Carleton University
Department of Systems and Computer Engineering

SYSC 4600 **Digital Communication** **Winter 2022/23**

Assignment #2
Due on Wednesday, Febrary 15, 2023

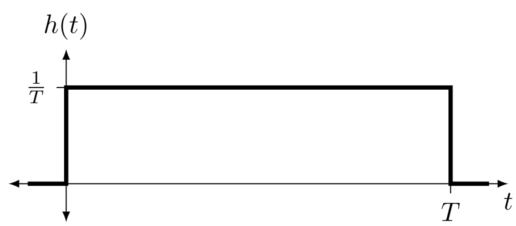
1. For two continuous random variables, X and Y, with joint probability density function $f_{X,Y}(x, y)$, prove that $\mathbf{E}[X + Y] = \mathbf{E}[X] + \mathbf{E}[Y]$.

2. Suppose $\{X_n | n \in \{1, 2, ..., N\}\}$ is a set of N independent Gaussian random variables with mean μ_n and variance σ_n^2 which are different for each n. If

$$Y = \sum_{n=1}^{N} X_n$$

express the mean and variance of Y in terms of $\{\mu_n\}$ and $\{\sigma_n^2\}$. Write the probability density function of Y.

3. Suppose $x(t)$ is a zero-mean stationary Gaussian random process with power spectral density $\Phi_x(f)$ is the input to a linear filter whose impulse response show below. A sample, Y, is taken of the output of the filter at time T.



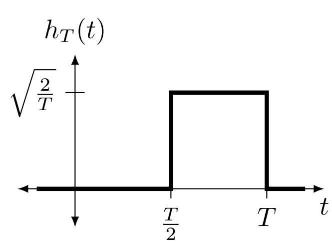
(a) Express the mean and variance of Y in terms of $\Phi_x(f)$ and T.

(b) Upper bound the variance under the condition $\Phi_x(f) \le S$ for all f.

4. Suppose the normalized pulse shape, $h_T(t)$, shown below, is used to generate the pulse train

$$v(t) = \sum_{n=-\infty}^{\infty} v_n h_T(t - nT)$$

where $\{v_n\}$ are the transmitted symbols, with each v_n selected independently from $\{0, +1\}$ with equal probability.



(a) Find the Fourier transform of $h_T(t)$.

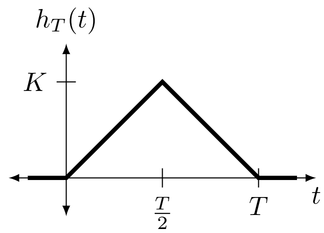
(b) Find a simple mathematical expression for the power spectral density of $v(t)$.

(c) Sketch the PSD over the frequency range $[-5/T, 5/T]$ Hz.

5. Suppose the normalized pulse shape, $h_T(t)$, shown below, is used to generate the pulse train

$$v(t) = \sum_{n=-\infty}^{\infty} v_n h_T(t - nT)$$

where $\{v_n\}$ are the transmitted symbols, with each v_n selected independently from $\{-1, +1\}$ with equal probability.



(a) Find K so that $h_T(t)$ is normalized to have unit energy.

(b) Find a simple mathematical expression for the power spectral density of $v(t)$.

(c) Plot the PSD (in dB) using MATLAB, using $T = 1$ second, over the frequency range $[-5, 5]$ Hz. The y-axis should cover the range $[-50, 10]$ dB.

6. Suppose the signals $s_0(t)$ and $s_1(t)$ shown below are used to represent a '0' and a '1', respectively. If they are used to transmit a long sequence of bits (with zeros and ones equally likely to be transmitted), find and sketch the power spectral density of the transmitted pulse train.