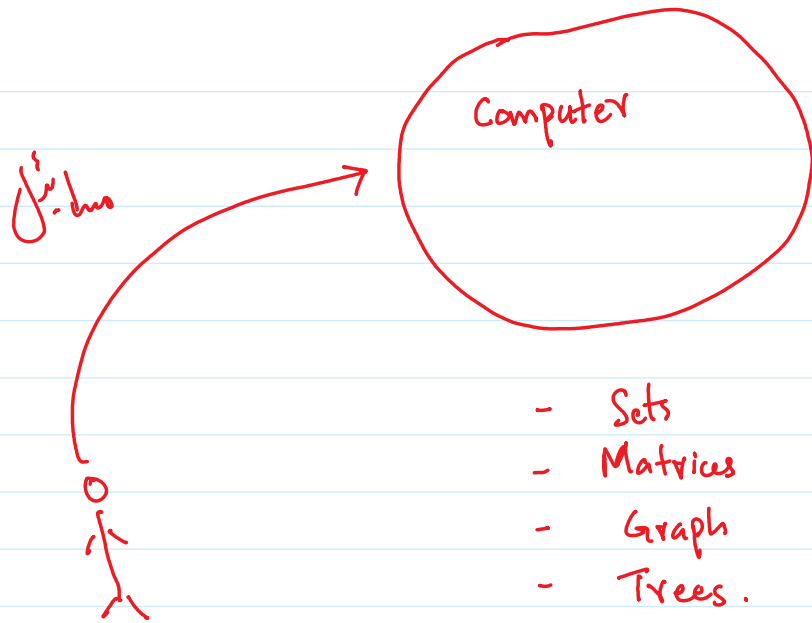
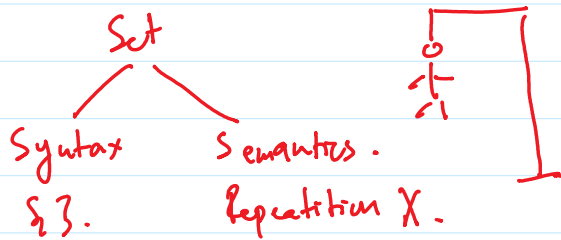


lec # 10:-



Relation:- A binary relation on Set A & B
تعلق is a subset of $A \times B$.

Background Knowledge:-



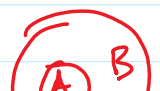
$A = \{a, b\}$
 $B = \{1, 2\}$

$A \times B = \{ \underline{(a, 1)}, (a, 2), (b, 1), (b, 2) \}$
↓
tuple $(1, a) \neq (a, 1)$.

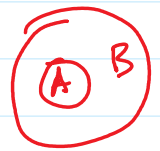
Cardinality $|A| = 2$
 $|B| = 2$.

$|A \times B| = |A| \times |B| = 2 \times 2 = 4$.

Subset $A \subseteq B$ if $\forall (a \in A \rightarrow a \in B)$.



Subset $A \subseteq B$ if $\forall (a \in A \rightarrow a \in B)$.



All Subsets of a Set.

Power Set = All Subsets of a Set.

$$A = \{a, b\}.$$

$$\text{pow}(A) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}.$$

Order does not matter in Sets.

$$|\text{pow}(A)| = 2^{|A|} = 2^2.$$

$$A \times B = \{(a, 1), (a, 2), (b, 1), (b, 2)\}.$$

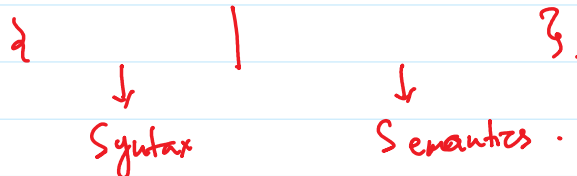
$$\begin{aligned} \text{pow}(A \times B) = & \{ \emptyset, \{(a, 1)\}, \{(a, 2)\}, \{(b, 1)\}, \{(b, 2)\}, \\ & \{(a, 1), (a, 2)\}, \{(a, 1), (b, 1)\}, \{(a, 1), (b, 2)\}, \\ & \text{Remaining 8 at home} \}. \end{aligned}$$

$$|\text{pow}(A \times B)| = 2^{|A \times B|} = 2^{|A| \times |B|} = 2^{2 \times 2} = 2^4 = 16.$$

How many Relations on $A \times A$ if $|A| = 5$.

$$|\text{pow}(A \times A)| = 2^{|A \times A|} = 2^{|A| \times |A|} = 2^{5 \times 5} = 2^{25}$$

Set builder Notation.



$$A \times A =$$

Ex 5 $R = \{(a, b) \in A \times A \mid a \leq b\}$ $A = \{1, 2, 3, 4\}$.

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$$A \times A = \{(1, 1), (1, 2), (1, 3), (1, 4),$$

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$R_2 = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4)\}$.

$A \times A = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4)\}$.

$R_2 = \{(a,b) \mid a > b\}$.

$R_3 = \{(a,b) \mid a \geq b\}$.

$R_4 = \{(a,b) \mid a \geq b+1\}$.

$R_5 = \{(a,b) \mid a = b+2\}$.

$R_6 = \{(a,b) \mid a+b \leq 3\}$.

H.W.

Properties of Relations.

i) Reflexive: $\forall a \in A \quad (a,a) \in R$.

$\rightarrow (1,1) \in R \checkmark \quad (2,2) \in R \checkmark \quad (3,3) \in R \checkmark \quad (4,4) \in R \checkmark \quad A = \{1,2,3,4\}$.

$R = \{(1,1)\}$

$R = \{(1,1)\} \times X$

$R = \{(1,1), (2,2), (3,3), (4,4), (1,2)\} \checkmark$.

Ex:

$A = \{a,b\}$.

H.W.

All possible Reflexive.

Step 1: $P(A \times A) = 2^{16}$ elements.

Symmetric: $\forall a,b \in A \quad \text{if } (a,b) \in R \rightarrow (b,a) \in R$.

symmetric. $\forall a, b \in A$ $(a, b) \in R \Rightarrow (b, a) \in R$.

$A = \{1, 2, 3, 4\}$.

$$R = \emptyset \quad \checkmark$$

$$R = \{(1, 2)\} \quad \text{X}$$

$$R = \{(1, 1), (1, 2)\} \quad \text{X}$$

$$R = \{(1, 1), (2, 2), (3, 3), (4, 4)\} \quad \checkmark$$

$$R = \{(1, 2), (2, 1), (3, 2), (2, 3)\} \quad \text{X}$$

$$A = \emptyset$$

$$\text{pow}(A \times A) = \{\emptyset\}$$

$$R = \emptyset \quad \checkmark \quad \text{reflexive. symmetric.}$$

$$1) \text{ Reflexive: } \forall a \in A \quad (a, a) \in R.$$

$$2) \text{ Symmetric:}$$