



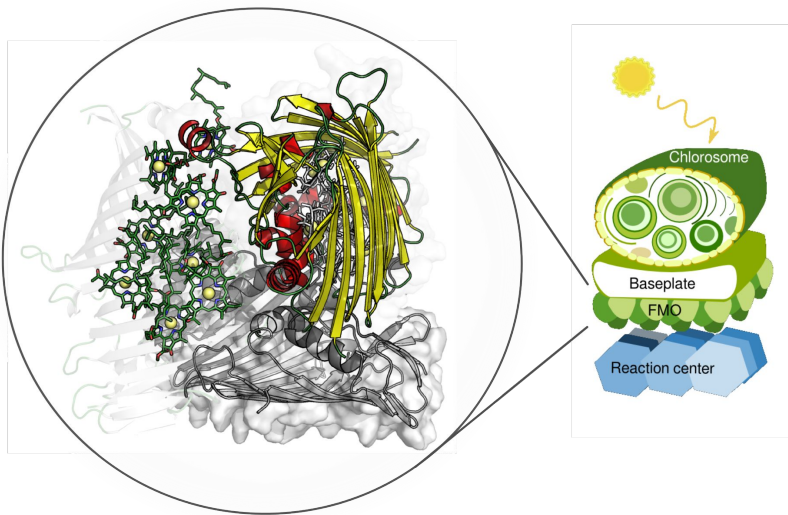
# Implementation and Parallelization of Redfield Equations

CS 205 Final Project  
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# Background: Exciton Transfer

- Important for improving efficiency of solar cells
- Current methods limited by:
  - small number of excitonic sites ( $<30$ )  $\rightarrow$  Scaling:  $O(N^6)$ ,  $N$  = number of sites
  - short time scales
- Redfield Equation

David Glowacki



# Redfield Equation

*Bottleneck:* matrix-matrix multiplications

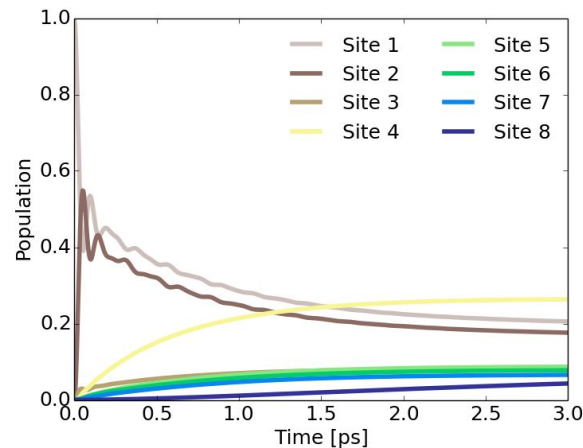
$$\frac{d\rho(t)}{dt} = \underbrace{-\frac{i}{\hbar}[H, \rho(t)]}_{\text{System contribution}} + \underbrace{\sum_{k,m,n} \gamma(\omega_{mn}) \left( V_k(\omega_{mn}) \rho(t) V_k^\dagger(\omega_{mn}) - \frac{1}{2} V_k^\dagger(\omega_{mn}) V_k(\omega_{mn}) \rho(t) - \frac{1}{2} \rho(t) V_k^\dagger(\omega_{mn}) V_k(\omega_{mn}) \right)}_{\text{Contribution from coupling between system and environment}},$$

System contribution

Contribution from coupling between system and environment

Dimension: N sites + loss state + target state = N+2 total states

- Dynamics of excited states described by time evolution of density matrix
- Current methods do not scale well beyond systems with > 30 excitonic states.
- Can we do better using different parallelization models? (Long term goal: run a full-scale simulation of large systems)

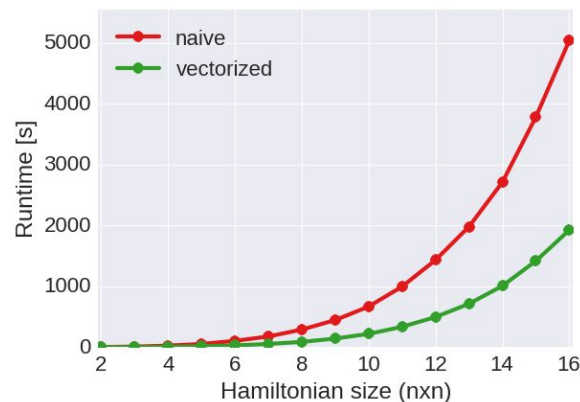
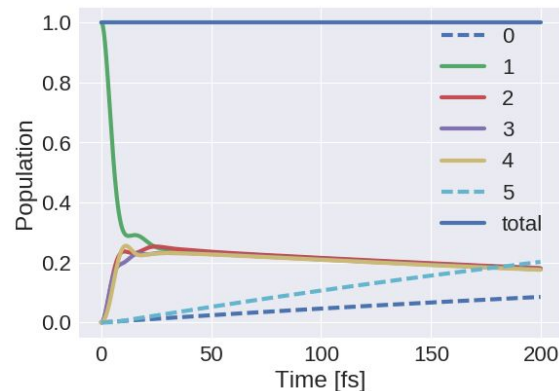


# Approaching the problem: Python implementation

1. Identified bottleneck (Lindblad term)

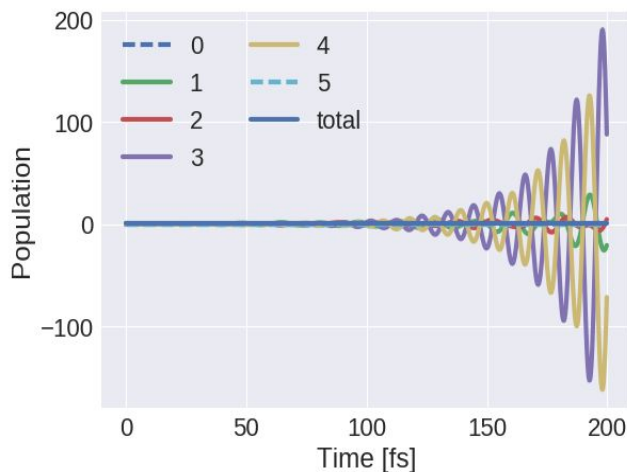
2. Preprocessing the problem ...

- Naive:
  - $H$ ,  $V$ ,  $\rho$  complex-valued
- Vectorized:
  - $H$ ,  $V$  real-valued (can ignore complex phase)
  - $H$  diagonalized to further reduce computational cost
  - $V$  and  $\chi$  independent of current time step so precomputed and stored in memory for later usage

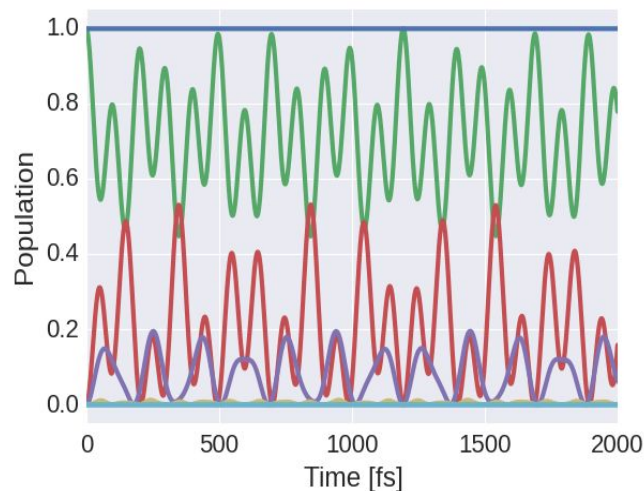


# Increasing Efficiency and Accuracy

- Efficiency: translate code to C to further improve runtimes
- Accuracy: Runge-Kutta 4 (vs. Euler)



Euler integration (Python implementation)



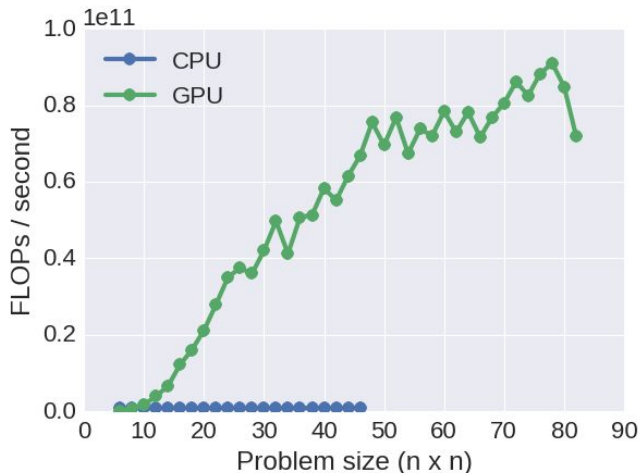
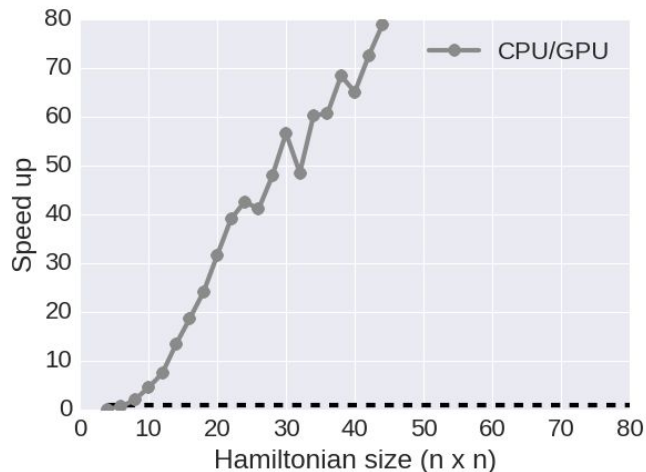
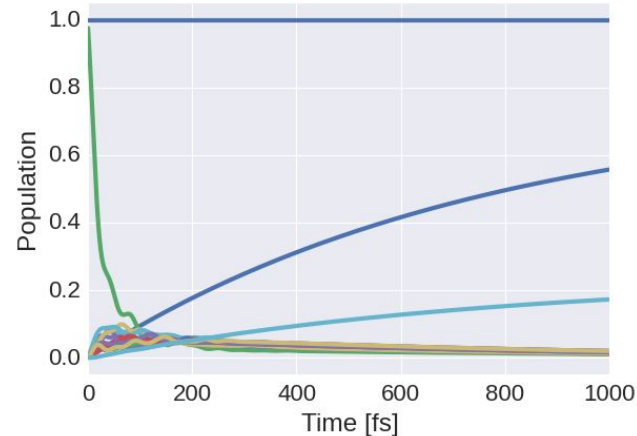
4th order Runge Kutta integration (C implementation)

# Computing Redfield in SIMT Model: OpenACC on Tesla K80

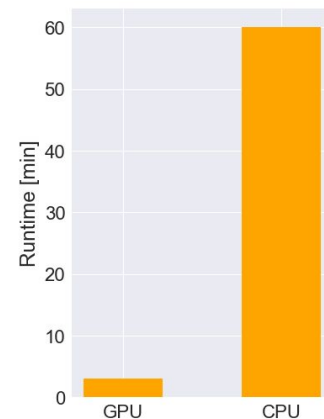
## Features:

- Blocked matrix matrix multiplications
- Reducing the algorithm to the bare essentials

$$\frac{d\rho(t)}{dt} = -\frac{i}{\hbar}[H, \rho(t)] + \sum_{k,m,n} \gamma(\omega_{mn}) \left( V_k(\omega_{mn}) \rho(t) V_k^\dagger(\omega_{mn}) - \frac{1}{2} V_k^\dagger(\omega_{mn}) V_k(\omega_{mn}) \rho(t) - \frac{1}{2} \rho(t) V_k^\dagger(\omega_{mn}) V_k(\omega_{mn}) \right)$$



## 16 site population dynamics:

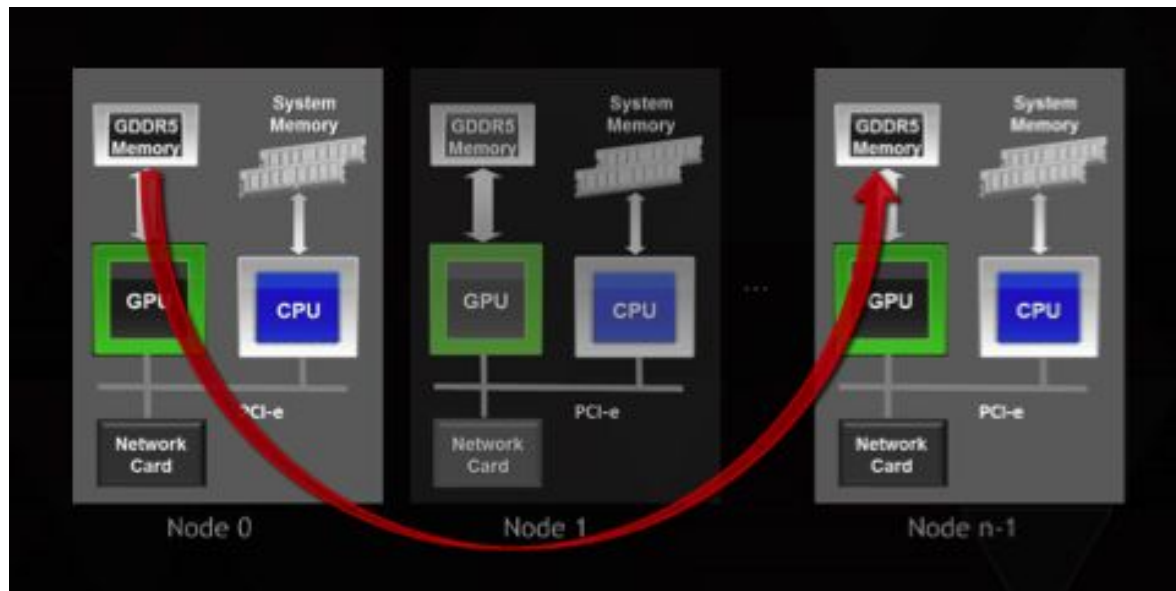


# Parallel Implementation: MPI + OpenAcc

$$\frac{d\rho(t)}{dt} = -\frac{i}{\hbar}[H, \rho(t)] + \sum_{k,m,n} \gamma(\omega_{mn}) \left( V_k(\omega_{mn}) \rho(t) V_k^\dagger(\omega_{mn}) - \frac{1}{2} V_k^\dagger(\omega_{mn}) V_k(\omega_{mn}) \rho(t) - \frac{1}{2} \rho(t) V_k^\dagger(\omega_{mn}) V_k(\omega_{mn}) \right),$$

Contribution from coupling between system and environment

- Current status:
  - Memory Overheads
  - Corruptions in memory



# Advanced Feature: Stronger Scaling via OpenMP

$$\frac{d\rho(t)}{dt} = -\frac{i}{\hbar}[H, \rho(t)] + \sum_{k,m,n} \gamma(\omega_{mn}) \left( V_k(\omega_{mn}) \rho(t) V_k^\dagger(\omega_{mn}) - \frac{1}{2} V_k^\dagger(\omega_{mn}) V_k(\omega_{mn}) \rho(t) - \frac{1}{2} \rho(t) V_k^\dagger(\omega_{mn}) V_k(\omega_{mn}) \right)$$

- Bottleneck: matrix-matrix multiplication operations in Lindblad term
  - $15 \cdot N^3$  operations but Runge-Kutta implementation so 4 of these sets per time step
- Approaches: blocking + multithreading
- Better speedups with more threads
- Better scaling as system size increases

