Numerical Methods, Fall 2018

Final Project

Due by 9:00 AM on December 14 th, 2018.

Send PDF of your report to t.zaki@jhu.edu

Consider the 2-D Laplace equation

$$(u_{xx} + u_{yy}) = 0$$

for $0 \leq x, y \leq 2\pi$ with the following boundary conditions

$$u(0, y) = sin(4y)$$

$$u(2\pi, y) = 0$$

$$u(x, 0) = sin(4x)$$

$$u(x, 2\pi) = 0$$

(a) We will adopt a second-order central-difference scheme with a cell-centered arrangement. Develop a multigrid code to solve the above problem using V-cycles and the following options for relaxation:

  (i) Point Gauss-Seidel iteration

  (ii) Line Gauss-Seidel with alternating directions

(b) Draw a flow chart of your solver.

(c) Perform the multigrid solution on grids with 64×64 grid cells, and also 128×128 grid cells. If possible, attempt 256×256 grid cells as well. Use a random number generator to provide the starting guess in the range of $[-1, 1]$.

(d) Sanity Check: Demonstrate that your multigrid is giving the correct result, i.e. is the final solution correct? Is the residual decreasing as expected?

(e) Discuss the observed performance of your multigrid solver. Consider questions such as:

  (i) How does the convergence change with number of coarse grid levels?

  (ii) How does the CPU time increase with number of grid points with and without multigrid. Is the observed trend expected?

(f) Include a printout of your program as an Appendix to your report.

(g) For extra credits examine the effect of one or more of the following:

  (i) Performance of Red-Black Gauss-Seidel iteration

  (ii) Influence of over-relaxation parameter on stability and convergence with Multigrid method

  (iii) Comparison of the performance of different multigrid cycles (for example W-cycle or Full Mutigrid)

Note 1: Report needs to by typed. Equations may be written neatly by hand and incorporated if needed, but are preferably typed. Graphs and plots need to have clear captions. All symbols should be clearly defined. Points are not based on the length of the report but the quality of your analysis and discussion.

Note 2: Question (d) above requires numerical "experimentation". As in any good experiment, it is a bad idea to change more than one independent variables at a time.