MIDDLE EAST TECHNICAL UNIVERSITY

DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

EE564 Project #1

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1 Inductor Design

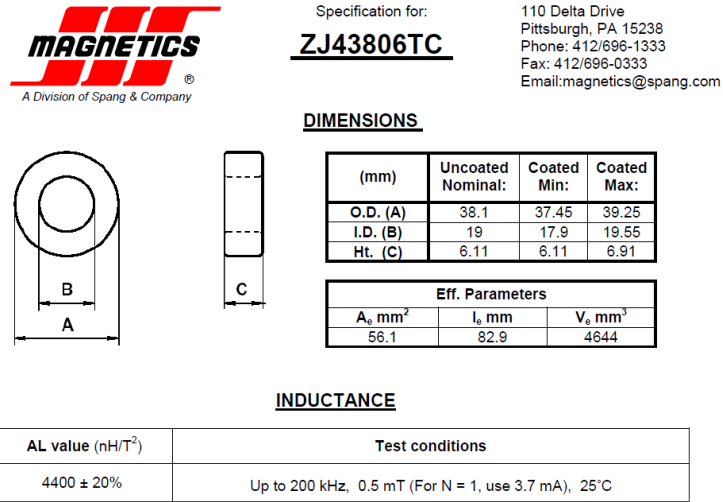In this part, I have designed an inductor with a toroidal core, whose datasheet is given in Figure 1.

Figure 1: Datasheet of the Toroidal Core

I chose the number of turns as 15, and Dc current as 0.3 A, in order to obtain N.I= 4.5 A.turns, which makes the inductor operate in linear region, but close to saturation. These choices are verified by calculating, calculations are as follows:

$$\frac{B * L_{eff}}{\mu_0 * \mu_r} = N * I$$

$$B = \frac{4.5 * 5000 * 4 * \pi * 10^{-7}}{0.0829} = 0.341T$$

Analytical calculations of inductance (with MATLAB code) are given below. It should be considered that BH curve data used in non-linear material calculations does not belong to the material that I have chosen. Its relative permeability is around 4000, where that of my toroid of choice is 5000. An additional difference is also expected in analytical calculations because of this fact.

Part A

Parameters

```
in_dia= 19; %mm
in_rad=in_dia/2*1e-3;
out_dia= 38.1; %mm
out_rad=out_dia/2*1e-3;
ht= 6.11; %mm

Leff=82.9; %mm
Aeff=56.1; %mm^2

N=15 ;% turns
I=0.3; % A

mu_r=5000;
mu_0=4*pi*1e-7;

AL=4400; %nH/turns^2
```

Analytical Calculations

```
%
% # 1

reluctance=Leff*1e-3/(Aeff*1e-6*mu_r*mu_0);

L=N^2/reluctance; %H

fprintf('Inductance assuming homogeneous distribution is %d H. \n', L);

% # 2
% In this part, H.dl should be integrated over over the cross-section of
% coaxial circles with radii from in_dia/2 to out_dia/2.
% H*2*pi*(out_rad-in_rad)=N.I

index=linspace(in_rad,out_rad,500);
for i=1:numel(index)

    H(:,i)=N*I/(2*pi*index(:,i));
    B(:,i)=mu_0*mu_r*H(:,i); %T
    Phi(:,i)=B(:,i)*ht*1e-3*(out_rad-in_rad)/500; %Wb
end
```

```matlab
tot_phi=0;

for k=1:500

    tot_phi=tot_phi+Phi(:,i);

end

L_2=tot_phi*N/I;

fprintf('Inductance assuming non-homogeneous distribution is %d H. \n', L_2);
%
% # 3_1
%
I_2=I*1.5;

H_2=N*I_2/(Leff*1e-3);

B_2=interp1(B_nl,H_nl,H_2);

phi_2=Aeff*1e-6*B_2;

L_3=N*phi_2/I_2;

fprintf('Inductance assuming homogeneous distribution & non-homogeneous material is %d H

%
% # 3_2
%


index=linspace(in_rad,out_rad,500);
for i=1:numel(index)

    H_3(:,i)=N*I_2/(2*pi*index(:,i));
    B_3(:,i)=interp1(B_nl,H_nl,H_3(:,i));
    Phi(:,i)=B_3(:,i)*ht*1e-3*(out_rad-in_rad)/500; %Wb
end

tot_phi=0;
```

```matlab
for k=1:500

    tot_phi=tot_phi+Phi(:,i);

end

L_4=tot_phi*N/I_2;

fprintf('Inductance assuming non-homogeneous distribution & non-homogeneous material is

%
% # 4
%

rel_gap=2e-3/(mu_0*Aeff*1e-6);
rel_core=(Leff-2)*1e-3/(Aeff*1e-6*mu_r*mu_0);
rel_tot=rel_gap+rel_core;

L_5=N^2/rel_tot;

fprintf('Inductance of the gapped core assuming homogeneous distribution is %d H. \n', L

%
% # 5
%

% In this part, we may assume the fringing flux is considerable for an area
% of 2 mm (the length of airgap) to the left and to the right of the
% airgap. Therefore we may assume the equivalent reluctance of the magnetic
% circuit as R_core in series with (R_gap_left||R_gap||R_gap_right). This
% approach is not a reliable one, nevertheless it may give us an idea.

rel_gap_side=2e-3/(mu_0*2e-3*ht*1e-3);
rel_tot_2=rel_core+(1/rel_gap_side+1/rel_gap+1/rel_gap_side)^(-1);

L_6=N^2/rel_tot_2;

fprintf('Inductance of the gapped core including fringing flux is %d H. \n', L_6);


Inductance assuming homogeneous distribution is 9.566889e-04 H.
Inductance assuming non-homogeneous distribution is 6.891791e-04 H.
Inductance assuming homogeneous distribution & non-homogeneous material is 5.383088e-04
```

4

```
Inductance assuming non-homogeneous distribution & non-homogeneous material is 5.027784e
Inductance of the gapped core assuming homogeneous distribution is 7.867304e-06 H.
Inductance of the gapped core including fringing flux is 1.125535e-05 H.
```

Part B

1. Mesh plots, B field distribution, distribution of B vectors and flux line distribution of the toroid, assuming linear magnetic material properties, are given below on Figures 2-5. Note that thin copper layer inside the toroid is excited with positive 4.5 A.t and the one that is at the outside of the toroid is excited with negative 4.5 A.t.
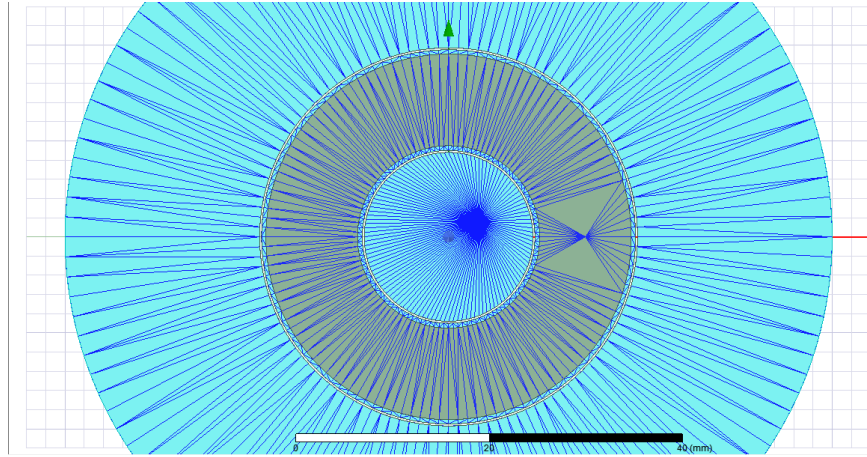

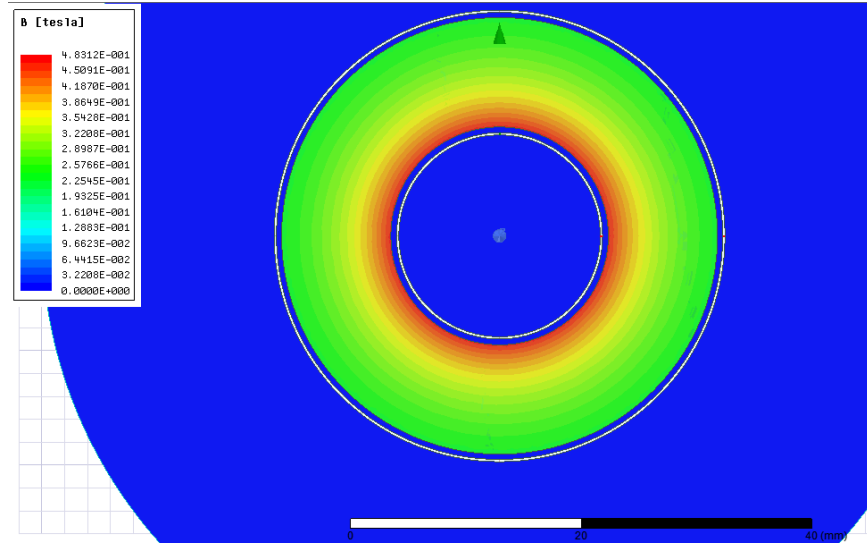Figure 2: Mesh Operations on the Toroid
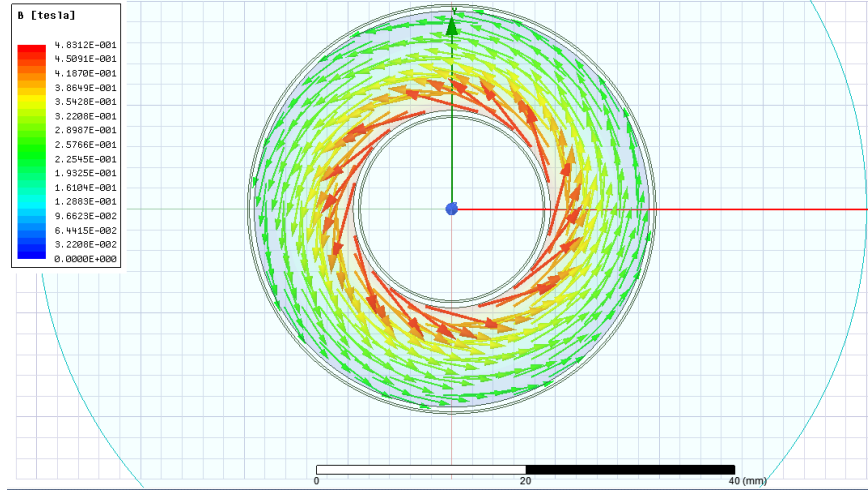

Figure 3: B Field Distribution in the Inductor
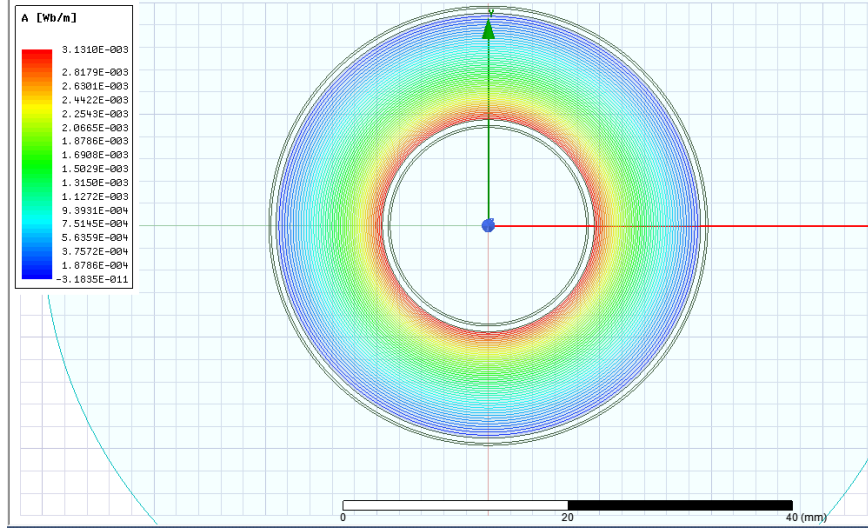
Figure 4: B Vectors in the Inductor

Figure 5: Flux Lines in the Toroidal Core

2. Inductance is calculated on the line from inner radius to outer radius, using the formula given below:

$$L = \frac{N * B * A}{I}$$

where A is denoted by the line, since the simulation is 2D.

To calculate the leakage inductance, same approach is used, (with a longer line crossing both inner and outer sides of the inductor) however no leakage is observed in this case. Inductance values are given on Figures 6 and 11, below.
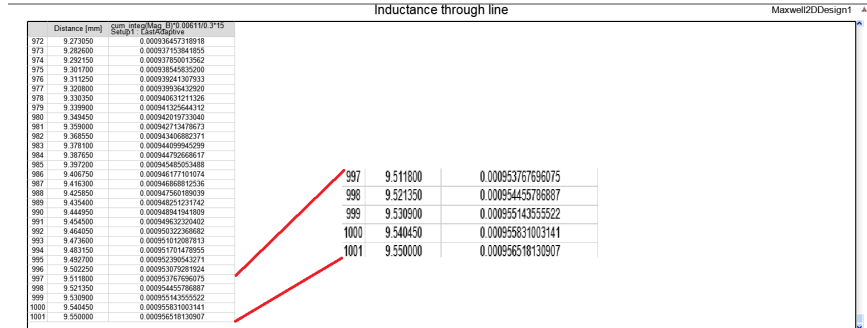

Figure 6: Inductance with Linear Material Properties

3. Here, the major observation is the saturation of the core. Also, there is a considerable drop in inductance.
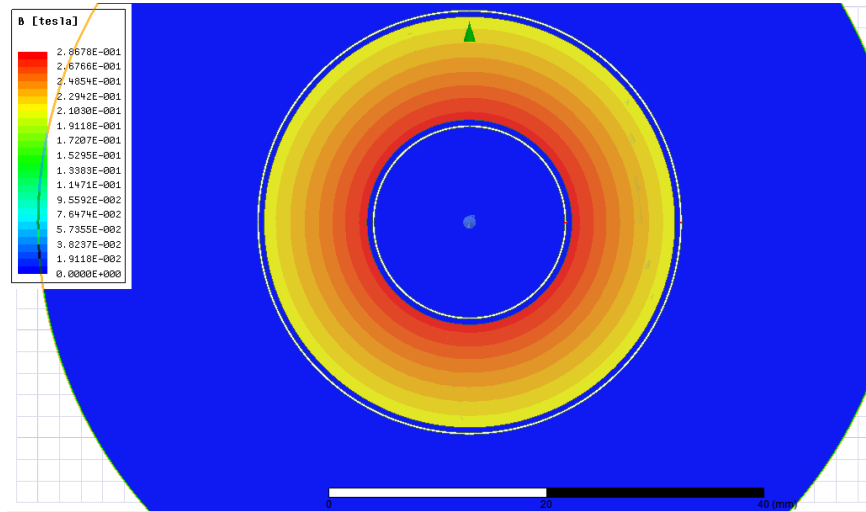

Figure 7: B Field Distribution in the Inductor with Non-Linear Material Properties
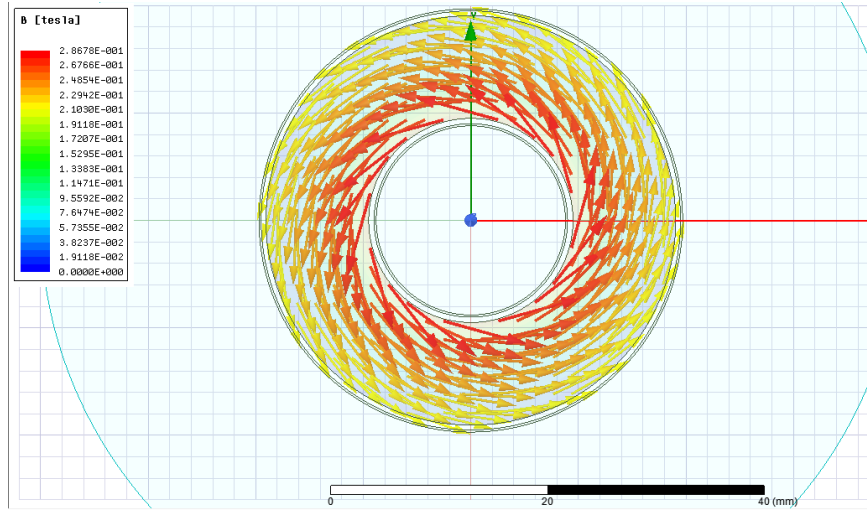
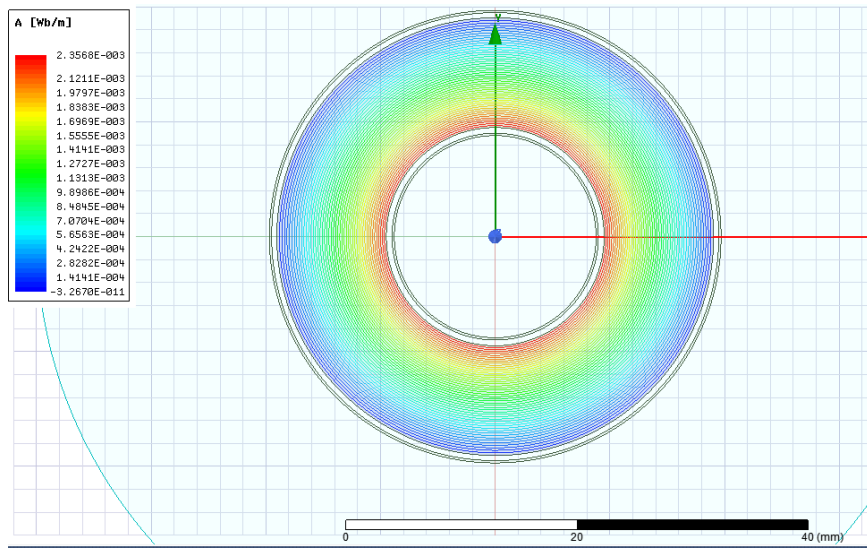Figure 8: B Vectors in the Inductor with Non-Linear Material Properties

Figure 9: Flux Lines in the Toroidal Core with Non-Linear Material Properties

Figure 10: Inductance with Non-Linear Material Properties
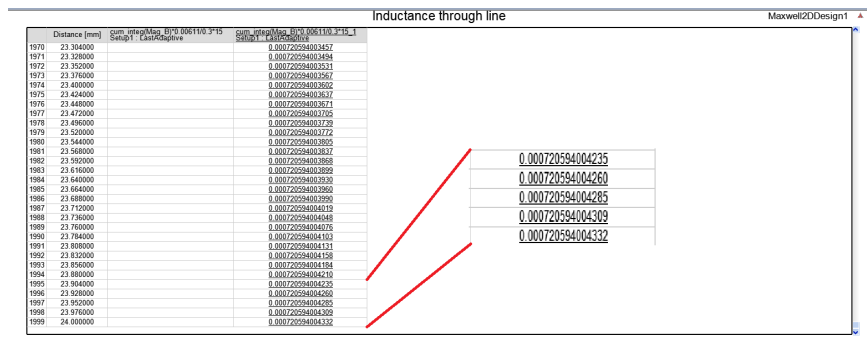
Figure 11: Leakage Included Inductance with Linear Material Properties

4. In this case, airgap introduces a significant increase in reluctance to the system, therefore inductance decreases considerably. Further, we can observe the effect of leakage and fringing flux on Figures 12, 13 and 14, which creates leakage inductance. Leakage inductance can be calculated bu subtracting inductance value on Figure 15 from overall inductance given in Figure 16.
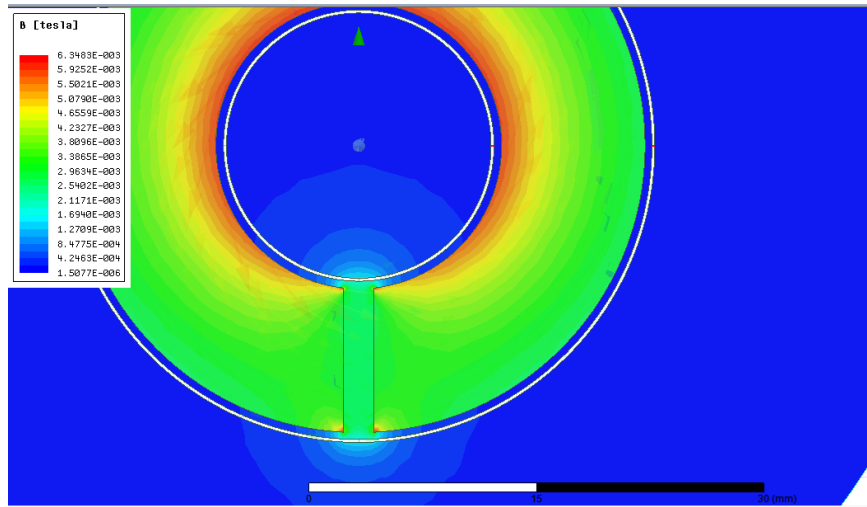
Figure 12: B Field Distribution in the Gapped Inductor with Non-Linear Material Properties

Figure 13: B Vectors in the Gapped Inductor with Non-Linear Material Properties

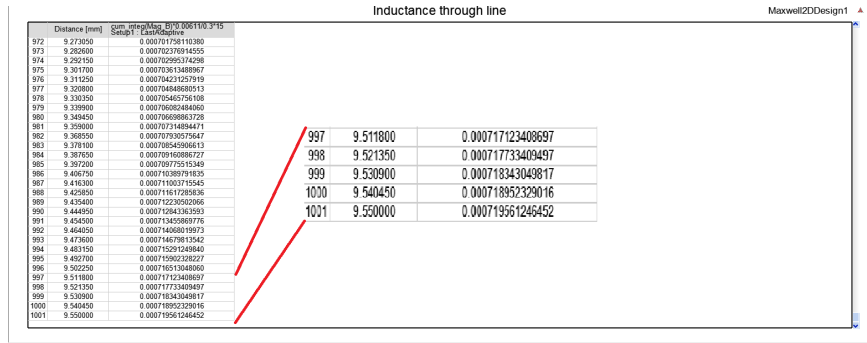Figure 14: Flux Lines in the Gapped Toroidal Core with Non-Linear Material Properties

Figure 15: Inductance of the Gapped Core with Non-Linear Material Properties
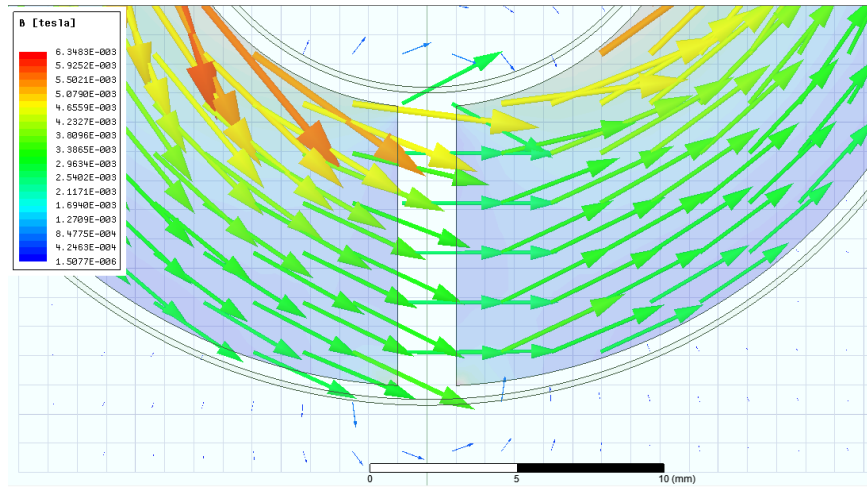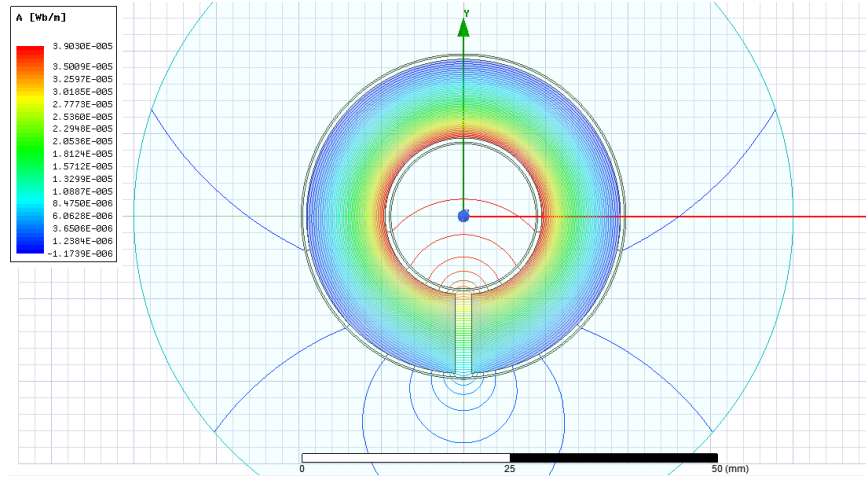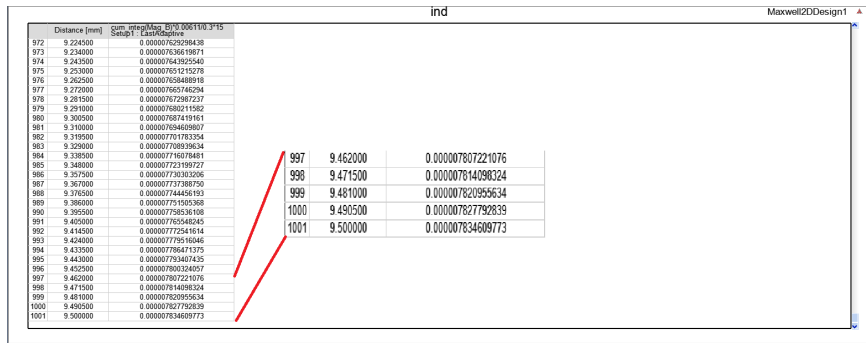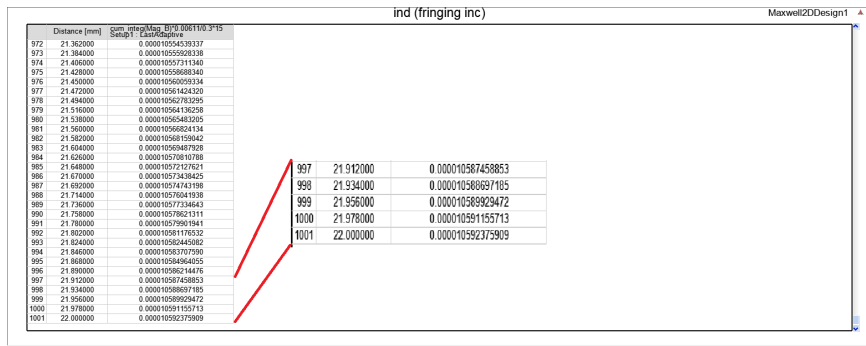

Figure 16: Leakage Included Inductance of the Gapped Core with Non-Linear Material Properties

Part C

Below, on Table 1, inductance calculation results of both methods are present.

Table 1: Comparison of Inductance Calculations

	FEA	Analytical
Linear	0.956 mH	0.957 mH
Non-Linear	0.720 mH	0.538 mH
With Airgap	7.83 μH	7.87 μH

Possible discrepancies between these calculations are caused by approximations made in analytical approach. In FEA, as calculations are made for infinitesimal elements and then integrated, it gives us more precise results. However for analytical approach, this is not possible. Hence, for calculations such as non-linear or non-homogeneous material properties, FEA technique is more reliable.

If we used 3D FEA instead of 2D, we could also observe the leakage and fringing flux through the cross-section of the core.

2 Transformer Design

With given parameters, a transformer is designed and optimized. MATLAB routine for this operation is present in the next section of the report.

Remarks and assumptions made in the optimization process are as follows:

-

Figure 17: Cost Analysis
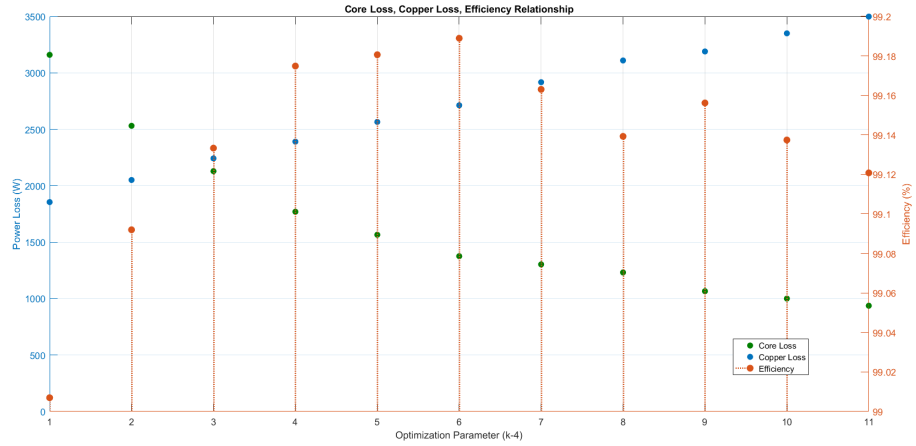
Figure 18: Relationship Between Core Loss, Copper Loss and Efficiency
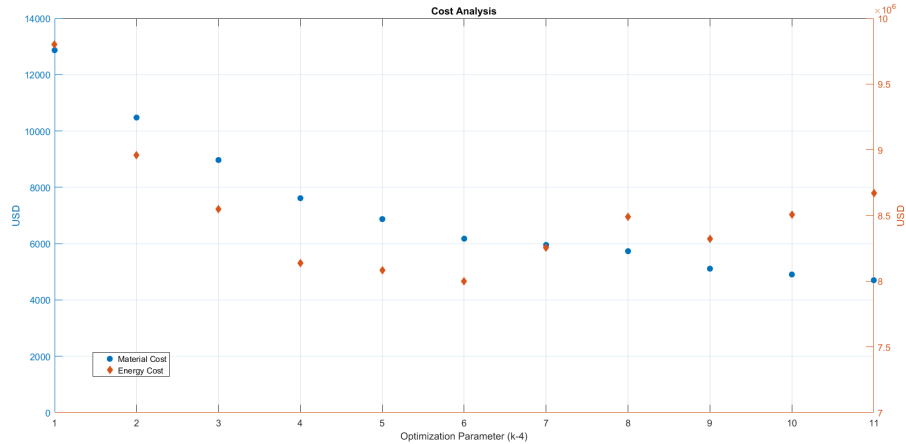
10	0.1019	2.58826	5.26	0.9989	3.276392	15	2600 Hz
11	0.0907	2.30378	4.17	1.26	4.1328	12	3200 Hz
12	0.0808	2.05232	3.31	1.588	5.20864	9.3	4150 Hz

Figure 19: Cable Sizes in AWG System

Grade	THICKNESS		TYPICAL CORE LOSS AT				GUARANTEED CORE LOSS AT		TYPICAL POLARIZATION AT
powercore®	mm	inch	1.5 T 50 Hz W/kg	1.7 T 50 Hz W/kg	1.5 T 60 Hz W/lb	1.7 T 60 Hz W/lb	1.7 T 50 Hz W/kg	1.7 T 60 Hz W/lb	800 A/m typ. T
C 120-23	0.23	0.009	0.77	1.18	0.46	0.71	1.20	0.72	1.83
C 120-27	0.27	0.011	0.80	1.18	0.48	0.71	1.20	0.72	1.83
C 130-27	0.27	0.011	0.83	1.23	0.50	0.74	1.30	0.78	1.83
C 120-30	0.30	0.012	0.82	1.18	0.49	0.71	1.20	0.72	1.83
C 130-30	0.30	0.012	0.84	1.23	0.50	0.74	1.30	0.78	1.83
C 150-30	0.30	0.012	0.93	1.43	0.56	0.85	1.50	0.89	1.78
C 165-35	0.35	0.014	1.00	1.48	0.60	0.88	1.65	0.99	1.78

Figure 20: Properties of Steel Laminations

Code for Transformer Design and Optimization

Parameters

```
B_op=1.5; %T
mu=1.83/800;
f=50; %Hz
P1= 500e3; %W
V1=34500; %V
V2=25000; %V
I1=P1/V1; %A
J=4; %A/mm^2
a_cable=I1/J; %mm^2
fprintf('Cable area should be around %d mm^2.\n',a_cable);

% This value is close to AWG11 size.

a_cable=4.1684; %mm^2
dia_cable=2.30378; %mm
res_cable=4.1328; %Ohms/km
```

Cable area should be around 3.623188e+00 mm^2.

Sizing

```
% Recalling the equation: V_ind=2*pi/sqrt(2)*f*B*A*N
% A*N is constant and they are dependent on each other. Optimum values of
% them will be found with an optimization parameter "k".

ff=0.5; %fill factor
dens_steel=7650; %kg/m^3
dens_copper=8940; %kg/m^3
core_loss_dens=0.77; %W/kg
price_steel=3; %$/kg
price_copper=7; %$/kg

i=1;
for k=5:15

    N1(:,i)=69*k;
    N2(:,i)=50*k;
    A(:,i)=V2*sqrt(2)/(2*pi*f*B_op*N1(:,i)); %m^2
```

```matlab
    % Window area

    x1(:,i)=dia_cable*23/ff/1000; %m
    x2(:,i)=dia_cable*3*k/ff/1000; %m
    x3(:,i)=ceil(sqrt(A(:,i)*10000))/100; %m

    w1(:,i)=x1(:,i);
    w2(:,i)=2*x2(:,i)+0.03;

%     fprintf('Window area is %d m^2.\n',w1(:,i)*w2(:,i));

    % Overall dimensions

    e1(:,i)=w1(:,i)+2*x3(:,i);
    e2(:,i)=w2(:,i)+2*x3(:,i);

%     fprintf('Dimensions of the transformer is %d x %d x %d m.\n',e1(:,i),e2(:,i),x3(:,

    vol(:,i)=(e1(:,i)*e2(:,i)-w1(:,i)*w2(:,i))*x3(:,i);

    m_steel(:,i)=dens_steel*vol(:,i);

%     fprintf('Steel mass is %d kg.\n',m_steel(:,i));

    core_loss(:,i)=core_loss_dens*m_steel(:,i);

%     fprintf('Core loss is %d Watts.\n',core_loss(:,i));

    % Cable length

    mean_length(:,i)=2*(x2(:,i)/2+x3(:,i))+pi*x3(:,i)*sqrt(2); %m
    l1(:,i)=mean_length(:,i)*N1(:,i);
    l2(:,i)=mean_length(:,i)*N2(:,i);

    r1(:,i)=l1(:,i)*res_cable/1000; %Ohms
    r2(:,i)=r1(:,i)*(N2(:,i)/N1(:,i))^2; %Ohms

    vol_copper(:,i)=2*l1(:,i)*a_cable/1000000; %m^3
    m_copper(:,i)=vol_copper(:,i)*dens_copper; %kg

%     fprintf('Copper mass is %d kg.\n',m_copper(:,i));
```

```matlab
    copper_loss(:,i)=I1^2*r1(:,i)*2; %W

%     fprintf('Copper loss is %d Watts.\n',copper_loss(:,i));

    % Inductances

    % Assuming L1 and L2 are 0.02 pu;

    ind1=V1^2/(P1*2*pi*f)*0.02; %H
    ind2=ind1*(N2/N1)^2; %H

    Leff(:,i)=2*(w1(:,i)+w2(:,i)+2*x3(:,i)); %m

    ind_m(:,i)=N1(:,i).^2*mu*A(:,i)/Leff(:,i); %H

    % Efficiency

    eff(:,i)=P1/(P1+core_loss(:,i)+copper_loss(:,i))*100; %percent

    % Cost

    cost(:,i)=price_copper*m_copper(:,i)+price_steel*m_steel(:,i); %$

    %Unit price of electricity: 0.4482 TL = 0.1125 USD

    lost_power(:,i)=P1*(100-eff(:,i))/1000; %kW

    lost_energy(:,i)=lost_power(:,i)*24*365*20*0.1125; %$

    cost_actual(:,i)=cost(:,i)+lost_energy(:,i); %$


    i=i+1;
end
```

Conclusion

```matlab
% Considering the optimizaton results, optimum case for the design seems to
% be i=6, k=10. Transformer parameters are as follows:

k=10;
i=6;
```

```
N1(:,i)=69*k;
N2(:,i)=50*k;
A(:,i)=V2*sqrt(2)/(2*pi*f*B_op*N1(:,i)); %m^2



% Window area

x1(:,i)=dia_cable*23/ff/1000; %m
x2(:,i)=dia_cable*3*k/ff/1000; %m
x3(:,i)=ceil(sqrt(A(:,i)*10000))/100; %m

w1(:,i)=x1(:,i);
w2(:,i)=2*x2(:,i)+0.03;



% Overall dimensions

e1(:,i)=w1(:,i)+2*x3(:,i);
e2(:,i)=w2(:,i)+2*x3(:,i);

vol(:,i)=(e1(:,i)*e2(:,i)-w1(:,i)*w2(:,i))*x3(:,i);

m_steel(:,i)=dens_steel*vol(:,i);

core_loss(:,i)=core_loss_dens*m_steel(:,i);

% Cable length

mean_length(:,i)=2*(x2(:,i)/2+x3(:,i))+pi*x3(:,i)*sqrt(2); %m
l1(:,i)=mean_length(:,i)*N1(:,i);
l2(:,i)=mean_length(:,i)*N2(:,i);

r1(:,i)=l1(:,i)*res_cable/1000; %Ohms
r2(:,i)=r1(:,i)*(N2(:,i)/N1(:,i))^2; %Ohms

vol_copper(:,i)=2*l1(:,i)*a_cable/1000000; %m^3
m_copper(:,i)=vol_copper(:,i)*dens_copper; %kg

copper_loss(:,i)=I1^2*r1(:,i)*2; %W
```

```matlab
% Inductances

% Assuming L1 and L2 are 0.02 pu;

ind1=V1^2/(P1*2*pi*f)*0.02; %H
ind2=ind1*(N2/N1)^2; %H

Leff(:,i)=2*(w1(:,i)+w2(:,i)+2*x3(:,i)); %m

ind_m(:,i)=N1(:,i).^2*mu*A(:,i)/Leff(:,i); %H

% Efficiency

eff(:,i)=P1/(P1+core_loss(:,i)+copper_loss(:,i))*100; %percent

% Cost

cost(:,i)=price_copper*m_copper(:,i)+price_steel*m_steel(:,i); %$


%Unit price of electricity: 0.4482 TL = 0.1125 USD

lost_power(:,i)=P1*(100-eff(:,i))/1000; %kW

lost_energy(:,i)=lost_power(:,i)*24*365*20*0.1125; %$

cost_actual(:,i)=cost(:,i)+lost_energy(:,i); %$

fprintf('Turns ratio is %d : %d. \n',N1(:,i),N2(:,i));

fprintf('Window area is %d m^2.\n',w1(:,i)*w2(:,i));

fprintf('Dimensions of the transformer is %d x %d x %d m.\n',e1(:,i),e2(:,i),x3(:,i)

fprintf('Steel mass is %d kg.\n',m_steel(:,i));

fprintf('Core loss is %d Watts.\n',core_loss(:,i));

fprintf('R1= %d Ohms, R2= %d Ohms. \n',r1(:,i),r2(:,i));

fprintf('Copper mass is %d kg.\n',m_copper(:,i));
```

```
    fprintf('Copper loss is %d Watts.\n',copper_loss(:,i));

    fprintf('L1= %d H, L2= %d H, Lm= %d H. \n',ind1,ind2,ind_m(:,i));

    fprintf('Efficiency is %d percent. \n',eff(:,i));

    fprintf('Material cost is %d USD. Lost money in 20 years is %d USD. \n',cost(:,i),co
```

```
Turns ratio is 690 : 500.
Window area is 3.247608e-02 m^2.
Dimensions of the transformer is 7.659739e-01 x 9.664536e-01 x 3.300000e-01 m.
Steel mass is 1.786846e+03 kg.
Core loss is 1.375872e+03 Watts.
R1= 6.457173e+00 Ohms, R2= 3.390660e+00 Ohms.
Copper mass is 1.164488e+02 kg.
Copper loss is 2.712528e+03 Watts.
L1= 1.515473e-01 H, L2= 7.957747e-02 H, Lm= 5.521107e+01 H.
Efficiency is 9.918895e+01 percent.
Material cost is 6.175681e+03 USD. Lost money in 20 years is 7.999056e+06 USD.
```