```
assume(wO, Type::Real):
assume(weA, Type::Real):
assume(we, Type::Real):
assume(wV, Type::Real):
assume(wA, Type::Real):
assume(a1,Type::Real):
assume(a3,Type::Real):
assume(a4,Type::Real)
assume(uhat0, Type::Real):
l1:=wO+sqrt(-1)*wV^2/we
```

$$wO + \frac{wV^2\, i}{we}$$

```
l2:=-(wV^2*wO/we^2)+sqrt(-1)*we
```

$$-\frac{wO\, wV^2}{we^2} + we\, i$$

```
l3:=(wV^2*wO/we^2)+sqrt(-1)*we
```

$$\frac{wO\, wV^2}{we^2} + we\, i$$

```
l4:=-wO+sqrt(-1)*wV^2/we
```

$$-wO + \frac{wV^2\, i}{we}$$

```
a1:=uhat0
```

$$uhat0$$

```
a3:=uhat0*(wV^2+wO^2)
```

$$uhat0\,(wO^2 + wV^2)$$

```
a4:=sqrt(-1)*uhat0*wV^2*we
```

$$uhat0\, wV^2\, we\, i$$

```
num1:=collect(expand((a4-a3*(l1+l3+l4)-a1*l3*l4*l1)),sqrt(-1))
```

$$\left(-\frac{uhat0\, wV^4}{we} - \frac{2\, uhat0\, wO^2\, wV^2}{we}\right) i + \frac{uhat0\, wO\, wV^6}{we^4} - \frac{uhat0\, wO\, wV^4}{we^2}$$

```
den1:=collect(expand((l2-l1)*(l2-l3)*(l2-l4)),sqrt(-1))
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$$\left(\frac{4\, wO^2\, wV^4}{we^3} - \frac{4\, wO^2\, wV^6}{we^5}\right) i + 2\, wO\, wV^2 - \frac{4\, wO\, wV^4}{we^2} + \frac{2\, wO\, wV^6}{we^4} + \frac{2\, wO^3\, wV^2}{we^2} - \frac{2\, wO^3\, wV^6}{we^6}$$

```
cden1:=collect(conjugate(den1),sqrt(-1))
```

$$\left(\frac{4\, wO^2\, wV^6}{we^5} - \frac{4\, wO^2\, wV^4}{we^3}\right) i + 2\, wO\, wV^2 - \frac{4\, wO\, wV^4}{we^2} + \frac{2\, wO\, wV^6}{we^4} + \frac{2\, wO^3\, wV^2}{we^2} - \frac{2\, wO^3\, wV^6}{we^6}$$

```
factorout(expand(den1*cden1),we^12*wO^2*wV^4)
```

$$(wO^2\ wV^4\ we^{12})\left(\frac{8\ wO^2}{we^2}+\frac{4\ wO^4}{we^4}-\frac{16\ wV^2}{we^2}+\frac{24\ wV^4}{we^4}-\frac{16\ wV^6}{we^6}+\frac{4\ wV^8}{we^8}-\frac{16\ wO^2\ wV^2}{we^4}+\frac{16\ wO^2\ wV^4}{we^6}-\frac{16\ wO^2\ wV^6}{we^8}-\frac{8\ wO^4\ wV^4}{we^8}+\frac{8\ wO^2\ wV^8}{we^{10}}+\frac{4\ wO^4\ wV^8}{we^{12}}+4\right)\Big/ we^{12}$$

```
collect(factorout(num1*cden1,we^10*wV^4*wO^2*uhat0),sqrt(-1))
```

$$\left(-uhat0\ wO^2\ wV^4\left(\frac{4\ wO^3}{we^3}+\frac{4\ wO}{we}-\frac{6\ wO\ wV^2}{we^3}+\frac{6\ wO\ wV^6}{we^7}-\frac{4\ wO\ wV^8}{we^9}+\frac{2\ wV^2}{wO\ we}-\frac{4\ wV^4}{wO\ we^3}+\frac{2\ wV^6}{wO\ we^5}-\frac{4\ wO^3\ wV^4}{we^7}\right)\right)i-uhat0\ wO^2\ wV^4\left(\frac{2\ wV^2}{we^2}-\frac{2\ wV^4}{we^4}+\frac{2\ wV^6}{we^6}-\frac{2\ wV^8}{we^8}+\frac{10\ wO^2\ wV^2}{we^4}-\frac{10\ wO^2\ wV^4}{we^6}-\frac{2\ wO^2\ wV^6}{we^8}+\frac{2\ wO^2\ wV^8}{we^{10}}\right)$$

```
DEN:=4*wO^2*wV^4
```

$$4\ wO^2\ wV^4$$

```
NUM:=-2*uhat0*(wO^2*wV^6/we^2)-4*uhat0*(wO^3*wV^4/we)*sqrt(-1)
```

$$-\frac{2\ uhat0\ wO^2\ wV^6}{we^2}-\frac{4\ uhat0\ wO^3\ wV^4\ i}{we}$$

```
B:=expand(NUM/DEN)
```

$$-\frac{uhat0\ wV^2}{2\ we^2}-\frac{uhat0\ wO\ i}{we}$$

```
s1:=collect(factorout(B*exp(-we*t)*(cos((wV^2*wO/we^2)*t)-sqrt(-1)*sin((wV'
```

$$\frac{uhat0\ e^{-t\ we}\left(wV^2\ \sigma_1-2\ wO\ we\ \sigma_2\right)}{2\ we^2}\ i-\frac{uhat0\ e^{-t\ we}\left(wV^2\ \sigma_2+2\ wO\ we\ \sigma_1\right)}{2\ we^2}$$

where

$$\sigma_1=\sin\left(\frac{t\ wO\ wV^2}{we^2}\right)$$

$$\sigma_2=\cos\left(\frac{t\ wO\ wV^2}{we^2}\right)$$

```
num2:=collect(expand((a4-a3*(l1+l2+l4)-a1*l2*l4*l1)),sqrt(-1))
```

$$\left(-\frac{uhat0\ wV^4}{we}-\frac{2\ uhat0\ wO^2\ wV^2}{we}\right)i+\frac{uhat0\ wO\ wV^4}{we^2}-\frac{uhat0\ wO\ wV^6}{we^4}$$

```
den2:=collect(expand((l3-l1)*(l3-l2)*(l3-l4)),sqrt(-1))
```

$$\left(\frac{4\ wO^2\ wV^4}{we^3}-\frac{4\ wO^2\ wV^6}{we^5}\right)i+\frac{4\ wO\ wV^4}{we^2}-2\ wO\ wV^2-\frac{2\ wO\ wV^6}{we^4}-\frac{2\ wO^3\ wV^2}{we^2}+\frac{2\ wO^3\ wV^6}{we^6}$$

```
cden2:=collect(conjugate(den2),sqrt(-1))
```

$$\left(\frac{4\ wO^2\ wV^6}{we^5}-\frac{4\ wO^2\ wV^4}{we^3}\right)i+\frac{4\ wO\ wV^4}{we^2}-2\ wO\ wV^2-\frac{2\ wO\ wV^6}{we^4}-\frac{2\ wO^3\ wV^2}{we^2}+\frac{2\ wO^3\ wV^6}{we^6}$$

```
factorout(expand(den2*cden2),we^12*wO^2*wV^4)
```

$$(wO^2\ wV^4\ we^{12})\left(\frac{8\ wO^2}{we^2}+\frac{4\ wO^4}{we^4}-\frac{16\ wV^2}{we^2}+\frac{24\ wV^4}{we^4}-\frac{16\ wV^6}{we^6}+\frac{4\ wV^8}{we^8}-\frac{16\ wO^2\ wV^2}{we^4}+\frac{16\ wO^2\ wV^4}{we^6}-\frac{16\ wO^2\ wV^6}{we^8}-\frac{8\ wO^4\ wV^4}{we^8}+\frac{8\ wO^2\ wV^8}{we^{10}}+\frac{4\ wO^4\ wV^8}{we^{12}}+4\right)\Big/ we^{12}$$

```
collect(factorout(num2*cden2,we^10*wV^4*wO^2*uhat0),sqrt(-1))
```

$$\left(uhat0\ wO^2\ wV^4\left(\frac{4\ wO^3}{we^3}+\frac{4\ wO}{we}-\frac{6\ wO\ wV^2}{we^3}+\frac{6\ wO\ wV^6}{we^7}-\frac{4\ wO\ wV^8}{we^9}+\frac{2\ wV^2}{wO\ we}-\frac{4\ wV^4}{wO\ we^3}+\frac{2\ wV^6}{wO\ we^5}-\frac{4\ wO^3\ wV^4}{we^7}\right)\right)i-uhat0\ wO^2\ wV^4\left(\frac{2\ wV^2}{we^2}-\frac{2\ wV^4}{we^4}+\frac{2\ wV^6}{we^6}-\frac{2\ wV^8}{we^8}+\frac{10\ wO^2\ wV^2}{we^4}-\frac{10\ wO^2\ wV^4}{we^6}-\frac{2\ wO^2\ wV^6}{we^8}+\frac{2\ wO^2\ wV^8}{we^{10}}\right)$$

```
DEN2:=4*wO^2*wV^4
```

$$4\ wO^2\ wV^4$$

```
NUM2:=-2*uhat0*(wO^2*wV^6/we^2)+4*uhat0*(wO^3*wV^4/we)*sqrt(-1)
```

$$-\frac{2\ uhat0\ wO^2\ wV^6}{we^2}+\frac{4\ uhat0\ wO^3\ wV^4\ i}{we}$$

```
C:=expand(NUM2/DEN2)
```

$$-\frac{uhat0\ wV^2}{2\ we^2}+\frac{uhat0\ wO\ i}{we}$$

```
s2:=collect(factorout(C*exp(-we*t)*(cos((wV^2*wO/we^2)*t)+sqrt(-1)*sin((wV'
```

$$\left(-\frac{uhat0\ e^{-t\ we}\left(wV^2\ \sigma_1-2\ wO\ we\ \sigma_2\right)}{2\ we^2}\right)i-\frac{uhat0\ e^{-t\ we}\left(wV^2\ \sigma_2+2\ wO\ we\ \sigma_1\right)}{2\ we^2}$$

where

$$\sigma_1=\sin\left(\frac{t\ wO\ wV^2}{we^2}\right)$$

$$\sigma_2=\cos\left(\frac{t\ wO\ wV^2}{we^2}\right)$$

```
s1+s2
```

$$-\frac{uhat0\ e^{-t\ we}\left(wV^2\cos\left(\frac{t\ wO\ wV^2}{we^2}\right)+2\ wO\ we\sin\left(\frac{t\ wO\ wV^2}{we^2}\right)\right)}{we^2}$$

```
wO:=Omg*kz/k
```

$$\frac{Omg\ kz}{k}$$

```
we:=eta*k^2
```

$$eta\ k^2$$

```
wV:=Va*kz
```

$$\mathrm{Va}\ \mathrm{kz}$$

```
wO/we
```

$$\frac{\mathrm{Omg}\ \mathrm{kz}}{\mathrm{eta}\ k^3}$$

```
wV^2/we^2
```

$$\frac{\mathrm{Va}^2\ \mathrm{kz}^2}{\mathrm{eta}^2\ k^4}$$