```
assume(wO, Type::Real):
assume(weA, Type::Real):
assume(we, Type::Real):
assume(wV, Type::Real):
assume(wA, Type::Real):
assume(a1,Type::Real):
assume(a3,Type::Real):
assume(a4,Type::Real)
assume(uhat0, Type::Real):
l1:=wO+sqrt(-1)*wV^2/we
```

$$wO + \frac{wV^2\, i}{we}$$

```
l2:=-(wV^2*wO/we^2)+sqrt(-1)*we
```

$$-\frac{wO\, wV^2}{we^2} + we\, i$$

```
l3:=(wV^2*wO/we^2)+sqrt(-1)*we
```

$$\frac{wO\, wV^2}{we^2} + we\, i$$

```
l4:=-wO+sqrt(-1)*wV^2/we
```

$$-wO + \frac{wV^2\, i}{we}$$

```
a1:=uhat0
```

$$uhat0$$

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a3:=uhat0*(wV^2+wO^2)
```

$$uhat0\,(wO^2 + wV^2)$$

```
a4:=sqrt(-1)*uhat0*wV^2*we
```

$$uhat0\, wV^2\, we\, i$$

```
num1:=collect(expand((a4-a3*(l2+l3+l4)-a1*l1l3*l4*l2)),sqrt(-1))
```

$$\left(\frac{uhat0\, wO^2\, wV^6}{we^5} - \frac{uhat0\, wV^4}{we} - \frac{uhat0\, wO^2\, wV^2}{we} - 2\, uhat0\, wO^2\, we\right) i + uhat0\, wO^3 + uhat0\, wO\, wV^2$$

$$- uhat0\, wO\, we^2 - \frac{uhat0\, wO^3\, wV^4}{we^4}$$

```
den1:=collect(expand((l1-l2)*(l1-l3)*(l1-l4)),sqrt(-1))
```

$$\left(\frac{4\, wO^2\, wV^2}{we} - 4\, wO^2\, we\right) i + 4\, wO\, wV^2 - 2\, wO\, we^2 + 2\, wO^3 - \frac{2\, wO\, wV^4}{we^2} - \frac{2\, wO^3\, wV^4}{we^4}$$

```
cden1:=collect(conjugate(den1),sqrt(-1))
```

$$\left(4\, wO^2\, we - \frac{4\, wO^2\, wV^2}{we}\right) i + 4\, wO\, wV^2 - 2\, wO\, we^2 + 2\, wO^3 - \frac{2\, wO\, wV^4}{we^2} - \frac{2\, wO^3\, wV^4}{we^4}$$

```
factorout(expand(den1*cden1),we^12*wO^2)
```

$$(wO^2\ we^{12})\left(\frac{8\ wO^2}{we^2}+\frac{4\ wO^4}{we^4}-\frac{16\ wV^2}{we^2}+\frac{24\ wV^4}{we^4}-\frac{16\ wV^6}{we^6}+\frac{4\ wV^8}{we^8}-\frac{16\ wO^2\ wV^2}{we^4}+\frac{16\ wO^2\ wV^4}{we^6}-\frac{16\ wO^2\ wV^6}{we^8}-\frac{8\ wO^4\ wV^4}{we^8}+\frac{8\ wO^2\ wV^8}{we^{10}}+\frac{4\ wO^4\ wV^8}{we^{12}}+4\right)\Big/ we^8$$

```
collect(factorout(num1*cden1,we^13*wO^1*wV*uhat0),sqrt(-1))
```

$$\left(uhat0\ wO\ wV\ we^4\left(\frac{2\ wV^3}{we^3}-\frac{4\ wV^5}{we^5}+\frac{2\ wV^7}{we^7}+\frac{2\ wO^2\ wV}{we^3}-\frac{6\ wO^4\ wV}{we^5}-\frac{6\ wO^2\ wV^3}{we^5}+\frac{6\ wO^2\ wV^7}{we^9}+\frac{8\ wO^4\ wV^5}{we^9}-\frac{2\ wO^2\ wV^9}{we^{11}}-\frac{2\ wO^4\ wV^9}{we^{13}}\right)\right) i + uhat0\ wO\ wV\ we^4\left(\frac{2\ wO}{wV}-\frac{6\ wO\ wV}{we^2}+\frac{10\ wO\ wV^3}{we^4}+\frac{2\ wO^3\ wV}{we^4}-\frac{6\ wO\ wV^5}{we^6}+\frac{4\ wO^3}{wV\ we^2}+\frac{2\ wO^5}{wV\ we^4}-\frac{2\ wO^3\ wV^3}{we^6}-\frac{10\ wO^3\ wV^5}{we^8}-\frac{4\ wO^5\ wV^3}{we^8}+\frac{6\ wO^3\ wV^7}{we^{10}}+\frac{2\ wO^5\ wV^7}{we^{12}}\right)$$

```
DEN:=4*wO^2*we^4
```

$$4\ wO^2\ we^4$$

```
NUM:=uhat0*wO^2*we^4*2+sqrt(-1)*uhat0*wO*wV*we^4*(2*wO^2*wV/we^3)
```

$$2\ i\ uhat0\ wO^3\ wV^2\ we + 2\ uhat0\ wO^2\ we^4$$

```
A:=NUM/DEN
```

$$\frac{2\ i\ uhat0\ wO^3\ wV^2\ we + 2\ uhat0\ wO^2\ we^4}{4\ wO^2\ we^4}$$

```
s1:=collect(factorout(A*exp(-(wV^2/we)*t)*(cos(wO*t)+sqrt(-1)*sin(wO*t)),uh
```

$$\frac{uhat0\ e^{-\frac{t\ wV^2}{we}}\left(\sin(t\ wO)+\frac{wO\ wV^2\cos(t\ wO)}{we^3}\right)}{2}\ i + \frac{uhat0\ e^{-\frac{t\ wV^2}{we}}\left(\cos(t\ wO)-\frac{wO\ wV^2\sin(t\ wO)}{we^3}\right)}{2}$$

```
num2:=collect(expand((a4-a3*(l2+l3+l1)-a1*l3*l1*l2)),sqrt(-1))
```

$$\left(\frac{uhat0\ wO^2\ wV^6}{we^5}-\frac{uhat0\ wV^4}{we}-\frac{uhat0\ wO^2\ wV^2}{we}-2\ uhat0\ wO^2\ we\right) i + uhat0\ wO\ we^2 - uhat0\ wO\ wV^2 - uhat0\ wO^3 + \frac{uhat0\ wO^3\ wV^4}{we^4}$$

```
den2:=collect(expand((l4-l3)*(l4-l2)*(l4-l1)),sqrt(-1))
```

$$\left(\frac{4\ wO^2\ wV^2}{we}-4\ wO^2\ we\right) i + 2\ wO\ we^2 - 4\ wO\ wV^2 - 2\ wO^3 + \frac{2\ wO\ wV^4}{we^2}+\frac{2\ wO^3\ wV^4}{we^4}$$

```
cden2:=collect(conjugate(den2),sqrt(-1))
```

$$\left(4\ wO^2\ we-\frac{4\ wO^2\ wV^2}{we}\right) i + 2\ wO\ we^2 - 4\ wO\ wV^2 - 2\ wO^3 + \frac{2\ wO\ wV^4}{we^2}+\frac{2\ wO^3\ wV^4}{we^4}$$

```
factorout(expand(den2*cden2),we^12*wO^2)
```

$$(wO^2\ we^{12})\left(\frac{8\ wO^2}{we^2}+\frac{4\ wO^4}{we^4}-\frac{16\ wV^2}{we^2}+\frac{24\ wV^4}{we^4}-\frac{16\ wV^6}{we^6}+\frac{4\ wV^8}{we^8}-\frac{16\ wO^2\ wV^2}{we^4}+\frac{16\ wO^2\ wV^4}{we^6}-\frac{16\ wO^2\ wV^6}{we^8}-\frac{8\ wO^4\ wV^4}{we^8}+\frac{8\ wO^2\ wV^8}{we^{10}}+\frac{4\ wO^4\ wV^8}{we^{12}}+4\right)\Big/ we^8$$

```
collect(factorout(num2*cden2,we^13*wO^1*wV*uhat0),sqrt(-1))
```

$$\left(-uhat0\ wO\ wV\ we^4\left(\frac{2\ wV^3}{we^3}-\frac{4\ wV^5}{we^5}+\frac{2\ wV^7}{we^7}+\frac{2\ wO^2\ wV}{we^3}-\frac{6\ wO^4\ wV}{we^5}-\frac{6\ wO^2\ wV^3}{we^5}+\frac{6\ wO^2\ wV^7}{we^9}+\frac{8\ wO^4\ wV^5}{we^9}-\frac{2\ wO^2\ wV^9}{we^{11}}-\frac{2\ wO^4\ wV^9}{we^{13}}\right)\right) i + uhat0\ wO\ wV\ we^4\left(\frac{2\ wO}{wV}-\frac{6\ wO\ wV}{we^2}+\frac{10\ wO\ wV^3}{we^4}+\frac{2\ wO^3\ wV}{we^4}-\frac{6\ wO\ wV^5}{we^6}+\frac{4\ wO^3}{wV\ we^2}+\frac{2\ wO^5}{wV\ we^4}-\frac{2\ wO^3\ wV^3}{we^6}-\frac{10\ wO^3\ wV^5}{we^8}-\frac{4\ wO^5\ wV^3}{we^8}+\frac{6\ wO^3\ wV^7}{we^{10}}+\frac{2\ wO^5\ wV^7}{we^{12}}\right)$$

```
DEN2:=4*wO^2*we^4
```

$$4\ wO^2\ we^4$$

```
NUM2:=uhat0*wO^2*we^4*2-sqrt(-1)*uhat0*wO*wV*we^4*(2*wO^2*wV/we^3)
```

$$2\ uhat0\ wO^2\ we^4 - 2\ uhat0\ wO^3\ wV^2\ we\ i$$

```
Df:=NUM2/DEN2
```

$$\frac{2\ uhat0\ wO^2\ we^4 - 2\ uhat0\ wO^3\ wV^2\ we\ i}{4\ wO^2\ we^4}$$

```
s2:=collect(factorout(Df*exp(-(wV^2/we)*t)*(cos(wO*t)-sqrt(-1)*sin(wO*t)),u
```

$$\left(-\frac{uhat0\ e^{-\frac{t\ wV^2}{we}}\left(\sin(t\ wO)+\frac{wO\ wV^2\cos(t\ wO)}{we^3}\right)}{2}\right) i + \frac{uhat0\ e^{-\frac{t\ wV^2}{we}}\left(\cos(t\ wO)-\frac{wO\ wV^2\sin(t\ wO)}{we^3}\right)}{2}$$

```
s1+s2
```

$$uhat0\ e^{-\frac{t\ wV^2}{we}}\left(\cos(t\ wO)-\frac{wO\ wV^2\sin(t\ wO)}{we^3}\right)$$

```
wO:=2*Omg*kz/k
```

$$\frac{2\ Omg\ kz}{k}$$

```
wV:=Va*kz
```

$$Va\ kz$$

```
we:=eta*k^2
```

$$eta\ k^2$$

```
wO*wV^2/we^3
```

$$\frac{2\ Omg\ Va^2\ kz^3}{eta^3\ k^7}$$