Lec # 17 Equivalence Classes.

$[a] = \{ s \mid (a,s) \in R \}$.

Ex 9 :-
P496

$R = \{ (a,b) \mid a \equiv b \mod m \}$ $\quad A = \mathbb{Z}$. $\quad m > 0$.

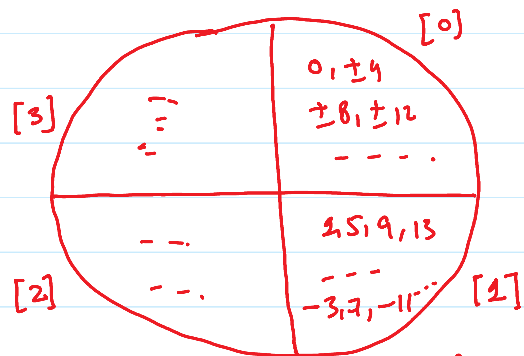
$[0] = \{ 0, \pm 4, \pm 8, \pm 12, \ldots \}$. $\quad m = 4$.

$[1] = \{ 1, 5, 9, 13, \ldots$
$-3, -7, -11, -15, \ldots \}$.

$[2] = \{ \quad \}$

$[3] = \{ \quad \}$.

Categorization.



PARTITION let $P = \{ P_1, P_2, P_3, \ldots P_n \}$ be a family of Sets. P creates a Partition of S.

1) $\forall i \; P_i \neq \phi$.

2) $\forall i,j \quad P_i \cap P_j = \phi \quad i \neq j$.

3). $\bigcup_{i=1}^{\infty} P_i = S$.

Ex 13
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$A_1 = \{1, 2, 3\}$ $\quad A_2 = \{4, 5\}$, $\quad A_3 = \{6\}$.

Ex13
499

$A_1 = \{1,2,3\}$ $A_2 = \{4,5\}$, $A_3 = \{6\}$.

$S = \{1,2,3,4,5,6\}$. Does A_1, A_2, A_3 Creates a partition of S.

1) $\forall i \; P_i \neq \phi$.

$P_1 \neq \phi \wedge P_2 \neq \phi \wedge P_3 \neq \phi$.

$\{1,2,3\} \neq \phi \wedge \{4,5\} \neq \phi \wedge \{6\} \neq \phi$.

$T \wedge T \wedge T = T$ ✓.

2) $\forall i,j \quad P_i \cap P_j = \phi \quad i \neq j$.

$P_1 \cap P_2 = \phi \wedge P_1 \cap P_3 = \phi \wedge P_2 \cap P_3 = \phi$.

$\{1,2,3\} \cap \{4,5\} = \phi \wedge \{1,2,3\} \cap \{6\} = \phi \wedge \{4,5\} \cap \{6\} = \phi$.

$T \wedge T \wedge T = T$ ✓.

3). $\bigcup_{i=1}^{\infty} P_i = S$.

$P_1 \cup P_2 \cup P_3 = S$.

$\{1,2,3\} \cup \{4,5\} \cup \{6\} = \{1,2,\cdots 6\}$.

$\{1,2,\cdots 6\} = \{1,2\cdots 6\}$.

T ✓.

Equivalence classes Creates a Partition.

ER $\rightarrow$ EC $\rightarrow$ Partition.

Partition in the Above Example.

$P = \{ \{1,2,3\}, \{4,5\}, \{6\} \}$.

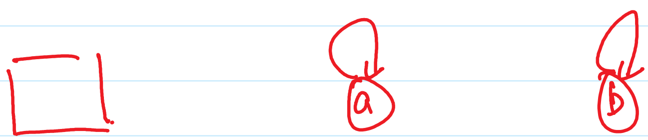
Let $[1] = \{1,2,3\}. = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3)$
$(3,1), (3,2), (3,3)\}$.

Let $[1] = \{1,2,3\}. = \{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)\}.$

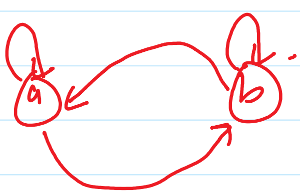
$[4] = \{4,5\}. = \{(4,4),(4,5),(5,4),(5,5)\}.$

$[6] = \{6\}. = \{(6,6)\}.$

$R = \{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3),(4,4),(4,5),(5,4),(5,5),(6,6)\}$



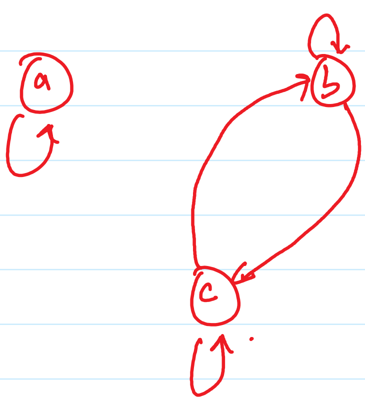
$P = \{\{a\},\{b\}\}.$



$P = \{\{a,b\}\}.$

$[a] = [b] = \{a,b\}.$

$R = \{(a,a),(a,b),(b,a),(b,b)\}.$



$[\ ].$ $\quad [0].$ $\quad [1]$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$ $\quad \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$$\begin{array}{c c} & \begin{array}{ccc} a & b & c \end{array} \\ \begin{array}{c} a \\ b \\ c \end{array} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \end{array}$$

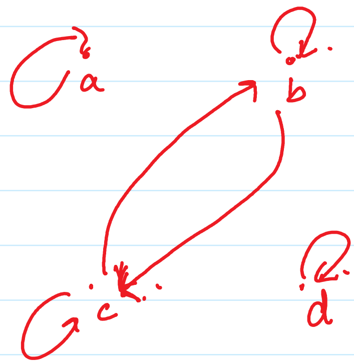
$[a] = \{a\}.$

$[b] = \{b,c\}.$

Quiz #3:-

20-OCT-2023.

a).



Find partition.

b). $R = \{(a,b) \mid a \equiv b \bmod m\}$. $m = 6$.

$[-3] = \{$? $\}$.