**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans :

[ x = 60 min(1 hr) – 10 min = 50 min]

Z = 0.62

Using the stats.norm,cdf( 50,45,8) 🡪 0.73

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans : A. Probability of being older than 44 :

Calculate z-score for x = 44

z = 1

stats.norm.cdf(44,38,6) 🡪 0.84 This is the probability that an employee is older

than 44 .

Probability of being between 38 and 44 :

Calculate z-score for x = 38 and x= 44

z = 0 z = 1

Using stats.norm.cdf((38,44),38,6) 🡪 Output : (0.5,0.84)

Calculate the probability between these Z-scores:

(0.84-0.5) = 0.34 This is the probability that an employee’s age is between 38 and

44.

Probability of being older than 44 is appr. 0.84

Probability of being between 38 and 44 is appr. 0.34

So, the statement is true .

B ) Probability of being under the age of 30 :

Z = -1.33

Using stats.norm.cdf (30,38,6) 🡪 0.09 This is the probability that an employee is

under the age of 30.

Expected number of employees under the age of 30 :

Expected number = 0.09\*400 = 36

So, the statement is true .

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans : Both X1 & X2 are independent and identically distributed normal random variables.

X1~N(µ, σ2)

X2~N(µ, σ2)

2X1~N(2µ , 22 σ2) => 2X1~N(2µ , 4 σ2) =>

X1+X2~N(µ +µ , σ2 + σ2) => X1+X2~N(µ +µ , 2σ2)

2X1-(X1+X2)~N(2µ - 2µ , 4 σ2- 2 σ2 ) = > 2X1 –(X1+X2)~N(0,2 σ2)

2X1 AND (X1+X2) have the same mean.

Variance of 2X1 is twice the variance of (X1+X2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78 s
5. 48.5, 151.5
6. 90.1, 109.9

Ans : - X ~N(100,202) = X~N(µ, σ2)

99 % = +- 2.576

a = 151.76 b = 48.48

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Ans :- Total mean = 12 $ million = 12 \*45 = 540 million

Total variance = 25 $ million

Variance = 5 \* 45 = 225 million

95% 🡪 Z-score 🡪approximately 1.96

1. Rupee Range = Mean +- (Z\* Standard Deviation )

= 540 +- (1.96 \* 225)

= (981,99)

So, the rupee range that contains 95% probability for the annual profit of the company

is approximately Rs 99 million to Rs. 981 million .

1. To specify the 5th percentile of profit for the company, we need to find the value at which 5 % of the profits are lower .

5% 🡪 Z-value 🡪 -1.645

5th percentile = mean+ (z \* standard deviation )

= 540 + (-1.645 \* 225)

= 169.875 million

1. To determine which of the two divisions has a larger probability of making a loss in a given year , compare the mean and standard deviations of profit1 and profit2 . A division is more likely to make a loss of its mean profit is lower and its standard deviation is higher .

|  |  |  |
| --- | --- | --- |
|  | Mean | Variance |
| Profit 1 | 5 | 9 |
| Profit 2 | 7 | 16 |

Profit1 has a lower mean but a lower variance compared to profit2 . Since profit1 has

A lower mean , it is more likely to make a loss in a given year compared to profit2 .