lec # 9 :- Rules of Inference

$$\left.\begin{array}{l} P1 \\ P2 \\ P3 \\ \vdots \\ \underline{PN} \\ \therefore C \end{array}\right\} \text{ Argument}$$

Valid Argument.

$$((P1 \wedge P2 \wedge \cdots \wedge PN) \rightarrow C)$$

Tautology.
Fallacy.

Observation :-
1) Order
2) lots of Rules.

Principle of Resolution.

literal :- A propositional Variable or its Negation.
$\neg p,\ p,\ r,\ \neg r,\ \cdots$

Clause :- A disjunction of literals.
$\neg p \vee r \ ,\quad p \vee r \vee \neg q \ ,\quad p \vee \neg p .$

P1	C1
P2	C2
P3	$\vdots$
$\vdots$	$\vdots$
PN	CM.
$\therefore C$	$\neg C$

$\neg\ \vee$
$\wedge$.
$\rightarrow$
$\leftrightarrow$

How to make clauses.

$$p \wedge q \left.\right\} \begin{array}{l} p \\ q \end{array}$$

$$p \rightarrow q \equiv \neg p \vee q .$$

$$p \leftrightarrow q \equiv p \rightarrow q \wedge q \rightarrow p .$$
$$(\neg p \vee q) \wedge (\neg q \vee p)$$

$\neg p \vee q$
$\neg q \vee p$.

Ex
P65

P1 :- P
P2 :- $P \rightarrow Q$
C :- $\therefore Q$.

C1 :- P ✓
C2 :- $\neg P \vee Q$ ✓
C3 :- $\neg Q$ ✓

Resolution.

P q

C2:- $\neg P \vee Q$ ✓
C3:- $\neg Q$ ✓

C4:- Q From C1, C2 ✓
C5:- $\square$ " C3, C4.

Resolution.

$p \vee q$
$\neg q \vee r$
$\therefore p \vee r$.

$\overset{p}{F} \vee \overset{q}{P}$
$\neg P \vee Q$
$\therefore P \vee Q = Q$.

Ex11:- P65

P1:- $T \to M \vee E$
P2:- $S \to \neg E$
P3:- $T \wedge S$
C:- $\therefore M$

C1:- $\neg T \vee M \vee E$. ✓
C2:- $\neg S \vee \neg E$ ✓
C3:- T ✓
C4:- S ✓
C5:- $\neg M$. ✓

C6:- $\neg T \vee M \vee \neg S$ from C1, C2. ✓
C7:- $M \vee \neg S$ " C3, C6 ✓
C8:- M " C7, C4 ✓
C9:- $\square$ " C5, C8

Ex6:- P62

P1:- $\neg p \wedge q$
P2:- $r \to p$
P3:- $\neg r \to s$
P4:- $s \to t$
C $\therefore t$.

C1:- $\neg p$ ✓
C2:- q → Redundant.
C3:- $\neg r \vee p$ ✓
C4:- $r \vee s$ ✓
C5:- $\neg s \vee t$ ✓
C6:- $\neg t$ ✓
C7:- $\neg r$ from C1, C3. ✓
C8:- s " C4, C7 ✓
C9:- t " C5, C8. ✓
C10:- $\square$ " C6, C9

Ex7:- P62

P1:- $p \to q$
P2:- $\neg p \to r$
P3:- $r \to s$
C $\therefore \neg q \to s$

$\neg(\neg q \to s)$

P62 P2: $\neg p \to r$
P3: $r \to s$
C. $\therefore \neg q \to s$.

$\neg(\neg q \to s)$.
$\neg(q \vee s)$.
$\neg q \wedge \neg s$.
$\neg q$
$\neg s$.

C1: $\neg p \vee q$. ✓
C2: $p \vee r$ ✓
C3: $\neg r \vee s$. ✓
C4: $\neg q$ ✓
C5: $\neg s$ ✓
C6: $q \vee r$ from C1, C2. ✓
C7: $q \vee s$ " C3 & 6. ✓
C8: s " C7, C4 ✓
C9: $\square$ ", C8, C5

Ex9 (Ad...).
p62

T
F F

p	q	$p \vee q$	$\vee$	$p \vee q$	
T	T	T		T	T
T	F	T		T	T
F	T	T		T	T
F	F	F		F	F

The disjunction of two Contingencies Can be a tautology.

there exist x, x is Contingency, there exist y,
y is Contingency, the disjunction of x & y Can be a tautology.

$P(x,y) =$?

$\exists x \exists y \; P(x,y)$.

$x, y \in$ Set of Contingency.

$p(x)$ $x + 3 = 5$

$\exists x \, p(x) = p(1) \vee p(2) \vee p(3)$
$(1+3=5) \vee (2+3=5) \vee (3+3=5)$.

$F \vee T \vee F = T$.