**CSE512 – Assignment 2**

Q1.

**I: Convex hulls and linear separability**

We are given the convex hull of S defined by following:

ConvexHull(S) = **---------- (a)**

We are also given that ------------------- **(b)**

The convex hull using **(a)** and **(b)** for the point T hence becomes ConvexHull(

If 2 points are linearly separable then, where and are 2 vectors and .

In order to prove that the convex hull of the above 2 points don’t intersect, for this we find linear discriminant:

**-----------** **(c)**

Using **(a)** and **(c):**

because is a scalar quantity**-----------** **(d)**

Now, because and , for the other observations we have:

**-----------** **(e)**

In case both convex hulls intersect there must exist a point which is common between and

The linear discriminant of that point from equations **(d)** and **(e)**

**-----------** **(f)**

However, we know for the linear separability the linear discriminants should be:

> 0,

**-----------** **(g)**

The equations **(f)** and **(g)** lead to a contradiction as the equation **(g)** must be bigger and less than 0 at the same time. This from equation **(f)** we have proved that for the points S and T to be linearly separable their convex hulls must be non-intersecting.

**III: Non-equivalence of “hard” and “soft” SVM**

The statement that the hard and soft SVM would return the exact same hypothesis is wrong.

Let’s begin by taking 2 integers a and b such that a > 1 and b > 0.

Now, let , where .

For and let

Let

For hard SVM, we have , we have,

For soft SVM, w = (0,1). As such that

The inequality holds because we see that there is for every , if is small enough then soft SVM neglects .

**IV: Kernel construction**

and are valid kernels on a domain X hence both have gram matrix which is symmetric as well as positive-definitive.

A valid kernel has 2 properties:

1. Symmetry **---------- (a)**
2. Positive semi-definiteness **---------- (b)**

and become kernel matrices when we restrict them to finite set of points.

If are finite set of points then,

**---------- (c)**

is positive, so is a positive semi-definite. If we multiply a constant to each element of the matrix the Gram matrix would not get altered. Since observes both the properties **(a)** and **(b)** hence it is a valid kernel. Similarly, is a valid kernel.

From **(c)** we have where and are symmetric gram matrices. K is a matrix that is symmetric as well as positive semi-definite because of adding of 2 matrices that are symmetric and positive semi-definite.

Thus, the properties of a valid kernel are satisfied as seen above and hence we have:

**---------- (d)**

From Mercer’s theorem, being valid kernels must have an inner product representation. Let i and j denote feature vectors of respectively, then,

**i** and **j** are functions producing M-Dimensional and N-Dimensional matrices respectively.

Now, if f(z) = M.N dimensional matrix then

**---------- (e)**

Thus, we have proven using the Mercer’s theorem that the validity of the kernel function **(e).**