lec # 13. Relations.

AntiSymmetric:

$\forall a,b \in A$ if $(a,b) \in R \wedge (b,a) \in R \rightarrow a = b$.

Ex7 p462

$R = \{\}$. ✓ $A = \{1,2,3,4\}$.

$R = \{(1,1)\}$. ✓

$R = \{(1,1), (2,1)\}$. ✓

$R = \{(1,1), (2,1), (1,2)\}$,

$A = \{x,y\}$.

Determine all AntiSymmetric Relations on A.

$A \times A = \{(x,x), (x,y), (y,x), (y,y)\}$.

$pow(A \times A) = \{\Phi, \{(x,x)\}, \{(x,y)\},$

$\ldots \{(x,x), (x,y), (y,x), (y,y)\}$

Transitive:

$\forall x,y,z \in A$ if $(x,y) \in R \wedge (y,z) \in R \rightarrow (x,z) \in R$.

Ex7 p462

$R = \{\}$. ✓ $A = \{1,2,3,4\}$.

$R = \{(1,1)\}$ ✓

$R = \{ (1,2), (2,2) \}$ ✓

$R = \{ (1,2), (2,3), (2,2), (2,4), (4,6) \}$. X.

Ex12 $R = \{ (a,b) \mid a \text{ divides } b \}$ $A = \mathbb{Z}^+$
P463

$a \geq b$ $a \leq b$
$a + b = 0$ $-a \geq b$

Reflexive $\forall a \in A$ $(a,a) \in R$.

$\forall a \in \mathbb{Z}^+$ a divides a. ✓

Symmetric $\forall a,b \in A$ if $(a,b) \in R \rightarrow (b,a) \in R$.

$\forall a,b \in \mathbb{Z}^+$ if a divides b $\rightarrow$ b divides a. X.
$(2,4) \in R \rightarrow (4,2) \notin R$.

Anti Symmetric $\forall a,b \in A$ if $(a,b) \in R \wedge (b,a) \in R \rightarrow a = b$.

$\forall a,b \in \mathbb{Z}^+$ if a divides b $\wedge$ b divides a $\rightarrow a = b$.
✓

Transitive $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$.

$\forall a,b,c \in \mathbb{Z}^+$ if a divides b $\wedge$ b divides c $\rightarrow$ a divides c.

Q6 467:- $A = \mathbb{R}$.
Ex

$R = \{ (a,b) \mid a = \pm b \}$.
$R = \{ (a,b) \mid a = b \text{ or } a = -b \}$
$R = \{ (a,b) \mid ab \geq 0 \}$.

$R^{-1} = \{ (b,a) \mid (a,b) \in R \}$.
$\overline{R} = \{ (a,b) \mid (a,b) \notin R \}$.

$R = \{ (1,3), (2,4) \}$.
$R^{-1} = \{ (3,1), (4,2) \}$.

$\bar{R} = \{(a,b) \mid (a,b) \notin R\}$.

$R^{-1} = \{(3,1), (4,2)\}$.

$A = \{1,2\}$.

$A \times A = \{(1,1), \cancel{(1,2)}, (2,1), (2,2)\}$.

$\bar{R} = A \times A - R = \{(1,1), (2,1), (2,2)\}$.

$R = \{(1,2)\}$.

Ex 17
P 465

$A = \{1,2,3\}$. $B = \{1,2,3,4\}$,

$R_1 = \{(1,1), (2,2), (3,3)\}$. $A \times B$.

$R_2 = \{(1,1), (1,2), (1,3), (1,4)\}$.

$R_1 \cap R_2 = \{(1,1)\}$.

$R_1 \cup R_2 = \{(1,1), (2,2), (3,3), (1,2), (1,3), (1,4)\}$.

$R_1 - R_2 = \{(2,2), (3,3)\}$.

$R_2 - R_1 = \ ?$

Composit. R $A \times B$ (a,b)

S $B \times C$. (b,c).

$S \circ R$ (a,c).

$(a,c) \in S \circ R$ if $(a,b) \in R \wedge (b,c) \in S$. $\rightarrow$ $(a,c) \in S \circ R$.

Ex 20
465

$A = \{1,2,3\}$.
$B = \{1,2,3,4\}$,
$C = \{0,1,2\}$.

$R = \{(1,1), (1,4), (2,3), (3,1), (3,4)\}$.

$S = \{(1,0), (2,0), (3,1), (3,2), (4,1)\}$.

$S \circ R = \{(1,0), (1,1), (2,1), (2,2), (3,0), (3,1)\}$.

$S \circ R = \{(1,0), (1,1), (2,1), (2,2), (3,0), (3,1)\}.$

$S \circ R \neq R \circ S.$ (HW).

$R \circ R = R^2$

$R^2 \circ R = R^3$

$\vdots$

$R^{n-1} \circ R = R^n$

HW P466–468 (1–60). Ex.