

# **ECE1388 VLSI Design Methodology**

## **Lecture 10: Combinational Circuit Design**

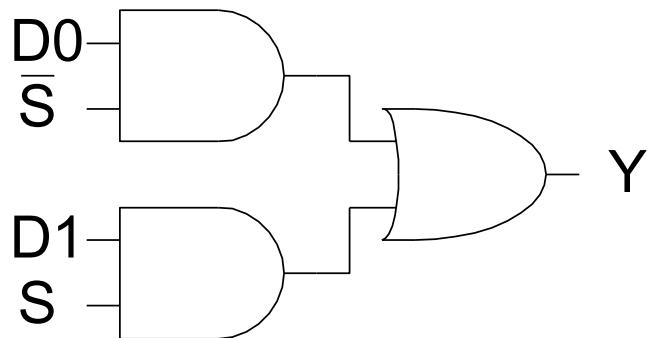
# Outline

- ❑ Bubble Pushing
- ❑ Compound Gates
- ❑ Logical Effort Example
- ❑ Input Ordering
- ❑ Asymmetric Gates
- ❑ Skewed Gates
- ❑ Best P/N ratio

# Example 1

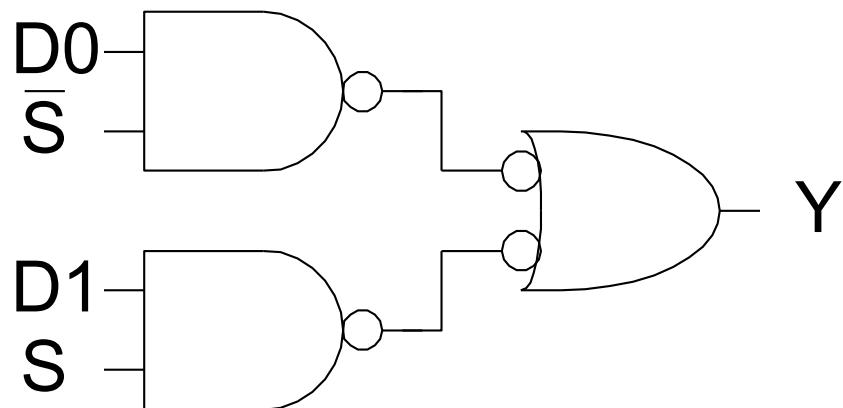
```
module mux(input  s, d0, d1,  
           output y);  
  
  assign y = s ? d1 : d0;  
endmodule
```

- 1) Sketch a design using AND, OR, and NOT gates.  
Assume  $\sim S$  is available.



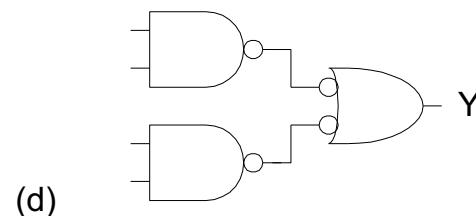
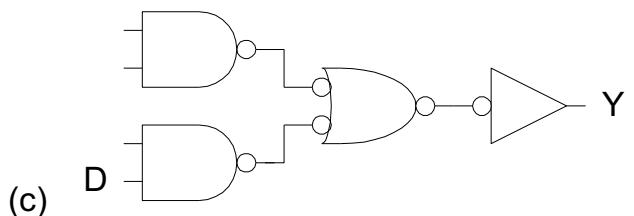
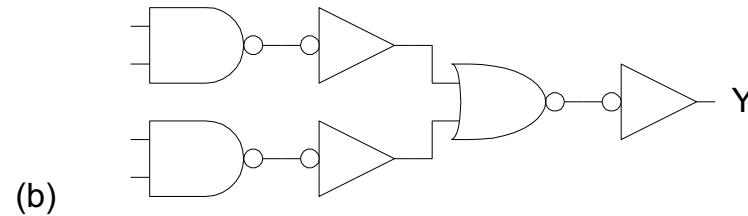
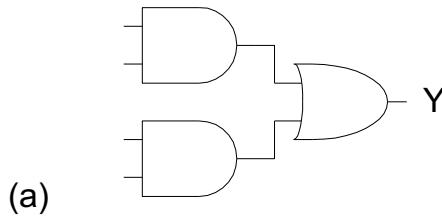
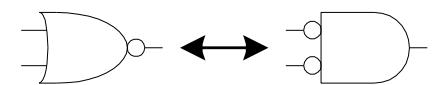
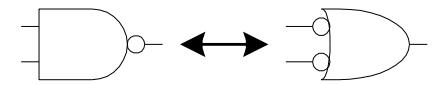
# Example 2

- 2) Sketch a design using NAND, NOR, and NOT gates.  
Assume  $\sim S$  is available.



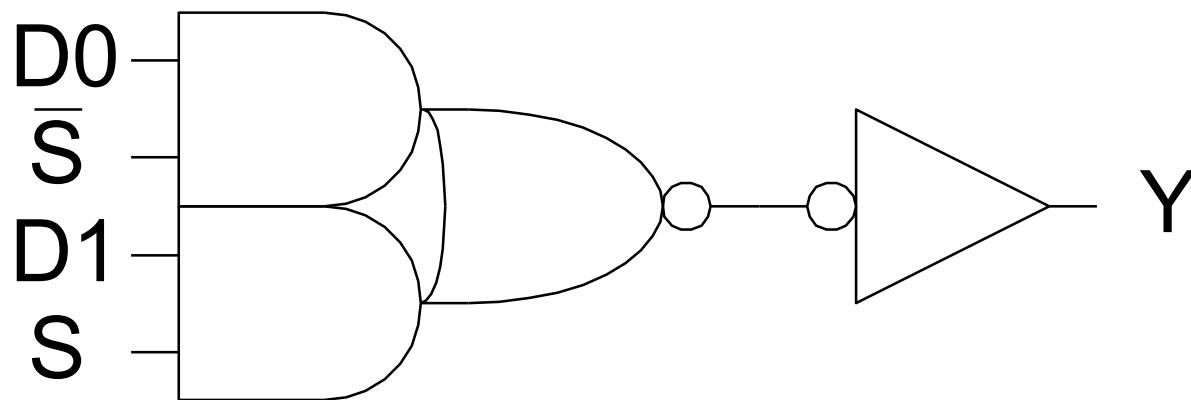
# Bubble Pushing

- Start with network of AND / OR gates
- Convert to NAND / NOR + inverters
- Push bubbles around to simplify logic
  - Remember DeMorgan’s Law



# Example 3

- 3) Sketch a design using one compound gate and one NOT gate. Assume  $\sim S$  is available.

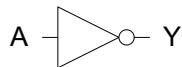


# Compound Gates

## □ Logical Effort of compound gates

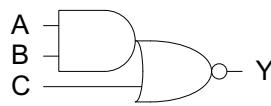
unit inverter

$$Y = \overline{A}$$



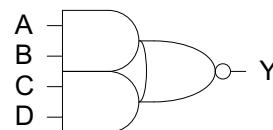
AOI21

$$Y = \overline{A \cdot B + C}$$



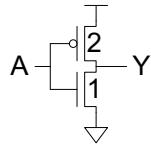
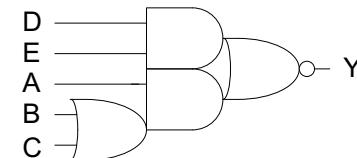
AOI22

$$Y = \overline{A \cdot B + C \cdot D}$$



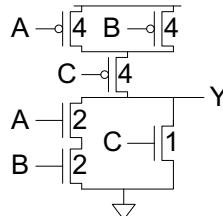
Complex AOI

$$Y = \overline{A \cdot (B + C) + D \cdot E}$$



$$g_A = 3/3$$

$$p = 3/3$$

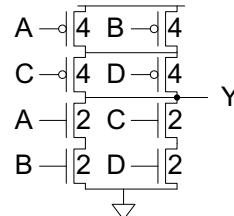


$$g_A = 6/3$$

$$g_B = 6/3$$

$$g_C = 5/3$$

$$p = 7/3$$



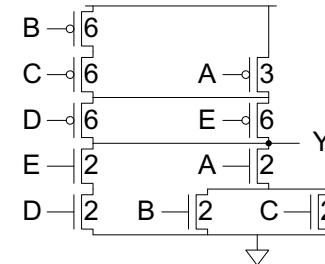
$$g_A = 6/3$$

$$g_B = 6/3$$

$$g_C = 6/3$$

$$g_D = 6/3$$

$$p = 12/3$$



$$g_A = 5/3$$

$$g_B = 8/3$$

$$g_C = 8/3$$

$$g_D = 8/3$$

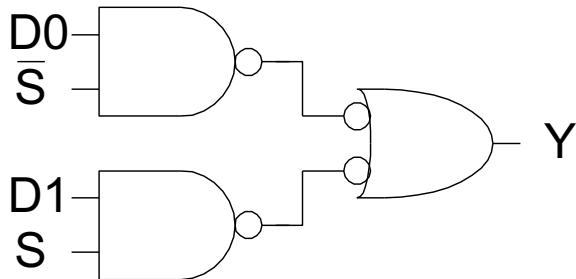
$$g_E = 8/3$$

$$p = 16/3$$

# Example 4

- The multiplexer has a maximum input capacitance of 16 units on each input. It must drive a load of 160 units. Estimate the delay of the two designs.

$$H = 160 / 16 = 10 \quad B = 1 \quad N = 2$$



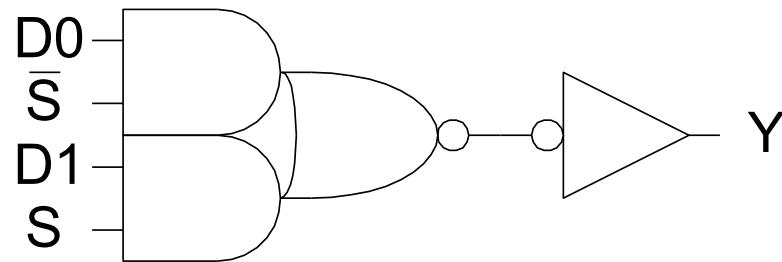
$$P = 2 + 2 = 4$$

$$G = (4/3) \cdot (4/3) = 16/9$$

$$F = GBH = (16/9) \cdot 1 \cdot 10 = 17.8$$

$$\hat{f} = \sqrt[3]{F} = 4.2$$

$$D = N\hat{f} + P = 12.4\tau$$



$$P = 4 + 1 = 5$$

$$G = (6/3) \cdot (1) = 2$$

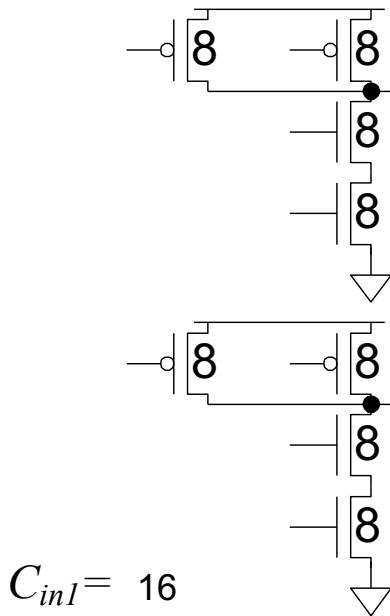
$$F = GBH = 20$$

$$\hat{f} = \sqrt[3]{F} = 4.5$$

$$D = N\hat{f} + P = 14\tau$$

# Example 5

- Annotate your designs with transistor sizes that achieve this delay.



$$C_{in1} = 16$$

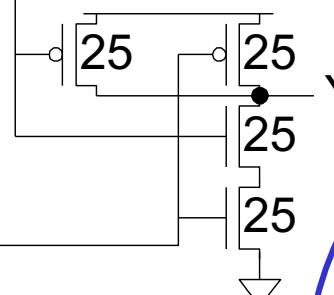
$$g = 4/3$$

$$G = (4/3) \cdot (4/3) = 16/9; B = 1; H = 160/16 = 10$$

$$F = GBH = (16/9) \cdot 1 \cdot 10 = 17.8$$

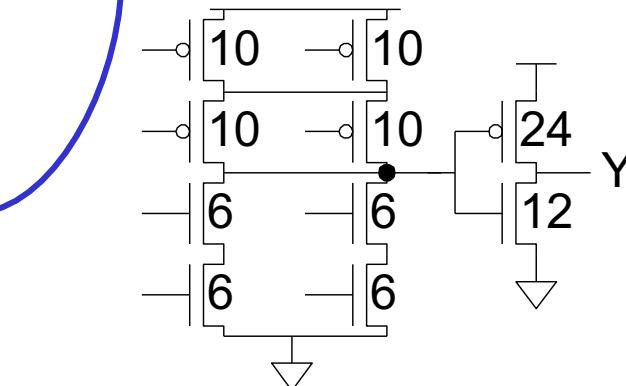
$$\hat{f} = \sqrt[4]{F} = 4.2$$

$$\hat{f} = gh = g \frac{C_{out}}{C_{in}} \Rightarrow C_{in_i} = \frac{C_{out_i} g_i}{\hat{f}}$$



$$C_{in2} = 160 * (4/3) / 4.2 = 50$$

*Repeated from the previous slide:*



$$C_{in1} = 16$$

$$g = 1$$

$$G = (6/3) \cdot (1) = 2; B = 1; H = 160/16 = 10$$

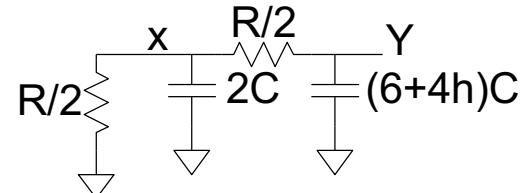
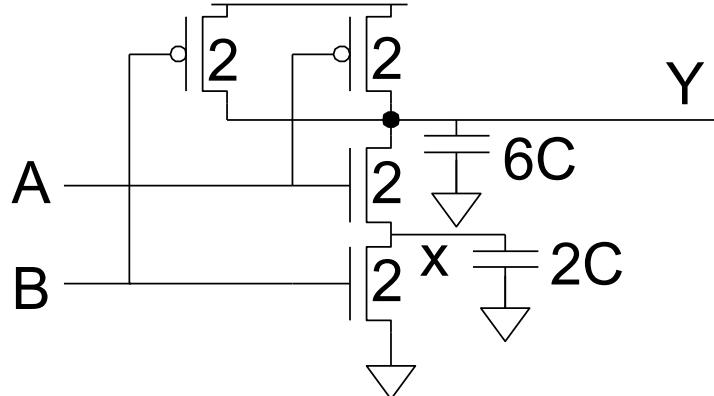
$$F = GBH = 20$$

$$\hat{f} = \sqrt[4]{F} = 4.5$$

$$C_{in2} = 60 * 1 / 4.5 = 36$$

*(10.66 rounded to 10,  
5.33 rounded to 6; Sum=16)*

# Recall: Input Order



- Calculate *parasitic delay* for Y falling

- If A arrives latest?  $2\tau$  (*node x already discharged*)

$$t_{pdf} = [(6+4h)C] \left( \frac{R}{2} + \frac{R}{2} \right) = (6+4h)RC$$

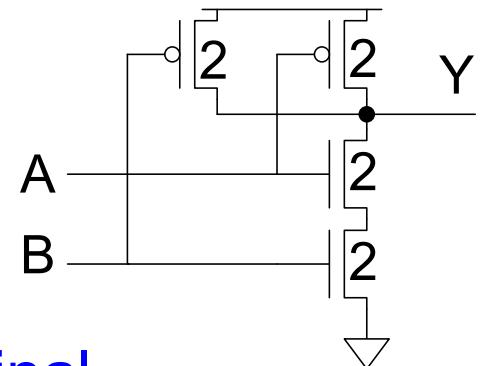
- If B arrives latest?  $2.33\tau$  (*node x not discharged*)

$$t_{pdf} = (2C) \left( \frac{R}{2} \right) + [(6+4h)C] \left( \frac{R}{2} + \frac{R}{2} \right) = (7+4h)RC$$

$$d = t_{pdf} / 3RC \text{ and } h = 0$$

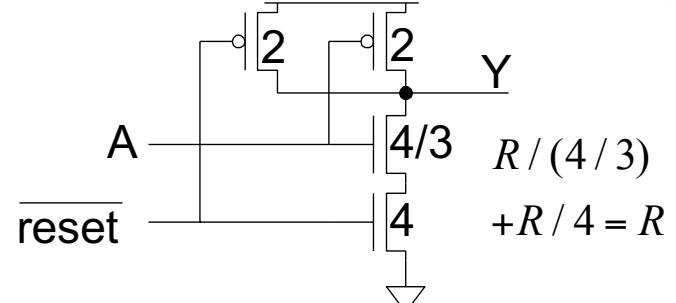
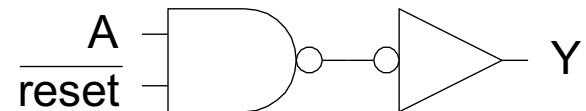
# Inner & Outer Inputs

- ❑ *Inner* input is closest to output (A)
- ❑ *Outer* input is closest to rail (B)
- ❑ If input arrival time is known
  - Connect latest input to inner terminal



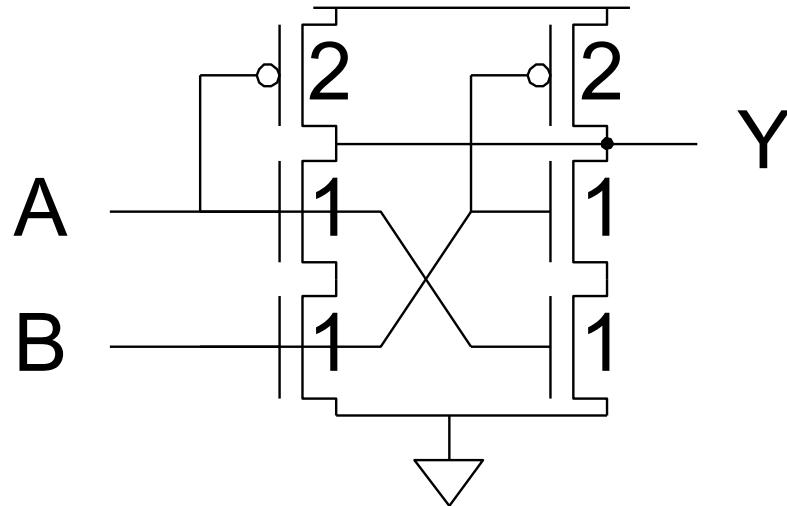
# Asymmetric Gates

- ❑ Asymmetric gates favor one input over another
- ❑ Ex: suppose input A of a NAND gate is most critical
  - Use smaller transistor on A (less capacitance)
  - Boost size of noncritical input
  - So total resistance is same
- ❑  $g_A = (2+4/3)/3 = 10/9$  (<  $4/3 = 12/9$  for symmetric NAND2)
- ❑  $g_B = 6/3 = 2$  (for reset input)
- ❑  $g_{\text{total}} = g_A + g_B = 28/9$   
(>  $4/3 + 4/3 = 24/9$ )
- ❑ Asymmetric gate can approach  $g = 1$  on critical input
- ❑ But total logical effort goes up



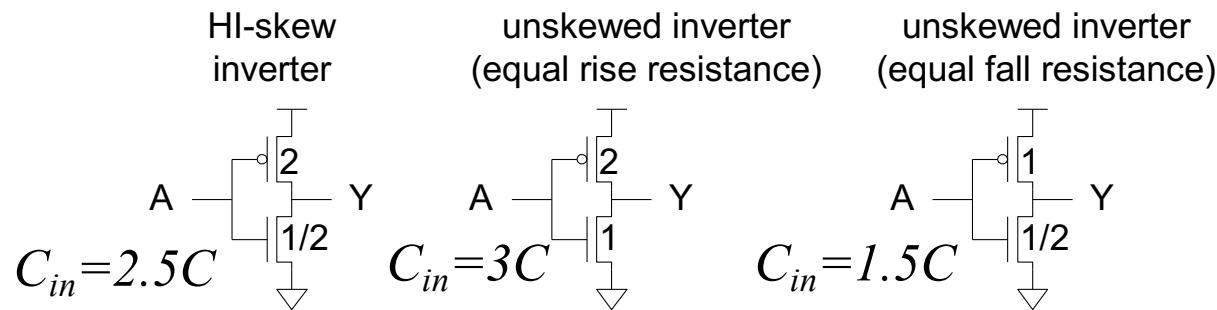
# Symmetric Gates

- Inputs can be made perfectly symmetric



# Skewed Gates

- Skewed gates favor one edge over another
- Ex: suppose **the rising output of inverter is most critical**
  - Downsize noncritical nMOS transistor

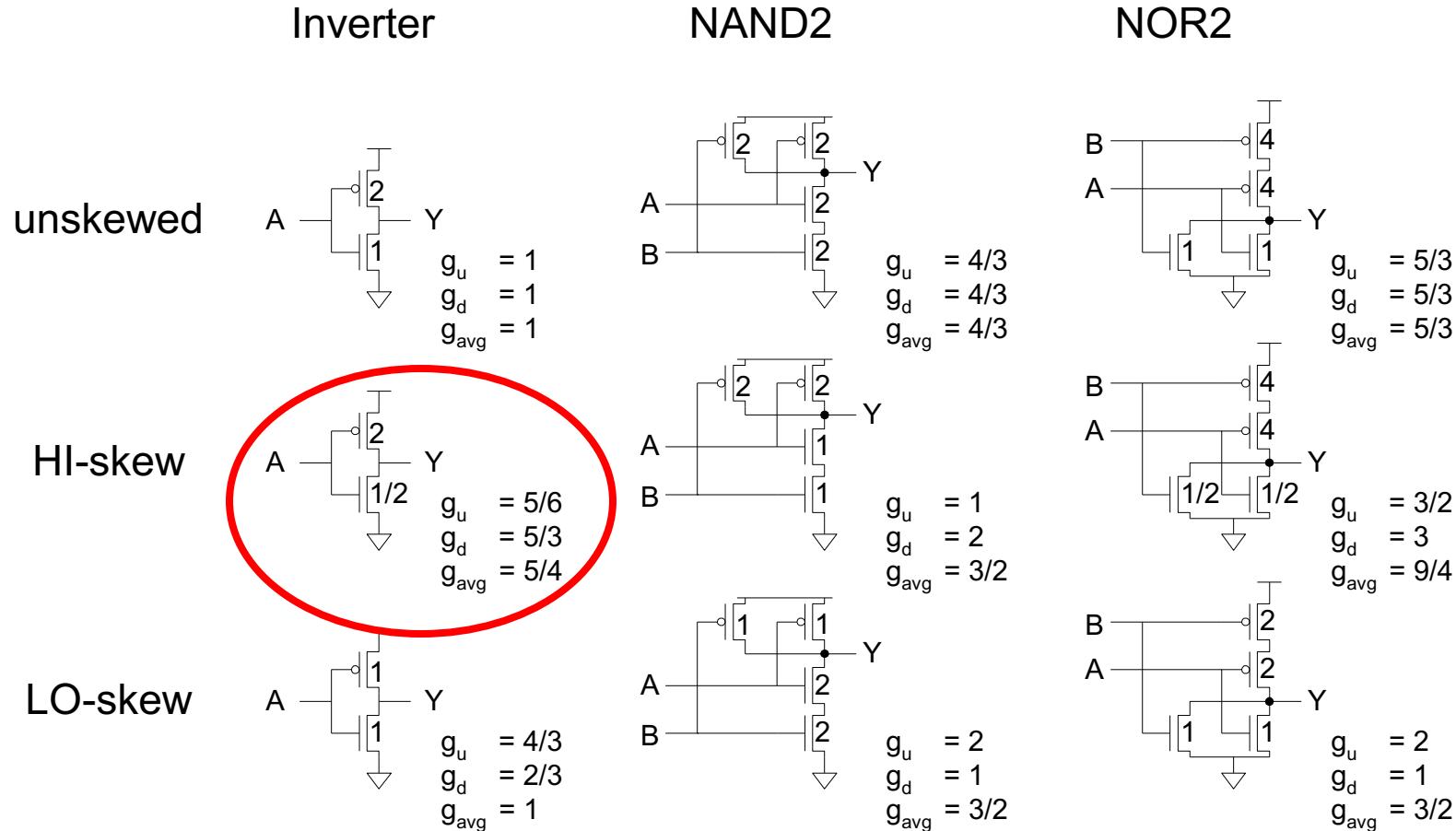


- Calculate logical effort by comparing to *unskewed* inverter with same effective resistance on that edge.
  - $g_u = 2.5 / 3 = 5/6 < 1 \rightarrow$  improved vs unit inverter
  - $g_d = 2.5 / 1.5 = 5/3 > 1 \rightarrow$  worse vs unit inverter
  - $g_{ave} = (5/6+5/3)/2=5/4 > 1 \rightarrow$  worse vs unit inverter

# HI- and LO-Skew

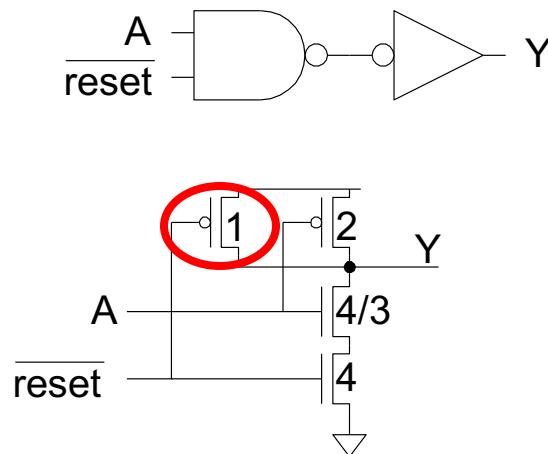
- Def: Logical effort of a skewed gate **for a particular transition** is the ratio of the input capacitance of that gate to the input capacitance of an unskewed inverter delivering the same output current **for the same transition**.
- Skewed gates reduce size of noncritical transistors
  - HI-skew gates favor rising output (small nMOS)
  - LO-skew gates favor falling output (small pMOS)
- **Logical effort is smaller for favored direction**
- But larger for the other direction

# Catalog of Skewed Gates



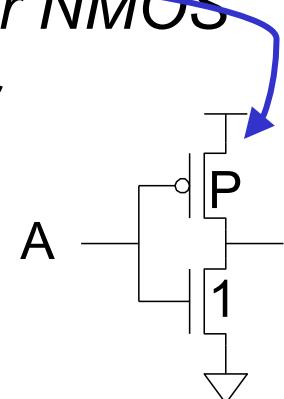
# Asymmetric Skew

- Combine asymmetric and skewed gates
  - Downsize noncritical transistor on unimportant input
  - Reduces parasitic delay for critical input



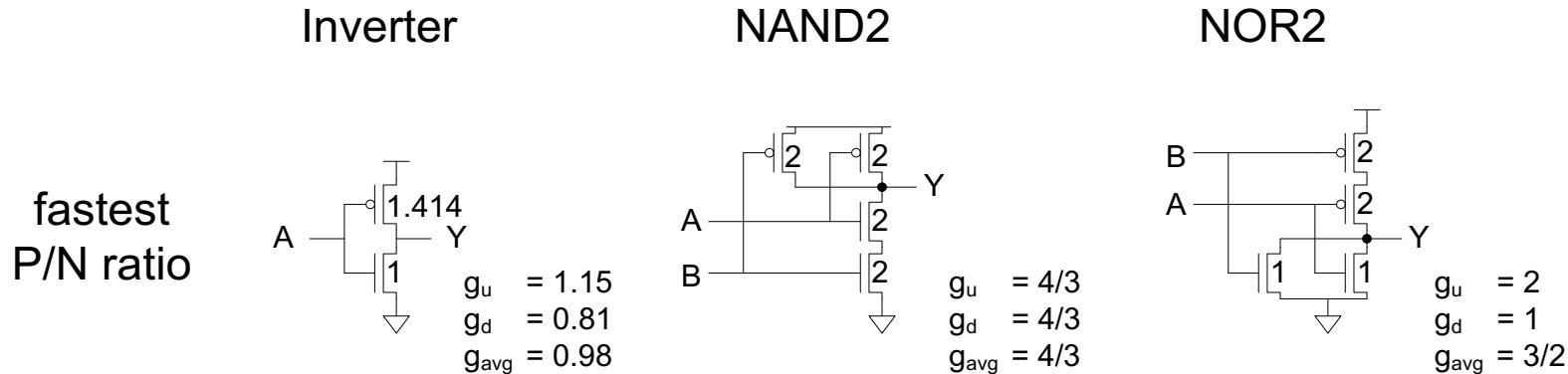
# Best P/N Ratio for Min Delay

- For best noise margins we have selected  $W_p/W_n = \mu_n/\mu_p$ 
  - e.g.,  $\mu_n/\mu_p = 2$  in textbook
- For least average delay, choose  $W_p/W_n = \sqrt{\mu_n/\mu_p}$
- Proof for inverter with no load
  - Define  $\mu = \mu_n/\mu_p$ ;  $P$  is size parameter (not paras. delay)
  - $t_{pdf} = RC_{out} = P+1$  - as  $C_{out} = P+1$  and  $R=1$  for NMOS
  - $t_{pdr} = (P+1)(\mu/P)$  - as  $xP$  wider but  $x\mu$  slower
  - $t_{pd} = (P+1)(1+\mu/P)/2 = (P + 1 + \mu + \mu/P)/2$
  - $dt_{pd} / dP = (1 - \mu/P^2)/2 = 0$
  - Least delay for  $P = \sqrt{\mu} = \sqrt{\mu_n/\mu_p}$



# Preferred P/N Ratios

- Often, best P/N ratio is chosen as  $W_p/W_n = \sqrt{\mu_n/\mu_p}$ 
  - Degrades noise margin somewhat
  - Only improves average delay *slightly* for inverter
  - But **significantly decreases area and power**
- Ex: **inverter** - best speed, good area/power/noise-tolerance  
**NAND**: best noise-tolerance, good speed/area/power  
**NOR**: good area/speed/power; lower noise-tolerance



# Observations

- For speed:
  - NAND vs. NOR
  - Many simple stages vs. fewer high fan-in stages
  - Delay-critical input
    - Should be connected closest to the output
- For area and power:
  - Many simple stages vs. fewer high fan-in stages