Lec # 18:-

PARTIAL ORDER.

→ Reflexive
→ Anti Symmetric
→ Transitive.

✓

Ex 1 P 504

$R = \{(a,b) \mid a \geq b\}$. $A = \mathbb{Z}$.

Reflexive:- $\forall a \in A \quad (a,a) \in R$.

$\forall a \in \mathbb{Z} \quad a \geq a$

AntiSymmetric $\forall a,b \in A$ if $(a,b) \in R \wedge (b,a) \in R \rightarrow a = b$.
$\forall a,b \in \mathbb{Z}$ if $a \geq b \wedge b \geq a \rightarrow a = b$.

Transitive $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$.
$\forall a,b,c \in \mathbb{Z}$ if $a \geq b \wedge b \geq c \rightarrow a \geq c$.

Generalized Notation for Partial Order.

$(S, \preceq)$ POSET.

$(\mathbb{Z}, \geq)$ $\{R = \{(a,b) \mid a \geq b\} \quad A = \mathbb{Z}$.
$(\mathbb{Z}, \leq)$
$(\mathbb{Z}, \div)$.

$(a,b) \in R$.
$a \preceq_R b$.

Comparable:- Two elements $a,b \in S$ are Comparable in $(S, \preceq)$. when $a \preceq b$ or $b \preceq a$.

Ex:- $(\mathbb{Z}^+, \mid)$. 5 and 7 are Comparable. X.
Sol $5 \mid 7$ or $7 \mid 5$

Ex:- 504 $(\mathbb{Z}^+, |)$.

5 and 7 are Comparable. X.

$5 \preccurlyeq 7$ or $7 \preccurlyeq 5$.

$5 | 7$ or $7 | 5$

3 & 9. are Comparable ? ✓.

$3 \preccurlyeq 9$ or $9 \preccurlyeq 3$.

$3 | 9$ or $9 | 3$.

Total Order:- If all elements are Comparable in a poset.

Ex6 :- 505 $(\mathbb{Z}, \leq)$.

$5 \preccurlyeq 7$ or $7 \preccurlyeq 5$.

$\underset{T}{5 \leq 7}$ or $7 \leq 5$. ✓

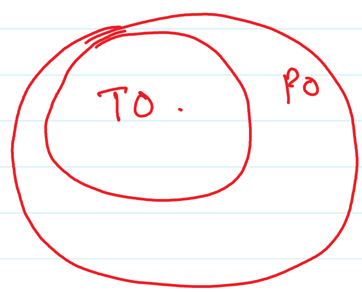
$$-\infty \leq -\infty+1 \leq \cdots -1 \leq 0 \leq 1 \leq 2 \leq \cdots \leq +\infty.$$

Ex7 $(\mathbb{Z}, |)$.

$7 \preccurlyeq 3$ or $3 \preccurlyeq 7$.

$7 | 3$ or $3 | 7$. X.

$\preccurlyeq$ $\prec$



Lexicographic Order.

$(A_1, \preccurlyeq_1)$ $(A_2, \preccurlyeq_2)$

$(A_1 \times A_2, \preccurlyeq)$.

$A_1 = \{a_1, a_2, \cdots a_n\}$.

$A_2 = \{b_1, b_2, \cdots b_n\}$.

$(a_1, a_2) \prec (b_1, b_2)$.

if $(a_1 \prec b_1)$ or $(a_1 = b_1 \wedge a_2 \prec b_2)$

Ex9 :- 506 $(\underset{\uparrow\, a_1}{3}, \underset{\uparrow\, a_2}{5}) \prec (\underset{\uparrow\, b_1}{4}, \underset{\uparrow\, b_2}{8})$ $(\mathbb{Z} \times \mathbb{Z}, \leq)$.

$(3 < 4) \vee (3 = 4 \wedge 5 < 8)$

$(3, 8) \prec (4, 5)$. ✓

$(3,8) \prec (4,5)$. ✓

$(A_1, \preceq_1) \quad (A_2, \preceq_2) \quad --- \quad (A_n, \preceq_n)$.

$(A_1 \times A_2 \times --- \times A_n, \prec)$.

$(a_1, a_2, a_3 ---, a_n) \prec (b_1, b_2, --- b_n)$.

if $a_1 \prec_1 b_1$ or $\exists_{i>0} \quad a_1 = b_1 \wedge a_2 = b_2 \wedge --- \wedge a_i = b_i \wedge a_{i+1} \prec_{i+1} b_{i+1}$

Ex16
506

$$(1, 2, 3, 5) \prec (1, 2, 4, 3). \qquad (\mathbb{Z} \times \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}, \leq).$$

$a_1 \; a_2 \; a_3 \; a_4 \qquad b_1 \; b_2 \; b_3 \; b_4$.

$i=2$

$(a_1 < b_1) \vee (a_1 = b_1 \wedge a_2 = b_2 \wedge a_3 < b_3)$.

$(1 < 1) \vee (1 = 1 \wedge 2 = 2 \wedge 3 < 4)$.

$F \vee (T \wedge T \wedge T)$ ✓

$F \vee T = T$

$(\underline{180}, \underline{16}0, 20, 30) \prec (\underline{180}, 15, 300, 10000)$.

Ex11
507

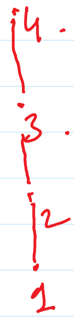
discreet $\prec$ discrete. ✓

Hasse Diagram. $(\{1,2,3,4\}, \leq)$.

$R = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4), (4,4)\}$.

4 .

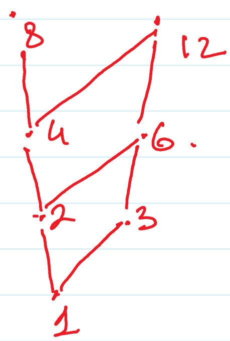
3 .



$(3,3), (3,4),$
$(4,4)\}.$

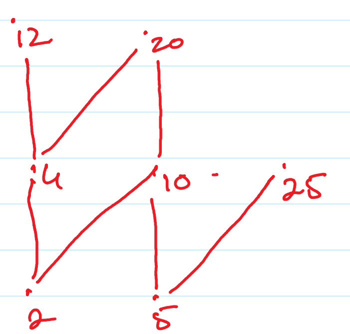
Ex12
P508

$(\{\cancel{1}, 2, 3, 4, 6, \cancel{8}, \cancel{12}\}, |).$



Ex14

$(\{\cancel{2}, 4, 5, \cancel{10}, \cancel{12}, \overset{20}{\cancel{25}}\}, |).$



$S \times S$ X.

Ex13
509

$(P(S), \subseteq).$

$S = \{a, b, c\}$ $\quad 2^{|S|} = 2^3 = 8.$

$P(S) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}$
$\{a,b,c\}\}$

$P(S) = \{ \phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\},$
$\{a,b,c\} \}.$

$$P(S) \times P(S) = |P(S)| \times |P(S)|$$
$$= 8 \times 8 = 64.$$

$R = \{ (\phi,\phi), (\phi,\{a\}), (\phi,\{b\})$ - - - - - ,
$(\{a\},\{a\}), (\{a\},\{a,b\}), (\{a\},\{a,c\}), (\{a\},\{a,b,c\}),$
$(\{b\},\{b\}), (\{b\},\{a,b\}), (\{b\},\{b,c\}), (\{b\},\{a,b,c\}),$
$(\{c\},\{c\}), (\{c\},\{a,c\}), (\{c\},\{a,c\}), (\{c\},\{a,b,c\}),$

$(\{a,b\},\{a,b\}), (\{a,b\},\{a,b,c\}),$
$(\{b,c\},\{b,c\}), (\{b,c\},\{a,b,c\}),$
$(\{a,c\},\{a,c\}), (\{a,c\},\{a,b,c\}),$
$(\{a,b,c\},\{a,b,c\}) \}.$