lec 5

Propositional Equivalences.

1) Tautology.

A Compound Proposition with truth Value of T

كا بيان لينا (1
كانون كو كهلا رفيس (2
مسئله (3
T تاسعین کهلیں (4

2) Contradiction.

A Compound Proposition with truth Value of F

3) Contingency.   Sometimes T and other times F.

p	¬p	$p \lor \neg p$	$p \land \neg p$
T	F	T	F
F	T	T	F

Tautology.   Contradict

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

→ Contingency.

logical Equivalence. $\equiv$

Two Compound Propositions p & q

& logically Equivalent if $p \leftrightarrow q$ results in a

tautology.

Q22

Ex2

$\neg(p \cup q)$ and $\neg p \land \neg q$ are equivalent.

so $\neg(p \land q)$ and $\neg p \land \neg q$ are equivalent.

Ex 2

p	q	$p \lor q$	$\neg(p \lor q)$	$\neg p$	$\neg q$	$\neg p \land \neg q$	$\neg(p \lor q) \leftrightarrow \neg p \land \neg q$
T	T	T	F	F	F	F	T
T	F	T	F	F	T	F	T
F	T	T	F	T	F	F	T
F	F	F	T	T	T	T	T

↓

Tautology.

Ex3 & Ex4   H.W.

p 22-23.

Table 6

p24

1) $p \land T \equiv p$ Identity law
        $p \lor F \equiv p$

2) $p \lor T \equiv T$ Domination law.
        $p \land F \equiv F$

3) $p \land p \equiv p$ Idempotent law.
        $p \lor p \equiv p$.

4) $\neg(\neg p) \equiv p$ Double negation.

5) $p \land q \equiv q \land p$ $p \to q \not\equiv q \to p$.
        $p \lor q \equiv q \lor p$.

⋮

8) $\neg(p \land q) \equiv \neg p \lor \neg q$.
        $\neg(p \lor q) \equiv \neg p \land \neg q$.

PREDICATES :-

$$P(x) = \quad x + 3 = 5 \qquad x \in \{1, 2, 3\}.$$

$$P(1) = \quad 1 + 3 = 5 \quad F$$
$$P(2) = \quad 2 + 3 = 5 \quad T$$
$$P(3) = \quad 3 + 3 = 5 \leftarrow . F.$$

$$P(x) = \boxed{x} \boxed{>} 5 \qquad x \in \{1, 2, 3\}.$$

↓ predicate.

Subject.

Domain.

Ex1 :-
P31

$$P(x) = \quad x > 3$$
$$P(2) = \quad 2 > 3 \quad F$$
$$P(4) = \quad 4 > 3 \quad T$$

Ex2
P31

$$P(x) = \text{"Computer } x \text{ is under attack"}$$

CS2 & MATH1 is under attack.

$$P(CS1) = ? \quad F$$
$$P(CS2) = ? \quad T.$$

Ex3
P31

$$Q(x, y) = \quad x = y + 3$$

$$Q(1, 2) = \quad 1 = 2 + 3 \quad F$$
$$Q(3, 0) = \quad 3 = 0 + 3 \quad T$$

Ex4, 5   HW.
P31

Quantifiers.

Universal $\forall$

for all,
For every,
For any,

$$\forall x \, P(x) = P(1) \wedge P(2) \wedge P(3) \\ --- \wedge P(N).$$

$x \in \{1, 2, 3, \cdots N\}.$

Existential $\exists$

there exist
for Some.
for atleast one,

$$\exists_x P(x) = P(1) \vee P(2) \vee P(3) \vee --- \vee P(N).$$

$x \in \{1, 2, 3, \cdots N\}.$

Ex 9
P34

$P(x) = x > 2$ 　　　　$x \in \mathbb{R}.$

$\forall_x P(x) = ? \, \mathbb{F}.$ 　　$3 > 2$ 　T
　　　　　　　　　　　$-1 > 2$ 　F.

Ex 10　　　　$P(x) = x^2 > 0$ 　　　　$x \in \mathbb{Z}.$

$\forall_x P(x) = 0 > 0$ 　F

$\forall_x P(x) = F$

Ex 17
P36

$\exists_z \, P(z) = z^2 = 2.$ 　　　$z \in \mathbb{R}.$
　　　　　　　　　$\mathbb{F}$

Questions　　　1)　Converse, Inverse, Contra, Implicah.
　　　　　　　2)　System　Speerfication.

　　　　　　　3)　Knave, Knight.

Questions

1) Converse, Inverse, Contra, Implicat.

2) System Specification.

3) Knaver Knight.

4) Quantifier.

5) ?