ass3

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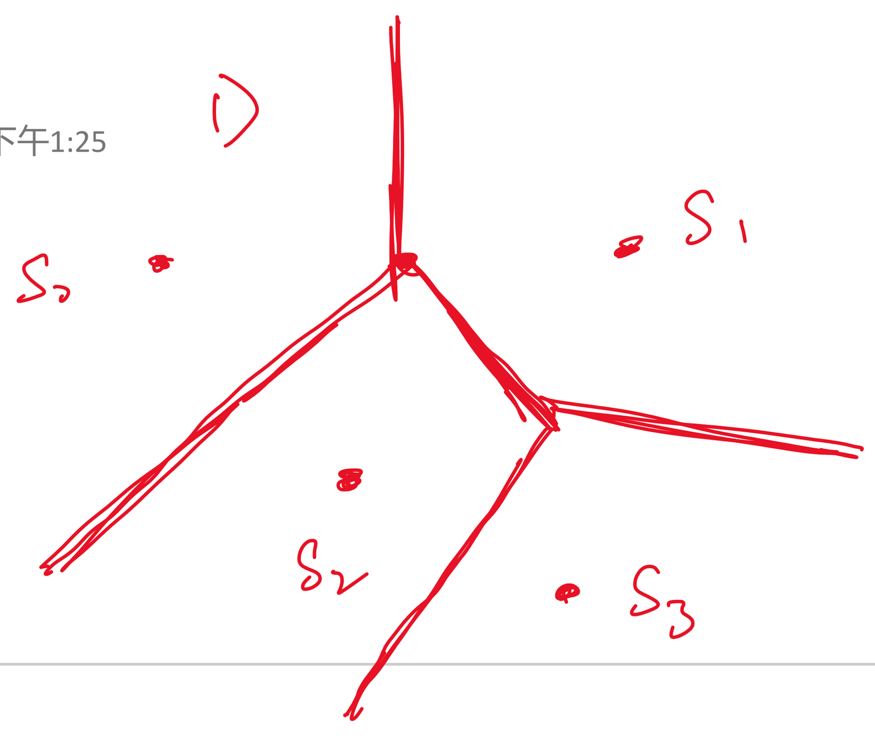


ass3

Assignment #3

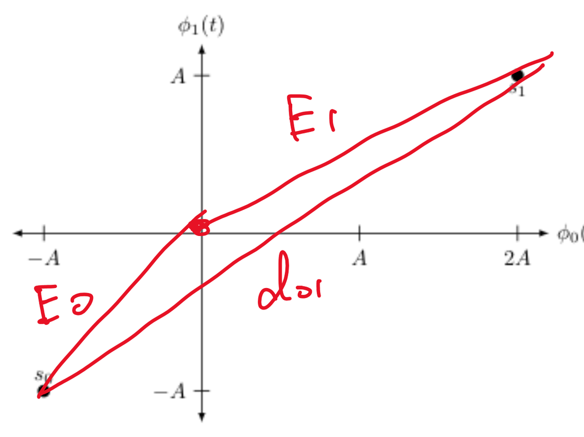
Due on Monday, March 27, 2023

1. Suppose a signal constellation consists of the following four points: $s_0 = (A, -A)$, $s_1 = (2A, A)$, $s_2 = (0, 0)$, and $s_3 = (-A, A)$. Draw the signal space diagram and carefully draw the boundaries of the decision regions. Use a ruler, and draw to scale.

2. Suppose the signal constellation shown below was used for communication over an AWGN channel with a power spectral density of $\mathcal{N}_0$.

   (a) Find the probability of error, expressed in terms of the energy per bit, $\mathcal{E}_b$, and $\mathcal{N}_0$.

   (b) Evaluate the probability of error if $\mathcal{E}_b/\mathcal{N}_0 = 8$ dB.



$S_1\ (2A, A)$
$S_0\ (-A, -A)$

3. Suppose that for some binary communication system the output of the matched filter is distributed with likelihood functions of

$$f_R(r \mid 0 \text{ sent}) = \frac{r}{\sigma^2} \exp\left\{-\frac{r^2}{2\sigma^2}\right\}$$

$$f_R(r \mid 1 \text{ sent}) = \frac{r}{\mathcal{E}_b + \sigma^2} \exp\left\{-\frac{r^2}{2(\mathcal{E}_b + \sigma^2)}\right\}$$

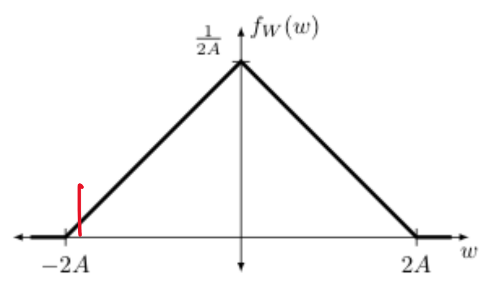
for $r \geq 0$, where $\mathcal{E}_b$ is the transmitted energy per bit and σ^2 is the variance of additive noise. Assume that 0 and 1 are equally likely to be transmitted. Derive an expression for the optimal decision rule. Provide the simplest expression you can.

4. Suppose the input to the decision device at the receiver is modelled as

$$R = \begin{cases} A + W, & \text{if a 0 was transmitted} \\ -A + W, & \text{if a 1 was transmitted} \end{cases},$$

where the noise variable, W, is characterized by the probability density function shown below:

$W = R - A$



$$f_W(w) = \begin{cases} \frac{1}{4A^2}(2A - |w|), & |w| \leq 2A \\ 0, & |w| > 2A \end{cases}$$

2.

i) $d_{01} = \sqrt{(3A)^2 + (2A)^2}$

$= \sqrt{9A^2 + 4A^2}$

$= \sqrt{13A^2}$

$= \sqrt{13}A$

$P = Q\left(\frac{d_{01}}{\sqrt{2N_0}}\right)$

$= Q\left(\frac{\sqrt{13}A}{\sqrt{2N_0}}\right)$

$= Q\left(\frac{\sqrt{13} \times \sqrt{\frac{2}{7}E_b}}{\sqrt{2N_0}}\right)$

$= Q\left(\frac{\sqrt{\frac{26}{7}} \times \sqrt{E_b}}{\sqrt{2} \times \sqrt{N_0}}\right)$

$= Q\left(\sqrt{\frac{13}{7}}\sqrt{\frac{E_b}{N_0}}\right)$

$E_0 = (-A)^2 + (-A)^2 = 2A^2$

$E_1 = (2A)^2 + A^2 = 5A^2$

$E_s = \frac{1}{2}(E_0 + E_1)$

$= \frac{1}{2}(2A^2 + 5A^2)$

$= \frac{7}{2}A^2$

$\frac{E_s}{N_0} = \frac{E_b}{N_0}\log_2(M)$

$\frac{E_s}{N_0} = \frac{E_b}{N_0}\log_2(2) = 1$

$\frac{E_s}{\cancel{N_0}} = \frac{E_b}{\cancel{N_0}}$

$E_b = E_s = \frac{7}{2}A^2$

$A^2 = \frac{2}{7}E_b$

$A = \sqrt{\frac{2}{7}E_b}.$

2) if $\frac{E_b}{N_0} = 8\text{dB}.$

$8 = 10\log_{10}(x)$

$x = 6.31$

$P = Q\left(\sqrt{\frac{13}{7}}\sqrt{6.31}\right)$

$= Q(3.42)$

Suppose that the *a priori* probability that a 0 was sent is given by p, for some $p < \frac{1}{2}$. Find the decision rule which leads to the minimum probability of error, and express it in the simplest form possible. Hint: A sketch of the *a posteriori* probabilities as functions of R would make it easier to verify if your answer is reasonable.

3. choose $\hat{m} = 0$:

$f(R \mid 0\ \text{sent}) > f(R \mid 1\ \text{sent})$

$\frac{r}{\sigma^2}\exp\left(-\frac{r^2}{2\sigma^2}\right) > \frac{r}{\varepsilon_b + \sigma^2}\exp\left(-\frac{r^2}{2(\varepsilon_b + \sigma^2)}\right)$

$\log\left(\frac{r}{\sigma^2}\right) - \frac{r^2}{2\sigma^2} > \log\left(\frac{r}{\varepsilon_b + \sigma^2}\right) - \frac{r^2}{2(\varepsilon_b + \sigma^2)}$

$-\frac{r^2}{2\sigma^2} + \frac{r^2}{2(\varepsilon_b + \sigma^2)} > \log\left(\frac{r}{\varepsilon_b + \sigma^2}\right) - \log\left(\frac{r}{\sigma^2}\right)$

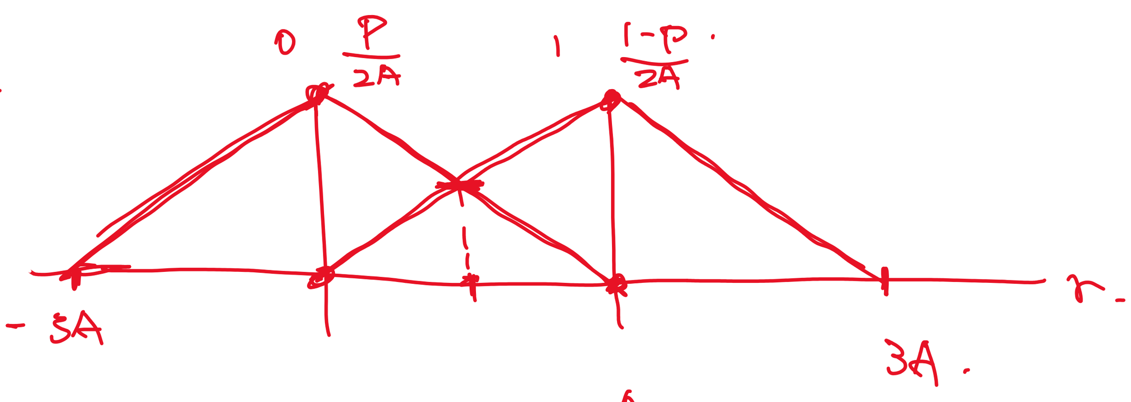
$\frac{-2r^2\varepsilon_b}{4\sigma^2(\varepsilon_b + \sigma^2)} > \log\left(\frac{\sigma^2}{\varepsilon_b + \sigma^2}\right)$

$r > \sqrt{\log\left(\frac{\sigma^2}{\varepsilon_b + \sigma^2}\right)\frac{4\sigma^2(\varepsilon_b + \sigma^2)}{-2\varepsilon_b}}.$

if $r \leq$ - - - - ~

choose $\hat{m} = 1$

4.



$f(x) = ax + b.$

$\begin{cases} (-A, 0) \\ (A, \frac{1-P}{2A}) \end{cases}$ $\begin{cases} 0 = -Aa + b \rightarrow b = Aa. \\ \frac{1-P}{2A} = Aa + b. \end{cases}$

$\Rightarrow \frac{1-P}{2A} = Aa + Aa.$

$\frac{1-P}{2A} = 2Aa.$

$a = \frac{1-P}{4A^2}.$

$f_1(r) = \frac{1-P}{4A^2}r + \frac{1-P}{4A}$

$\begin{cases} (-A, \frac{P}{2A}) \\ (A, 0) \end{cases}$ $\begin{cases} \frac{P}{2A} = -Aa + b. \\ 0 = Aa + b. \rightarrow b = -Aa. \end{cases}$

$\Rightarrow. \frac{P}{2A} = -Aa - Aa.$

$\frac{P}{2A} = -2Aa.$

$a = -\frac{P}{4A^2}.$

$f_0(r) = -\frac{P}{4A^2}r + \frac{P}{4A}.$

$f_0(r) = f_1(r)$

$-\frac{P}{4A^2}r + \frac{P}{4A} = \frac{1-P}{4A^2}r + \frac{1-P}{4A}.$

$-\frac{P}{4A^2}r - \frac{1-P}{4A^2}r = \frac{1-P}{4A} - \frac{P}{4A}.$

$-\frac{P+1-P}{4A^2}r = \frac{1-P-P}{4A}.$

$-\frac{r}{4A^2} = \frac{1-2P}{4A}.$

$r = -\frac{1-2P}{\cancel{4A}} \times \cancel{4}A^2$

$r = (2p-1)A.$

if $r > (2p-1)A$. choose $\hat{m} = 1$

if $r < (2p-1)A$. choose $\hat{m} = 0$.