lec #14:-

N-ary Relations.

$A = \{1,2\}.$

$B = \{a,b\}.$

$C = \{x,y\}.$

$|A \times B \times C| = |A| \times |B| \times |C|$

$= 2 \times 2 \times 2$

$= 8.$

$|pow(A \times B \times C)| = 2^{|A \times B \times C|}$

$= 2^{|A| \times |B| \times |C|}$

$= 2^{2 \times 2 \times 2} = 2^8 = 256$

$pow(A \times B \times C) = \{\phi, \{(1,a,x),$

$(1,a,y), --- \}.$

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Ex1:- $R = \{(a,b,c) \mid a < b < c\}.$

$N \times N \times N$

$= \{(1,1,1), ----$

$(\infty,\infty,\infty)\}.$

$(1,2,3) \in R$ ✓

$(2,4,3) \in R.$ X.

Ex2:-

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$R = \{(a,b,c) \mid b = a+K, c = a+2K\}.$

$\exists K \in Z.$

$Z \times Z \times Z$

$(1,3,5) \in R.$ ✓

$3 = 1+K$ $K = 2.$

$5 = 1+2.2$

$5 = 5.$

$(2,5,9) \in R.$ X.

$5 = 2+K \Rightarrow K = 3.$

$9 = 2+2.3$

$9 = 2+6$

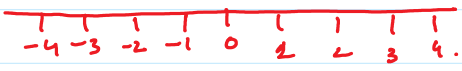
$9 \neq 8.$

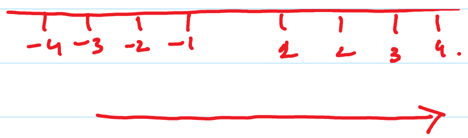
Ex3

469.

23 mod 4.

$$\begin{array}{r} 5 \\ 4\overline{)\,23} \\ \pm 20 \\ \hline \textcircled{3} \end{array} \qquad \begin{array}{r} -6 \\ 4\overline{)\,-23} \\ \mp 24 \\ \hline +1 \end{array}$$





$R = \{(a,b,c) \mid a \equiv b \bmod c\}$ $\quad Z \times Z \times Z^+$.

$(8,2,3) \in R$. ✓ $\quad 21 \equiv 5 \bmod 4$. ✓

$8 \equiv 2 \bmod 3$.

$$\begin{array}{r} 1 \\ 4\overline{)5} \\ -4 \\ \hline 1 \end{array} \qquad \begin{array}{r} 5 \\ 4\overline{)21} \\ -20 \\ \hline 1. \end{array}$$

$(-1, 9, 5) \in R$ (HW).

$-1 \equiv 9 \bmod 5$.

$$\begin{array}{r} 0 \\ 3\overline{)2} \\ 0 \\ \hline 2 \end{array} \qquad \begin{array}{r} 2 \\ 3\overline{)8} \\ -6 \\ \hline 2. \end{array}$$

Ex4
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$A \times N \times S \times D \times T$.

A = Set of Airlines.
N = " " Flight #'s
S = " " Sources
D = " " Destination.
T = " " Hours.

A	N	S	D	T.
Qatar	QR-256	PEW	DXB	22:00.

Representing Relations. [ ]

Size rows x Col.

Capital letter for denoting Matrix = M
for individual element = m_{ij}

R is defined on $A \times B$. $\quad A = \{a_1, a_2, --- a_m\}$.
$B = \{b_1, b_2, --- b_n\}$.
$|A \times B| = |A| \times |B| = m \times n$.

M_R row x col.
rows = $|A|$
Col = $|B|$

m_{11}

$$m_{ij} = \begin{cases} 1 & \text{if } (a_i, b_j) \in R \\ 0 & \text{if } (a_i, b_j) \notin R. \end{cases}$$

Ex1: $R = \{(2,1), (3,1), (3,2)\}$ 476 Find its matrix representation.

$$A = \{\overset{a_1}{1}, \overset{a_2}{2}, \overset{a_3}{3}\}.$$

$$B = \{\underset{b_1}{1}, \underset{b_2}{2}\}.$$

$$M_R = \begin{bmatrix} \overset{0}{\underline{m_{11}}} & \overset{0}{\underline{m_{12}}} \\ \overset{1}{\underline{m_{21}}} & \overset{0}{\underline{m_{22}}} \\ \overset{1}{\underline{m_{31}}} & \overset{1}{\underline{m_{32}}} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}.$$

$$\begin{array}{c|cc} & x & y \\ \hline a & & \\ b & & \\ c & & \end{array}$$

[

Properties:-

1) Reflexive. $\forall a \quad (a,a) \in R.$

$\forall a_i \quad (a_i, a_i) \in R.$

$\forall i \quad m_{ii} = 1.$

$[\ ]$ ✓ $\qquad R = \{\emptyset\}. \qquad |A| = |B| = 0 \qquad |A \times B| = 0.$

$|2^{|A \times B|}| = 1.$

$[0]$ X $\qquad |A| = |B| = 1$

$[1]$ ✓

$A = \{a, b\}. \qquad A \times A = ?$

$Pow(A \times A)$

$= \{\emptyset, \{(a,a)\}$

$$\begin{array}{c|cc} & a & b \\ \hline a & 0 & 0 \end{array} \quad \begin{array}{c|cc} & a & b \\ \hline a & 1 & 0 \end{array} \quad \begin{bmatrix} 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 \end{bmatrix}$$

$\overset{X}{\begin{array}{c|cc} & a & b \\ \hline a & 0 & 0 \\ b & 0 & 0 \end{array}}$ $\overset{X}{\begin{array}{c|cc} & a & b \\ \hline a & 1 & 0 \\ b & 0 & 0 \end{array}}$ $\overset{X}{\begin{bmatrix}0&1\\0&0\end{bmatrix}}$ $\overset{X}{\begin{bmatrix}0&0\\1&0\end{bmatrix}}$ $\overset{X}{\begin{bmatrix}0&0\\0&1\end{bmatrix}}$

$\overset{X}{\begin{bmatrix}1&1\\0&0\end{bmatrix}}$ $\overset{X}{\begin{bmatrix}1&0\\1&0\end{bmatrix}}$ $\overset{\checkmark}{\begin{bmatrix}1&0\\0&1\end{bmatrix}}$ $\overset{X}{\begin{bmatrix}0&1\\1&0\end{bmatrix}}$ $\overset{X}{\begin{bmatrix}0&1\\0&1\end{bmatrix}}$ $\overset{X}{\begin{bmatrix}0&0\\1&1\end{bmatrix}}$

$\overset{X}{\begin{bmatrix}1&1\\1&0\end{bmatrix}}$ $\overset{\checkmark}{\begin{bmatrix}1&0\\1&1\end{bmatrix}}$ $\overset{\checkmark}{\begin{bmatrix}1&1\\0&1\end{bmatrix}}$ $\overset{X}{\begin{bmatrix}0&1\\1&1\end{bmatrix}}$ $\overset{\checkmark}{\begin{bmatrix}1&1\\1&1\end{bmatrix}}$.

$Pow(A \times A)$
$= \{\phi, \{(a,a)\},$
$\vdots$
$\}$

Symmetric $\forall a,b$ if $(a,b) \in R \rightarrow (b,a) \in R$.
$\forall a_i, b_j$ if $(a_i, b_j) \in R \rightarrow (b_j, a_i) \in R$.

$\forall ij$ if $m_{ij} = 1 \rightarrow m_{ji} = 1$.

$[\ ]$ ✓
$[0]$ ✓
$[1]$ ✓

$\overset{\checkmark}{\begin{bmatrix}0&0\\0&0\end{bmatrix}}$ $\overset{\checkmark}{\begin{bmatrix}1&0\\0&0\end{bmatrix}}$ $\overset{X}{\begin{bmatrix}0&1\\0&0\end{bmatrix}}$ $\overset{X}{\begin{bmatrix}0&0\\1&0\end{bmatrix}}$ $\overset{\checkmark}{\begin{bmatrix}0&0\\0&1\end{bmatrix}}$

$\overset{X}{\begin{bmatrix}1&1\\0&0\end{bmatrix}}$ $\overset{X}{\begin{bmatrix}1&0\\1&0\end{bmatrix}}$ $\overset{\checkmark}{\begin{bmatrix}1&0\\0&1\end{bmatrix}}$ $\overset{\checkmark}{\begin{bmatrix}0&1\\1&0\end{bmatrix}}$ $\overset{X}{\begin{bmatrix}0&1\\0&1\end{bmatrix}}$ $\overset{X}{\begin{bmatrix}0&0\\1&1\end{bmatrix}}$

$\overset{\checkmark}{\begin{bmatrix}1&1\\1&0\end{bmatrix}}$ $\overset{X}{\begin{bmatrix}1&0\\1&1\end{bmatrix}}$ $\overset{X}{\begin{bmatrix}1&1\\0&1\end{bmatrix}}$ $\overset{\checkmark}{\begin{bmatrix}0&1\\1&1\end{bmatrix}}$ $\overset{\checkmark}{\begin{bmatrix}1&1\\1&1\end{bmatrix}}$.

Anti Symmetric $\forall a,b \in A$ if $(a,b) \in R \wedge (b,a) \in R \rightarrow a = b$.
$\forall a_i, b_j \in A$ if $(a_i, b_j) \in R \wedge (b_j, a_i) \in R \rightarrow i = j$.

$\forall ij$ if $m_{ij} = 1 \wedge m_{ji} = 1 \rightarrow i = j$.

$[\ ]$ ✓
$[0]$ ✓

$[1]$ ✓

$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ ✓ $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ ✓ $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ ✓ $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ ✓ $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ ✓

$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ ✓ $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ ✓ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ✓ $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ ✓ $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$ ✓ $\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$ ✓

$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ X $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ ✓ $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ ✓ $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ X $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ X.

$$\begin{bmatrix} 1 & 0 & \overset{X}{0} \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

✓

X.

✓