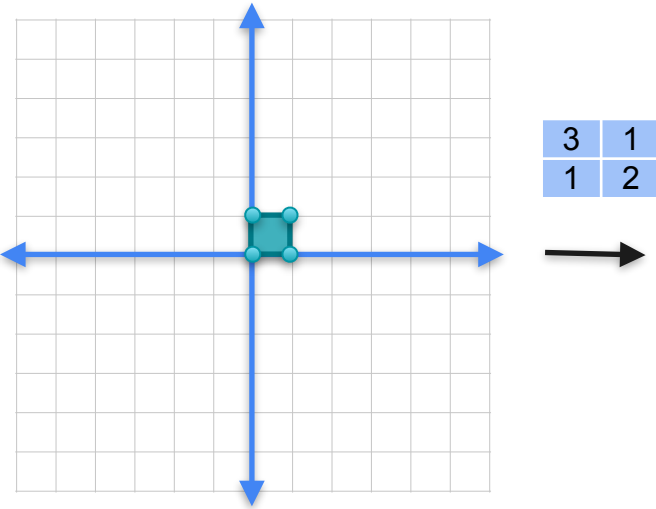
Matrix inverse

Matrix inverses

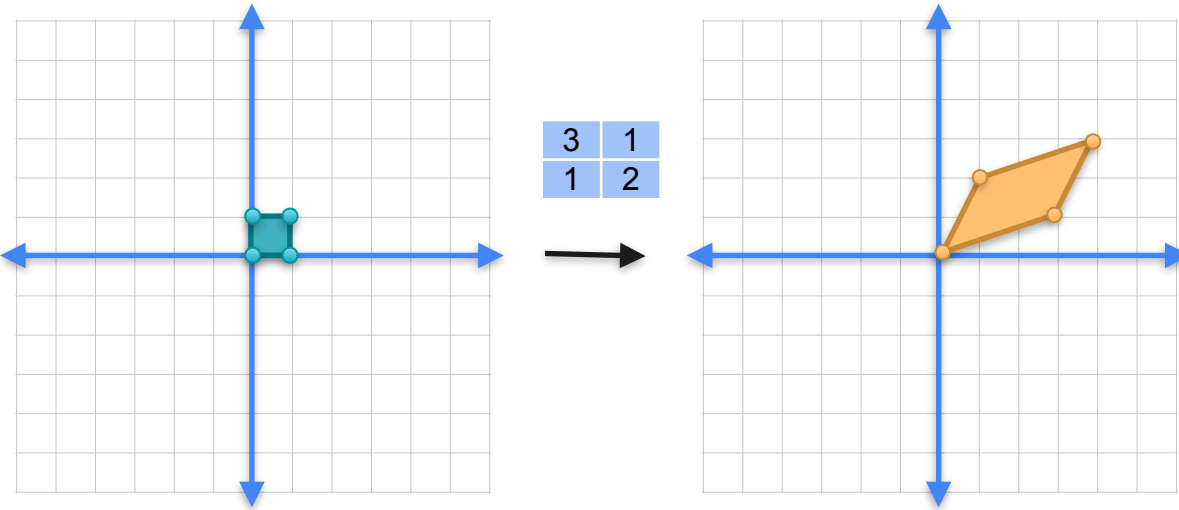
Matrix inverses



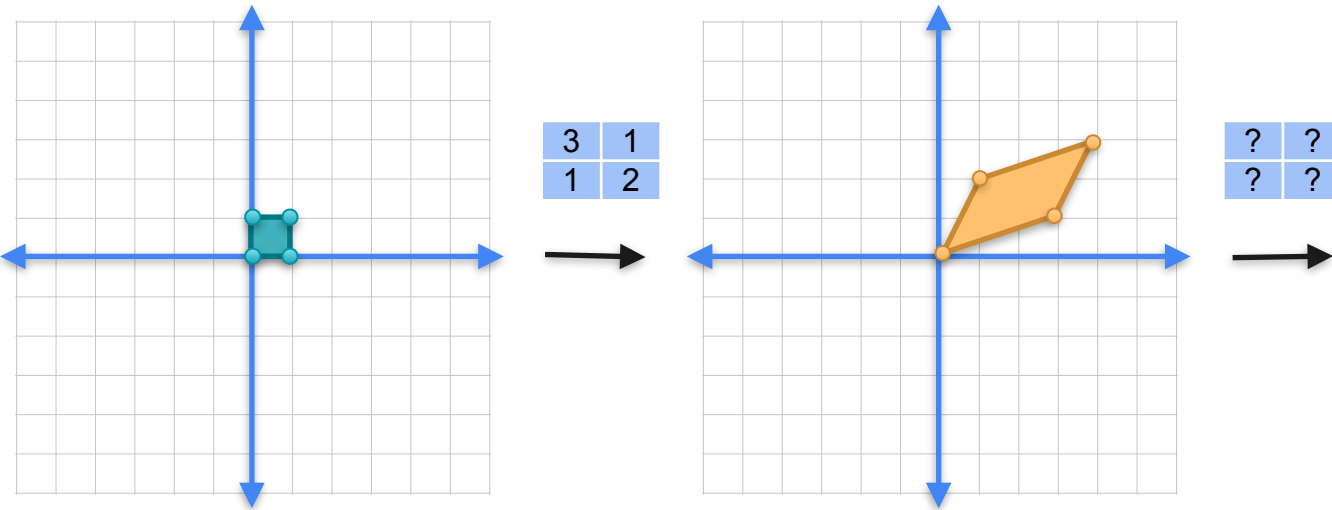
Matrix inverses



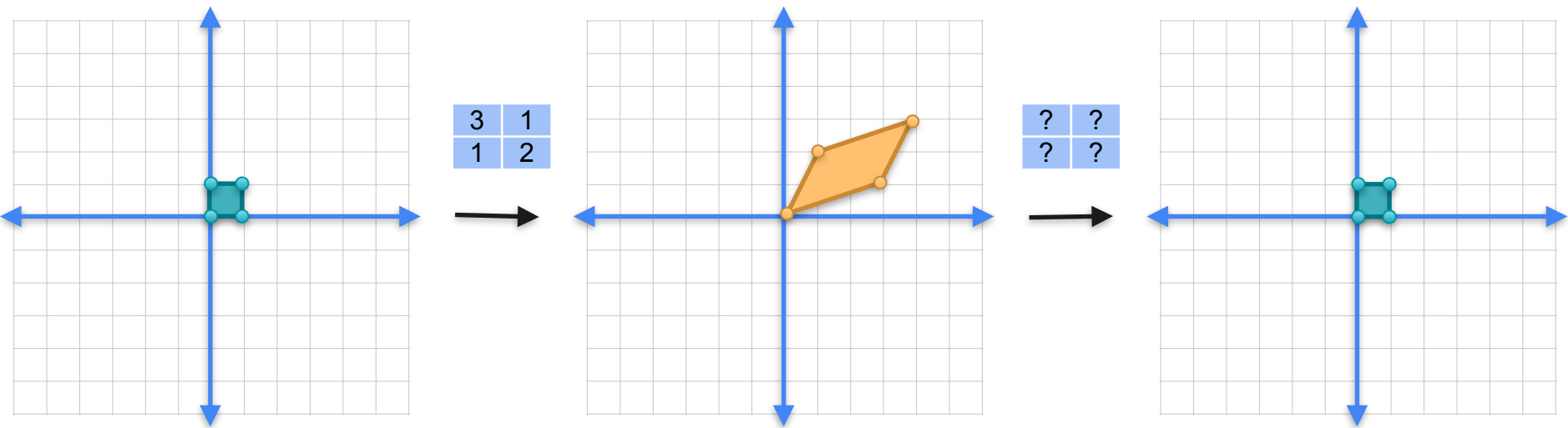
Matrix inverses



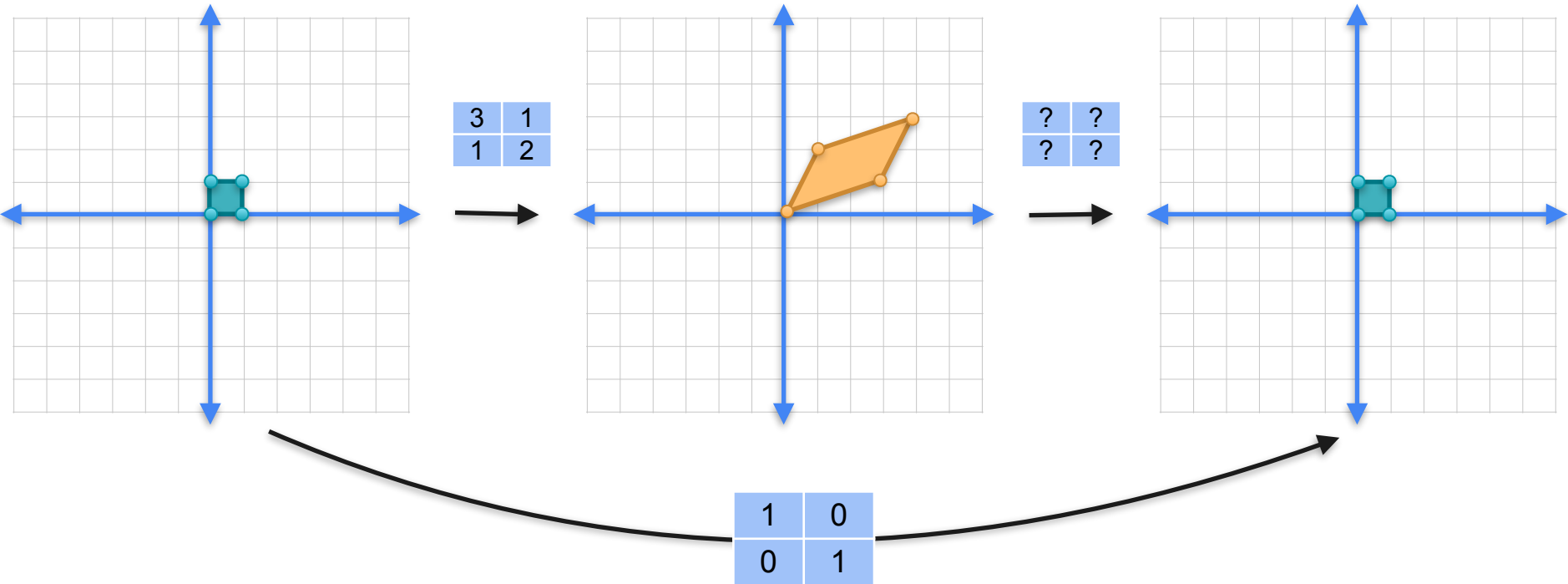
Matrix inverses



Matrix inverses



Matrix inverses



Multiplying matrices

Multiplying matrices

a	b
c	d

Multiplying matrices


$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$$

Multiplying matrices

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Multiplying matrices

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1}$$

Multiplying matrices

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 2/5 & -1/5 \\ -1/5 & 3/5 \end{bmatrix}$$

The diagram illustrates the relationship between a matrix and its inverse. A blue arrow points from the matrix $\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$ to the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ in the first equation, indicating that $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is the inverse of $\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$.

How to find an inverse?

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

How to find an inverse?

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = 1$$

$$\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 0$$

$$\begin{bmatrix} c & d \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} c & d \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 1$$

How to find an inverse?

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = 1$$

$$3a + 1b = 1$$

$$\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 0$$

$$1a + 2b = 0$$

$$\begin{bmatrix} c & d \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = 0$$

$$3c + 1d = 0$$

$$\begin{bmatrix} c & d \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 1$$

$$1c + 2d = 1$$

How to find an inverse?

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = 1$$

$$3a + 1b = 1$$

$$a = \frac{2}{5}$$

$$\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 0$$

$$1a + 2b = 0$$

$$b = -\frac{1}{5}$$

$$\begin{bmatrix} c & d \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = 0$$

$$3c + 1d = 0$$

$$c = -\frac{1}{5}$$

$$\begin{bmatrix} c & d \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 1$$

$$1c + 2d = 1$$

$$d = \frac{3}{5}$$

Quiz

- Find the inverse of the following matrix. If you find that the task is impossible, feel free to click on “I couldn’t find it”

5	2
1	2

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 1$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 0$$

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{array}{l} \begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 1 \\ \begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 0 \\ \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 0 \\ \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 1 \end{array}$$

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\bullet 5a + 2c = 1$$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\bullet 5b + 2d = 0$$

$$\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\bullet a + 2c = 0$$

$$\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\bullet b + 2d = 1$$

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 1$$

$$\bullet 5a + 2c = 1$$

$$\bullet a = 1/4$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 0$$

$$\bullet 5b + 2d = 0$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 0$$

$$\bullet a + 2c = 0$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 1$$

$$\bullet b + 2d = 1$$

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 1$$

$$\bullet 5a + 2c = 1$$

$$\bullet a = 1/4$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 0$$

$$\bullet 5b + 2d = 0$$

$$\bullet b = -1/4$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 0$$

$$\bullet a + 2c = 0$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 1$$

$$\bullet b + 2d = 1$$

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 1$$

$$\bullet 5a + 2c = 1$$

$$\bullet a = 1/4$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 0$$

$$\bullet 5b + 2d = 0$$

$$\bullet b = -1/4$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 0$$

$$\bullet a + 2c = 0$$

$$\bullet c = -1/8$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 1$$

$$\bullet b + 2d = 1$$

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 1$$

$$\bullet 5a + 2c = 1$$

$$\bullet a = 1/4$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 0$$

$$\bullet 5b + 2d = 0$$

$$\bullet b = -1/4$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 0$$

$$\bullet a + 2c = 0$$

$$\bullet c = -1/8$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 1$$

$$\bullet b + 2d = 1$$

$$\bullet d = 5/8$$

Quiz

- Find the inverse of the following matrix. If you find that the task is impossible, feel free to click on “I’m reaching a dead end”

1	1
2	2

Solutions

- The inverse doesn't exist!

We need to solve the following system of linear equations:

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$a + c = 1$$

$$2b + 2d = 1$$

$$2a + 2c = 0$$

$$b + d = 0$$

This is clearly a contradiction, since equation 1 says $a+c=1$, and equation 3 says $2a+2c=0$.



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Vectors and Linear Transformations

**Which matrices have an
inverse?**

Which matrices have inverses?

Which matrices have inverses?

$$5^{-1} = 0.2$$

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Non-singular matrix

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Singular matrix

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix
Invertible

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Singular matrix

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix
Invertible

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix
Invertible

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Singular matrix

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

3	1
1	2

⁻¹ =

0.4	-0.2
-0.2	0.6

Non-singular matrix
Invertible

5	2
1	2

⁻¹ =

0.25	-0.25
-0.125	0.625

Non-singular matrix
Invertible

1	1
2	2

 =

?	?
?	?

Singular matrix
Non-invertible

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

3	1
1	2

⁻¹ =

0.4	-0.2
-0.2	0.6

Non-singular matrix
Invertible

$$\text{Det} = 5$$

5	2
1	2

⁻¹ =

0.25	-0.25
-0.125	0.625

Non-singular matrix
Invertible

1	1
2	2

 =

?	?
?	?

Singular matrix
Non-invertible

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

3	1
1	2

⁻¹ =

0.4	-0.2
-0.2	0.6

Non-singular matrix
Invertible

$$\text{Det} = 5$$

5	2
1	2

⁻¹ =

0.25	-0.25
-0.125	0.625

Non-singular matrix
Invertible

$$\text{Det} = 8$$

1	1
2	2

 =

?	?
?	?

Singular matrix
Non-invertible

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix
Invertible

$$\text{Det} = 5$$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix
Invertible

$$\text{Det} = 8$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Singular matrix
Non-invertible

$$\text{Det} = 0$$

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix
Invertible

$$\text{Det} = 5$$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix
Invertible

$$\text{Det} = 8$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Singular matrix
Non-invertible

$$\text{Det} = 0$$

Non-zero determinants

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix
Invertible

$$\text{Det} = 5$$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix
Invertible

$$\text{Det} = 8$$

Non-zero determinants

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Singular matrix
Non-invertible

$$\text{Det} = 0$$

Zero determinant



DeepLearning.AI

Vectors and Linear Transformations

**Neural networks and
matrices**



Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Scores:

Lottery: ____ points

Win: ____ points

Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Scores:

Lottery: ____ points

Win: ____ points

Examples

Lottery: 3 point

Win: 2 points

“Win, win the lottery!” : 7points

Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Scores:

Lottery: ____ points

Win: ____ points

Examples

Lottery: 3 point

Win: 2 points

“Win, win the lottery!” : 7points

Rule:

If the number of points of the sentence is bigger than ____, then the email is spam.

Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Scores:

Lottery: ____ points

Win: ____ points

Examples

Lottery: 3 point

Win: 2 points

“Win, win the lottery!” : 7points

Rule:

If the number of points of the sentence is bigger than ____, then the email is spam.

Goal: Find the best points and threshold

Lottery: ____ point

Win: ____ point

Threshold: ____ points

Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Score	> 1.5?
2	Yes
3	Yes
0	No
2	Yes
1	No
1	No
4	Yes
2	Yes
3	Yes

Solution:

Lottery: 1 point

Win: 1 point

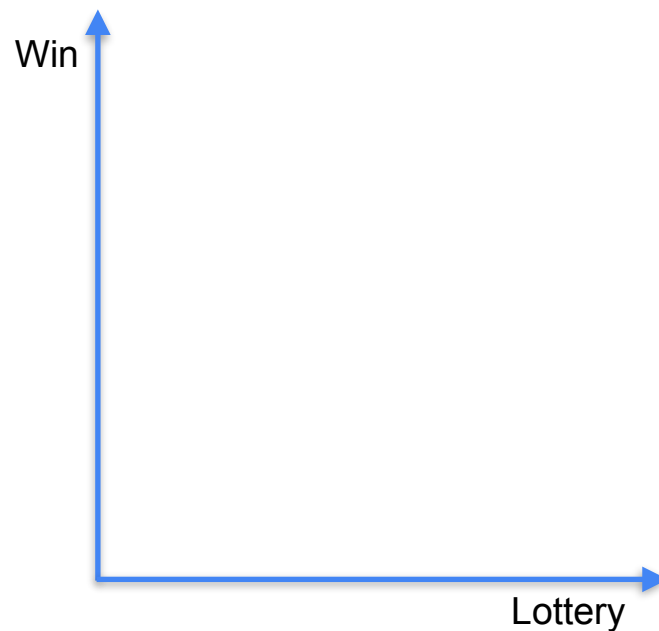
Threshold: 1.5 points

Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

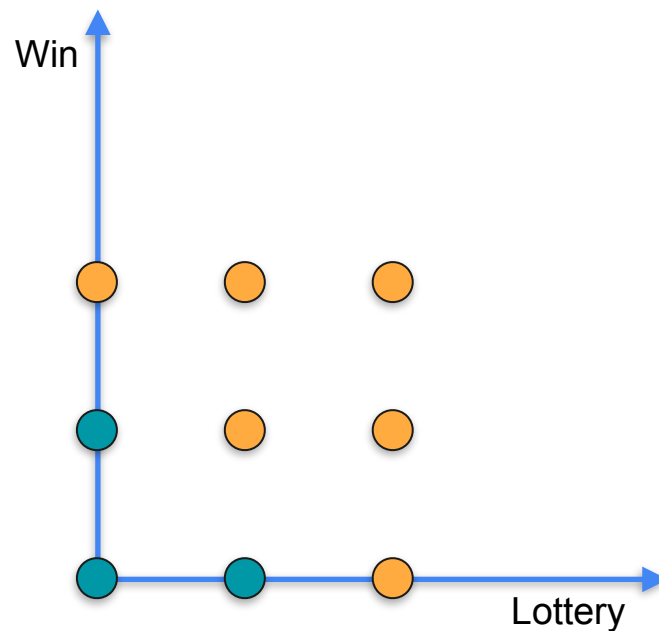
Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2



Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

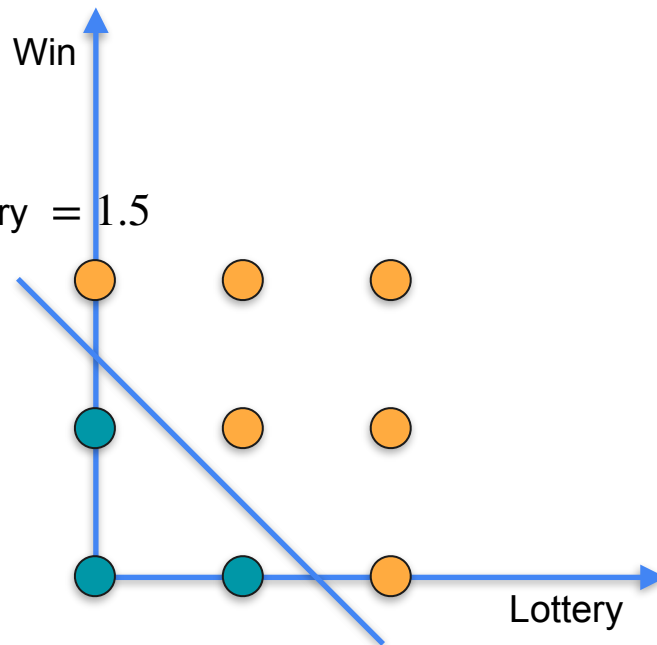


Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Line:

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} = 1.5$$

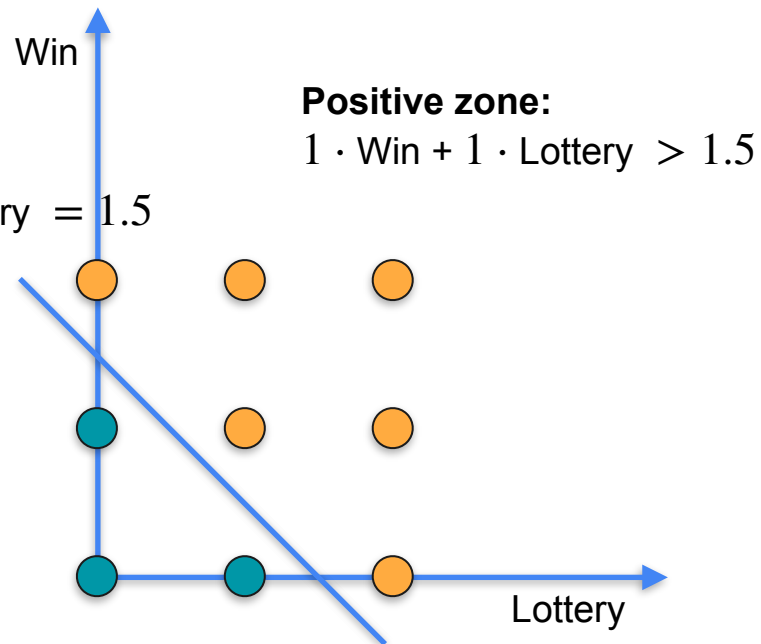


Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Line:

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} = 1.5$$



Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Line:

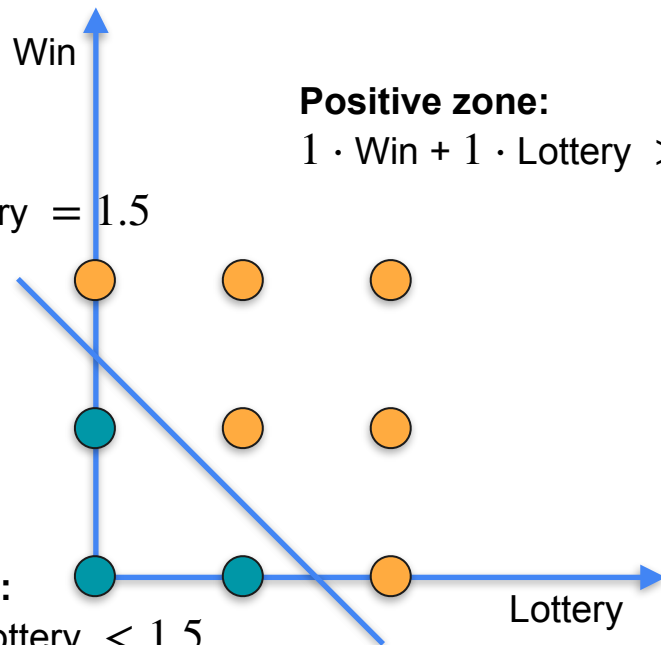
$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} = 1.5$$

Negative zone:

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} < 1.5$$

Positive zone:

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} > 1.5$$



Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

Check: > 1.5 ?

Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

2	1
---	---

Model
1
1

Check: > 1.5?

Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

2

1

Model
1
1

= 3

Check: > 1.5?

Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

2

1

Model
1
1

= 3

Check: > 1.5?



Spam

Dot product between vectors

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

Check: > 1.5 ?

Dot product between vectors

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

0	1
---	---

Model
1
1

Check: > 1.5 ?

Dot product between vectors

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

0

1

Model
1
1

= 1

Check: > 1.5?

Dot product between vectors

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

0

1

Model
1
1

= 1

Check: > 1.5?



Not spam

Matrix multiplication

Spam	Lottery	Win	<div>Model</div> <div>1</div> <div>1</div>
Yes	1	1	
Yes	2	1	
No	0	0	
Yes	0	2	
No	0	1	
No	1	0	
Yes	2	2	
Yes	2	0	
Yes	1	2	

Matrix multiplication

Spam	Lottery	Win			Prod
Yes	1	1			2
Yes	2	1			3
No	0	0			0
Yes	0	2			2
No	0	1			1
No	1	0			1
Yes	2	2			4
Yes	2	0			2
Yes	1	2			3

Model
1
1

=

2
3
0
2
1
1
4
2
3

Matrix multiplication

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

=

Prod
2
3
0
2
1
1
4
2
3

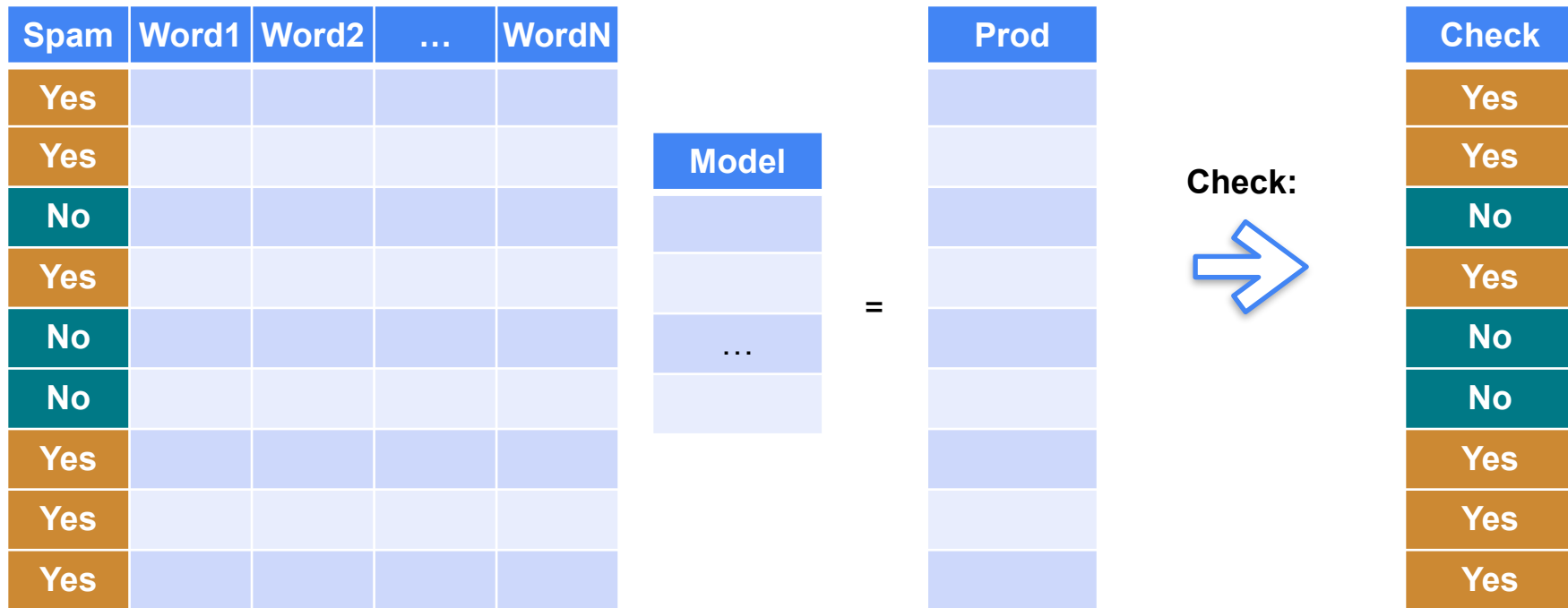
Check: >1.5?



Matrix multiplication

Spam	Lottery	Win	Model	=	Prod	Check: >1.5?	Check
Yes	1	1					
Yes	2	1	1		2		Yes
No	0	0	1		3		Yes
Yes	0	2			0		No
No	0	1			2		Yes
No	1	0			1		No
Yes	2	2			1		No
Yes	2	0			4		Yes
Yes	1	2			2		Yes
					3		Yes

Perceptrons



Threshold and bias

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

Check: > 1.5 ?

Threshold and bias

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Check

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

 Threshold

Model
1
1

Check: > 1.5?

Threshold and bias

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Check

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

Threshold

Bias

Model
1
1

Check: > 1.5?

Threshold and bias

Spam	Lottery	Win	Bias
Yes	1	1	1
Yes	2	1	1
No	0	0	1
Yes	0	2	1
No	0	1	1
No	1	0	1
Yes	2	2	1
Yes	2	0	1
Yes	1	2	1

Check

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

Threshold

Bias

Model
1
1

Check: > 1.5?

Threshold and bias

Spam	Lottery	Win	Bias
Yes	1	1	1
Yes	2	1	1
No	0	0	1
Yes	0	2	1
No	0	1	1
No	1	0	1
Yes	2	2	1
Yes	2	0	1
Yes	1	2	1

Check

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

Threshold

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

Bias

Model
1
1
-1.5

Check: > 1.5?

Bias

Threshold and bias

Spam	Lottery	Win	Bias
Yes	1	1	1
Yes	2	1	1
No	0	0	1
Yes	0	2	1
No	0	1	1
No	1	0	1
Yes	2	2	1
Yes	2	0	1
Yes	1	2	1

Check

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

Threshold

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

Bias

Model
1
1
-1.5

Check: > 0?

Bias

The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

Model
1
1

The AND operator

AND	x	y		Dot prod
No	0	0		0
No	1	0		1
No	0	1		1
Yes	1	1		2

Model
1
1

=

Dot prod
0
1
1
2

The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

Model
1
1

=

Dot prod
0
1
1
2

Check: >1.5?



The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

Model
1
1

=

Dot prod
0
1
1
2

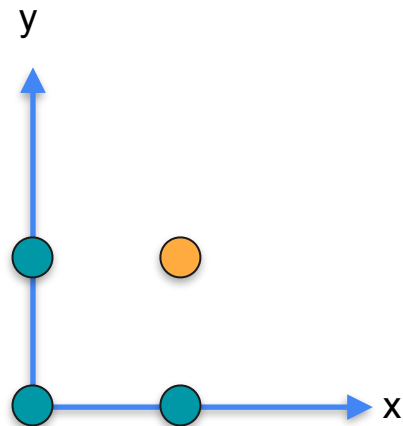
Check: >1.5?



Check
No
No
No
Yes

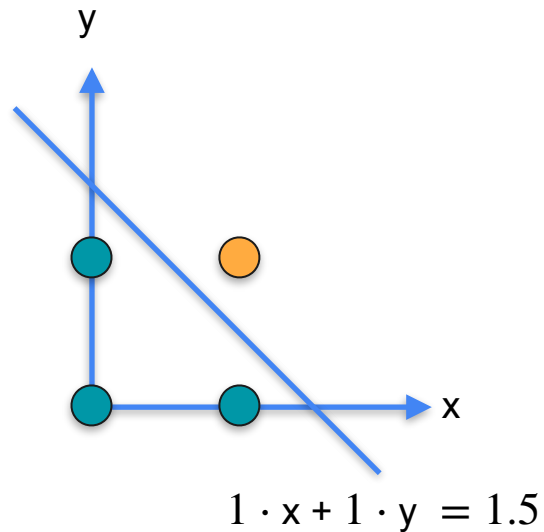
The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

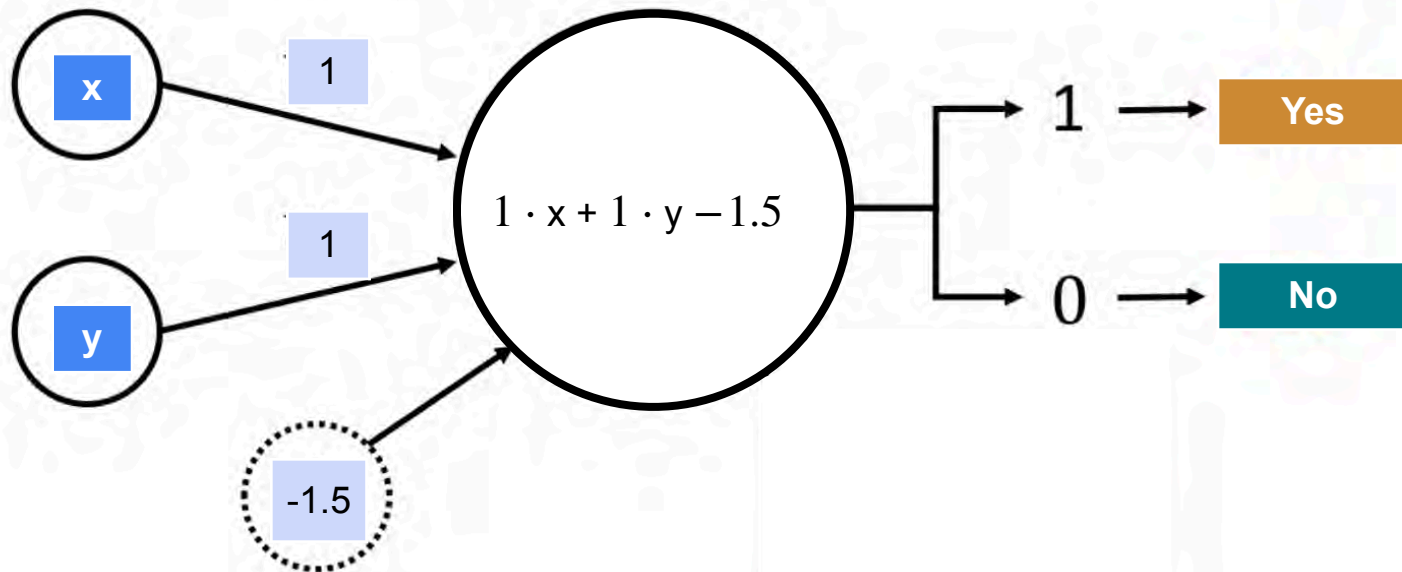


The AND operator

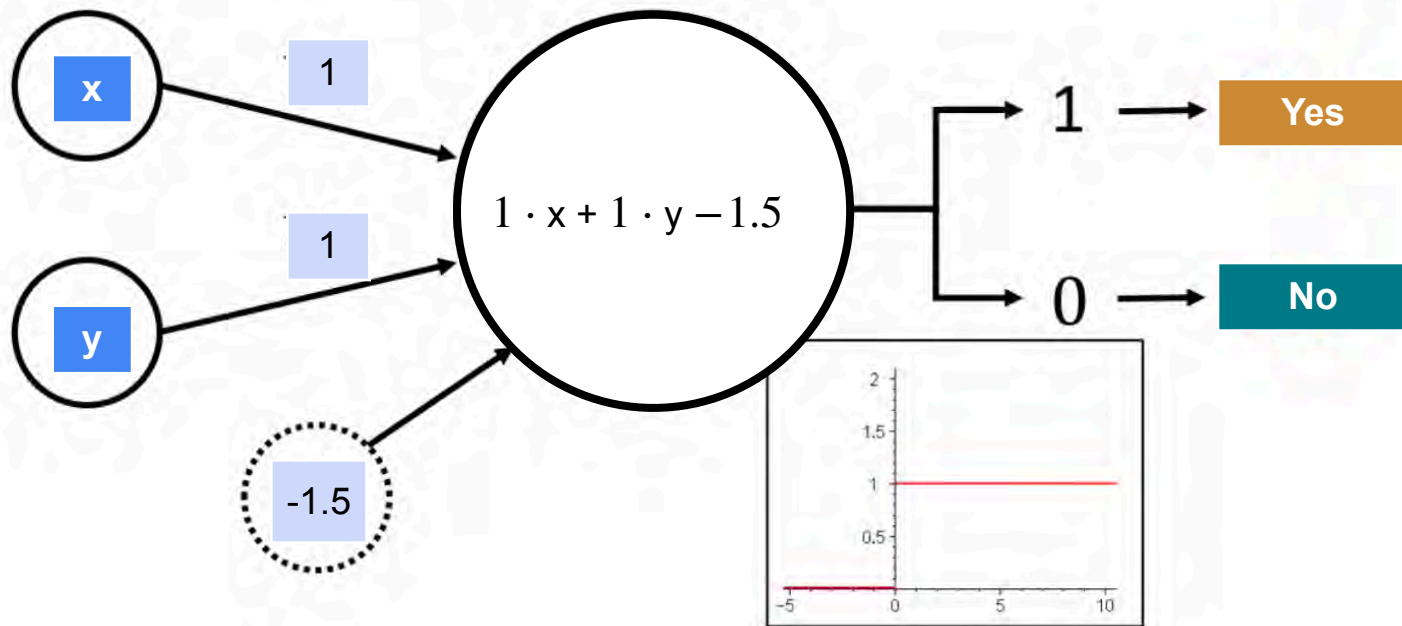
AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

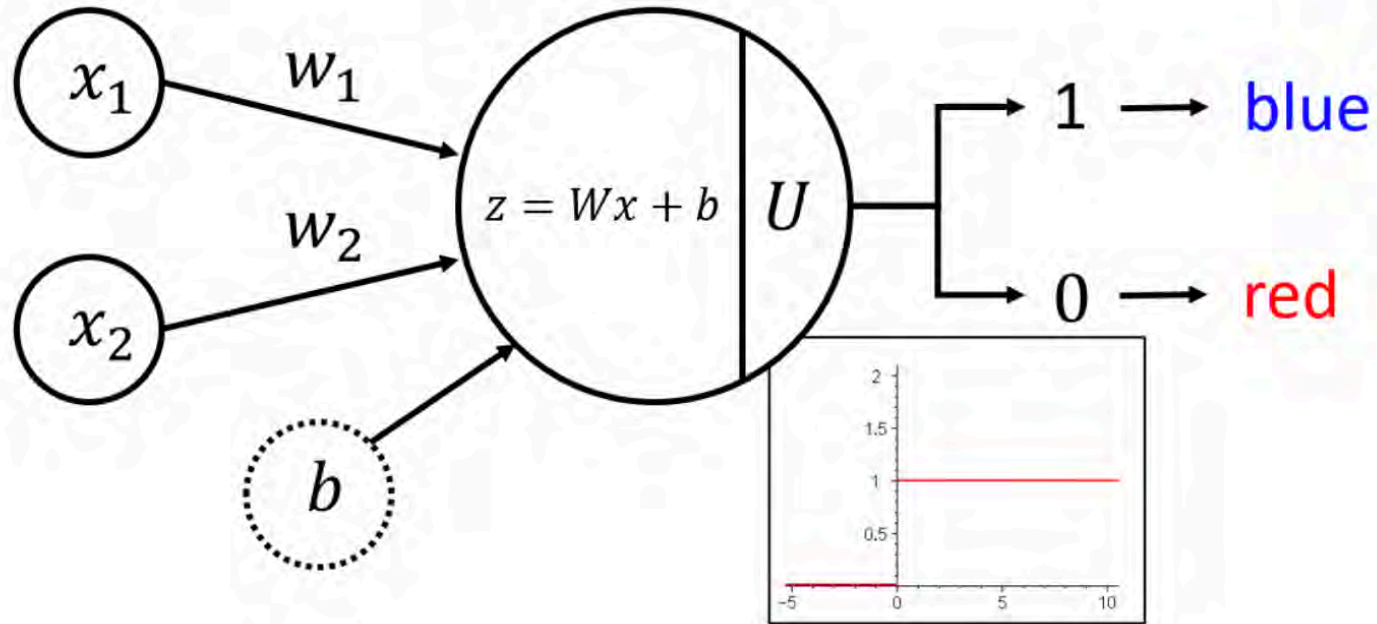


The perceptron



The perceptron







DeepLearning.AI

Vectors and Linear Transformations

Conclusion