lec # 16:-

Closures.

Transitive

$(1,2), (2,3), (2,4), (3,1).$

$R = \{(1,3),(1,4),(2,1),(3,2)\}.$ $A = \{1,2,3,4\}.$

missing.

$R = \{(1,3),(1,4),(2,1),(3,2)\} \cup \{(1,2), (2,3), (2,4), (3,1)\}.$

$R \cup \text{missing} = \{(1,3),(1,4),(2,1),(3,2),(1,2),(2,3),(2,4),(3,1)\}$?

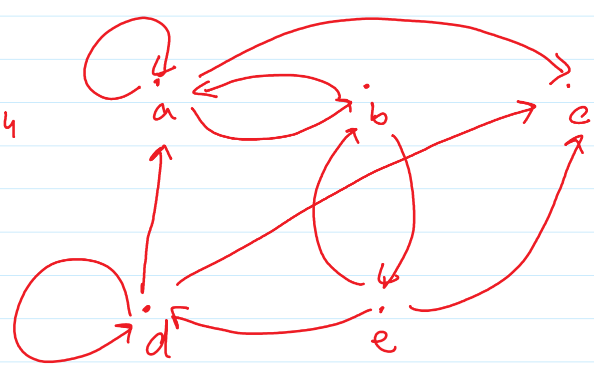
b c. (3,4) x. a b.

loop {
R.
missing
R ∪ missing.
}.

Path: ∃ a path from a to b if. ∃ a sequence of edges. Such that

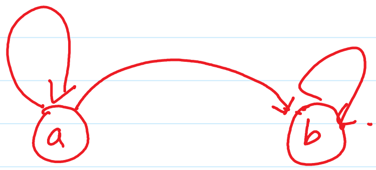
$(a,x_1),(x_1,x_2),(x_2,x_3),\text{----}(x_{n-1},x_n),(x_n,b).$

Ex3
p484



	Length Path
a to e.	
(a,b) (b,e).	2.
a b e	3-1=2.
a b e d a	5-1=4.
a b a	3-1=2.
a a	2-1=1.
aaaaaaaaaaa	11-1=10.

theorem:- Let R be a relation on A. ∃ a path of length n, $(n>0)$ from a to b iff $(a,b) \in R^n$.



$R^2 = R \circ R$.
$R^3 = R^2 \circ R^1$.
$\vdots$
$R^n = R^{n-1} \circ R^1$.

$R = \{(a,a), (a,b), (b,b)\}$.

Connectivity Relation. Let R be a relation of A.

R^* it contain (a,b) if $\exists$ a path from a to b in R.

$$R^* = \bigcup_{i=1}^{\infty} R^i$$

Ex4:- 485 $R = \{(a,b) \mid a \text{ has met } b\}$. $A =$ Set of all people.

What is R^n & R^*.

$\overset{R}{S} \circ R$

R	$(a, \overset{x_1}{b})$	$A \times \overset{A}{B}$
S	$(\overset{x_1}{b}, \overset{b}{c})$.	$\overset{A}{B} \times \overset{A}{C}$.

$(a,c) \in S \circ R$ $\quad (a,b) \in R \wedge (b,c) \in S$.

$(a,b) \in R \circ R$ $\quad (a,x_1) \in R \wedge (x_1,b) \in R$.
$(a,b) \in R^2$ $\quad$ a has met x_1 $\wedge$ x_1 has met b.

$(a,b) \in R^3$ $\quad$ a has met $x_1 \wedge x_1$ has met $x_2 \wedge x_2$ has met b.
$\vdots$

$(a,b) \in R^n$ $\quad \exists$ persons $x_1, x_2, \ldots x_{n-1}$ Such that
a met x_1
x_1 " x_2
$\vdots$

$n-2$ 4 x_{n-1}.
x_{n-1} a   b.

Transitive Closure:- Transitive Closure $= R^*$.

Ex 6 $R = \{(a,b) \mid a$ has a Common border with $b\}$.

R^4

R^*.

Equivelence Relation:-

1) Reflexive
2) Symmetric
3) Transitive

Ex 2 / 494 $R = \{(a,b) \mid a-b \in \mathbb{Z}\}$. $A = \mathbb{Z}$.

Reflexive:- $\forall a \in A \; (a,a) \in R$.

$\forall a \in \mathbb{Z}$ $a-a \in \mathbb{Z}$.
$0 \in \mathbb{Z}$. ✓

Symmetric $\forall a,b \in A$ if $(a,b) \in R \rightarrow (b,a) \in R$.
$\forall a,b \in \mathbb{Z}$ if $a-b \in \mathbb{Z} \rightarrow b-a \in \mathbb{Z}$.

Transitive $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$.
$\forall a,b,c \in \mathbb{Z}$ if $a-b \in \mathbb{Z} \wedge b-c \in \mathbb{Z} \rightarrow a-c \in \mathbb{Z}$ ✓

Ex 3 / 494 $R = \{(a,b) \mid a \equiv b \bmod m\}$. $m > 0$.

Reflexive:- $\forall a \in A \quad (a,a) \in R$.

$\forall a \in Z \quad a \equiv a \bmod m$ ✓

Symmetric $\forall a,b \in A \quad$ if $(a,b) \in R \rightarrow (b,a) \in R$.

$\forall a,b \in Z \quad$ if $\quad a \equiv b \bmod \underline{\underline{m}} \rightarrow b \equiv a \bmod \underline{\underline{m}}$.

Transitive $\forall a,b,c \in A \quad$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$.

$\forall a,b,c \in Z \quad$ if $\quad a \equiv b \bmod m \wedge b \equiv c \bmod m \rightarrow a \equiv c \bmod m$.

$\{ \quad | \quad \}$.

Syntax.

$[a] = \{ s \mid (a,s) \in R \}$.

Ex8:- $\quad R = \{ (a,b) \mid a = b \text{ or } a = -b \} \qquad A = \mathbb{Z}$.

$(7, \quad 7=7 \quad 7=-(-7)$.

$[7] = \{7, -7\} = [-7]$.