ECE 1513: Introduction to Machine Learning

Lecture 10: Convolutional Neural Networks and Sequence Data

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Recap: *Building and Training NN*

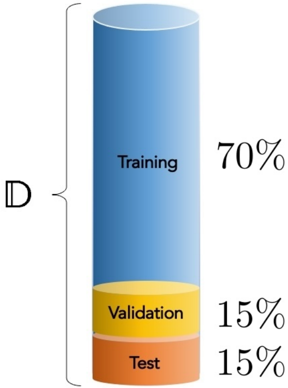
`TrainingLoop():`

1: Build NN $\boldsymbol{y} = f_{\mathbf{w}}(\boldsymbol{x})$ with some hyperparameters and *initial weights*
2: Split training set to mini-batches
3: Specify the loss function $\mathcal{L}$
4: **for** *epochs* = $1, \ldots, E$ **do**
5: Keep applying mini-batch SGD
6: **end for**
7: Return final weights $\mathbf{w}^*$, average training loss, and accuracy on training set

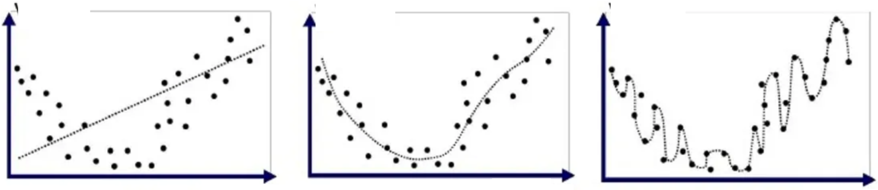
Recap: *Generalization*

? *How can we measure generalization?*

! *We try samples that we did not use for training*



Recap: *Underfitting and Overfitting*



- *Left* $\rightsquigarrow$ *Underfitting*
- *Middle* $\rightsquigarrow$ *Learning fairly*
- *Right* $\rightsquigarrow$ *Overfitting*

Recap: *Bias and Variance*

Generalization Error Components

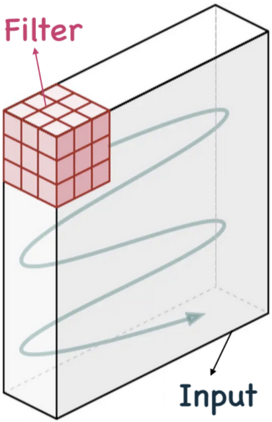
Generalization error is proportional to the model bias and variance

We can only minimize the bias and variance of our model output

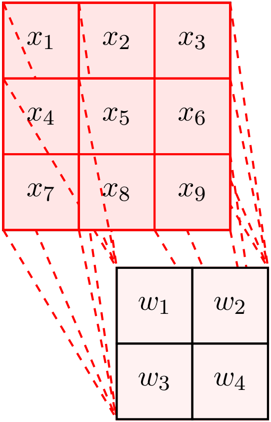
- *If we set the model to give us zero bias* $\rightsquigarrow$ *unbiased estimator*
  - ↳ *But this may lead to higher variance*
- *There is always a minimum error that we cannot beat* $\rightsquigarrow$ *Bayes error*

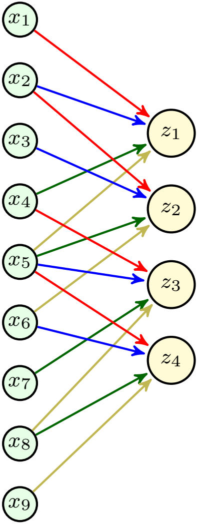
Recap: *Convolution*

In convolution, we slide a filter over the input sample



Recap: *Convolution as Sparse Linear Transform*





Today's Agenda: *CNNs and Sequence Data*

Today, we use the convolution to build more efficient FNNs called

Convolutional Neural Networks

In this way, we learn

- *Convolutional layers*
- *Pooling layers*
- *Architecture of deep CNNs*

We then start our final journey which explores briefly

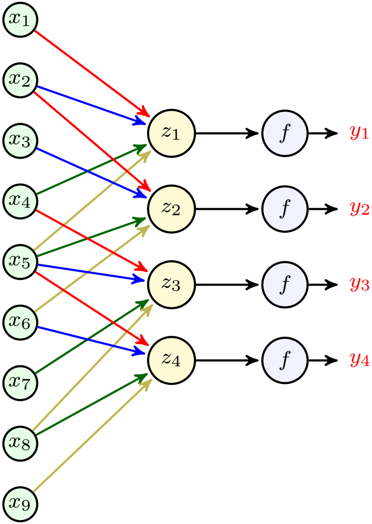
Advanced Topics in Machine Learning

In today's episode, we talk about

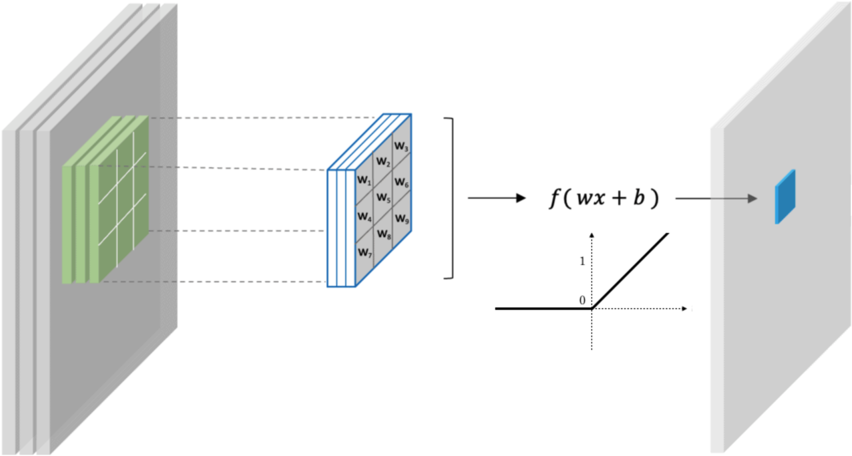
- *Sequence Data and Seq2Seq Models*

Building Neural Layers with Convolution

We can activate the convolution output to make a partially-connected layer

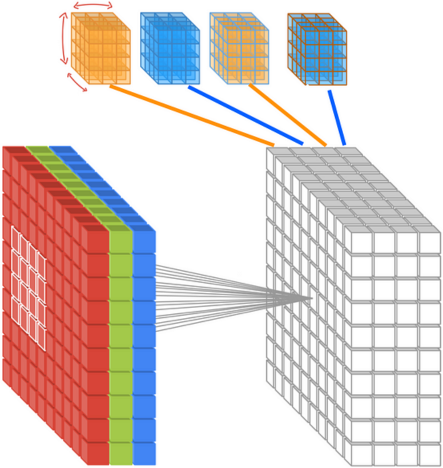


Convolutional Layer: *Single-Channel Output*



Convolutional Layer: *Multi-Channel Output*

To have multiple output channels, we can use multiple filters



Pooling: *Max-Pooling*

Pooling

Pooling is a convolution-like operation that computes a fix function in each slide

Example: *Max-Pooling*

$$\mathbf{X} = \begin{bmatrix} X_{1,1} & X_{1,2} & \cdots & \cdots & X_{1,M} \\ X_{2,1} & X_{2,2} & \cdots & \cdots & X_{2,M} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ X_{N,1} & X_{N,2} & \cdots & \cdots & X_{N,M} \end{bmatrix}$$

$$Z = \max\left\{\text{window}\right\}$$

we pool the maximum

$$\mathbf{Z} = \begin{bmatrix} Z_{1,1} & Z_{1,2} & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix}$$

Max Pooling: *Numerical Example*

0.4	2.3	0	1.3	0
1.9	3.2	2.1	1.5	0
2.3	0	3.8	0	4.3
1.5	2.3	1.5	0.7	2.1

3.2	3.2	2.1	1.5
3.2	3.8	3.8	4.3
2.3	3.8	3.8	4.3

Mean-Pooling

Mean-pooling is another approach in which we compute the average

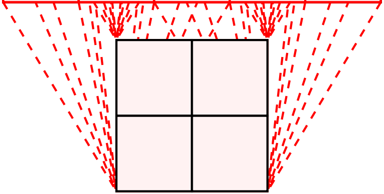
$$\mathbf{X} = \begin{bmatrix} X_{1,1} & X_{1,2} & \cdots & \cdots & X_{1,M} \\ X_{2,1} & X_{2,2} & \cdots & \cdots & X_{2,M} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ X_{N,1} & X_{N,2} & \cdots & \cdots & X_{N,M} \end{bmatrix} \qquad \texttt{mean}\,\{\quad\}$$

In each window, we pool the average

$$\mathbf{Z} = \begin{bmatrix} Z_{1,1} & Z_{1,2} & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix}$$

Mean-Pooling: *Numerical Example*

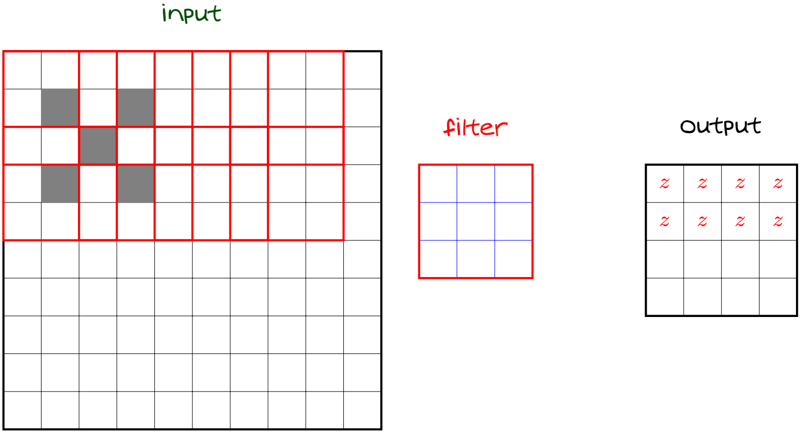
0.4	2.3	0	1.3	0
1.9	3.2	2.1	1.5	0
2.3	0	3.8	0	4.3
1.5	2.3	1.5	0.7	2.1



2.05	1.9	1.225	0.7
1.85	2.275	1.85	1.45
1.525	1.9	1.5	1.775

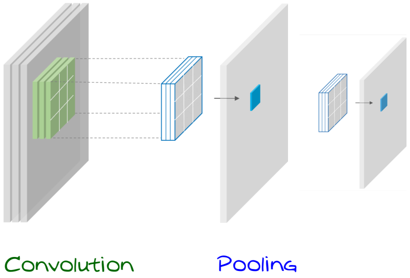
Convolution *with Stride*

We can perform all operations with *stride*, *e.g.*, *stride 2*



Convolutional Unit: *Complex Convolutional Layer*

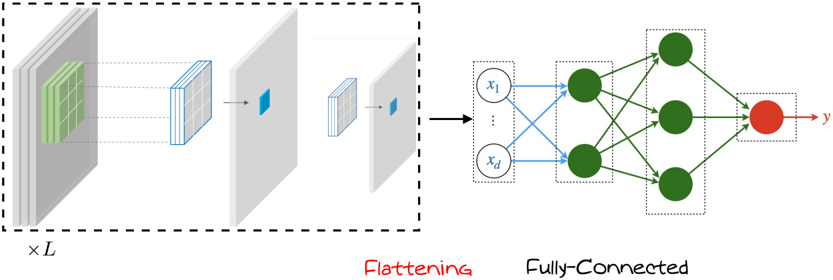
A convolutional unit is made as



- *The convolution performs less complex linear operation*
- *Pooling make the output smooth, i.e., less varying*

CNN: General Architecture

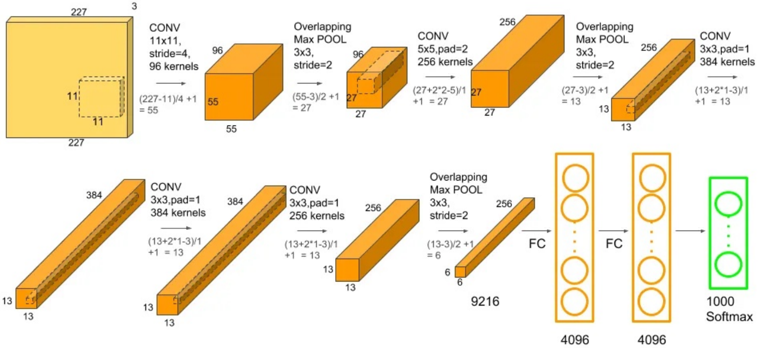
A CNN consists of multiple convolutional units and a fully-connected NN



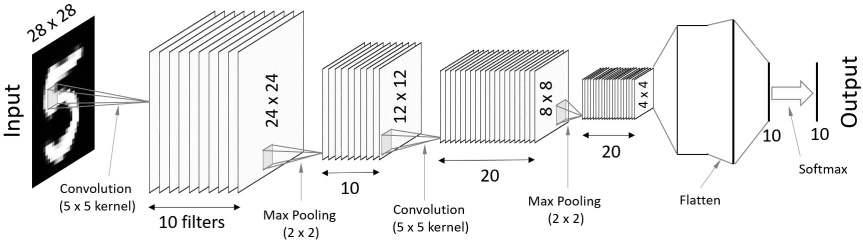
We can repeat the convolutional unit multiple times

Example: *AlexNet*

Let's look at the winner of the ImagNet challenge in 2012



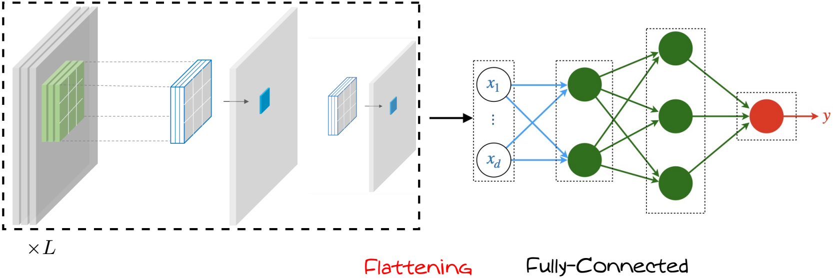
Example: *Custom CNN for MNIST Classification*



In this network, we do the following

- *We apply convolution with 10 filters*
  - ↳ *We apply pooling with stride 2*
- *We apply convolution with 20 filters*
  - ↳ *We apply pooling with stride 2*
- *We flatten and pass through one fully-connected layer*
  - ↳ *We compute Softmax for classification*

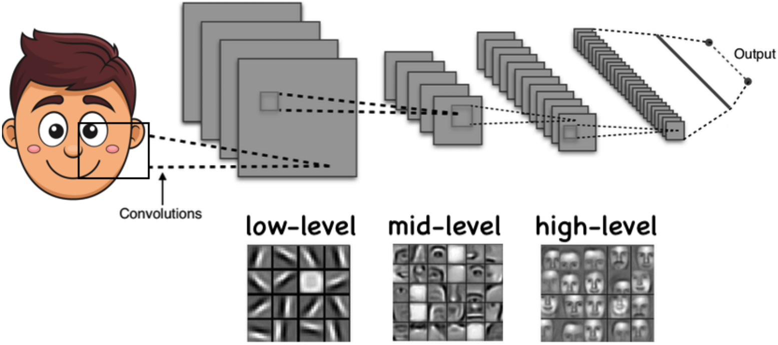
Backpropagation



It is easy to see that backward pass is dual to the forward pass

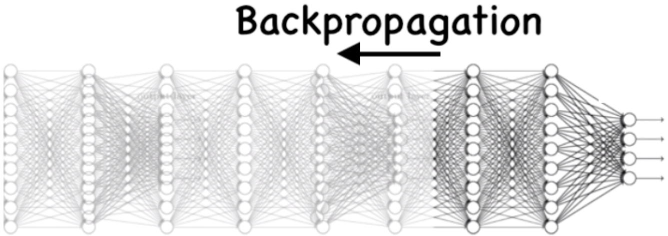
- *We can compute sample gradients with backpropagation*
- *Using mini-batch SGD, we can efficiently train CNNs*

Gradual Feature Extraction



As we go deeper, CNN extracts higher levels of features

Vanishing or Exploding Gradient

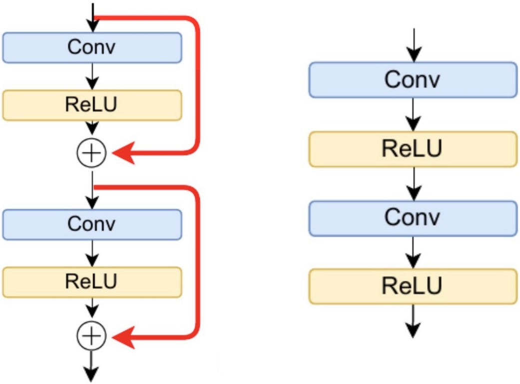


As we go very deep in FNNs, we could experience

- *Decrease in gradient values through depth*
  ↳ *This results in vanishing gradient*
- *Increase in gradient values through depth*
  ↳ *This results in exploding gradient*

ResNet: *Skip Connection*

ResNet uses ***skip connections*** *to overcome this issue*



Further Read

- Goodfellow
  - ↳ Chapter 9 ***CNNs***

CNNs are further discussed in details in

- *ECE1508: Applied Deep Learning*
  - ↳ *Given in both Fall and Winter Semesters*

Sequence Data: *Many Applications*

 → "This is the first lecture on ..."

 → Classic/Jazz/Pop/Hip-hop

"This product is useless!" → ★☆☆☆☆

 → A cute puppy in snow

 → Sport/Drama/Documentary

Sequence Learning Problem

Basic FNNs cannot be used in practice: *we need a huge input and/or output*

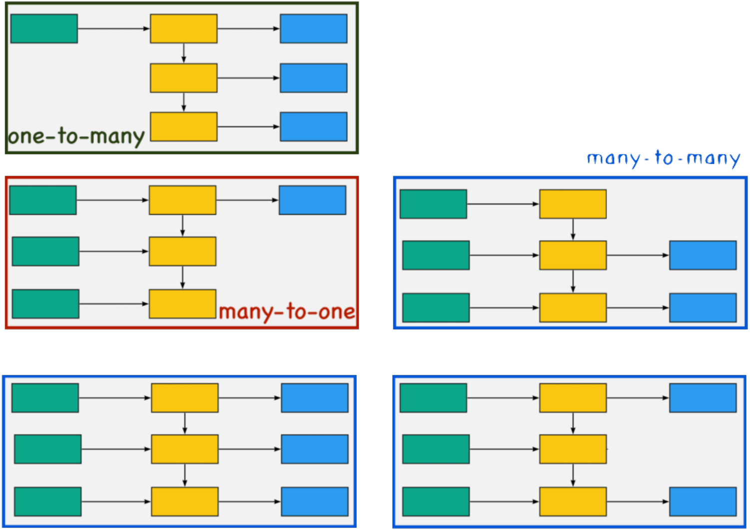
Sequence Learning Model

Models that get sequence inputs and return sequence outputs

There are various approaches in the literature

- *Recurrent Neural Networks (RNNs)*
- *Encoder-Decoder Architecture (Seq2Seq Models)*
- *Transformers*

Types of Sequence Problems

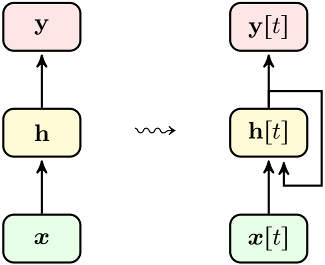


Recurrent Neural Networks

We can feed a sequence to a neural network, if we include recurrence

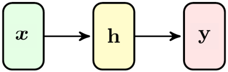
Recurrence

NN takes a state as input and returns the next state as output



Example: *Turning Shallow FNN to Basic RNN*

Say we have a shallow fully-connected FNN



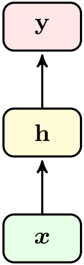
- *It computes hidden feature from the input* $\boldsymbol{x}$

$$\mathbf{h} = f\left(\mathbf{W}_1 \boldsymbol{x}\right)$$

- *It computes the output from the hidden features*

$$\mathbf{y} = f\left(\mathbf{W}_2 \mathbf{h}\right)$$

Example: *Turning Shallow FNN to Basic RNN*



The model in this case gives us

$$\mathbf{y} \propto P\left(v | \boldsymbol{x}\right)$$

and when we train it, we maximize its likelihood

Example: *Turning Shallow FNN to Basic RNN*

We can make an RNN by using the previous feature in each time: at time t

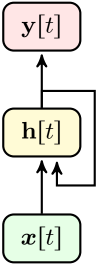
- *compute new features from the input* $\boldsymbol{x}[t]$ *and previous features* $\mathbf{h}[t-1]$

$$\mathbf{h}[t] = f\left(\mathbf{W}_1 \boldsymbol{x}[t] + \mathbf{W}_0 \mathbf{h}[t-1]\right)$$

- *computes the output from the new features*

$$\mathbf{y}[t] = f\left(\mathbf{W}_2 \mathbf{h}[t]\right)$$

Example: *Turning Shallow FNN to Basic RNN*



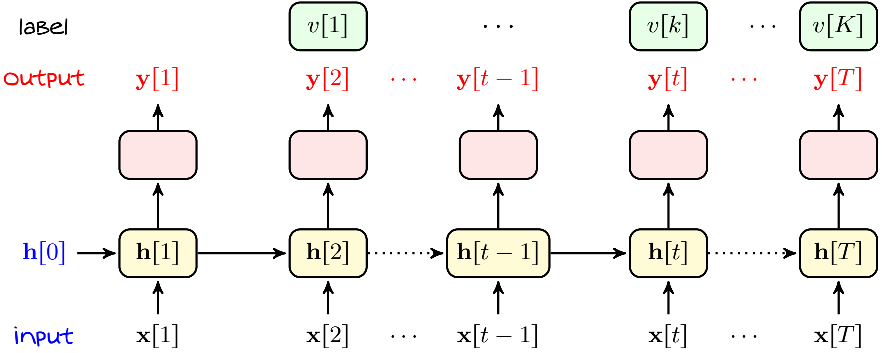
This recursive model determines for us

$$\mathbf{y}[t] \propto P\left(v[t] | \mathbf{h}[t-1], \boldsymbol{x}[t]\right) \equiv P\left(v[t] | \boldsymbol{x}[1], \ldots, \boldsymbol{x}[t]\right)$$

and we should maximize the likelihood over time

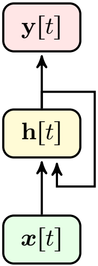
↳ *information about* $\boldsymbol{x}[1], \ldots, \boldsymbol{x}[t-1]$ *is somehow encoded in* $\mathbf{h}[t-1]$

Example: *Turning Shallow FNN to Basic RNN*



Generic RNN

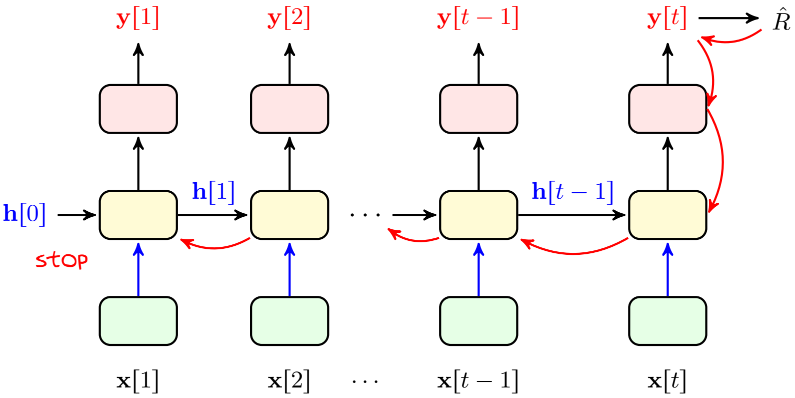
We can do the same thing with any FNN, including CNNs



$\mathbf{h}[t]$ *is the output of a ny number of hidden layers*

Training RNNs

We can apply the same risk minimization; this time over time



To train RNNs, we need to backpropagate through time

Main Challenge: *Vanishing Gradient Through Time*

With long sequences

backpropagation through time $\approx$ *backpropagation through deep* NNs

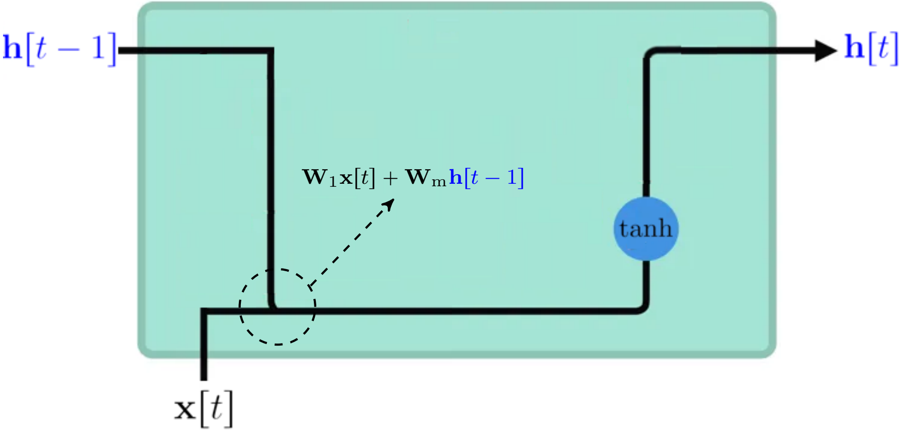
- *Exploding Gradients*
- *Vanishing Gradients*
  - ↳ *Gradients become small over time*
  - ↳ *Weights of RNN are updated only with most recent time instances*
  - ↳ *RNN has a limited memory!*

Classical Remedies

There are various approaches to handle these issues

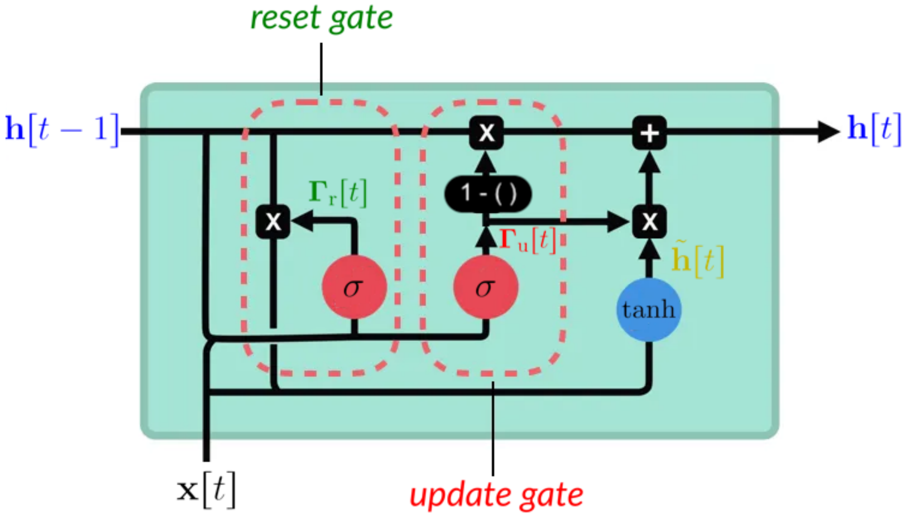
- *Clipping gradients when exploding*
  - ↳ *Usually used to deal with exploding gradient*
- *Truncated backpropagation through time*
  - ↳ *Updated multiple times in the middle to carry memory forward*
- *Using gated units*
  - ↳ *Use the so-called gate to control better the memory*

Basic RNN as a Unit



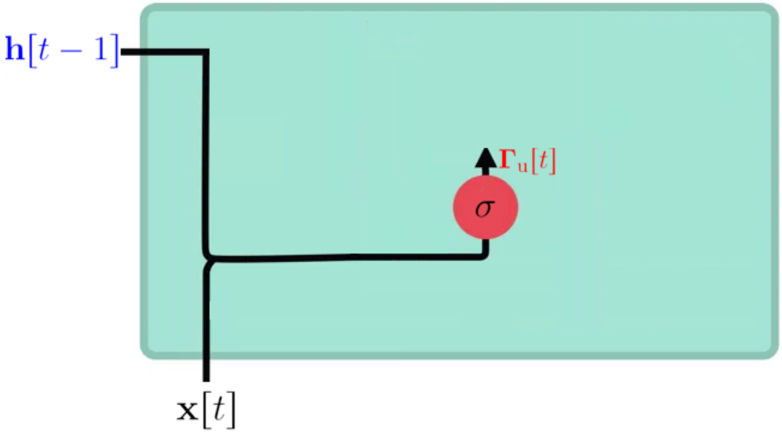
Gated Recurrent Unit: *GRU*

This is what's going on in a GRU cell



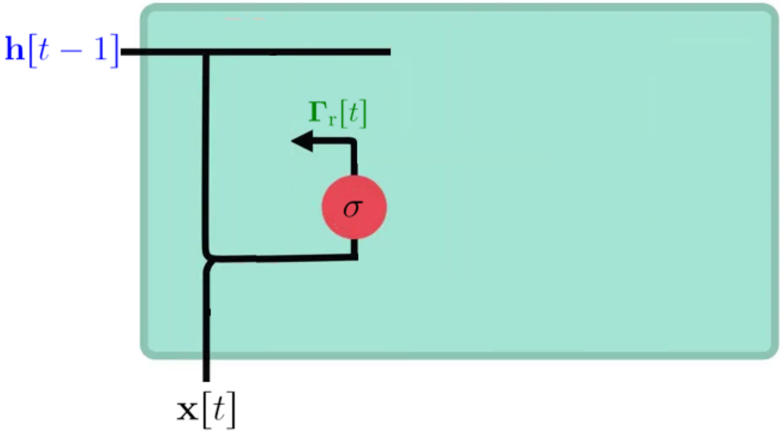
Gated Recurrent Unit: *GRU*

Compute update gate $\mathbf{\Gamma}_{\mathrm{u}}[t] = \sigma\left(\mathbf{W}_{\mathrm{u,in}}\mathbf{x}[t] + \mathbf{W}_{\mathrm{u,m}}\mathbf{h}[t-1]\right)$



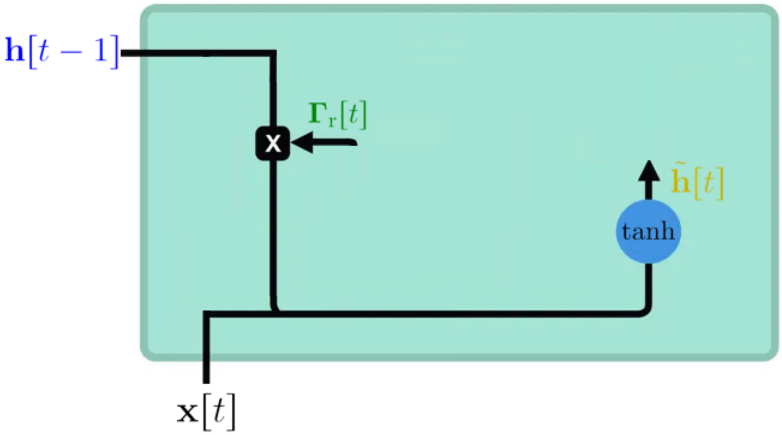
Gated Recurrent Unit: *GRU*

Compute reset gate $\mathbf{\Gamma}_{\mathrm{r}}[t] = \sigma\left(\mathbf{W}_{\mathrm{r,in}}\mathbf{x}[t] + \mathbf{W}_{\mathrm{r,m}}\mathbf{h}[t-1]\right)$



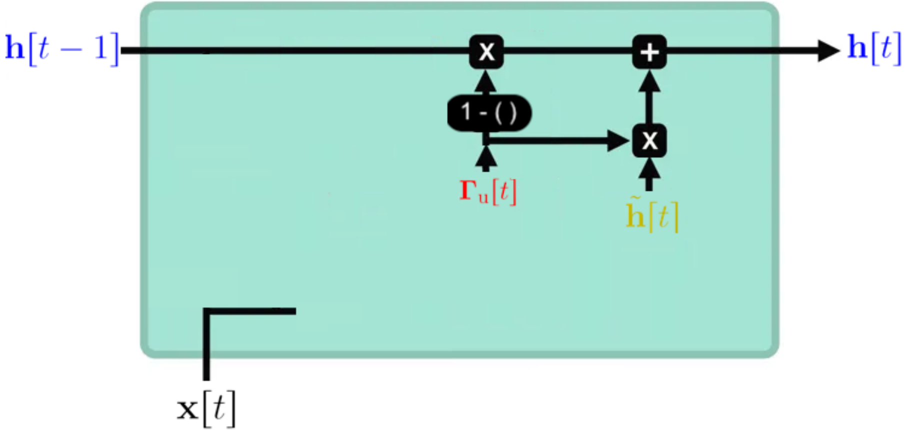
Gated Recurrent Unit: *GRU*

Compute actual memory $\tilde{\mathbf{h}}[t] = f(\mathbf{W}_1 \mathbf{x}[t] + \mathbf{W}_\mathrm{m} \boldsymbol{\Gamma}_\mathrm{r}[t] \odot \mathbf{h}[t-1])$



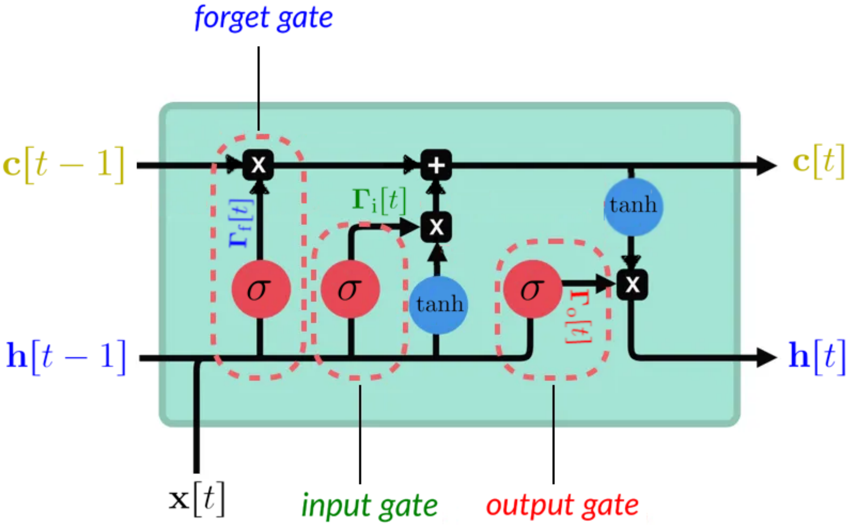
Gated Recurrent Unit: *GRU*

Update hidden state as $\mathbf{h}[t] = (1 - \mathbf{\Gamma}_{\mathrm{u}}[t]) \odot \mathbf{h}[t-1] + \mathbf{\Gamma}_{\mathrm{u}}[t] \odot \tilde{\mathbf{h}}[t]$



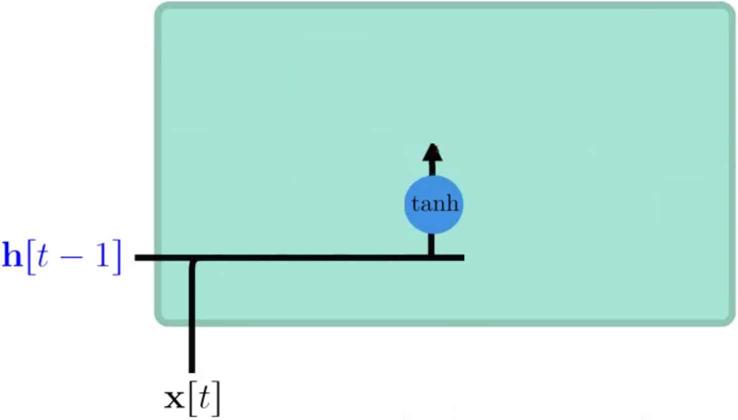
Long Short-Term Memory: *LSTM*

This is how inside an LSTM unit looks like



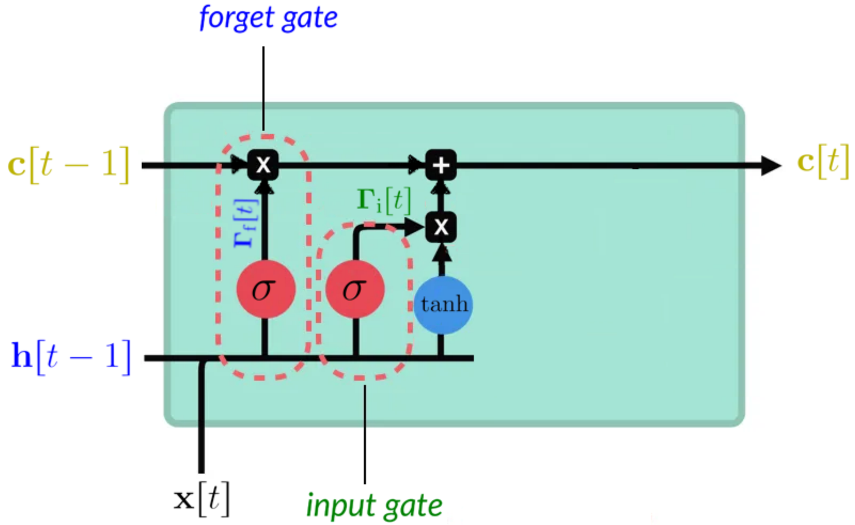
Long Short-Term Memory: *LSTM*

Actual cell state $\tilde{\mathrm{c}}[t] = f(\mathbf{W}_1 \mathbf{x}[t] + \mathbf{W}_\mathrm{m} \mathbf{h}[t-1])$



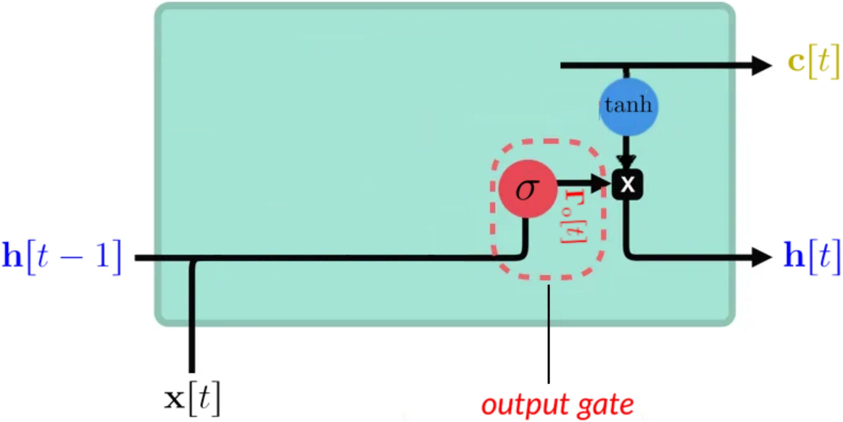
Long Short-Term Memory: *LSTM*

We use forget gate and update gate to update cell state



Long Short-Term Memory: *LSTM*

We use output gate to control fellow of memory to the hidden state



Gating is Helpful

- *Using gating we can control vanishing and exploding gradient*
- *LSTM used to be a robust sequence-based model*
  - ↳ *We can still use it for simple tasks like price prediction*
  - ↳ *It was one of the first models used for text generation*
- *At the end of the day, RNNs will still carry limited memory*
  - ↳ *This is why Attention Mechanism is developed*
  - ↳ *We can do whole sequence processing based on Attention*
  - ↳ *This is the idea of Transformers*

Further Read

- Goodfellow
  - ↳ Chapter 10 ***RNNs***

RNNs and Transformers are discussed in

- *ECE1508: Applied Deep Learning*
  - ↳ *Given in both Fall and Winter Semesters*
- *ECE1786: Creative Applications of NLP*
  - ↳ *Given in Fall Semesters*