Lec # 7

Nested Quantifiers.

$x \in \{1, 2, 3, \cdots N\}$.

$\forall x P(x) = P(1) \wedge P(2) \wedge \cdots \wedge P(N)$

$\exists x P(x) = P(1) \vee P(2) \vee \cdots \vee P(N)$.

$\forall x \forall y \, P(x,y)$ $\qquad x, y \in \{1, 2, 3, \cdots N\}$.

$= \forall x \left(P(x,1) \wedge P(x,2) \wedge P(x,3) \wedge \cdots \wedge P(x,N) \right)$.

$= \underline{\forall x P(x,1)} \wedge \underline{\forall x P(x,2)} \wedge \forall x P(x,3) \wedge \cdots \wedge \underline{\forall x P(x,N)}$.

$\Downarrow$

$= \big(P(1,1) \wedge P(2,1) \wedge P(3,1) \wedge \cdots \wedge P(N,1) \big) \wedge$

$\big(P(1,2) \wedge P(2,2) \wedge P(3,2) \wedge \cdots \wedge P(N,2) \big) \wedge$

$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$

$\big(P(1,N) \wedge P(2,N) \wedge P(3,N) \wedge \cdots \wedge P(N,N) \big)$.

$\forall x \exists y \, P(x,y) = \forall x \left(P(x,1) \vee P(x,2) \vee P(x,3) \vee \cdots \vee P(x,N) \right)$.

$= \underline{\forall x P(x,1)} \vee \forall x P(x,2) \vee \forall x P(x,3) \vee \cdots \vee \forall x P(x,N)$.

$\Downarrow$

$= \big(P(1,1) \wedge P(2,1) \wedge P(3,1) \wedge \cdots \wedge P(N,1) \big) \vee$

$\big(P(1,2) \wedge P(2,2) \wedge P(3,2) \wedge \cdots \wedge P(N,2) \big) \vee$

$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$

$\big(P(1,N) \wedge P(2,N) \wedge P(3,N) \wedge \cdots \wedge P(N,N) \big)$.

Homework.

$\exists x \forall y \, P(x,y) = ?$ $\qquad \exists x \exists y \, P(x,y) = ?$

$\forall y \exists x\ P(y,x)$? $\quad\quad$ $\forall y \exists x \neg P(x,y)$. ?

Ex 4 p48 let $Q(x,y)$: $x + y \geq 0$. $\quad x, y \in \mathbb{R}$.

find $\exists y \forall x\ Q(x,y)$? F.

$\forall x \exists y\ Q(x,y)$? T

Ex 5 p49 $Q(x,y,z)$: $x + y = z$ $\quad x, y, z \in \mathbb{R}$.

$\forall x \forall y \exists z\ Q(x,y,z)$? T

$\exists z \forall x \forall y\ Q(x,y,z)$? F

Ex 15 (c) p 56 — Every one | in this class | has taken (predicate) | at least one | CS Course

for all x, x is student in this class, there exist y, y is a CS Course, x has taken y.

let $P(x,y)$: x has taken y. $\quad x \in$ Set of Students in this class. $\quad y \in$ Set of CS Courses.

$\forall x \exists y\ P(x,y)$.

Q 15 (f) p 56. there is | a student in this class | who has been to every | room | in every | building of this Campus |

there exist x, x is a student in this class, for all y,
y is a room in a building on Campus, for all z,
z is a building on Campus, x has been to y in z.

Let $P(x,y,z)$ = x has been to y in z.

$x \in$?
$y \in$?
$z \in$?

$\exists x \forall y \forall z \; P(x,y,z)$.

Q29 (c) P57 $\exists x \forall y \; P(x,y)$ $\quad x,y \in \{1,2\}$.

$\wedge, \vee, \neg$.

$= \exists x (P(x,1) \wedge P(x,2))$.

$= \exists x \, P(x,1) \wedge \exists x \, P(x,2)$.

$= (P(1,1) \vee P(2,1)) \wedge (P(1,2) \vee P(2,2))$.

HW. $\forall x \exists y \forall z \; P(x,y,z)$ $\quad x,y,z \in \{1,2\}$.

$\wedge, \vee, \neg$.

Everyone (loves) Someone.

for all x, x is a person, there exist y, y is a person.
x loves y.

$P(x,y) = ?$

$x \in$?
$y \in$?

$\forall x \exists y \, P(x,y)$.

Everyone has exactly one best friend.

For all x, x is a person, there exist y, y is a person,
x & y are two different persons, if x is a bestfriend of y.
Then, there should not exist z, such, x is a bestfriend of z.
z is a person.

x, y, z ∈ Persons.

$\forall x \exists y \exists z ((P(x,y) \wedge (x \neq y)) \rightarrow R(x,y))$.