ass1

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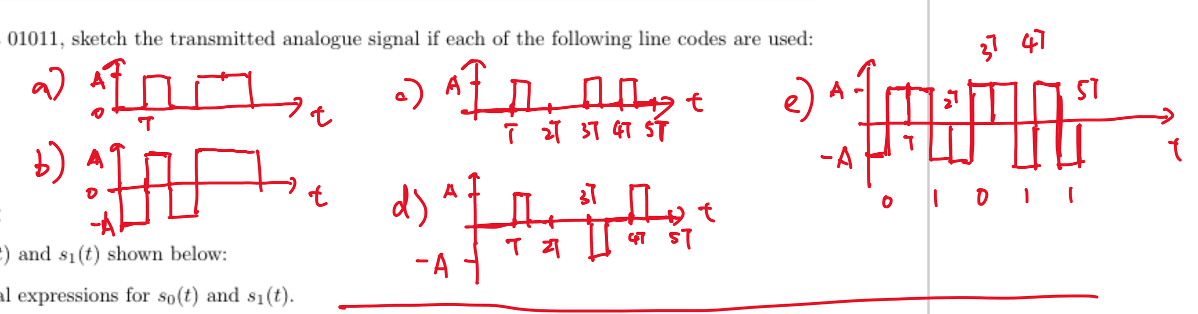
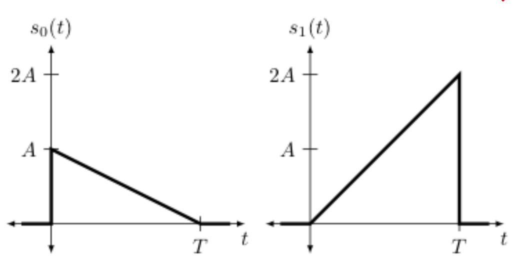
ass1

Carleton University
Department of Systems and Computer Engineering
SYSC 4600 Digital Communication Winter 2022/23
Assignment #1
Due on Wednesday, February 8, 2023

1. For the sequence of bits 01011, sketch the transmitted analogue signal if each of the following line codes are used:
   (a) unipolar NRZ
   (b) polar NRZ
   (c) unipolar RZ
   (d) bipolar RZ
   (e) Manchester coding



2. Suppose the signals $s_0(t)$ and $s_1(t)$ shown below:
   (a) Write mathematical expressions for $s_0(t)$ and $s_1(t)$.
   (b) Sketch the sum $v(t) = s_0(t) + s_1(t)$.
   (c) Sketch the product $v(t) = s_0(t)s_1(t)$.
   (d) Sketch the convolution $v(t) = s_0(t) \circledast s_1(t)$.
   (e) Find the inner product $s_{01} = \langle s_0(t), s_1(t)\rangle$.



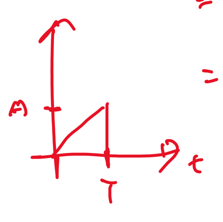
a) $S_0(t) = -\frac{A}{T}t$

$S_1(t) = \frac{2A}{T}t$

(b) $V(t) = S_0(t) + S_1(t)$

$= \frac{2A}{T}t - \frac{A}{T}t$

$= \frac{A}{T}t$

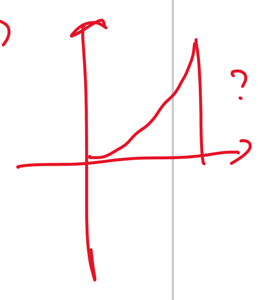


(c) $V(t) = S_0(t) S_1(t)$

$= \left(-\frac{A}{T}t\right)\left(\frac{2A}{T}t\right)$

$= -\frac{2A^2}{T^2}t$

(d)



e) $S_{01} = \langle S_0(t), S_1(t)\rangle$

$= \int_{-\infty}^{\infty} S_0(t) S_1(t)\, dt$

$= \int_0^T \left(-\frac{A}{T}t\right)\left(\frac{2A}{T}t\right) dt$

$= \int_0^T -\frac{2A^2}{T^2} t^2\, dt.$

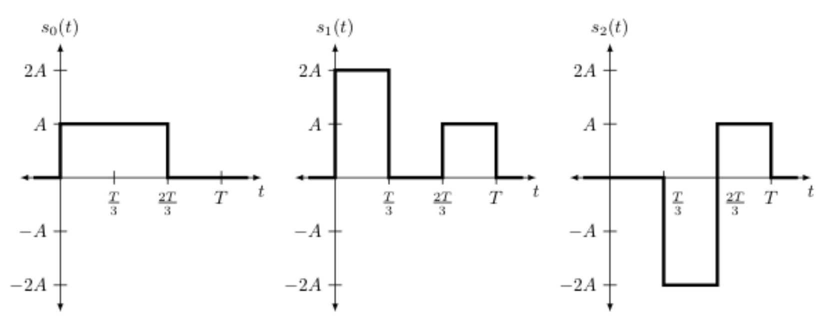
$= -\frac{2A^2}{T^2}\left[\frac{1}{3}t^3\right]_0^T$

$= -\frac{2A^2}{T^2}\left[\frac{1}{3}T^3 - 0\right]$

$= -\frac{2A^2}{3T^2}T^3$

$= -\frac{2}{3}A^2T$

3. For the signals $s_0(t)$, $s_1(t)$, and $s_2(t)$ shown below:
   (a) Use the Gram-Schmidt orthogonalization procedure to find a set of basis signals. Sketch the basis signals.
   (b) Express $s_0(t)$, $s_1(t)$, and $s_2(t)$ as linear combinations of the basis signals.
   (c) Show $s_0(t)$, $s_1(t)$, and $s_2(t)$ on a signal space diagram.
   (d) What is the average transmitted energy, if each signal is equally likely to be transmitted?



a)

1) Find 1st base signal

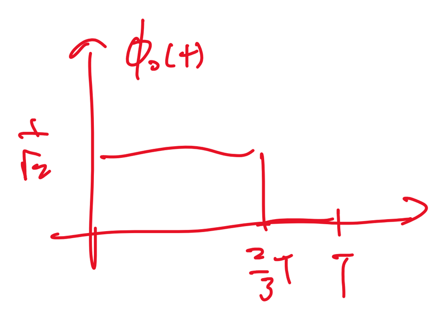
$\|S_0(t)\|^2 = \int_{-\infty}^{\infty} |S_0(t)|^2 dt.$

$= \int_0^{\frac{T}{3}} (A)^2 dt + \int_{\frac{1}{3}}^{\frac{2}{3}T} (A)^2 dt + \int_{\frac{2}{3}T}^{T} (0)^2 dt.$

$= A^2 + A^2 + 0$

$= 2A^2$

$\phi_0(t) = \frac{S_0}{\|S_0\|} = \frac{S_0}{\sqrt{2}A} = \begin{cases} \frac{1}{\sqrt{2}} & 0 < t \le \frac{2}{3}T \\ 0 & \frac{2}{3}T < t \le T \end{cases}$



$S_0(t) = \sqrt{2}A\, \phi_0(t)$

2). 2nd base signal

$S_{1,0} = \langle S_1, \phi_0 \rangle$

$= \int_{-\infty}^{\infty} S_1(t)\phi_0(t)\, dt.$

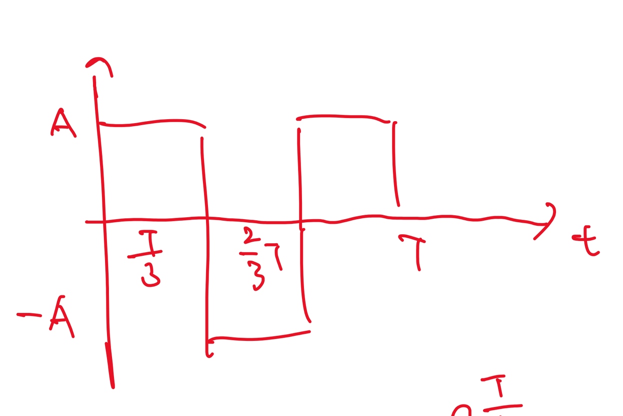
$= \int_0^{\frac{T}{3}} 2A \times \frac{1}{\sqrt{2}}\, dt + \int_{\frac{T}{3}}^{\frac{2}{3}T} 0 \times \frac{1}{\sqrt{2}}\, dt + \int_{\frac{2}{3}T}^{T} A \times 0\, dt.$

$= \sqrt{2}A + 0 + 0 = \underline{\sqrt{2}A}.$

error signal

$g_1(t) = S_1(t) - S_{1,0}\,\phi_0(t)$

$= \begin{cases} 2A - \sqrt{2}A \times \frac{1}{\sqrt{2}} = 2A - A = A & 0 \le t \le \frac{T}{3} \\ 0 - \sqrt{2}A \times \frac{1}{\sqrt{2}} = -A & \frac{T}{3} \le t \le \frac{2}{3}T \\ A - \sqrt{2}A \times 0 = A - 0 = A. & \frac{2T}{3} \le t \le T \end{cases}$

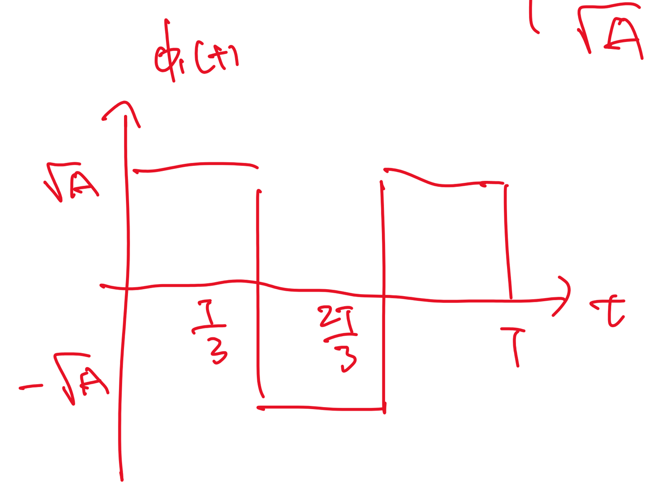


$\|g(t)\|^2 = \int_0^{\frac{T}{3}} A\, dt + \int_{\frac{T}{3}}^{\frac{2}{3}T} -A\, dt + \int_{\frac{2}{3}T}^{T} A\, dt.$

$= A - A + A$

$= A.$

$\phi_1(t) = \frac{g(t)}{\|g(t)\|} = \begin{cases} \sqrt{A} & 0 \le t \le \frac{T}{3} \\ -\sqrt{A} & \frac{T}{3} \le t \le \frac{2}{3}T \\ \sqrt{A} & \frac{2T}{3} \le t \le T \end{cases}$



$S_1(t) = S_{0,1}\,\phi_0(t) + \|g(t)\|\,\phi_1(t)$

$= \sqrt{2}A\,\phi_0(t) + \sqrt{A}\,\phi_1(t)$

check:

$S_1(t) = \begin{cases} \sqrt{2}A \times \frac{1}{\sqrt{2}} + \sqrt{A} \times \sqrt{A} = A + A = 2A \checkmark \\ \sqrt{2}A \times \frac{1}{\sqrt{2}} + \sqrt{A} \times -\sqrt{A} = 0 - A = -A \checkmark \\ \sqrt{2}A \times 0 + \sqrt{A} \times \sqrt{A} = 0 + A = A \checkmark \end{cases}$

match ✓

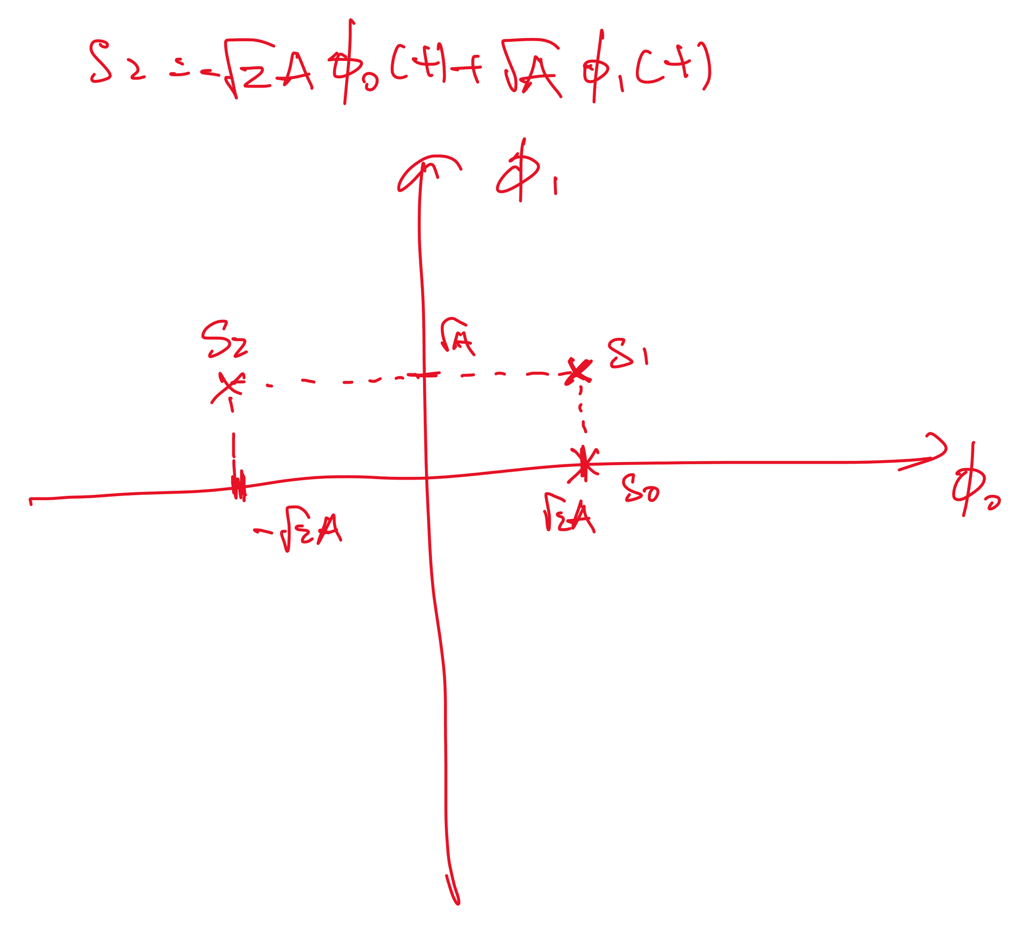
$S_2 = -\sqrt{2}A\,\phi_0 + \sqrt{A}\,\phi_1$

Summary:

$S_0 = \sqrt{2}A\,\phi_0(t)$

$S_1 = \sqrt{2}A\,\phi_0(t) + \sqrt{A}\,\phi_1(t)$

$S_2 = -\sqrt{2}A\,\phi_0(t) + \sqrt{A}\,\phi_1(t)$



d) Avg. E.

$E_0 = (\sqrt{2}A)^2 = 2A^2$

$E_1 = (\sqrt{2}A)^2 + (\sqrt{A})^2 = 2A^2 + A$

$E_2 = (-\sqrt{2}A)^2 + (\sqrt{A})^2 = 2A^2 + A$

$E_{Avg.} = \frac{1}{3}(2A^2 + 2A^2 + A + 2A^2 + A)$

$= \frac{1}{3}(6A^2 + 2A)$

$= 2A^2 + \frac{2}{3}A.$