Laboratory #2

Probability of Error for BPSK and 16-QAM

SYSC 4600 Digital Communications

Department of Systems and Computer Engineering
Faculty of Engineering
Carleton University

Purpose and Objectives

The purpose of this laboratory experiment is to investigate the probability of bit error for different digital bandpass modulation schemes. It builds on Laboratory #1, which you should review. You are encouraged to refer back to that lab manual while working on this lab.

Laboratory

Step 1: BER of BPSK

To estimate the probability of a bit error for BPSK, modify your simulator from Lab #1 by removing the instructions for calculating the PSD, and use the following system parameters:

Message word length	$N_a = 1000$ bits
Number of message words to test	$N_f = 10$ words
Symbol duration	$T = 0.01$ seconds
Sampling ratio	$\eta = 64$ samples per symbol
Carrier frequency	$f_c = 400$ Hz
Signal-to-noise ratio (SNR)	$E_b/N_0 = 5$ dB
Symbol Map	`SM = [+1 -1];`

Use a randomly-generated message. Keep track of the total number of message bit errors that occur over all N_f transmitted message words.

The AWGN bandpass channel model is

$$r_c(t) = v_c(t) + w_c(t)$$

where $v_c(t)$ is the transmitted bandpass signal, and $w_c(t)$ is the noise, which is modeled as a Gaussian random process with zero mean and a double-sided noise PSD of $N_0/2$. To simulate the bandpass noise in MATLAB, use

```
rct = vct + sqrt(1/Ts * N0/2) * randn(1, length(vct));
```

where `rct` is the received bandpass signal, `vct` is the transmitted bandpass signal, and `Ts` is the sampling period ($T_s = T/\eta$). The value for N_0 is calculated from $N_0 = E_b \times 10^{-SNRdB/10}$ where *SNRdB* is the desired SNR (E_b/N_0) in dB, and $E_b = E[|v_n|^2]$ which can be calculated from the symbol map. Note that the theoretical value of the probability of error for BPSK is

$$P_b = Q\left(\sqrt{2\frac{E_b}{N_0}}\right) = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

where $Q(\cdot)$ is the Q function and erfc( ) is the complementary error function (use either `qfunc` or `erfc` in MATLAB), and E_b/N_0 is on a linear scale (not dB).

Question 1. Start by simulating $N_f = 10$ message words of $N_a = 1000$ bits each, at a signal-to-noise ratio of $E_b/N_0 = 5$ dB. What is the total number of bit errors that occurred? What is your estimate of the probability of a bit error? What is the theoretical probability of error for BPSK? How to these values compare?

Question 2. Run your simulator for different values of E_b/N_0 ranging from 0 dB to 10 dB with a step size of 1 dB. Generate a graph of the estimated probability of bit error vs. E_b/N_0. Use a logarithm scale for the y-axis. In the same graph, show the theoretical BER of BPSK.

Question 3. To get more accurate results, increase N_f to 1000. Rerun your simulator for E_b/N_0 ranging from 0 dB to 10 dB with a step size of 1 dB, and generate a graph of the estimated probability of bit error vs. E_b/N_0. In the same graph, show the theoretical probability of error.

Step 2: Probability of Bit Error for 4-QAM

Modify your simulator to use a 4-QAM instead of BPSK. This will entail using complex-value variables for v_n, $v(t)$, $r_o(t)$, and r_n. The transmitted bandpass signal will be

$$v_c(t) = \Re\left\{v(t)\sqrt{2}e^{j2\pi f_c t}\right\}$$

where $\Re\{z\}$ denotes the real part of z. At the receiver the demodulated signal is

$$r_o(t) = r_c(t)\sqrt{2}e^{-j2\pi f_c t} \ .$$

The symbol mapper and decision device will be more difficult to implement.

After you have tested and debugged your simulator, use $N_f = 1000$ to estimate the probability of bit error for E_b/N_0 ranging from 0 dB to 10 dB with a step size of 1 dB.

Question 4. Generate a graph of the estimated probability of bit error vs. E_b/N_0. In the same graph, show the theoretical probability of error,

$$P_b = Q\left(\sqrt{2\frac{E_b}{N_0}}\right) = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right).$$

Step 3: Probability of Bit Error for 16-QAM

Modify your simulator to use 16-QAM instead of 4-QAM. This will only require changes to the symbol mapper and decision device.

After you have tested and debugged your simulator, use $N_f = 1000$ to estimate the probability of bit error for E_b/N_0 ranging from 0 dB to 14 dB with a step size of 1 dB.

Question 5. Generate a graph of the estimated probability of bit error vs. E_b/N_0. In the same graph, show the theoretical probability of error,

$$\begin{aligned} P_b &= \frac{3}{4}Q\left(\sqrt{\frac{4}{5}\frac{E_b}{N_0}}\right) + \frac{2}{4}Q\left(3\sqrt{\frac{4}{5}\frac{E_b}{N_0}}\right) - \frac{1}{4}Q\left(5\sqrt{\frac{4}{5}\frac{E_b}{N_0}}\right) \\ &= \frac{3}{8}\operatorname{erfc}\left(\sqrt{\frac{2}{5}\frac{E_b}{N_0}}\right) + \frac{2}{8}\operatorname{erfc}\left(3\sqrt{\frac{2}{5}\frac{E_b}{N_0}}\right) - \frac{1}{8}\operatorname{erfc}\left(5\sqrt{\frac{2}{5}\frac{E_b}{N_0}}\right). \end{aligned}$$