

Rules of Inference:-

$P \rightarrow Q$ If $\underbrace{\text{you have the current password}}_{P}$ $\underbrace{\text{Then you can log onto the network.}}_{Q}$

P you have the current password
therefore.

$\therefore Q$ you can log onto the network.

$P1$	$P \rightarrow Q$	} premises -	} Argument
$P2$	P		
C	$\therefore Q$		

General form of an argument. (Argument will be valid).
 $((P1 \wedge P2 \wedge P3 \wedge \dots \wedge PN) \rightarrow C)$ tautology.

$$(P1 \wedge P2) \rightarrow C$$

$$((P \rightarrow Q) \wedge P) \rightarrow Q$$

P	Q	$P \rightarrow Q$	$(P \rightarrow Q) \wedge P$	$((P \rightarrow Q) \wedge P) \rightarrow Q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Tautology.

Ex 2:-

$$\left[\text{If } \frac{1.41}{\sqrt{2}} > \frac{1.5}{2} \text{ then } (\sqrt{2})^2 > \left(\frac{3}{2}\right)^2 \right] \quad P \rightarrow Q$$

$$\frac{1.41}{\sqrt{2}} > \frac{1.5}{2} \quad \text{Then } (\sqrt{2})^2 > \left(\frac{3}{2}\right)^2$$

$$1.0 > 2.25 \quad \text{R}$$

2 P.

$$\overline{P60} \quad | \quad 0 \quad \overline{2} \quad | \quad \overline{(2)} \quad | \quad 2P.$$

$$\text{circled } \sqrt{2} > \frac{3}{2} \quad P$$

$$\therefore (\sqrt{2})^2 > \left(\frac{3}{2}\right)^2 \quad X.$$

$$P \rightarrow P \quad 2T$$

$$\text{circled } P$$

Rules of Inference.

1 $P \rightarrow q$ Modus Ponens $S- \frac{P}{\therefore P \vee q}$ Addition.

$$\frac{P}{\therefore q}$$

2 $P \rightarrow q$ Modus Tollens $6 \frac{P \wedge q}{\therefore P}$ Simplification.

$$\frac{\neg q}{\therefore \neg P}$$

3 $P \rightarrow q$ Hypothetical Syllogism $7- \frac{P}{\therefore P \wedge q}$ Conjunction.

$$\frac{q \rightarrow r}{\therefore P \rightarrow r}$$

4- $P \vee q$ Disjunctive Syllogism $8- \frac{P \vee q}{\therefore P \vee \neg q}$

$$\frac{\neg P}{\therefore q}$$

Ex 6 :-
 P62
 "It is not ^{P.}Sunny this afternoon and ^{q.}it is Colder than yesterday"
 "we will go swimming ^Sonly if ^{P.}it is Sunny"
 "If we do not go swimming then we will take a Canoe trip".
 S

(If) we do (not) go swimming (then) we will take a Canoe trip.
 $\neg S \rightarrow S$
 (If) we take a Canoe trip (then) we will be home by Sunset.
 $S \rightarrow T$

leads to Conclusion-

"we will be home by Sunset".
 T

P1	$\neg P \wedge Q$ ✓	2	$P \rightarrow Q$ Modus P forwards $\therefore Q$	5-	$\frac{P}{\therefore PVQ}$ Addition.
P2	$\neg Q \rightarrow P$ ✓				
P3	$\neg V \rightarrow S$ ✓	2	$P \rightarrow Q$ Modus $\neg Q$ Tollens $\therefore \neg P$	6	$\frac{P \wedge Q}{\therefore P}$ Simplification.
P4	$S \rightarrow T$ ✓				
<hr/>					
C	$\therefore T$				

P5: from P1 $\neg P$ by Simplification ✓	3	$P \rightarrow Q$ Hypothetical $Q \rightarrow S$ Syllogism. $\therefore P \rightarrow S$	7-	$\frac{P}{\therefore P \wedge Q}$ Conjunction.
P6: from P5, P2 $\neg S$ by MT ✓				
P7: from P6, P3 S by MP ✓	4-	$P \vee Q$ Disjunctive $\neg P$ Syllogism. $\therefore Q$	8-	$\frac{P \vee Q}{\therefore P \vee S}$
P8: from P7, P4 T by MP				

which is Conclusion.

Ex 7 :-
 P62

P1	$P \rightarrow Q$ ✓	2	$P \rightarrow Q$ Modus P forwards $\therefore Q$	5-	$\frac{P}{\therefore PVQ}$ Addition.
P2	$\neg P \rightarrow \neg V$ ✓				
P3	$\neg Q \rightarrow S$	2	$P \rightarrow Q$ Modus $\neg Q$ Tollens $\therefore \neg P$	6	$\frac{P \wedge Q}{\therefore P}$ Simplification.
C	$\therefore \neg Q \rightarrow S$				

or ~~from P.O. $\neg P \rightarrow S$ is not~~

~~P4~~ from ~~P2, P3~~ ~~$\neg P \rightarrow S$~~ by HS.

P4: $\neg q \rightarrow \neg p$ by CP. ✓

P5: from P4, P2 $\neg q \rightarrow r$ by HS.

P6: from P3, P5 $\neg q \rightarrow S$ by HS.

which is Conclusion.

$$\frac{\neg r}{\therefore \neg p} \text{ Tollens} \quad \therefore p$$

$$\begin{array}{ll} 3 & \frac{p \rightarrow q \quad q \rightarrow r}{\therefore p \rightarrow r} \text{ Hypothetical Syllogism.} \\ & \frac{p \quad p \wedge q}{\therefore p \wedge q} \text{ Conjunction.} \end{array}$$

$$\begin{array}{ll} 4. & \frac{p \vee q \quad \neg p}{\therefore q} \text{ Disjunctive Syllogism.} \\ & \frac{p \vee q \quad \neg q \vee r}{\therefore p \vee r} \end{array}$$

$$\begin{array}{ll} E \times 8 & \\ P1 & T \rightarrow MVE \quad \checkmark \\ P2 & S \rightarrow \neg E \quad \checkmark \\ P3 & \frac{TAS}{\therefore M.} \quad \checkmark \end{array}$$

$$\begin{array}{ll} 2 & \frac{p \rightarrow q \quad p}{\therefore q} \text{ Modus Ponens} \\ & \frac{p \quad p \vee q}{\therefore p \vee q} \text{ Addition.} \end{array}$$

$$\begin{array}{ll} 2 & \frac{p \rightarrow q \quad \neg q}{\therefore \neg p} \text{ Modus Tollens} \\ & \frac{p \wedge q \quad p}{\therefore p} \text{ Simplification.} \end{array}$$

P4 from T by Simplification ✓

P5 from S " " ✓

P6 from P4 & P2 MVE by MP.

P7 from P4, P5 $\neg E$ " " " ✓

P8 from P6, P7 M by Resolution

which is Conclusion.

$$\begin{array}{ll} 3 & \frac{p \rightarrow q \quad q \rightarrow r}{\therefore p \rightarrow r} \text{ Hypothetical Syllogism.} \\ & \frac{p \quad p \wedge q}{\therefore p \wedge q} \text{ Conjunction.} \end{array}$$

$$\begin{array}{ll} 8. & \frac{p \vee q \quad \neg q \vee r}{\therefore p \vee r} \text{ Disjunctive Syllogism.} \end{array}$$

11