Probability Queline:	PI
Sec Z. 17~76.	r: Answ
field. F-set, messurable F. Bo, B. T-field.	
J(A), J-tield generated by A.	
set tunction. / probability measure, on a tield.	
probability meesure space / probability space.	
support.	
discrete probability messure.	
inclusion-exclusion tormala.	
finite subadditivity (or Boole's inequality).	
Thus > \ A A A P. and magains on a field.	4
They (1) Cont. from below (2) from above (3) Comilloy show	oldierve.
Thin 2.2 Leb. meeure ? on Bo is a (countedly additive) probe	stility measure.
Sac & 2111 14 (Tristence and Extension)	
Thin 3. 1 A push were on a field bas a unique extension to	, ere generales o cres
onter measure. I inner measure 17.17 (2-1 ACIZITY CO)	*(EA)+p*(EA°), \ E]
4 propercies of P.	
Lemme   M is a field.	Jonly use
Lemme Z A1, Az, EM. E E Q => P*(E(UAn)) = \( \sum_{n} P^{*} \)	(EAn), the 4
Lemme 2 A1, A2, EM. E E Q = ) P*(E(UAn)) = \( \subseteq P^*\)  Lemme 3 M: \( \tau - \text{field} \) and \( P^* \) \( \text{counterbly radatitive} \).	elicit form of P*
Lemme 4 Pr := int ? J. (hen To = ) (, L	
Lemmas. P:= intland. (men f (1) + (1), VACVO	•
Uniqueness	5.0
TI- system / 7- system. Rmk: o-field =) 7- system. But	e not vice versa.

Lemme 6 &: both Ti-system and 7-system => A	: J-field. P.Z
Thm 3.2 (Dynkin's Ti-7 theorem).  P: Ti-system. L: 7-system. PEL.	The state of the s
=) o(p) ∈ L.	
Thm 3.3 (Uniqueness of extension)	
Pi, Pz: prob. mer. on o(P), where P: The system	<b>4</b>
$P_1 = P_2$ on $\mathcal{P}$ .	
=) P_=P2 on o(P).	
Monotone Classes	
monotone doss.	
Thm 3.4 (Halmos's monotone class cheanem)	
Fo: field. M: mondone cless.	
J. EM.	
=) σ(F <sub>0</sub> ) ∈ H.	
Sec 4. Demmerable Probabilities. (51~64)	
lim An, lim An, lan An=A.	
Thm 4.1	
(1) P(lin An) & lim P(An) & lim P(An) & P(lim An)	
$(2)$ $A_n \rightarrow A \Rightarrow P(A_n) \rightarrow P(A)$ .	
independence of enerce / independence of classes.	
Rrink: Pairwise indep. does not imply indep.	
Thm 4.2	
A, Az,, An: indep. Tr-systems.	
=) J(A1), J(Az),, J(An): indep.	
Cor 2 Suppose	Au Au are indep.
$A_0$ , $0 \in \Theta$ : indep. $\pi$ -systems. $F_i := \sigma$	-field generated by the ith row.
Daid , and	
Rmk: (by Thm 4.2)	F2, F3,: indep.
Rmk: (by Thm 4.2)  A, Az,, An: indep. =) P(A, Az Ac Akti An) = P(A, P(A, E))	) P(AE) P(AK) P(An), K=1, -, N.

Then u=u, on o(P).

See 11. July Measures.

Detorder messure u\* / u\* - messurable. / M (u\*)

Thm 11. 1 11\*: order meesure =) M(111\*); J-tield and M\*/M(111\*); meesure.

Thm 11.2 A measure on a field has an extension to the generated o-field.

Def semiring.

Thm 11.3

M: set tunction on A, A: semiring.

M(\$)=0, M: finitely additive and countably subadditive.

M: has values in [0, 20].

=) Il extends to a messure on o(\$4).

Ruk: 0 Thm 11.3 = ) Thm 11.2.

3 If 4: J-finite on A, then by Thin 10.3, the extension is unique.

3) The benefit of incroducing semiring is that we don't have to bother ourselves by extending first to tields.

Lemma 1

A, A, ..., An E A, d = semiring.

=) = disjoint C15., Cm ESS S.E. ANAIN... NA" = C, U-... U Cm.

Thm 11.4 (Approximation Theonem)

A: semiring

M: messure on F= o(A), M: T-finite on A.

Then, for BEF and E70,

(1) I completely many disjoint A1, A2, ... Ex se. B = DAn and u(UAn B) < E.

(11) if u(B) < 00, then I finitely many disjoint A1, ", An E & s.t. u(B & ( WAK)) < E.

Cor

M: finite measure on F, here F: O-field generated by a field Fo.

=) VAEF and Ero, JBEF. S.t. M(ABB) < E.

Cor 2

A : semiring, MI(A) EMZ(A) < D, HAED.

 $u_1, u_2$ : measures on  $\mathcal{F} = \sigma(\mathcal{A})$ Then u,(B) < Mz(B), & B & F. M, Mz: T-finite on &

.17.5

Lemma 2 (used in Sec 12). u: nonnepative, finitely additive, set function on <math>A, A: semiring.  $A, A_1, \dots, A_n \in A$ .

(i) If  $\bigcup_{k=1}^{n} A_k \subseteq A$ , and  $A_1, \dots, A_n: disjoint, then <math>\sum_{k=1}^{n} \mathcal{M}(A_k) \subseteq \mathcal{M}(A)$ .

(ii) If  $A \subseteq \bigcup_{k=1}^{n} A_k$ , then  $\mathcal{M}(A) \subseteq \sum_{k=1}^{n} \mathcal{M}(A_k)$ .

Te day's letter of

gift. Use supergrings for chore-line sopra particular to and with or the widing course of activities for on the unit to be intries for any factoring up the introduction has believed as a believe of incerving the believes as believes on board luck. The police were looking for a leaf and the first waite boarding up this widing me because the intervals and one fait and completely first found to be supposed to a characteristic for the provider of the characteristic of the context of th

rest chare when any hends told the ladge of type of built hunting with a paper may be stick.

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warring it. The state of the street of the state of the s

Bed of roses. A pleasant easy place, job, of position, an easy life (used as a neuri place). A code princy's job is graf a bed of roses.

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Best success to the suncto the constraint before another person has a chance to to it.

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directions i which de not out as marques.

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or handed down by another).