lec # 15 :-

# Representing Relations Using Graphs.

○ ⇉.

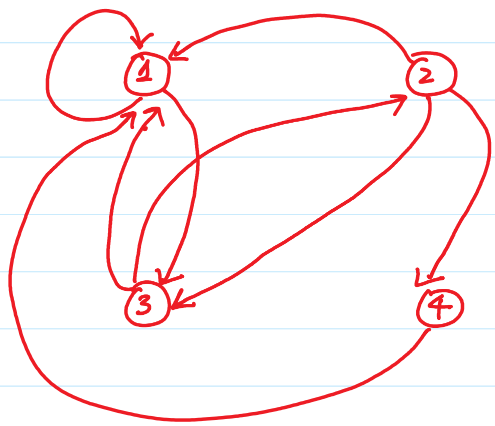
Set of Vertices — V — a

Set of Edges. — E — (a, b).

$R = E$ $\qquad$ $A = V$.

**Ex 8** :- 479

$R = \{(1,1), (1,3), (2,1), (2,3), (2,4), (3,1), (3,2), (4,1)\} = E$. $\qquad$ $A = \{1, 2, 3, 4\}$.



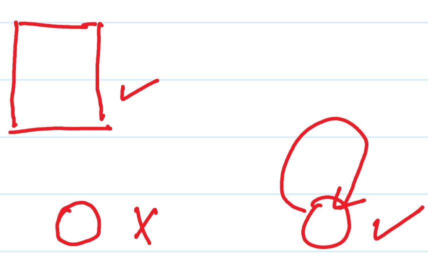

loop.

HW.

Construct the Corresponding Matrix.

Observation. Set Representation = Matrix Representation = Graph "

Reflexive :- $\forall a \in A$ $\quad (a, a) \in R$.



$A = \{\}$.

$A \times A = \{\}$.

$2^{|A| \times |A|} = 2^0 = 1$.

$A = \{ , \}$

$= \{a, b\}.$

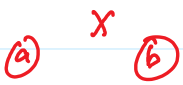
$A \times A = \{(a,a), (a,b), (b,a), (b,b)\}.$

$Pow(A \times A) = \{\phi, \{(a,a)\},$

$\_\ \_\ \_\ \_\_$

$\_\ \_.$

$\{(a,a), (a,b), (b,a), (b,b)\}\}$



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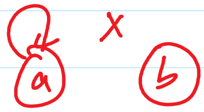

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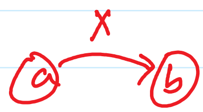

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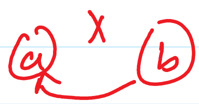

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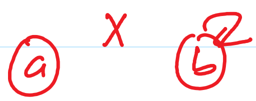

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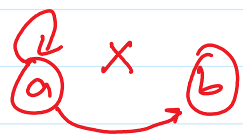

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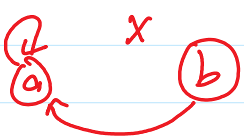

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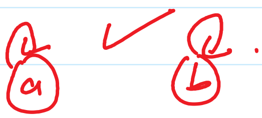

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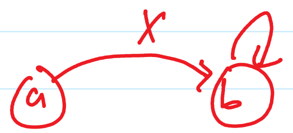

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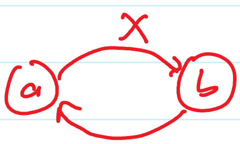

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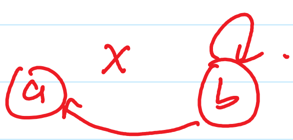

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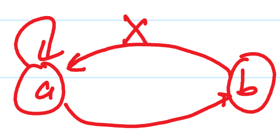

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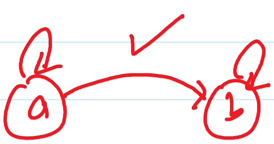

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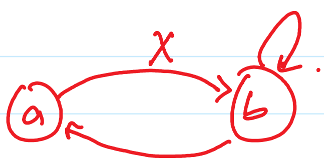

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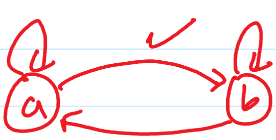
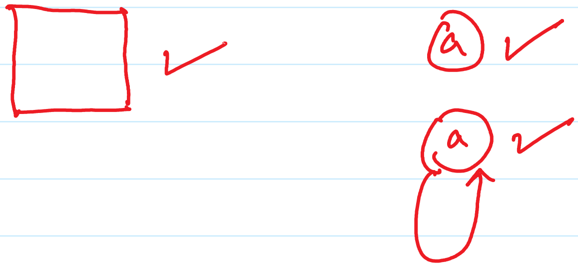
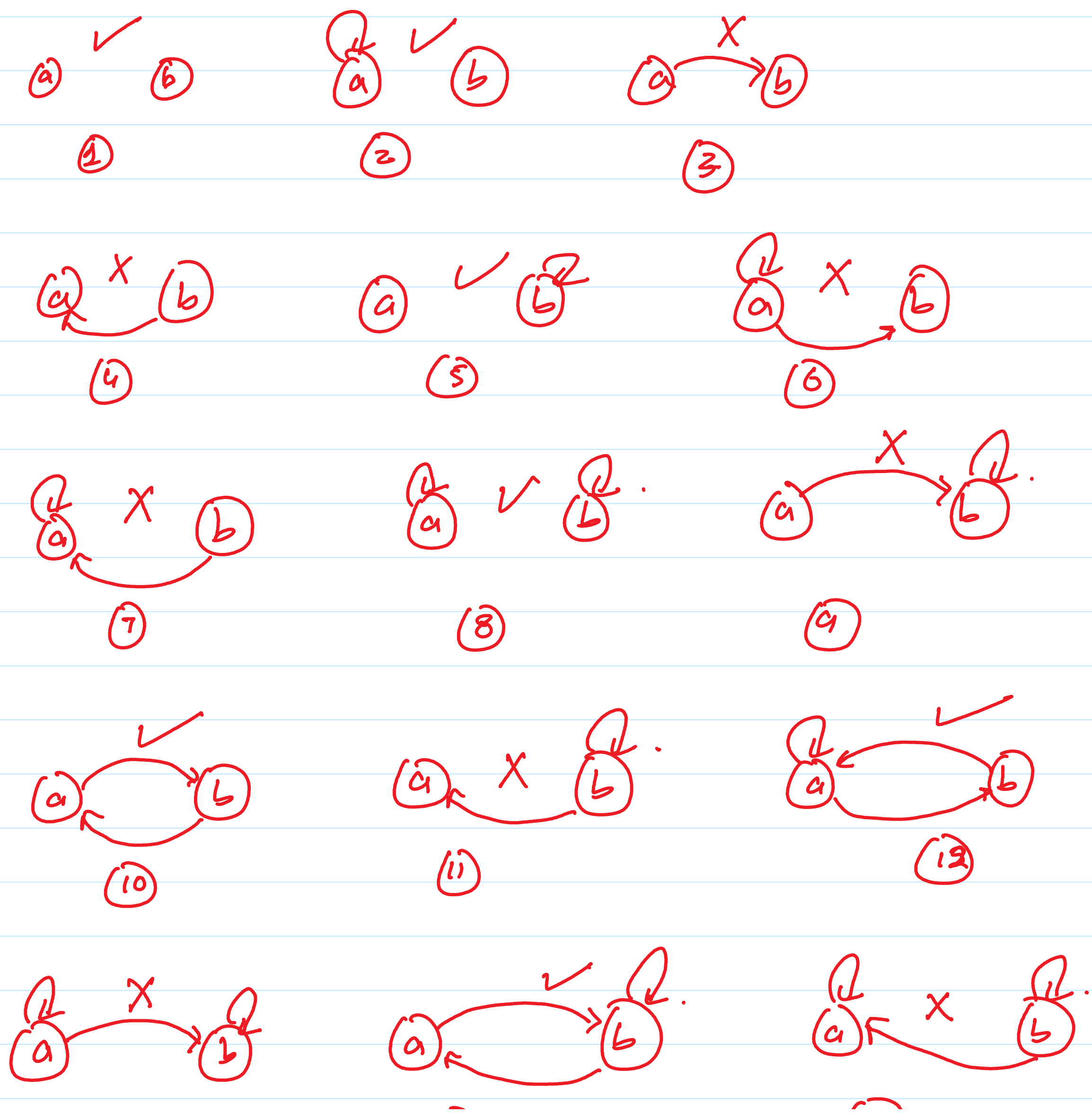


16

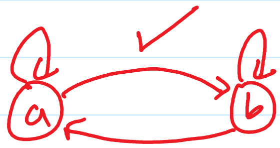
Symmetric

$\forall a,b \in A$ if $(a,b) \in R \rightarrow (b,a) \in R$.





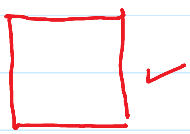
(13) (14) (15)

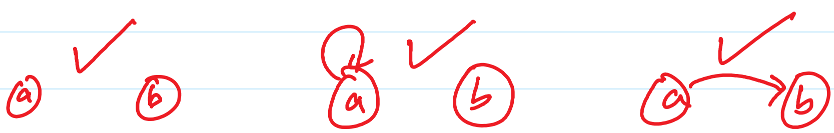


(16)

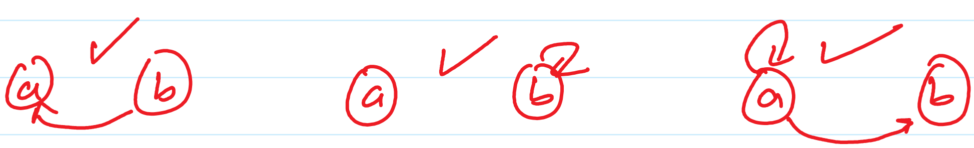
Anti Symmetric.

$\forall a,b \in A$ if $(a,b) \in R \wedge (b,a) \in R \rightarrow a = b$.

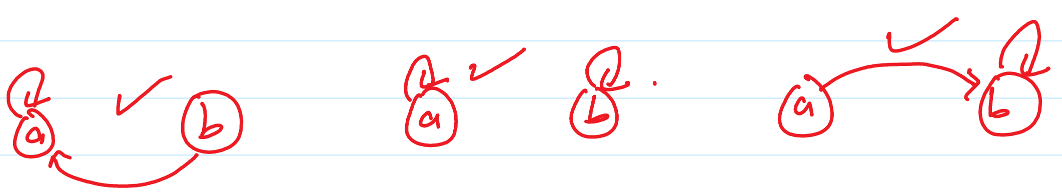




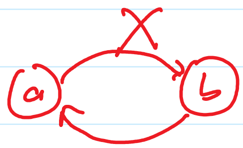
(1) (2) (3)



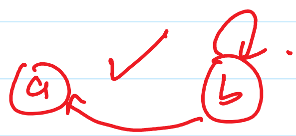
(4) (5) (6)



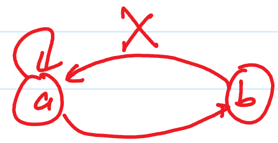
(7) (8) (9)



(10)



(11)



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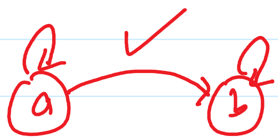

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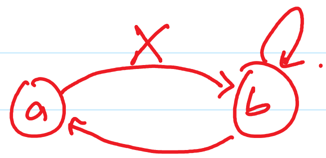
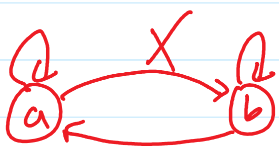


(14)

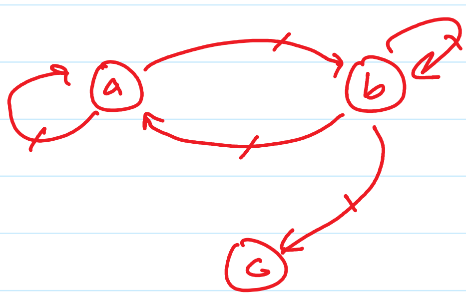
(15)



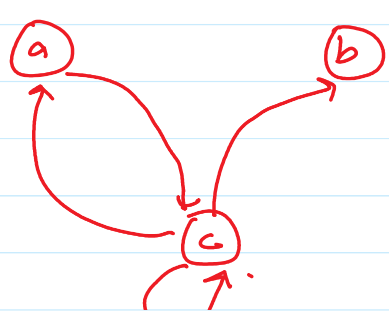
(16)

$R^{-1} = \{(b,a) \mid (a,b) \in R\}$.

$\bar{R} = \{(a,b) \mid (a,b) \notin R\}$.



$\bar{R}$.



Q9
581

Let $A = \{1, 2, 3, ---- 100\}$.

$R = \{(a,b) \mid a > b\}$.

How many Non-Zero entries in the matrix Representing R.

$$\begin{array}{c|c} & 1 \quad 2 \quad \cdots \quad 100 \\ \hline \begin{matrix}1\\2\\3\\ \vdots \\ 100\end{matrix} & \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & 0 \\ 1 & 1 & 0 \cdots & 0 \\ 1 & 1 & 1 \; 0 \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 1 & \cdots & \cdots & 0 \end{bmatrix} \end{array}$$

Total = $100 \times 100 = 10{,}000$.

$100 = 0's$

$10{,}000 - 100$

$= \frac{9{,}900}{2} = 4950.$

Total 0's = $4950 + 100 = 5{,}050$.

Total 1's = $10{,}000 - 5050 = 4950.$

$R = \{(a,b) \mid a + b = 1000\}$.

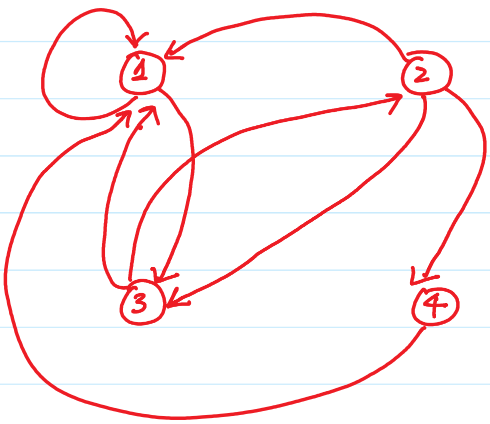
$R = \{(a,b) \mid a = b+1\}$.

$$\begin{array}{c|c} & 1 \quad 2 \quad \cdots \quad 100 \\ \hline \begin{matrix}1\\2\\3\\ \vdots \\ \vdots \\ 100\end{matrix} & \begin{bmatrix} 0 & 0 & \cdots & \cdots & 0 \\ 1 & 0 & \cdots & \cdots & 0 \\ 0 & 1 & 0 \cdots & \cdots & 0 \\ 0 & 0 & 1 \; 0 & \cdots & 0 \\ 0 & 0 & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix} \end{array}$$

99 1's.

R R?

$R$ $\qquad\qquad$ $R \circ R$.



Closure:- reflexive closure.

$R = \{(1,1), (1,2), (2,1), (3,2)\}$ $\qquad$ $A = \{1,2,3\}$.

$\Delta = \{(a,a) \mid a \in A\}$.
$= \{(1,1), (2,2), (3,3)\}$.

$R \cup \Delta = \{(1,1), (1,2), (2,1), (3,2)\} \cup \{(1,1), (2,2), (3,3)\}$.
$= \{(1,1), (1,2), (2,1), (3,2), (2,2), (3,3)\}$.

**Ex1** 483 $\qquad$ $R = \{(a,b) \mid a < b\}$. $\qquad$ $A = \mathbb{Z}$.

find reflexive closure of $R$.

$a = a$.

$\Delta = \{(a,a) \mid a \in \mathbb{Z}\}$.
$= \{(a,a) \mid a = a\}$.

$R \cup \Delta = \{(a,b) \mid a < b \text{ or. } a = b\}$.
$\{(a,b) \mid a \leq b\}$.

Symmetric $\qquad$ $R \cup R^{-1}$

Symmetric $R \cup R^{-1}$.

find. Symmetric Closure.

$R = \{(a,b) \mid a > b\}$.

$R^{-1} = \{(b,a) \mid (a,b) \in R\}$.

$= \{(b,a) \mid a > b\}$.

$\quad \downarrow \downarrow \quad \downarrow \downarrow$

$\quad a \; b \quad b \; a$.

$= \{(a,b) \mid b > a\}$. $\quad b > a = a < b$.

$= \{(a,b) \mid a < b\}$.

$R \cup R^{-1} = \{(a,b) \mid a > b \vee a < b\}$.

$R \cup R^{-1} = \{(a,b) \mid a \neq b\}$.