Friday, 8 September 2023 8:03 am

lec #6.

Negating Quantifiers.

$\neg\left(\forall x\, P(x)\right) = \neg\left(P(1) \wedge P(2) \wedge \cdots \wedge P(N)\right)$ $x \in \{1, 2, 3, \ldots N\}$.

$= \neg\forall x P(x) = \neg P(1) \vee \neg P(2) \vee \neg P(3) \vee \cdots \vee \neg P(N)$.

$= \exists x \neg P(x)$.

For all $\forall x P(x) = P(1) \wedge P(2) \wedge \cdots \wedge P(N)$ $x \in \{1, 2, \ldots N\}$

$\exists x P(x) = P(1) \vee P(2) \vee \cdots \vee P(N)$.

$\neg\forall x P(x) = \exists x \neg P(x)$. — (1)

$\neg(P \vee Q) = \neg P \wedge \neg Q$

$\neg(P_1 \vee P_2 \vee P_3 \cdots \vee P_N) = \neg P_1 \wedge \neg P_2 \wedge \neg P_3 \cdots \wedge \neg P_N$

$\neg(P \wedge Q) = \neg P \vee \neg Q$

$\neg(P_1 \wedge P_2 \wedge P_3 \wedge \cdots \wedge P_N) = \neg P_1 \vee \neg P_2 \vee \cdots \vee \neg P_N$

$\neg\exists x P(x) = \neg\left(P(1) \vee P(2) \vee \cdots \vee P(N)\right)$

$= \neg P(1) \wedge \neg P(2) \wedge \neg P(3) \wedge \cdots \wedge \neg P(N)$.

$= \forall x \neg P(x)$.

$\neg\exists x P(x) = \forall x \neg P(x)$. — (2)

$\neg\forall x P(x) = \exists x \neg P(x)$. — (1)

$\neg\exists x P(x) = \forall x \neg P(x)$. — (2)

$\forall x \exists y \neg \forall z \neg P(x, y, z)$. find Negation.

$= \neg \{\forall x \exists y \neg \forall z \neg P(x,y,z)\} \rightarrow P(x).$

$= \exists x \neg \exists y \{\neg \forall z \neg P(x,y,z)\} \rightarrow P(x).$

$= \exists x \forall y \neg \neg \forall z \neg P(x,y,z).$

$= \exists x \forall y \forall z \neg P(x,y,z).$ → Simplified.

Ex 9 :- P34

$P(x) = x < 2$

$\forall x P(x) = \forall x (x < 2).$

$\neg(<) = \geq.$

$\neg \forall x P(x) = \exists x \neg P(x).$

$= \exists x \neg (x < 2).$

$= \exists x (x \geq 2).$

Ex 20 :- P39

"there is an honest politician". find Negation.

"there exist x, x is a politician, x is honest".

let $P(x)$ = x is honest. $x \in$ Set of politician.

$\exists x P(x).$ Now find Negation.

$\neg \exists x P(x) = \forall x \neg P(x).$

for all x, x is a politician, x is not honest.

Ex 21) P39

$\forall x (x^2 > x).$

let $P(x) = x^2 > x.$

$\forall x P(x).$

$\neg \forall x P(x)$ Now take Negation.

$\neg(>) = <$

$\neg \forall x P(x)$ Now take Negation.

$\neg (?) \geq \leq$.

$= \exists x \neg P(x)$. $= \exists x (x^2 \leq x)$.

Ex 23 p40 Every Student in this class has studied Calculus.

For all x, x is a Student in this class, x has studied Calculus.

Let $P(x)$: x has studied Calculus. $x \in$ Set of Students in the class.

$\forall x P(x)$

$\neg \forall x P(x) \equiv \exists x \neg P(x)$. Now take Negation.

There exist x, x is a student in this class, x has not studied calculus.

HW p 43-46

Ex 1-40.

Q17 (c) $\exists x \neg P(x)$ $x \in \{0, 1, 2, 3, 4\}$.

$\wedge, \vee, \neg$.

$\neg P(0) \vee \neg P(1) \vee \neg P(2) \vee \neg P(3) \vee \neg P(4)$.

(e) $\neg \exists x P(x)$.

$= \forall x \neg P(x)$.

$= \neg P(0) \wedge \neg P(1) \wedge \neg P(2) \wedge \neg P(3) \wedge \neg P(4)$.

Q30 (b) $\forall y P(1, y)$ $x, y \in \{0, 1, 2\}$.

$= P(1,0) \wedge P(1,1) \wedge P(1,2)$.

Q32 (a) All dogs have fleas.

For all x, x is a dog, x has fleas. $x \in$ Set of dogs

Let $P(x)$: x has fleas.

$\forall x\, P(x)$

$\neg \forall x P(x) \equiv \exists x \neg P(x)$. Take Negation.

There exist x, x is a dog, x does not have fleas.

Q32 (b). No Rabbit learns Calculus. find Negation.

It is not the case. For all x, x is a rabbit, x learns Calculus.

Let $P(x)$: x learns Calculus. $x \in$ Set of Rabbits.

$\neg \forall x P(x)$

$\neg\neg \forall x P(x)$ Now take Negation.

$\forall x P(x)$: For all x, x is a rabbit, x learns Calculus

$\forall x\, P(x,y)$ $\quad x, y \in \{1,2,3\}$.

$\wedge, \vee,$

$= P(1,y) \wedge P(2,y) \wedge P(3,y)$.

Quiz 2 08- Sep- 2023.

Two types 1 0

(if 2+2=5 then 2+2=7) = True.

1 = They Speak truth & 0 = Speak truth

otherwise 1 = They Speak lies & 0 = " lies.

A person visited the place where there are only 1 & 0. He meet two people. A & B.

A says "I am 1". B Say "I am 0".

Determine A & B.

		p =	¬p =
		q =	¬q =
CASE 1: — , — .			
?	= ?	p = ?	¬p = ?
?	= ?	q = ?	¬q = ?