

lec # 4:

Applications of propositional logic.

- System Specification. ✓ Session 1.
- Logic puzzles. ✓ " 1.
- Searching.

Smullyan 1919

Variables.

$$2^2 = 4.$$

Knights & Truth  
Knaves & lies.

A, B.

Island.

A Says "B is a Knight"

B Says "the two of us are of opposite type".  
( $\neg(A \wedge B) \wedge (A \vee B)$ )

$\frac{A \text{ is a Knight}}{p} \wedge \frac{B \text{ is a Knave}}{\neg q} \vee \frac{A \text{ is a Knave}}{\neg p} \wedge \frac{B \text{ is a Knight}}{q}$

Determine the type of A & B?

Let  $p = "A \text{ is a Knight}"$   $\neg p = "A \text{ is a Knave}"$ .

$q = "B \text{ is a Knight}"$   $\neg q = "B \text{ is a Knave}"$ .

$\rightarrow q$

$$(p \wedge \neg q) \vee (\neg p \wedge q).$$

Case 1

A Knight B Knight.

$$p = T, q = T$$

	A	B
$\rightarrow$	Knight	Knight
$\rightarrow$	Knight	Knave
$\rightarrow$	Knave	Knight
$\rightarrow$	Knave	Knave

$$p = T, \neg p = F$$

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$$\textcircled{1} \quad q = T$$

$$\textcircled{2} \quad (P \wedge \neg q) \vee (\neg P \wedge q) = T$$

$$\textcircled{1} \Rightarrow T = T \quad \checkmark$$

$$\textcircled{2} \Rightarrow (T \wedge F) \vee (F \wedge T) = T$$

$$F \vee F \neq T$$

$F \neq T$  Does not hold.

$$P = T \quad \neg P = F$$

$$q = T \quad \neg q = F$$

CASE 2.      A      B  
         Knight      Knave

$$\textcircled{1} \Rightarrow q = T$$

$$P = T \quad \neg P = F$$

$$q = F \quad \neg q = T$$

$$\textcircled{2} \Rightarrow (P \wedge \neg q) \vee (\neg P \wedge q) = F$$

$F \neq T$  Does not hold.

CASE 3:-      A      B  
         Knave      Knight

$$\textcircled{1} \quad q = F$$

$$P = F \quad \neg P = T$$

$$q = T \quad \neg q = F$$

$$\textcircled{2} \quad (P \wedge \neg q) \vee (\neg P \wedge q) = T$$

$T \neq F$  Does not hold.

CASE 4      A      B  
         Knave      Knave

$$\textcircled{1} \quad q = F$$

$$P = F \quad \neg P = T$$

$$q = F \quad \neg q = T$$

$$\textcircled{2} \quad (P \wedge \neg q) \vee (\neg P \wedge q) = F$$

$$\textcircled{1} \Rightarrow P = F \quad \checkmark$$

$$\textcircled{2} \Rightarrow (F \wedge T) \vee (T \wedge F) = F$$

$P \vee P \supset P$   
 $P \supset P$

A is a Knave & B is a Knave.

A	B.
Knight	Knight
Knight	Knave
Knave	Knight
Knave	Knave.

No Case holds.  
 $\rightarrow$  Inconclusive.

A	B	C.
Knight	Knight	Knight.
Knave	Knave	Knave

Advice.

(Practice the Same by Assuming That)  
 Knight always lies &  
 Knave " speaks truth.  $\Rightarrow$  HW.

$P_1 \supset$  A is a Knight  
 $P_2 \supset$  A is a Knave.  
 $P_3 \supset$  A is a Spy.

$\therefore$  A is not a Knight

Knight  $\supset$  truth  
 Knave  $\supset$  lies  
 Spies.  $\supset$  ?

A is

$p_3 \equiv A \text{ is not a Knight}$

$A, B$

$\neg p_1 \equiv A \text{ is not a Knight}$

$\neg p_2 \equiv A \text{ is not a Knight}$

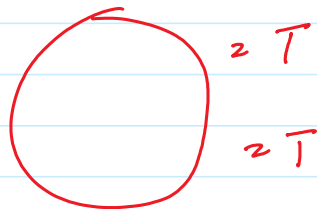
$\neg p_3 \equiv A \text{ is not a Spy}$

$q_1 \equiv \text{---}$   
 $q_2 \equiv \text{---}$   
 $q_3 \equiv \text{---}$

H.W.

$\neg q_1 \equiv \text{---}$   
 $\neg q_2 \equiv \text{---}$   
 $\neg q_3 \equiv \text{---}$

A Knight B Knight



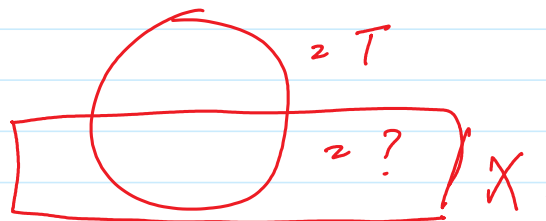
$p_1 \equiv T$   
 $p_2 \equiv F$   
 $p_3 \equiv F$

$\neg p_1 \equiv F$   
 $\neg p_2 \equiv T$   
 $\neg p_3 \equiv T$

$q_1 \equiv T$   
 $q_2 \equiv F$   
 $q_3 \equiv F$

$\neg q_1 \equiv F$   
 $\neg q_2 \equiv T$   
 $\neg q_3 \equiv T$

A Knight B Spy



$p_1 \equiv T$   
 $p_2 \equiv F$   
 $p_3 \equiv F$

$\neg p_1 \equiv F$   
 $\neg p_2 \equiv T$   
 $\neg p_3 \equiv T$

$q_1 \equiv F$   
 $q_2 \equiv F$   
 $q_3 \equiv T$

$\neg q_1 \equiv T$   
 $\neg q_2 \equiv T$   
 $\neg q_3 \equiv F$