Lec # 4:- Applications of propositional logic.

- System Specification. ✓ Session 1.
- Logic puzzles. ✓ " 1.
- Searching.

Smullyan 1919

Values Variables.

$2^2 = 4$.

Knights. = Truth
Knaves. = lies.

A, B.

Island.

A Says " B is a Knight"

B Says "the two of us are of opposite type".

(A ثابت ہے اور B سفید ہے) یا (A سفید ہے اور B ثابت ہے)

$\underbrace{A \text{ is a Knight}}_{p}$ $\underbrace{\text{and}}_{\wedge}$ $\underbrace{B \text{ is a Knave}}_{\neg q}$ $\underbrace{\text{or}}_{\vee}$ A is a Knave & B is a Knight.

Determine the type of A & B.?

Let p = " A is a Knight" $\neg p$ = " A is a Knave".

q = " B is a Knight" $\neg q$ = " B is a Knave".

$\rightarrow q$

$\rightarrow (p \wedge \neg q) \vee (\neg p \wedge q)$.

A	B.
→ Knight	Knight
→ Knight	Knave
→ Knave	Knight
→ Knave	Knave.

Case 1

A Knight B. Knight.

$\therefore q_1 = T$

$p = T$ $\neg p = F$

① $q = T$

② $(P \wedge \neg q) \vee (\neg P \wedge q) . = T$

$P = T \quad \neg P = F$
$q = T \quad \neg q = F$

① $\Rightarrow T = T$ ✓.

② $\Rightarrow (T \wedge F) \vee (F \wedge T) = T$

$F \vee F \neq T$.

$F \neq T$ Does not hold.

Case 2. A Knight B Knave

① $\Rightarrow q = T$

② $\Rightarrow . (P \wedge \neg q) \vee (\neg P \wedge q) = F$

$P = T \quad \neg P = F$
$q = F \quad \neg q = T$

$F \neq T$ Does not hold.

Case 3: A Knave B Knight

① $q = F$

② $(P \wedge \neg q) \vee (\neg P \wedge q) . = T$

$P = F \quad \neg P = T$
$q = T \quad \neg q = F$

$T \neq F$ Does not hold.

Case 4 A Knave B Knave

① $q = F$

② $(P \wedge \neg q) \vee (\neg P \wedge q) . = F$

$P = F \quad \neg P = T$
$q = F \quad \neg q = T$

① $\Rightarrow F = F$ ✓

② $\Rightarrow (F \wedge T) \vee (T \wedge F) = F$

$R \vee P \equiv P$

$P \equiv P$

A is a Knave & B is a Knave.

A	B.
Knight	Knight
Knight	Knave
Knave	Knight
Knave	Knave.

No Case holds.
→ Inconclusive.

A	B	C.
Knight	Knight	Knight.
Knave	Knave	Knave

Advice.

(Practice the Same by Assuming That
Knight always lies & Knave u Speaks truth.) ⇒ HW.

5%.

P_1 : A is a Knight
P_2 : A is a Knave.
P_3 : A is a Spy.

Knight : truth
Knaves : lies
Spies. : ?

p_3 : A is a Spy.

$\neg p_1$: A is not a Knight
$\neg p_2$: A is not a Knave.
$\neg p_3$: A is not a Spy.

A, B.

q_1 :
q_2 :
q_3 :

$\neg q_1$:
$\neg q_2$:
$\neg q_3$:

Hw.

A: Knight B: Knight

: T

: T

$p_1 = T$	$\neg p_1 = F$
$p_2 = F$	$\neg p_2 = T$
$p_3 = F$	$\neg p_3 = T$
$q_1 = T$	$\neg q_1 = F$
$q_2 = F$	$\neg q_2 = T$
$q_3 = F$	$\neg q_3 = T$.

A: Knight B: Spy.

: T

: ? X.

$p_1 = T$	$\neg p_1 = F$
$p_2 = F$	$\neg p_2 = T$
$p_3 = F$	$\neg p_3 = T$
$q_1 = F$	$\neg q_1 = T$
$q_2 = F$	$\neg q_2 = T$
$q_3 = T$	$\neg q_3 = F$.