Q3 – Optimization

**Part A:**

In this part, to optimize the designed machine, we have to define a cost function. As our aim is to maximize the torque with respect to rotor outer radius, the optimization problem can be identified as:

Where the torque function can be defined as:

In the previous parts, obtained was already a boundary value. Therefore, no further increase for magnetic loading is needed. Following parameters will be taken as same for this optimization problem:

* Number of slots, 24
* Current, 2.5 A
* Air-gap, 1 mm
* Magnet thickness, 4 mm
* Cable gauge, AWG20

Electric loading of the machine is also a function of rotor radius.

Number of turns per slot can be determined as long as slot dimensions are known. To determine the dimensions of the slot, we have to know the length of the back core. Assuming there will be a maximum flux density of 1.5 T in the back core, its length can be defined as a function of r as follows:

Each slot area can be calculated as a trapezoid (assuming equal tooth width with slot opening), with base lengths

Therefore, electric loading and torque becomes:

Using WolframAlpha, derivative of the torque function is calculated. Local maxima points appear at:

For these values, optimum rotor radius for maximum torque is 27 mm. The dimensions of the machine are as follows:

* Rotor radius: 27 mm
* Air-gap length: 1 mm
* Magnet thickness: 4 mm
* Back core length: 17 mm
* Slot height: 31 mm
* Stator inner radius: 32 mm
* Slot outer radius: 63 mm
* Slot ratio: 0.51

Solving for the equation, number of turns per slot becomes 455. For this number, electric loading of the machine can be calculated as follows:

Apparently, this value is too large. In practice, cooling this machine with natural convection would be impossible. Liquid cooling should be applied. If the machine should be cooled naturally, then penalty or barrier constraints can be added for electrical loading to optimization problem.

Magnetic loading is the same as in previous part. It can be calculated as:

As a result, stress, torque and output power can be calculated as follows:

In initial design, outer stator diameter was much larger, where a similar output power was obtained. The electric loading of this design is the main aspect that increases the power density.

**Part B:**

**Replacing the NdFeB magnets with ferrite decreases the magnetic loading of the machine and we can expect a significant drop in torque and output power of the machine. With the same magnet size and air-gap length, the new magnetic loading can be calculated as follows:**

**The other performance parameters will also decrease with the same ratio.**

**Part C:**

**Optimization of the machine with ferrite magnets introduces many design parameters to the problem. Therefore, several assumptions should be made. In ferrite machine, we know that the magnets will be thicker, teeth and back core will be thinner. We can start the design process with magnet thickness assumption.**

**Let magnet thickness be 10 mm, embrace be 0.8 and the air-gap length be 1 mm. Using magnetic equivalent circuit approach, air-gap flux density and magnetic loading can be calculated as follows:**

**Tooth flux density can reach up to 1.4-1.5 T. If the ratio of slot opening to tooth width becomes 5, the flux density at the tooth becomes 6 times of air-gap flux density and reaches 1.44 T. Back core length can be expressed as a function of rotor outer radius, which can be used as optimization variable, similar to the previous part.**

Electric loading of the machine can be expressed as:

Similar to the previous part, there will be a maximum flux density of 1.5 T in the back core, its length can be defined as a function of r as follows:

Each slot area can be calculated as a trapezoid (assuming equal tooth width with slot opening), with base lengths

Therefore, electric loading and torque becomes:

Using WolframAlpha, derivative of the torque function is calculated. Local maxima points appear at:

For these values, optimum rotor radius for maximum torque is 33.5 mm. The dimensions of the machine are as follows:

* Rotor radius: 33.5 mm
* Air-gap length: 1 mm
* Magnet thickness: 10 mm
* Back core length: 8.5 mm
* Slot height: 27 mm
* Stator inner radius: 44.5 mm
* Slot outer radius: 71.5 mm
* Slot ratio: 0.62

Solving for the equation, number of turns per slot becomes 575. For this number, electric loading of the machine can be calculated as follows:

Apparently, this value is also too large for natural convection. If the machine should be cooled naturally, then penalty or barrier constraints can be added for electrical loading to optimization problem.

As a result, stress, torque and output power can be calculated as follows: