ass2

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ass2

Carleton University
Department of Systems and Computer Engineering
SYSC 4600 Digital Communication Winter 2022/23
Assignment #2
Due on Wednesday, Febrary 15, 2023

1. For two continuous random variables, X and Y, with joint probability density function $f_{X,Y}(x, y)$, prove that $\mathbf{E}[X + Y] = \mathbf{E}[X] + \mathbf{E}[Y]$.

2. Suppose $\{X_n | n \in \{1, 2, ..., N\}\}$ is a set of N independent Gaussian random variables with mean μ_n and variance σ_n^2 which are different for each n. If

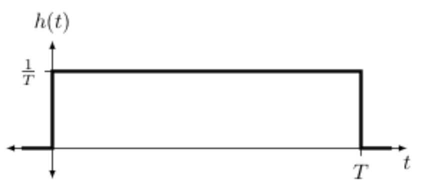
$Var = \sigma_n^2$ $E = \mu_n$

$$Y = \sum_{n=1}^{N} X_n$$

express the mean and variance of Y in terms of $\{\mu_n\}$ and $\{\sigma_n^2\}$. Write the probability density function of Y.

$\mu^2 = E(X) = 0.$

3. Suppose $x(t)$ is a zero-mean stationary Gaussian random process with power spectral density $\Phi_x(f)$ is the input to a linear filter whose impulse response show below. A sample, Y, is taken of the output of the filter at time T.

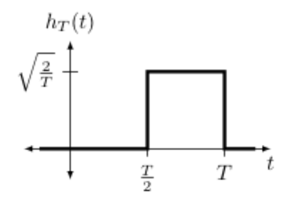


(a) Express the mean and variance of Y in terms of $\Phi_x(f)$ and T.
(b) Upper bound the variance under the condition $\Phi_x(f) \leq S$ for all f.

4. Suppose the normalized pulse shape, $h_T(t)$, shown below, is used to generate the pulse train

$$v(t) = \sum_{n=-\infty}^{\infty} v_n h_T(t - nT)$$

where $\{v_n\}$ are the transmitted symbols, with each v_n selected independently from $\{0, +1\}$ with equal probability.

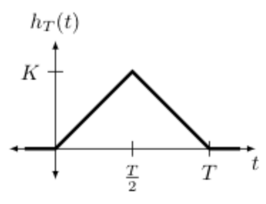


(a) Find the Fourier transform of $h_T(t)$.
(b) Find a simple mathematical expression for the power spectral density of $v(t)$.
(c) Sketch the PSD over the frequency range $[-5/T, 5/T]$ Hz.

5. Suppose the normalized pulse shape, $h_T(t)$, shown below, is used to generate the pulse train

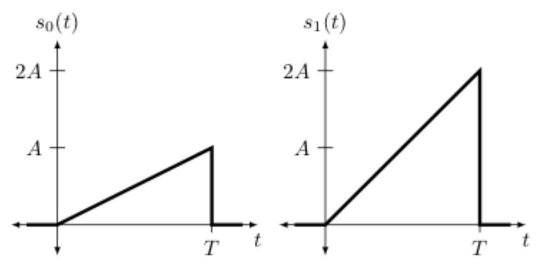
$$v(t) = \sum_{n=-\infty}^{\infty} v_n h_T(t - nT)$$

where $\{v_n\}$ are the transmitted symbols, with each v_n selected independently from $\{-1, +1\}$ with equal probability.



(a) Find K so that $h_T(t)$ is normalized to have unit energy.
(b) Find a simple mathematical expression for the power spectral density of $v(t)$.
(c) Plot the PSD (in dB) using MATLAB, using $T = 1$ second, over the frequency range $[-5, 5]$ Hz. The y-axis should cover the range $[-50, 10]$ dB.

6. Suppose the signals $s_0(t)$ and $s_1(t)$ shown below are used to represent a '0' and a '1', respectively. If they are used to transmit a long sequence of bits (with zeros and ones equally likely to be transmitted), find and sketch the power spectral density of the transmitted pulse train.



1) $E[X+Y] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x+y) f(x,y)\,dx\,dy.$

$= \iint x f(x,y)\,dy\,dx + \iint y f(x,y)\,dx\,dy.$

$= \int x \underbrace{\int f(x,y)\,dy}_{f_X(x)}\,dx + \int y \underbrace{\int f(x,y)\,dx}_{f_Y(y)}\,dy$

$= \int x f_X(x)\,dx + \int y f_Y(y)\,dy$

$= E[X] + E[Y].$

2) $pdf = \frac{1}{\sqrt{2\pi\sigma_x^2}} \exp\left\{-\frac{(x-\mu_x)^2}{2\sigma_x^2}\right\}$

3) a) mean $Y = E[Y(t)]$

$= E\left[\int_{-\infty}^{\infty} h(\alpha)\, x(t-\alpha)\,d\alpha\right]$

$= \int h(\alpha)\, E(x(t-\alpha))\,d\alpha$

$E(x(t)) = 0 \Rightarrow E(x(t-\alpha)) = 0.$

$= 0 \int h(\alpha)\,d\alpha$

$= 0. \longrightarrow 0$

Variance $Y(t) = E(y^2(t)) - E(y(t))^2$

$\sigma_Y^2 = E(y^2(t))$

5) a. $E = 2\int_0^{T/2} |h(t)|^2 dt.$

$= 2\int_0^{T/2} \left(\frac{2k}{T}t\right)^2 dt.$

$= 2 \times \frac{4k^2}{T^2}\int_0^{T/2} t^2\,dt.$

$= \frac{8k^2}{T^2}\left[\frac{1}{3}t^3\right]_0^{T/2}.$

$= \frac{\cancel{8}k^2}{\cancel{T^2}} \times \frac{1}{3} \times \frac{T^{\cancel{3}}}{\cancel{8}}$

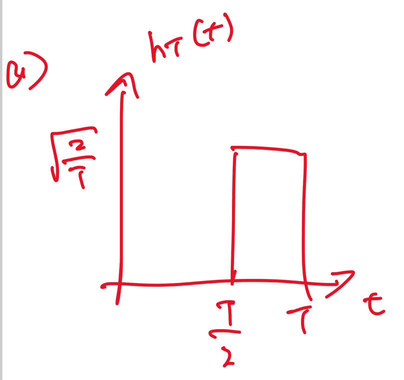
$= \frac{k^2 T}{3} = 1$

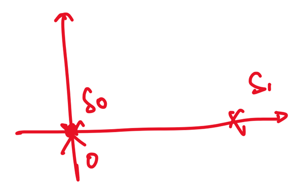
$k = \sqrt{\frac{3}{T}}$

b. $H_T(f) = \int_0^T h(t)\, e^{-j2\pi f t}\,dt$

$= \int$

4) a)





$H_T(f) = \int_{-\infty}^{\infty} h_T(t)\, e^{-j2\pi f t}\,dt.$

$= \int_{T/2}^{T} \sqrt{\frac{2}{T}}\, e^{-j2\pi f t}\,dt.$

$= \sqrt{\frac{2}{T}} \int e^{-j2\pi f t}\,dt$ $\int e^{ax}dx = \frac{1}{a}e^{ax}$

$= \sqrt{\frac{2}{T}}\left[\frac{-1}{j2\pi f} e^{-j2\pi f t}\right]_{T/2}^{T}$

$= \sqrt{\frac{2}{T}} \frac{(-1)}{j2\pi f}\left[e^{-j2\pi f T} - e^{-j2\pi f \frac{T}{2}}\right]$ $\sin A = \frac{1}{j2}\left[e^{jA} - e^{-jA}\right]$

$= \sqrt{\frac{2}{T}} \frac{-1}{j2\pi f}\left[e^{-j2\pi f \frac{T}{4}} - e^{+j2\pi f \frac{T}{4}}\right] e^{-j2\pi f \frac{3T}{4}}$

$= \sqrt{\frac{2}{T}} \frac{1}{\pi f} \sin 2\pi f \frac{T}{4}\; e^{-j2\pi f \frac{3T}{4}}$

$= \sqrt{\frac{2}{T}} \frac{T/2}{T/2} \frac{\sin \pi f \frac{T}{2}}{\pi f} e^{-j2\pi f \frac{3T}{4}}$

$= \sqrt{\frac{2}{T}} \frac{\sin \pi f \frac{T}{2}}{\pi f \frac{T}{2}} e^{-j2\pi f \frac{3T}{4}}$

$\rightarrow |\;|^2 = e^{+jA} \times e^{*jA} = 1$

$|H_T(f)|^2 = \frac{T}{2}\left(\frac{\sin \pi f \frac{T}{2}}{\pi f \frac{T}{2}}\right)^2$

$|e^{jn}|^2 = |e^{jA}|(e^{jA})^*$

$= e^{jA} e^{-jA} = e^{jA-jA}$

$= e^0 < 1$

$\frac{-1}{j2\pi t}\left[e^{-jA} - e^{+jA}\right]$

$= \frac{1}{j2\pi f}\left[e^{+jA} - e^{-jA}\right]$

$= \frac{1}{\pi f}\sin A$

$\forall \in [0,1]$

$\mu_v = E[V_n] = \frac{1}{2}(0) + \frac{1}{2}(1) = \frac{1}{2}$

$\sigma_v^2 = E[V_n^2] - \mu_v^2 = \frac{1}{2}(0)^2 + \frac{1}{2}(1)^2 - (\frac{1}{2})^2 = \frac{1}{4}$

$\frac{(\frac{1}{2})^2}{T^2} \times \frac{T}{2}$

$= \frac{1}{4T^{\cancel{2}}} \times \frac{\cancel{T}}{2}$

$= \frac{T}{8}$

b) $\Phi(f) = \frac{\sigma_v^2}{T}|H_T(f)|^2 + \frac{|\mu_v|^2}{T^2}\sum_{m=-\infty}^{\infty} |H_T(\frac{m}{T})|^2 \delta(f - \frac{m}{T})$

$= \frac{1/4}{T}\frac{T}{2}\left(\frac{\sin \pi f T/2}{\pi f T/2}\right)^2 + \frac{(1/2)^2}{T^2}\sum_m \frac{T}{2}\left(\frac{\sin \pi \frac{m}{T}\frac{T}{2}}{\pi \frac{m}{T}\frac{T}{2}}\right)^2 \delta(f - \frac{m}{T})$

$= \frac{1}{8}\left(\frac{\sin \pi f \frac{T}{2}}{\pi f \frac{T}{2}}\right)^2 + \frac{1}{8T}\sum_m \left(\frac{\sin \pi m/2}{\pi m/2}\right)^2 \delta(f - \frac{m}{T})$

NOT 0, so pulse

$\mathrm{sinc}(\frac{T}{2}\pi f)$

$\sin(\pi) = 0 \rightarrow 1\times\pi = \pi \rightarrow \frac{T}{2}f = 1 \rightarrow f = \frac{2}{T}$

$\sin(2\pi) = 0 \rightarrow \frac{T}{2}f = 2 \rightarrow f = \frac{4}{T}$