

## Read Me

October 12, 2019

This Geometric Algebra (GA) package was initially developed in January, 2017 using Mathematica 10 and has been extended using version 11. It is free to use, modify, and share but may not be used for commercial purposes. It performs GA operations in  $n$ -dimensions for any  $n$ . It was developed to investigate equations of GA objects. To this end, it is not a symbolic representation of GA but is rather an implementation tied to a specific orthonormal basis designated  $1, e_1, \dots, e_n, e_1e_2, \dots, e_1e_2\dots e_n$ . The commands allow a user to enter both numeric and symbolic coefficients. That is, one can perform specific numerical as well as abstract calculations. For example, you can check whether associativity holds for various operations or whether a particular vector identity extends to bivectors or from dimension 2 to dimension 8.

Unlike almost all packages of any kind written for Mathematica, this package uses standard mathematical notation. This is because Mathematica is diabolic in its treatment of subscripted variables. However, in this package, for example, 4-dimensional space-time is represented by a vector basis written  $\{e_1, e_2, e_3, e_4\}$  and the vector  $(1,2,3,4)$  is denoted  $e_1 + 2e_2 + 3e_3 + 4e_4$ . If spacetime (rather than space) is chosen, the convention is that  $e_1$  is the time axis. A palette is provided that allows the user to choose space vs spacetime and a quadratic form; that is, whether  $e_k^2 = \pm 1$  for all  $k$ . A palette also simplifies the entering of subscripted multivectors such as  $3a_0 + 2a_{1,3,4} e_1e_2e_4$ . The palette can be used for entering GA operations, and the results are always given in readable, standard mathematical notation. The palette, as well, provides definitions and examples of all provided GA functions.

Expressions and operations can also be entered directly without use of the palette.

Currently the package performs the following operations (all visible and easily entered from the palette):

- Geometric product (with allowance for multivectors that span multiple grades)
- Other GA products:
  - Wedge (a.k.a. exterior), Dot, Scalar, Contractions
- Hodge Dual
- Inverse, Reverse
- Norm, Gorm
- Operations involving grades: slices, max grade, list of grades
- Operations involving lists: conversion to a list and back, list of coefficients, list of subscripts, list of **atoms** (i.e., subscripted  $e_i\dots e_k$ ), signature of a list of atoms/coefficients/subscripts/etc.
- Grouping and expanding by atoms, and reducing of terms like  $e_1^2$

- Typing (i.e., inputting) vectors, bivectors, blades, multivector, rotors, even/odd/graded multivectors, pseudoscalar, atoms, complex and quaternionic GA expressions
- Switching dimensions, metric signature, space vs spacetime on-the-fly

The palette simplifies typing of subscripted multivectors by providing a number of expressions at the click of the mouse. If the desired expression is not there it still may be quickly entered by modifying one of the palette items. The palette also allows the user to enter the symbols for the various GA operations with a mouse click.

The author welcomes collaboration. The package has been extensively tested by the author but has not been beta tested, so feedback on errors or quirks would be welcome. In particular, Mathematica 10 introduced a nasty complication called “shadowing” errors and the author has incorporated a process to prevent such errors from popping up. Still, there may be certain actions a user can make to invoke these errors. If so, they can hopefully be remedied by following the instructions in the Documentation.

New users should next read the Installation Guide, then Quick Start, Examples, and Documentation (which is very short because not much is needed.) .