
**MIDDLE EAST TECHNICAL
UNIVERSITY**

**DEPARTMENT OF ELECTRICAL AND
ELECTRONICS ENGINEERING**

EE564 Project #1

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1 Inductor Design

In this part, I have designed an inductor with a toroidal core, whose datasheet is given in Figure 1.

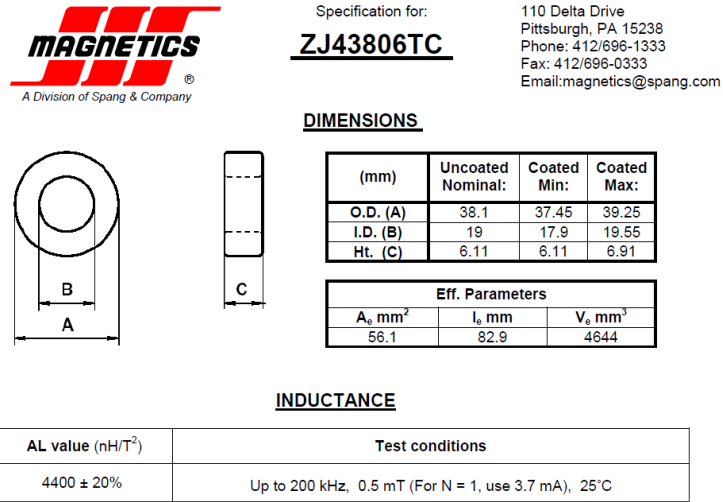


Figure 1: Datasheet of the Toroidal Core

I chose the number of turns as 15, and Dc current as 0.3 A, in order to obtain N.I= 4.5 A.turns, which makes the inductor operate in linear region, but close to saturation. These choices are verified by calculating, calculations are as follows:

$$\frac{B * L_{eff}}{\mu_0 * \mu_r} = N * I$$

$$B = \frac{4.5 * 5000 * 4 * \pi * 10^{-7}}{0.0829} = 0.341T$$

Analytical calculations of inductance (with MATLAB code) are given below. It should be considered that BH curve data used in non-linear material calculations does not belong to the material that I have chosen. Its relative permeability is around 4000, where that of my toroid of choice is 5000. An additional difference is also expected in analytical calculations because of this fact.

Part A

Parameters

```
in_dia= 19; %mm
in_rad=in_dia/2*1e-3;
out_dia= 38.1; %mm
out_rad=out_dia/2*1e-3;
ht= 6.11; %mm
```

```
Leff=82.9; %mm
Aeff=56.1; %mm^2
```

```
N=15 ;% turns
I=0.3; % A
```

```
mu_r=5000;
mu_0=4*pi*1e-7;
```

```
AL=4400; %nH/turns^2
```

Analytical Calculations

```
%
% # 1
```

```
reluctance=Leff*1e-3/(Aeff*1e-6*mu_r*mu_0);
```

```
L=N^2/reluctance; %H
```

```
fprintf('Inductance assuming homogeneous distribution is %d H. \n', L);
```

```
% # 2
```

```
% In this part, H.dl should be integrated over over the cross-section of
% coaxial circles with radii from in_dia/2 to out_dia/2.
%  $H \cdot 2 \cdot \pi \cdot (out\_rad - in\_rad) = N \cdot I$ 
```

```
index=linspace(in_rad,out_rad,500);
for i=1:numel(index)
```

```
    H(:,i)=N*I/(2*pi*index(:,i));
    B(:,i)=mu_0*mu_r*H(:,i); %T
    Phi(:,i)=B(:,i)*ht*1e-3*(out_rad-in_rad)/500; %Wb
```

```
end
```

```

tot_phi=0;

for k=1:500

    tot_phi=tot_phi+Phi(:,i);

end

L_2=tot_phi*N/I;

fprintf('Inductance assuming non-homogeneous distribution is %d H. \n', L_2);
%
% # 3_1
%
I_2=I*1.5;

H_2=N*I_2/(Leff*1e-3);

B_2=interp1(B_n1,H_n1,H_2);

phi_2=Aeff*1e-6*B_2;

L_3=N*phi_2/I_2;

fprintf('Inductance assuming homogeneous distribution & non-homogeneous material is %d H. \n', L_3);
%
% # 3_2
%

index=linspace(in_rad,out_rad,500);
for i=1:numel(index)

    H_3(:,i)=N*I_2/(2*pi*index(:,i));
    B_3(:,i)=interp1(B_n1,H_n1,H_3(:,i));
    Phi(:,i)=B_3(:,i)*ht*1e-3*(out_rad-in_rad)/500; %Wb
end

tot_phi=0;

```

```

for k=1:500

    tot_phi=tot_phi+Phi(:,i);

end

L_4=tot_phi*N/I_2;

fprintf('Inductance assuming non-homogeneous distribution & non-homogeneous material is

%
% # 4
%

rel_gap=2e-3/(mu_0*Aeff*1e-6);
rel_core=(Leff-2)*1e-3/(Aeff*1e-6*mu_r*mu_0);
rel_tot=rel_gap+rel_core;

L_5=N^2/rel_tot;

fprintf('Inductance of the gapped core assuming homogeneous distribution is %d H. \n', L

%
% # 5
%

% In this part, we may assume the fringing flux is considerable for an area
% of 2 mm (the length of airgap) to the left and to the right of the
% airgap. Therefore we may assume the equivalent reluctance of the magnetic
% circuit as R_core in series with (R_gap_left||R_gap||R_gap_right). This
% approach is not a reliable one, nevertheless it may give us an idea.

rel_gap_side=2e-3/(mu_0*2e-3*ht*1e-3);
rel_tot_2=rel_core+(1/rel_gap_side+1/rel_gap+1/rel_gap_side)^(-1);

L_6=N^2/rel_tot_2;

fprintf('Inductance of the gapped core including fringing flux is %d H. \n', L_6);

Inductance assuming homogeneous distribution is 9.566889e-04 H.
Inductance assuming non-homogeneous distribution is 6.891791e-04 H.
Inductance assuming homogeneous distribution & non-homogeneous material is 5.383088e-04

```

Inductance assuming non-homogeneous distribution & non-homogeneous material is 5.027784e-06 H.
Inductance of the gapped core assuming homogeneous distribution is 7.867304e-06 H.
Inductance of the gapped core including fringing flux is 1.125535e-05 H.

Part B

1. Mesh plots, B field distribution, distribution of B vectors and flux line distribution of the toroid, assuming linear magnetic material properties, are given below on Figures 2-5. Note that thin copper layer inside the toroid is excited with positive 4.5 A.t and the one that is at the outside of the toroid is excited with negative 4.5 A.t.

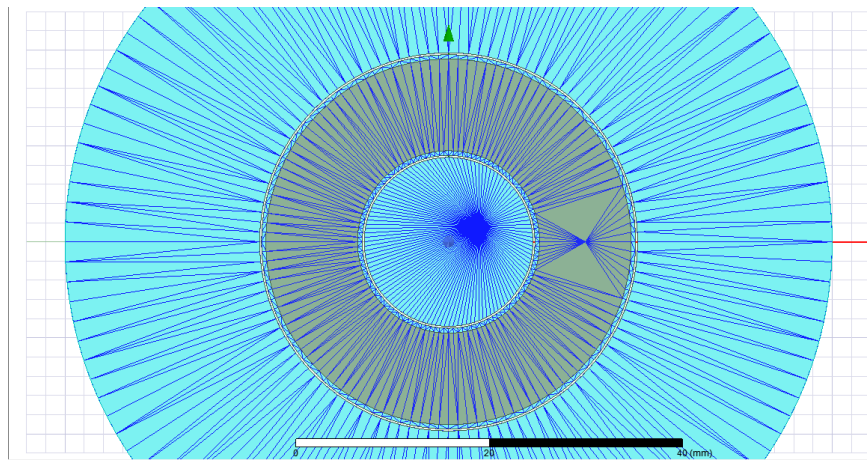


Figure 2: Mesh Operations on the Toroid

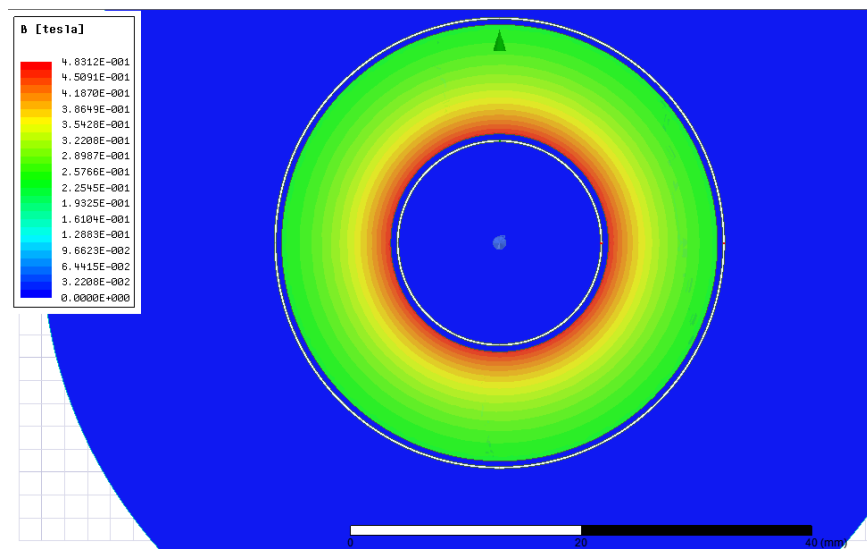


Figure 3: B Field Distribution in the Inductor

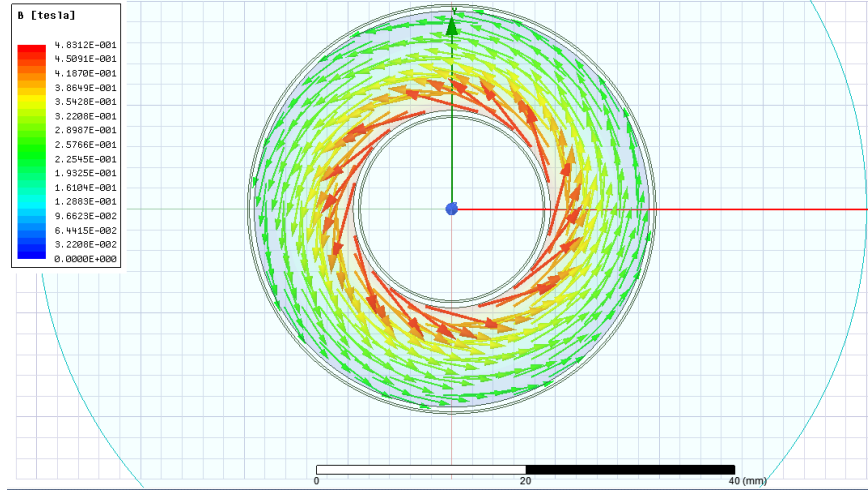


Figure 4: B Vectors in the Inductor

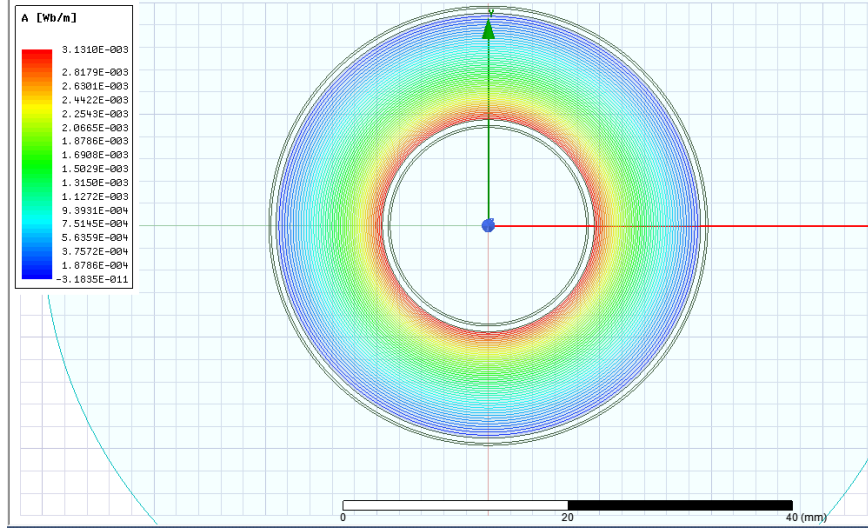


Figure 5: Flux Lines in the Toroidal Core

- Inductance is calculated on the line from inner radius to outer radius, using the formula given below:

$$L = \frac{N * B * A}{I}$$

where A is denoted by the line, since the simulation is 2D.

To calculate the leakage inductance, same approach is used, (with a longer line crossing both inner and outer sides of the inductor) however no leakage is observed in this case. Inductance values are given on Figures 6 and 11, below.

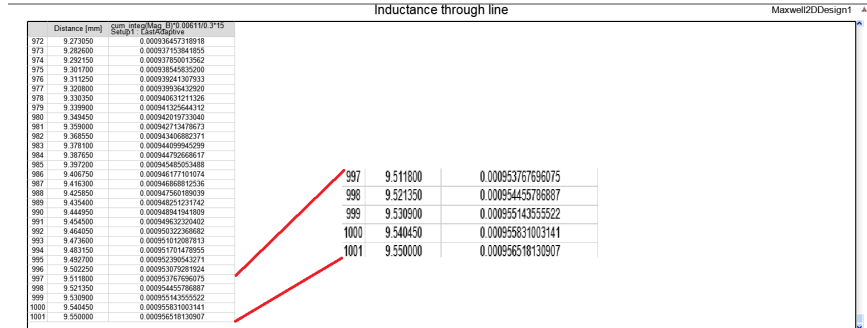


Figure 6: Inductance with Linear Material Properties

- Here, the major observation is the saturation of the core. Also, there is a considerable drop in inductance.

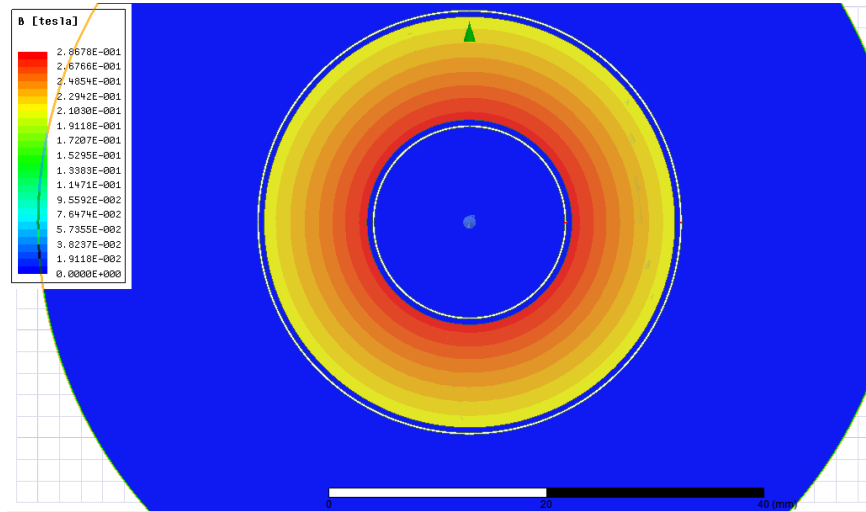


Figure 7: B Field Distribution in the Inductor with Non-Linear Material Properties

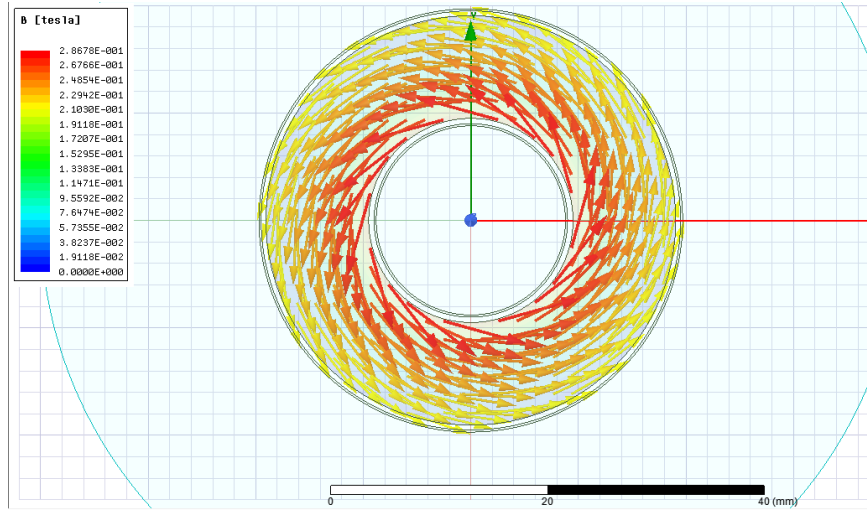


Figure 8: B Vectors in the Inductor with Non-Linear Material Properties

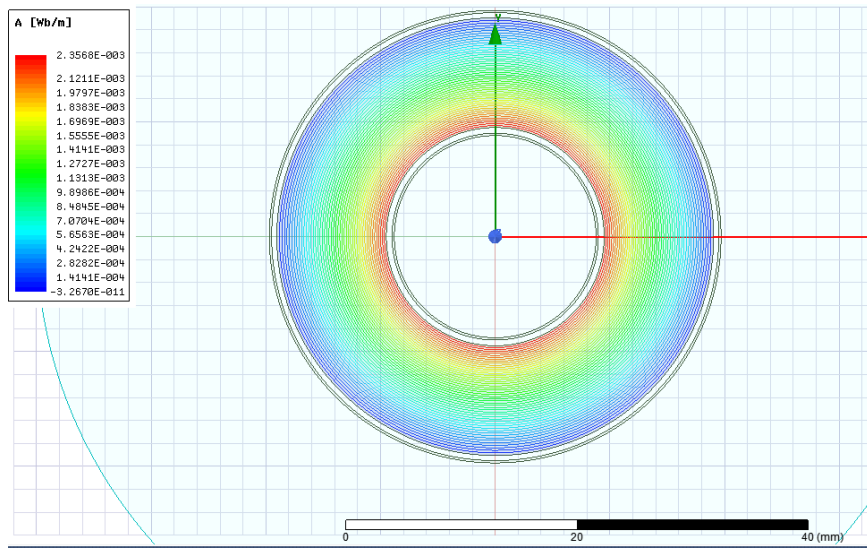


Figure 9: Flux Lines in the Toroidal Core with Non-Linear Material Properties

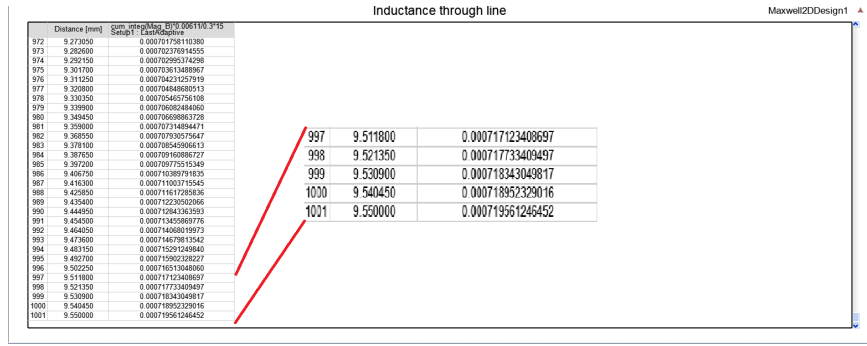


Figure 10: Inductance with Non-Linear Material Properties

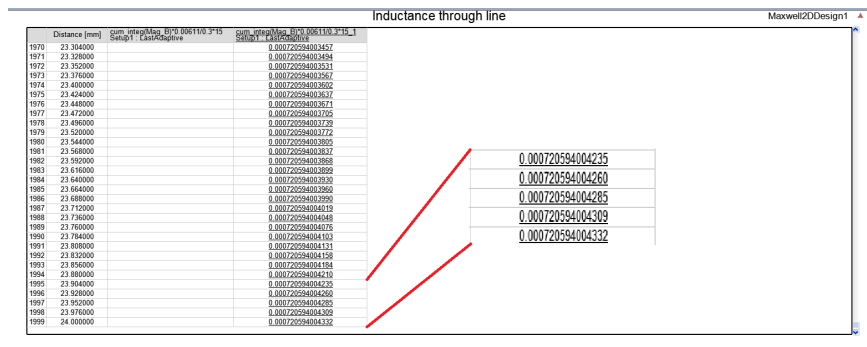


Figure 11: Leakage Included Inductance with Non-Linear Material Properties

- In this case, airgap introduces a significant increase in reluctance to the system, therefore inductance decreases considerably. Further, we can observe the effect of leakage and fringing flux on Figures 12, 13 and 14, which creates leakage inductance. Leakage inductance can be calculated by subtracting inductance value on Figure 15 from overall inductance given in Figure 16.

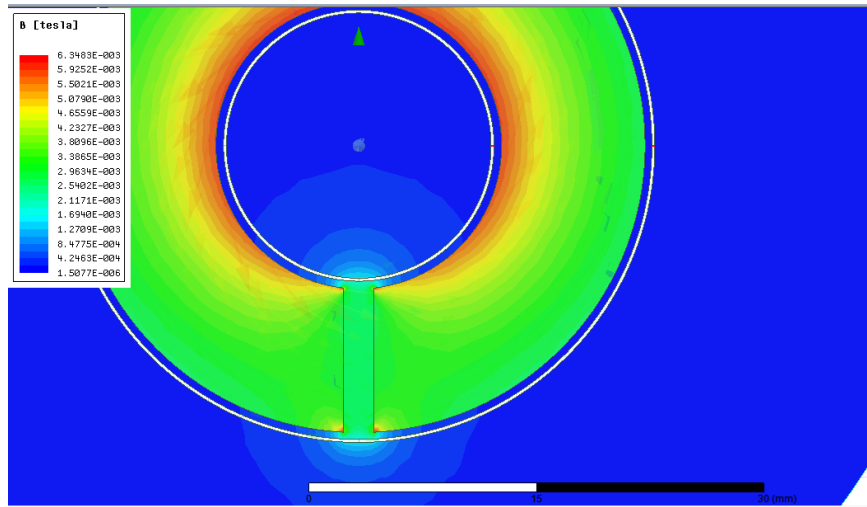


Figure 12: B Field Distribution in the Gapped Inductor with Non-Linear Material Properties

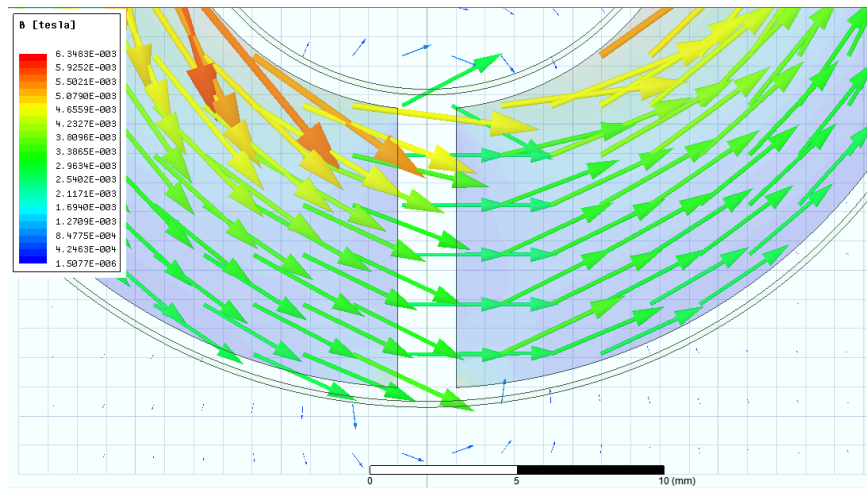


Figure 13: B Vectors in the Gapped Inductor with Non-Linear Material Properties

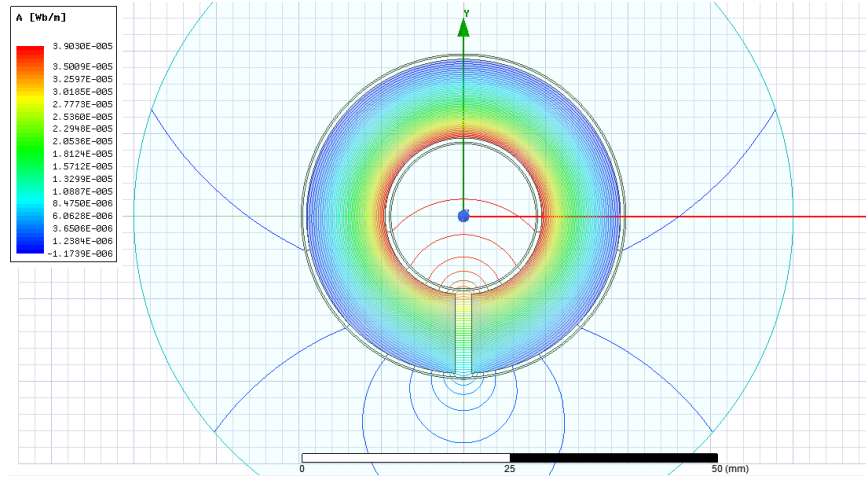


Figure 14: Flux Lines in the Gapped Toroidal Core with Non-Linear Material Properties

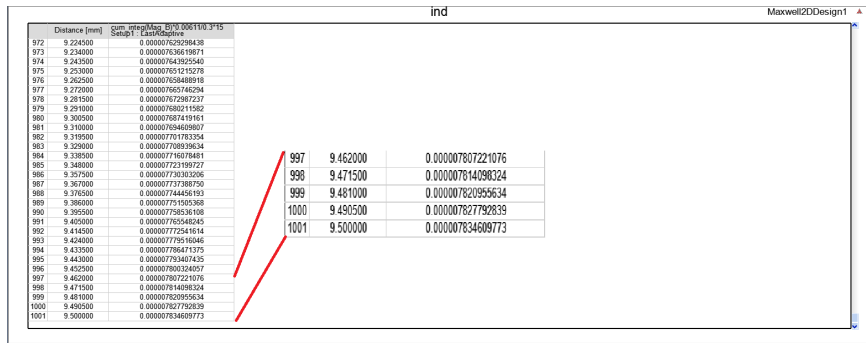


Figure 15: Inductance of the Gapped Core with Non-Linear Material Properties

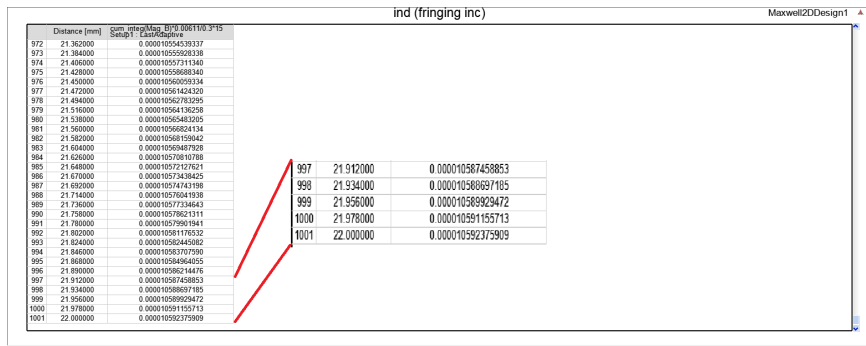


Figure 16: Leakage Included Inductance of the Gapped Core with Non-Linear Material Properties

Part C

Below, on Table 1, inductance calculation results of both methods are present.

Table 1: Comparison of Inductance Calculations

	FEA	Analytical
Linear	0.956 mH	0.957 mH
Non-Linear	0.720 mH	0.538 mH
With Airgap	7.83 μ H	7.87 μ H

Possible discrepancies between these calculations are caused by approximations made in analytical approach. In FEA, as calculations are made for infinitesimal elements and then integrated, it gives us more precise results. However for analytical approach, this is not possible. Hence, for calculations such as non-linear or non-homogeneous material properties, FEA technique is more reliable.

If we used 3D FEA instead of 2D, we could also observe the leakage and fringing flux through the cross-section of the core.

2 Transformer Design

With given parameters, a transformer is designed and optimized. MATLAB code for this operation is present in this section of the report.

Code for Transformer Design and Optimization

Parameters

```

B_op=1.5; %T
mu=1.83/800;
f=50; %Hz
P1= 500e3; %W
V1=34500; %V
V2=25000; %V
I1=P1/V1; %A
J=4; %A/mm^2
a_cable=I1/J; %mm^2
fprintf('Cable area should be around %d mm^2.\n',a_cable);

% This value is close to AWG11 size.

a_cable=4.1684; %mm^2
dia_cable=2.30378; %mm
res_cable=4.1328; %Ohms/km

```

Cable area should be around $3.623188e+00 \text{ mm}^2$.

Sizing

% Recalling the equation: $V_{ind}=2\pi f B A N$
% $A \cdot N$ is constant and they are dependent on each other. Optimum values of
% them will be found with an optimization parameter "k".

ff=0.5; %fill factor
dens_steel=7650; %kg/m³
dens_copper=8940; %kg/m³
core_loss_dens=0.77; %W/kg
price_steel=3; %\$/kg
price_copper=7; %\$/kg

i=1;

for k=5:15

 N1(:,i)=69*k;
 N2(:,i)=50*k;
 A(:,i)= $V^2 \cdot \sqrt{2} / (2 \cdot \pi \cdot f \cdot B_{op} \cdot N1(:,i))$; %m²

 % Window area

 x1(:,i)=dia_cable*23/ff/1000; %m
 x2(:,i)=dia_cable*3*k/ff/1000; %m
 x3(:,i)=ceil(sqrt(A(:,i)*10000))/100; %m

 w1(:,i)=x1(:,i);
 w2(:,i)=2*x2(:,i)+0.03;

% fprintf('Window area is %d m².\n',w1(:,i)*w2(:,i));

 % Overall dimensions

 e1(:,i)=w1(:,i)+2*x3(:,i);
 e2(:,i)=w2(:,i)+2*x3(:,i);

% fprintf('Dimensions of the transformer is %d x %d x %d m.\n',e1(:,i),e2(:,i),x3(:,i));

 vol(:,i)=(e1(:,i)*e2(:,i)-w1(:,i)*w2(:,i))*x3(:,i);

```

m_steel(:,i)=dens_steel*vol(:,i);

%   fprintf('Steel mass is %d kg.\n',m_steel(:,i));

core_loss(:,i)=core_loss_dens*m_steel(:,i);

%   fprintf('Core loss is %d Watts.\n',core_loss(:,i));

% Cable length

mean_length(:,i)=2*(x2(:,i)/2+x3(:,i))+pi*x3(:,i)*sqrt(2); %m
l1(:,i)=mean_length(:,i)*N1(:,i);
l2(:,i)=mean_length(:,i)*N2(:,i);

r1(:,i)=l1(:,i)*res_cable/1000; %Ohms
r2(:,i)=r1(:,i)*(N2(:,i)/N1(:,i))^2; %Ohms

vol_copper(:,i)=2*l1(:,i)*a_cable/1000000; %m^3
m_copper(:,i)=vol_copper(:,i)*dens_copper; %kg

%   fprintf('Copper mass is %d kg.\n',m_copper(:,i));

copper_loss(:,i)=I1^2*r1(:,i)*2; %W

%   fprintf('Copper loss is %d Watts.\n',copper_loss(:,i));

% Inductances

% Assuming L1 and L2 are 0.02 pu;

ind1=V1^2/(P1*2*pi*f)*0.02; %H
ind2=ind1*(N2/N1)^2; %H

Leff(:,i)=2*(w1(:,i)+w2(:,i)+2*x3(:,i)); %m

ind_m(:,i)=N1(:,i).^2*mu*A(:,i)/Leff(:,i); %H

% Efficiency

eff(:,i)=P1/(P1+core_loss(:,i)+copper_loss(:,i))*100; %percent

% Cost

```

```

cost(:,i)=price_copper*m_copper(:,i)+price_steel*m_steel(:,i); %$

%Unit price of electricity: 0.4482 TL = 0.1125 USD

lost_power(:,i)=P1*(100-eff(:,i))/1000; %kW

lost_energy(:,i)=lost_power(:,i)*24*365*20*0.1125; %$

cost_actual(:,i)=cost(:,i)+lost_energy(:,i); %$

i=i+1;
end

```

Conclusion

% Considering the optimization results, optimum case for the design seems to
 % be i=6, k=10. Transformer parameters are as follows:

```

k=10;
i=6;

N1(:,i)=69*k;
N2(:,i)=50*k;
A(:,i)=V2*sqrt(2)/(2*pi*f*B_op*N1(:,i)); %m^2

% Window area

x1(:,i)=dia_cable*23/ff/1000; %m
x2(:,i)=dia_cable*3*k/ff/1000; %m
x3(:,i)=ceil(sqrt(A(:,i)*10000))/100; %m

w1(:,i)=x1(:,i);
w2(:,i)=2*x2(:,i)+0.03;

% Overall dimensions

e1(:,i)=w1(:,i)+2*x3(:,i);
e2(:,i)=w2(:,i)+2*x3(:,i);

```



```

vol(:,i)=(e1(:,i)*e2(:,i)-w1(:,i)*w2(:,i))*x3(:,i);

m_steel(:,i)=dens_steel*vol(:,i);

core_loss(:,i)=core_loss_dens*m_steel(:,i);

% Cable length

mean_length(:,i)=2*(x2(:,i)/2+x3(:,i))+pi*x3(:,i)*sqrt(2); %m
l1(:,i)=mean_length(:,i)*N1(:,i);
l2(:,i)=mean_length(:,i)*N2(:,i);

r1(:,i)=l1(:,i)*res_cable/1000; %Ohms
r2(:,i)=r1(:,i)*(N2(:,i)/N1(:,i))^2; %Ohms

vol_copper(:,i)=2*l1(:,i)*a_cable/1000000; %m^3
m_copper(:,i)=vol_copper(:,i)*dens_copper; %kg

copper_loss(:,i)=I1^2*r1(:,i)*2; %W

% Inductances

% Assuming L1 and L2 are 0.02 pu;

ind1=V1^2/(P1*2*pi*f)*0.02; %H
ind2=ind1*(N2/N1)^2; %H

Leff(:,i)=2*(w1(:,i)+w2(:,i)+2*x3(:,i)); %m

ind_m(:,i)=N1(:,i).^2*mu*A(:,i)/Leff(:,i); %H

% Efficiency

eff(:,i)=P1/(P1+core_loss(:,i)+copper_loss(:,i))*100; %percent

% Cost

cost(:,i)=price_copper*m_copper(:,i)+price_steel*m_steel(:,i); %$

%Unit price of electricity: 0.4482 TL = 0.1125 USD

```

```

lost_power(:,i)=P1*(100-eff(:,i))/1000; %kW

lost_energy(:,i)=lost_power(:,i)*24*365*20*0.1125; %$

cost_actual(:,i)=cost(:,i)+lost_energy(:,i); %$

fprintf('Turns ratio is %d : %d. \n',N1(:,i),N2(:,i));

fprintf('Window area is %d m^2.\n',w1(:,i)*w2(:,i));

fprintf('Dimensions of the transformer is %d x %d x %d m.\n',e1(:,i),e2(:,i),x3(:,i))

fprintf('Steel mass is %d kg.\n',m_steel(:,i));

fprintf('Core loss is %d Watts.\n',core_loss(:,i));

fprintf('R1= %d Ohms, R2= %d Ohms. \n',r1(:,i),r2(:,i));

fprintf('Copper mass is %d kg.\n',m_copper(:,i));

fprintf('Copper loss is %d Watts.\n',copper_loss(:,i));

fprintf('L1= %d H, L2= %d H, Lm= %d H. \n',ind1,ind2,ind_m(:,i));

fprintf('Efficiency is %d percent. \n',eff(:,i));

fprintf('Material cost is %d USD. Lost money in 20 years is %d USD. \n',cost(:,i),co

```

Turns ratio is 690 : 500.

Window area is 3.247608e-02 m².

Dimensions of the transformer is 7.659739e-01 x 9.664536e-01 x 3.300000e-01 m.

Steel mass is 1.786846e+03 kg.

Core loss is 1.375872e+03 Watts.

R1= 6.457173e+00 Ohms, R2= 3.390660e+00 Ohms.

Copper mass is 1.164488e+02 kg.

Copper loss is 2.712528e+03 Watts.

L1= 1.515473e-01 H, L2= 7.957747e-02 H, Lm= 5.521107e+01 H.

Efficiency is 9.918895e+01 percent.

Material cost is 6.175681e+03 USD. Lost money in 20 years is 7.999056e+06 USD.

Here, for the design of the transformer, the main objective is to find an optimum point where $P_{core} = P_{copper}$, since this is the case where efficiency is maximum.

These two variables are dependent on each other, since induced voltage in the tranformer includes a constant term "N*A". This implies that number of turns of the transformer should be inversely proportional to the cross-section area of the core, which are actually the parameters that determines core and copper losses.

A graph that shows the relationship of core loss, copper loss and efficiency is given in Figure 17. The optimum point is chosen as the point where efficiency is maximum. However at this point, core loss and copper loss are not exactly equal to each other. This discrepancy is caused by (wire mean length) assumptions made in calculation of copper loss.

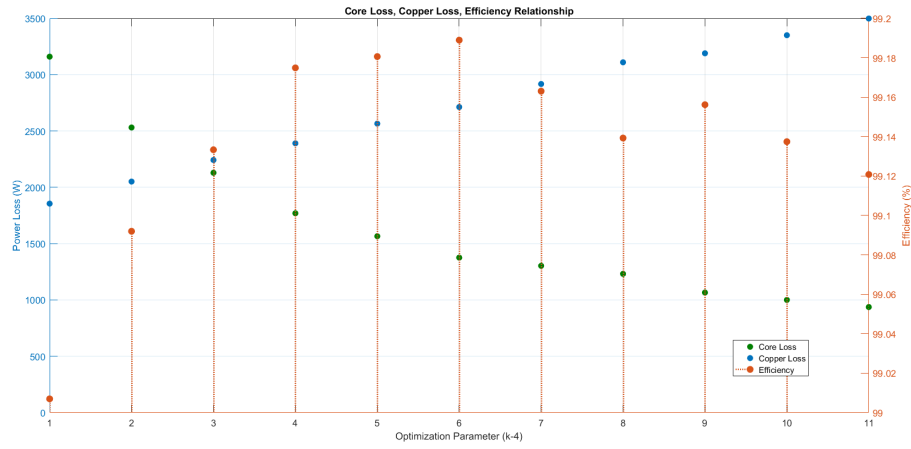


Figure 17: Relationship Between Core Loss, Copper Loss and Efficiency

Another concern in design of a transformer is obviously the price. What is important is not only the material price of the transformer, but also the money lost due to power lost while the transformer operates. Therefore cost should be considered in both cases. A graph showing this cost analysis is provided in Figure 18. Here, the most optimum case is also same as the previous one, since its efficiency is the highest.

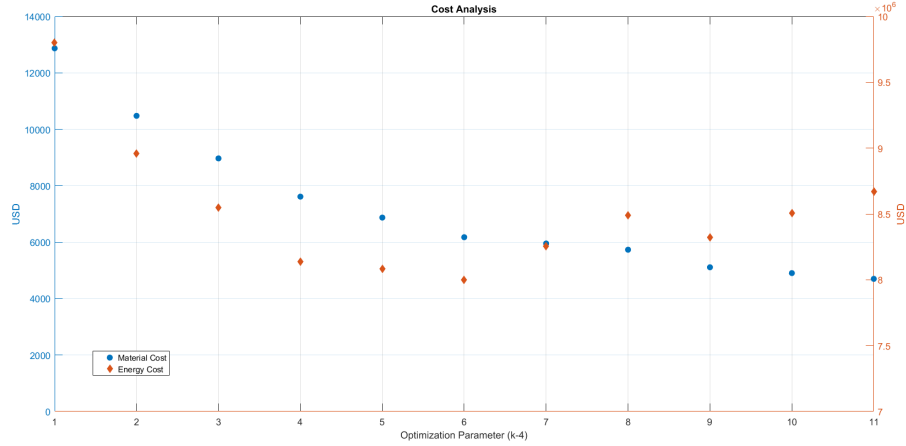


Figure 18: Cost Analysis

Different lamination types also effects the specifications of the transformer. As can be seen in Figure 19, core losses increase with increasing lamination thickness. This would have a significant effect on efficiency.

In this design, first lamination type (C120-23) is considered.

Grade	THICKNESS		TYPICAL CORE LOSS AT				GUARANTEED CORE LOSS AT		TYPICAL POLARIZATION AT
	mm	inch	1.5 T 50 Hz W/kg	1.7 T 50 Hz W/kg	1.5 T 60 Hz W/lb	1.7 T 60 Hz W/lb	1.7 T 50 Hz W/kg	1.7 T 60 Hz W/lb	
C 120-23	0.23	0.009	0.77	1.18	0.46	0.71	1.20	0.72	1.83
C 120-27	0.27	0.011	0.80	1.18	0.48	0.71	1.20	0.72	1.83
C 130-27	0.27	0.011	0.83	1.23	0.50	0.74	1.30	0.78	1.83
C 120-30	0.30	0.012	0.82	1.18	0.49	0.71	1.20	0.72	1.83
C 130-30	0.30	0.012	0.84	1.23	0.50	0.74	1.30	0.78	1.83
C 150-30	0.30	0.012	0.93	1.43	0.56	0.85	1.50	0.89	1.78
C 165-35	0.35	0.014	1.00	1.48	0.60	0.88	1.65	0.99	1.78

Figure 19: Properties of Steel Laminations