

# Quantum Transport in Nanoporous Graphene

## Bachelor defense

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# Outline

## Introduction

Project aim and nanoporous graphene

## Tight Binding

$\pi$ -orbitals,  $\pi$ -electrons and the TB approximation

## The Hamiltonian

Onsites, hops and the full TB Hamiltonian

## Green's functions and recursion

Green's matrix, recursion and LDOS

## Transmission

Device Green's functions, left/right geometry, rate matrices and spectral functions.

Transmission in 2D

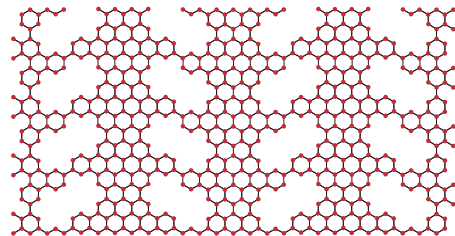
## Exploring GNR bridges

Para-O<sub>4</sub>-NPG, Para-(OH)<sub>4</sub>-NPG, Meta-O<sub>2</sub>-NPG, Meta-(OH)<sub>2</sub>-NPG

“Development of tight-binding routines in Python in order to understand electron transport in novel nanoporous graphene devices (NPGs)”

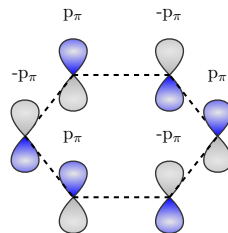
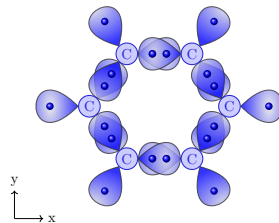
# Nanoporous graphene

- ▶ Planar graphene sheets
- ▶ Periodically removed atoms
- ▶ Ribbons and bridges
- ▶ Ballistic electron movement
- ▶ Potential for controlling currents on nanoscale



# $\pi$ -orbitals and $\pi$ -electrons

- ▶ In plane electrons are bound
- ▶ 1  $p_z$ -electron per site
- ▶ “Tightly bound” hops between sites



# TB approximation

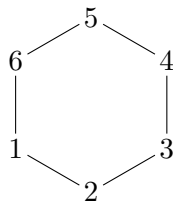
- ▶ Electrons tightly bound to sites
- ▶ Hops with potential
- ▶ Average electron energy on site
- ▶ The Hamiltonian is a hop matrix

$$V_{pp\pi} = \langle \phi_{\pi}(m) | \hat{\mathbf{H}} | \phi_{\pi}(n) \rangle$$

$$\epsilon_0 = \langle \phi_{\pi}(i) | \hat{\mathbf{H}} | \phi_{\pi}(i) \rangle$$

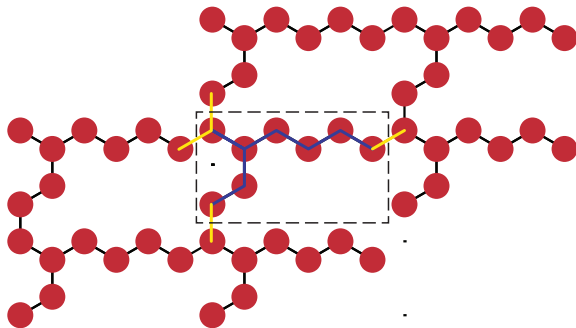
# Hamiltonian for benzene

$$\mathbf{H} = V_{pp\pi} \begin{array}{c} \begin{array}{cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} & \left( \begin{array}{cccccc} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{array} \right) \end{array} \end{array}$$



# Creating the first Hamiltonian

- ▶ Atom coordinates
- ▶ Interatomic distances
- ▶ Applying potential

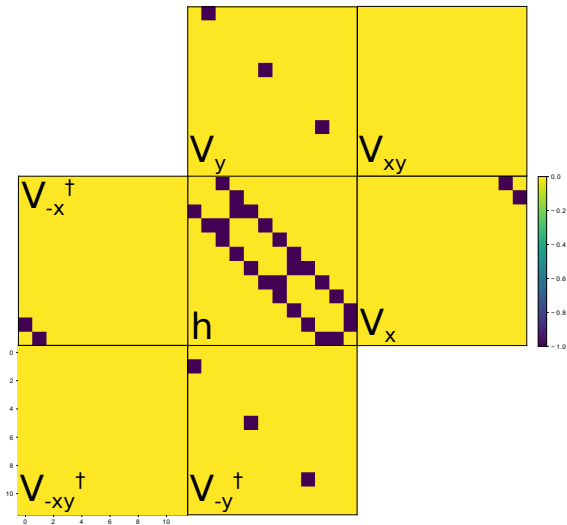


```
33 h = np.zeros((xyz.shape[0], xyz.shape[0])) # Empty matrix
34 for i in range(xyz.shape[0]): # Take an atomic coordinate
35     for j in range(xyz.shape[0]): # Take another atomic coordinate
36         h[i, j] = LA.norm(np.subtract(xyz[i], xyz[j])) # Measure distances
37 h = np.where(h < 1.6, Vppi, 0) # Replace distances under 1.6 angstrom with Vppi
38 h = np.subtract(h, Vppi * np.identity(xyz.shape[0])) # Remove the diagonal
```



# Hopping matrices

- Shift by lattice vector
- Resulting matrices:  
 $\mathbf{h}_0, \mathbf{V}, \mathbf{V}^\dagger$



# Full Hamiltonian and first band plots

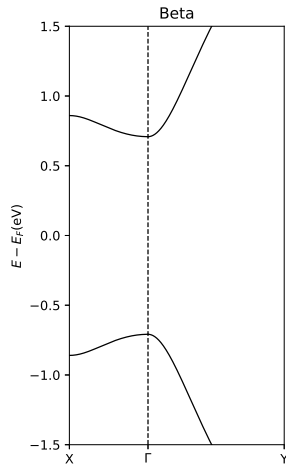
$$\mathbf{H}(k_x, k_y)\phi_k = \epsilon_n(k_x, k_y)\phi_k$$
$$\mathbf{H}(k_x, k_y) = \mathbf{h}_0 + (\mathbf{V}_x e^{-ik_x} + \mathbf{V}_x^\dagger e^{ik_x} + \mathbf{V}_y e^{-ik_y} + \mathbf{V}_y^\dagger e^{ik_y} \\ + \mathbf{V}_{xy} e^{-ik_x} e^{-ik_y} + \mathbf{V}_{xy}^\dagger e^{ik_x} e^{ik_y})$$

```
73 Ham = Ham + (V1 * np.exp(-1.0j * x) # Onsite hops and hops in x
74           + np.transpose(V1) * np.exp(1.0j * x) # Hops in -x
75           + V2 * np.exp(-1.0j * y) # Hops in y
76           + np.transpose(V2) * np.exp(1.0j * y) # Hops in -y
77           + V3 * np.exp(-1.0j * x) * np.exp(-1.0j * y) # Hops in both x and y
78           + np.transpose(V3) * np.exp(1.0j * x) * np.exp(1.0j * y)) # Hops in both -x and -y
79 e = LA.eigh(Ham)[0] # Eigen energies from the Hamiltonian
80 v = LA.eigh(Ham)[1] # Eigen vectors from the Hamiltonian
```

# Full Hamiltonian and first band plots

$$\text{X: } \mathbf{H}_X = \mathbf{h}_0 + (\mathbf{V}_x e^{ik_x} + \mathbf{V}_x^\dagger e^{-ik_x} + \mathbf{V}_y + \mathbf{V}_y^\dagger + \mathbf{V}_{xy} e^{ik_x} + \mathbf{V}_{xy}^\dagger e^{-ik_x})$$

$$\text{Y: } \mathbf{H}_Y = \mathbf{h}_0 + (\mathbf{V}_x + \mathbf{V}_x^\dagger + \mathbf{V}_y e^{-ik_y} + \mathbf{V}_y^\dagger e^{ik_y} + \mathbf{V}_{xy} e^{-ik_y} + \mathbf{V}_{xy}^\dagger e^{ik_y})$$



- Solution to the Scödinger Equation
- Propagator

$$[(E + i\eta)\mathbf{1} - \mathbf{H}]\mathbf{G}(E) = \mathbf{1}$$

↓

$$\mathbf{G}(E) = \mathbf{1}([(E + i\eta)\mathbf{1} - \mathbf{H}])^{-1}$$

► Semi-infinite chain

$$\begin{pmatrix} z\mathbf{1} - \mathbf{H}_c & -\mathbf{V}^\dagger \\ -\mathbf{V} & (z - \varepsilon')\mathbf{1} \end{pmatrix} \begin{pmatrix} \mathbf{X} & \mathbf{G}_{0c} \\ \mathbf{G}_{c0} & \mathbf{G}_{00} \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$$

$$\begin{aligned} \mathbf{G}_{00}(z) &= \left[ (z - \varepsilon') - \mathbf{V}(z\mathbf{1} - \mathbf{H}_c)\mathbf{V}^\dagger \right]^{-1} \\ &= (z - \varepsilon' - \Sigma(z))^{-1} \end{aligned}$$

where

$$z = E + i\eta$$

$$a_0 = \mathbf{V}^\dagger, \quad b_0 = \mathbf{V}$$

$$e_{s0} = \mathbf{h}_s, \quad e_0 = \mathbf{h}$$

in loop:

$$a_1 = a_0 \times g_0 \times a_0$$

$$b_1 = b_0 \times g_0 \times b_0$$

$$e_1 = e_0 + a_0 \times g_0 \times b_0 + b_0 \times g_0 \times a_0$$

$$e_{1s} = e_{0s} + a_0 \times g_0 \times b_0$$

$$g_1 = (z - e_1)^{-1}$$

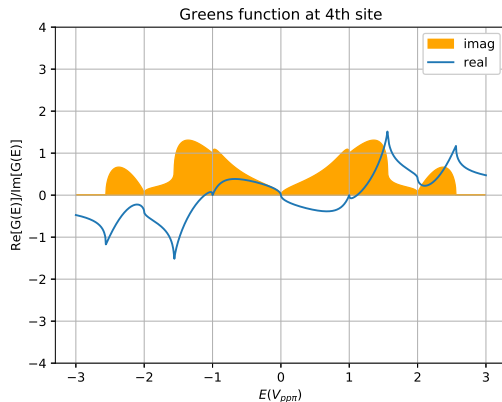
$$\Sigma_R = e_s - h$$

$$\Sigma_L = e - h - \Sigma_R$$

$$\mathbf{G}_{00} = (z - e_s)^{-1}$$

# Recursion

```
92 while np.max(np.abs(a0)) > 1e-6: # Loop with hop matrix as threshold
93     ag = a0 @ g0 # Product defined here and used multiple times
94     a1 = ag @ a0 # New hop matrix (transposed)
95     bg = b0 @ g0 # Product defined here and used multiple times
96     b1 = bg @ b0 # New hop matrix
97     e1 = e0 + ag @ b0 + bg @ a0 # New onsite for "other cells"
98     es1 = es0 + ag @ b0 # New onsite
99     g1 = LA.inv(z - e1) # New Green's function
100     a0 = a1 # Overwrite old variable
101     b0 = b1 # Overwrite old variable
102     e0 = e1 # Overwrite old variable
103     es0 = es1 # Overwrite old variable
104     g0 = g1 # Overwrite old variable
105     q = q + 1 # Counter (for diagnostic purposes)
106 e, es = e0, es0 # Define the onsite Hamiltonians
107 SelfER = es - h # Self-energy from the right
108 SelfEL = e - h - SelfER # Self-energy from the left
109 G00 = LA.inv(z - es) # Green's functions
```



```

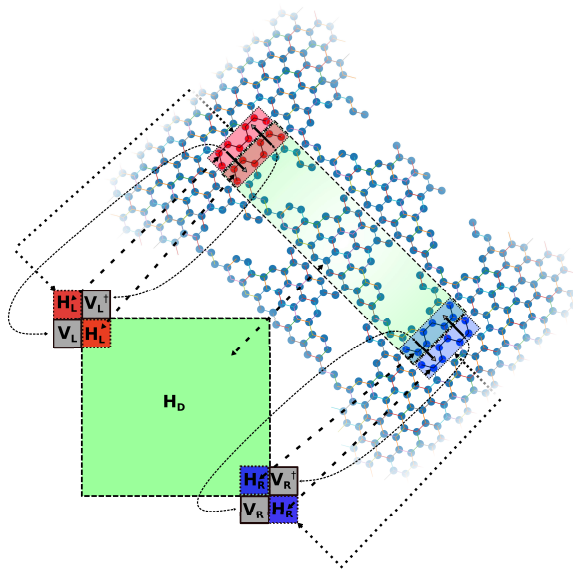
64  G00 = np.zeros((En.shape[0]), dtype=complex) # Empty data matrix for Green's functions
65  for i in range(En.shape[0]): # Loop iterating over energies
66      G, SelfER, SelfEL = RecursionRoutine(En[i], h, V, eta) # Invoking the RecursionRoutine
67      G = np.diag(G) # The Green's functions for each site is in the diagonal of the G matrix
68      G00[i] = G[4] # Chosen Green's function (here the 4th site)

```

Transmission is the probability of an electron being transported through a specific region for a specific range of energies.



# Translating from system to matrices



Introduction  
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Tight Binding  
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The Hamiltonian  
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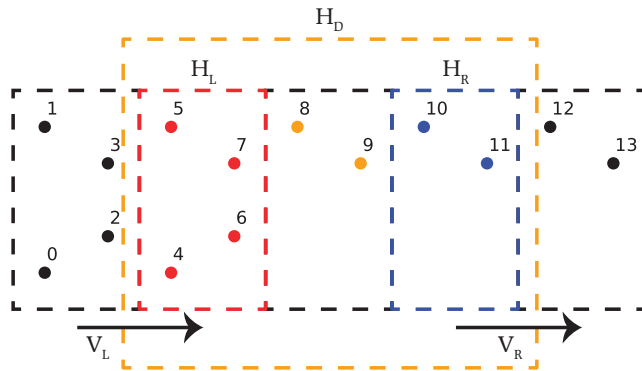
Green's functions and recursion  
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Transmission  
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Exploring GNR bridges  
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Questions  
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# The left and right self-energy



```

210     for i in En: # Iteration over each energy
211         gl, scrap, SEL = RecursionRoutine(i, HL, VL, eta=eta) # From the left
212         gr, SER, scrap = RecursionRoutine(i, HR, VR, eta=eta) # From the right
    
```

# Getting Transmission

$$\mathbf{G}_D = [\mathbf{1}(E + i\eta) - \mathbf{H}_D - \boldsymbol{\Sigma}_L(E) - \boldsymbol{\Sigma}_R(E)]^{-1}$$
$$\boldsymbol{\Gamma}_{L,R} = i\left(\boldsymbol{\Sigma}_{L,R} - \boldsymbol{\Sigma}_{L,R}^\dagger\right)$$

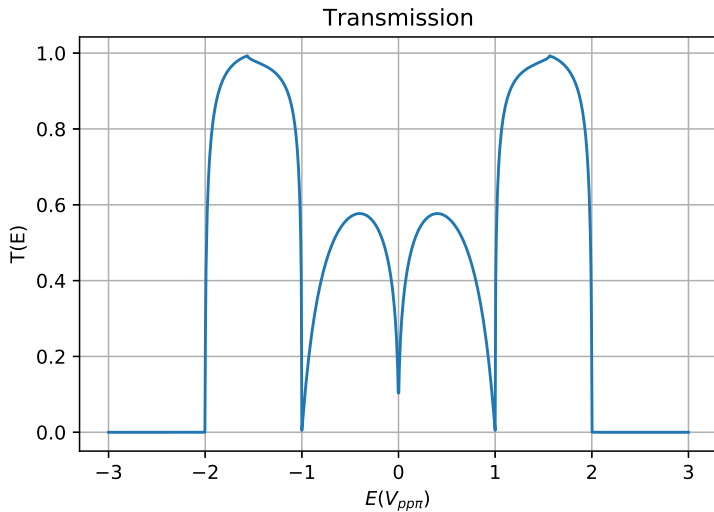
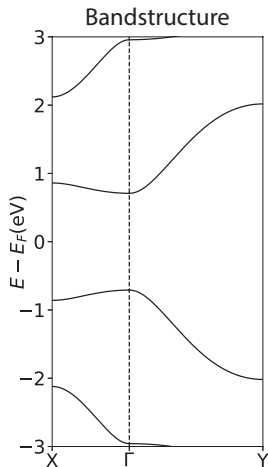
```
225 GD["GD{:d}".format(q)] = scp.linalg.inv( # Device Green's functions are saved
226     scp.identity(HD.shape[0]) * (i + eta) - HD - SEL - SER) # in a dictionary.
227 GammaL["GammaL{:d}".format(q)] = 1j * (SEL - SEL.conj().transpose()) # Rate matrices are
228 GammaR["GammaR{:d}".format(q)] = 1j * (SER - SER.conj().transpose()) # likewise saved.
```

# Getting Transmission

$$T(E) = \text{Tr} \left[ \mathbf{\Gamma}_R \mathbf{G}_D \mathbf{\Gamma}_L \mathbf{G}_D^\dagger \right] (E)$$

```
240     for i in range(En.shape[0]): # Iteration for every energy point.
241         T[i] = np.trace((GammaR["GammaR{:d}".format(i)] @ GD["GD{:d}".format(
242             i)] @ GammaL["GammaL{:d}".format(i)] @ GD["GD{:d}".format(i)].conj(
243             ).transpose()).todense()) # Calculate transmission (equation V.9).
```

# Getting Transmission



Introduction  
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Tight Binding  
ooo

The Hamiltonian  
ooo

Green's functions and recursion  
ooo

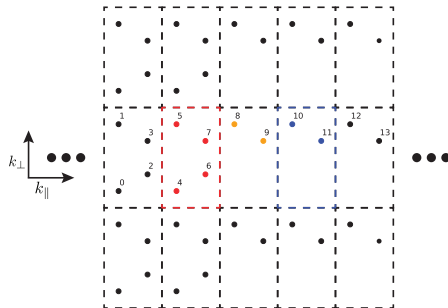
Transmission  
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Exploring GNR bridges  
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Questions  
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# Transmission in 2D

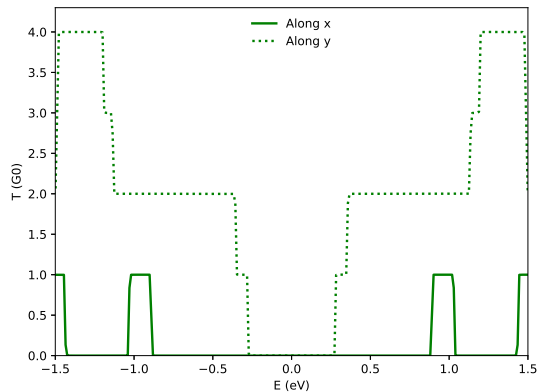
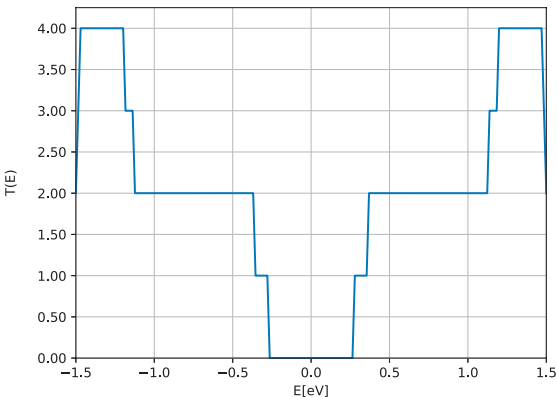
$$\mathbf{H} = \mathbf{h} + \mathbf{V}e^{ik_{\perp}} + \mathbf{V}^{\dagger}e^{i(-k_{\perp})}$$



```
250     h, p = Onsite(xyz=xyz, Vppi=-1, f=1) # Calculate onsite hops
251     V = Hop(xyz=xyz, xyz1=xyz + np.array([0, UY, 0]), Vppi=-1) # Calculate hops
252     print('Number of hopping elements: {}'.format(np.sum(np.abs(V)))) # Number of hops
253     Ham = h + V * np.exp(1j * i) + np.transpose(V) * np.exp(-1j * i) # Hamiltonian for ith kp
```

# Code validity

$$k = \frac{\pi}{2}$$



Introduction  
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Tight Binding  
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Green's functions and recursion  
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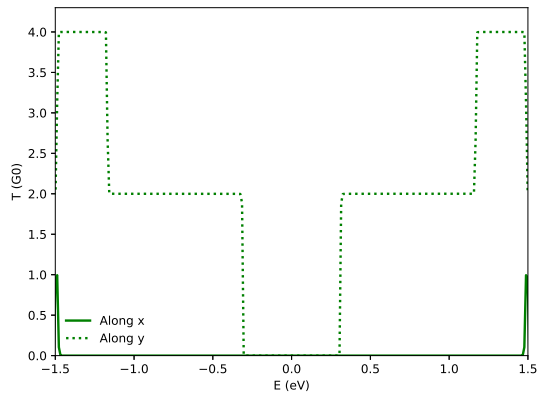
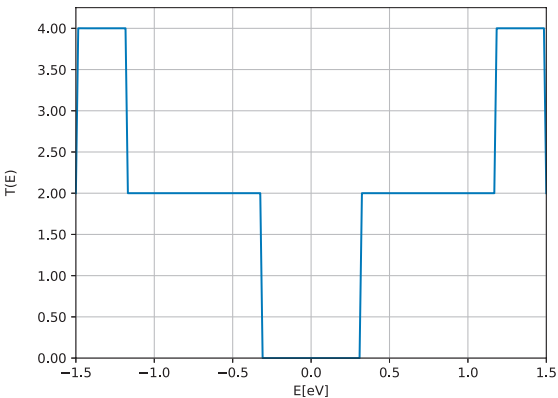
Transmission  
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Exploring GNR bridges  
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Questions  
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# Code validity

$$k = \pi$$



Introduction  
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Tight Binding  
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The Hamiltonian  
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Transmission  
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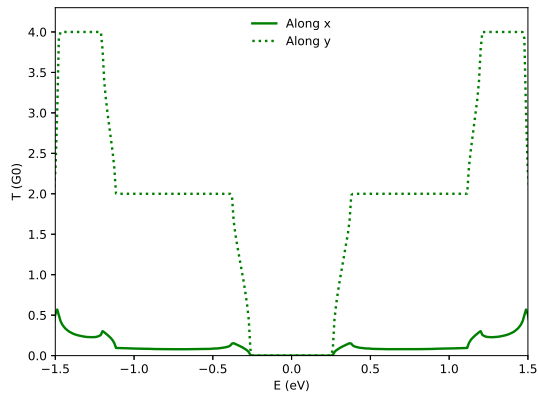
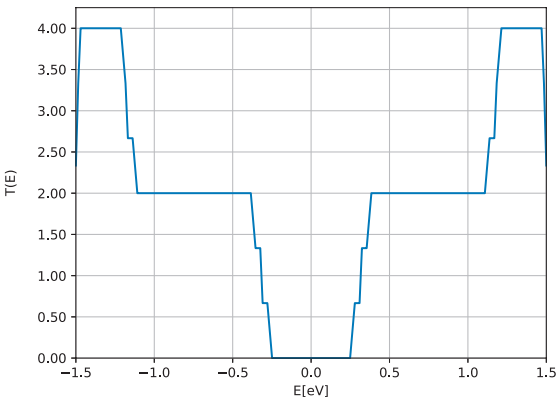
Exploring GNR bridges  
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Questions  
○



# Code validity

$$k = \text{AVG}\left(0, \frac{\pi}{2}, \pi\right)$$



Introduction  
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Tight Binding  
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The Hamiltonian  
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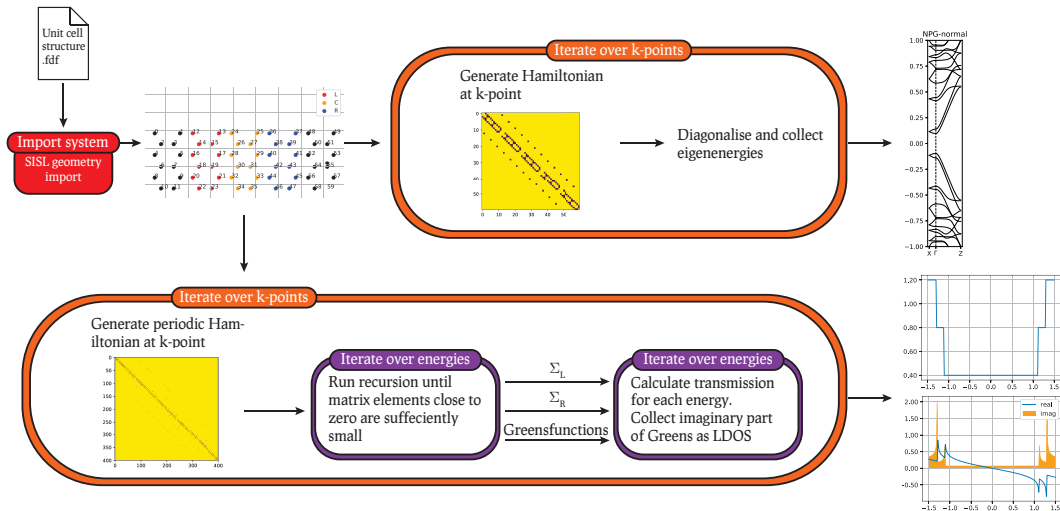
Green's functions and recursion  
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Transmission  
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Exploring GNR bridges  
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Questions  
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# Summary of code structure



Introduction  
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Tight Binding  
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The Hamiltonian  
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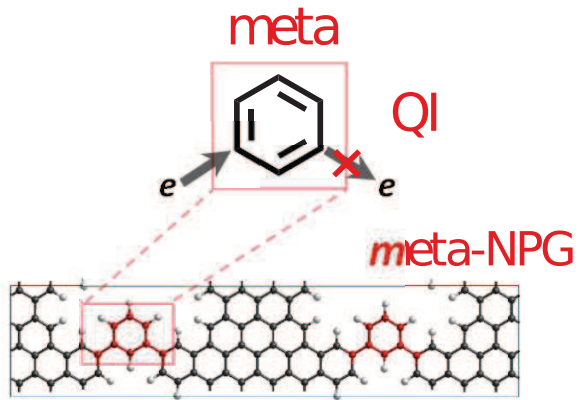
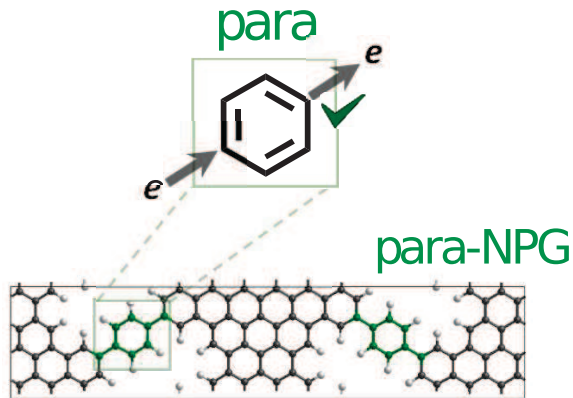
Green's functions and recursion  
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Transmission  
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Exploring GNR bridges  
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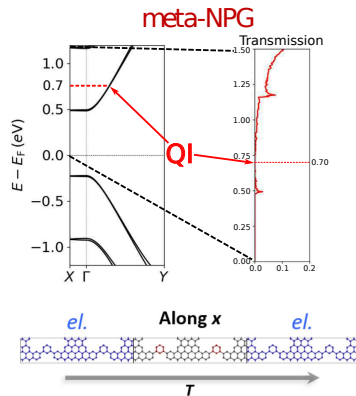
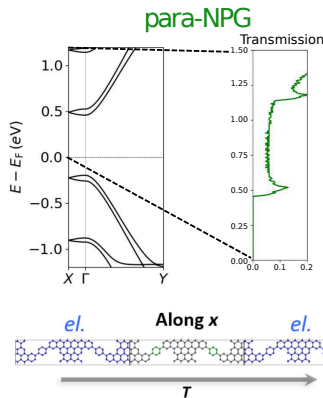
Questions  
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# Para and meta bridges



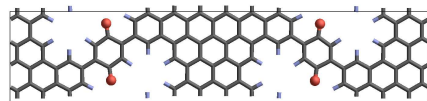
# Para and meta bridges

- ▶ Path length difference
- ▶ Quantum interference
- ▶ Transmission
- ▶ coupling/decoupling of GNR

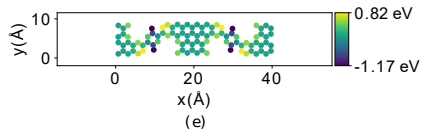
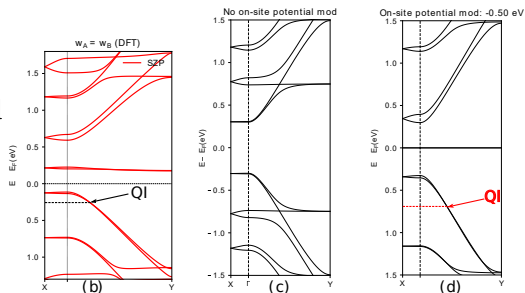


# Para-O<sub>4</sub>-NPG

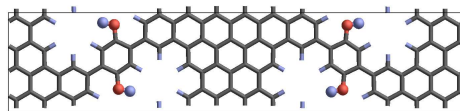
- Functionalisation with oxygen
- Quantum interference at valence band
- Reproducing DFT result
- Decoupling of GNR
- Bands at Fermi level



(a)

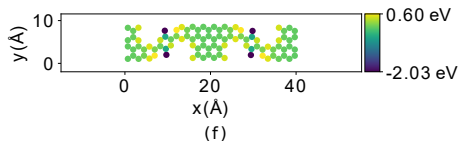
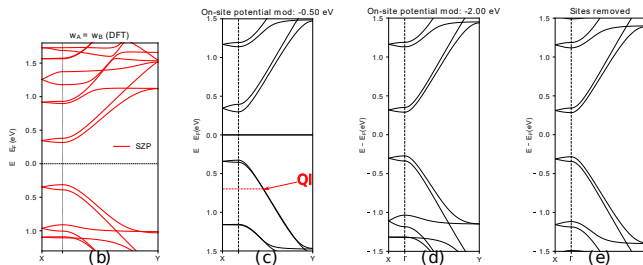


# Para-(OH)<sub>4</sub>-NPG



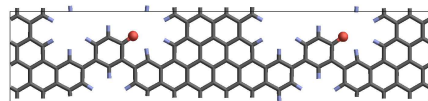
(a)

- Hydrogenation of oxygen
- Band splitting of DFT
- GNR coupling and resemblance with Para-NPG
- Reproducing DFT results

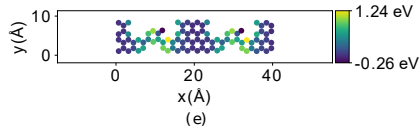
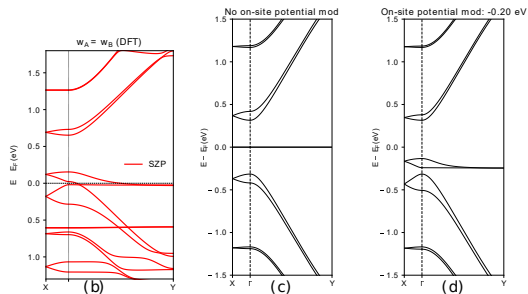


# Meta-O<sub>2</sub>-NPG

- Split in the valence for DFT
- Similarity in valence/conduction difference for meta/para
- Trying different potentials
- Flat bands at Fermi level



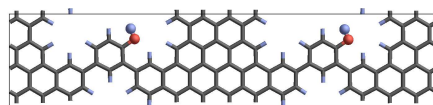
(a)



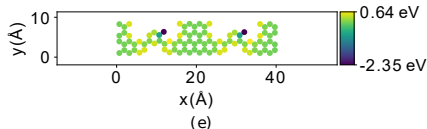
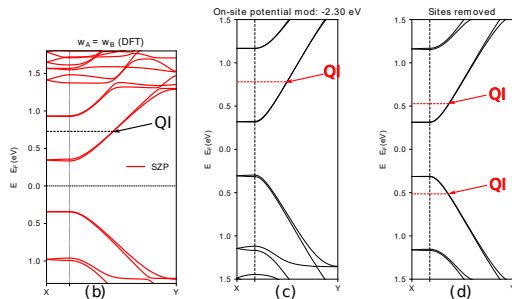
(e)

# Meta-(OH)<sub>2</sub>-NPG

- ▶ Hydrogenation lowering potential of oxygen
- ▶ Decoupling of oxygen = decoupling of GNR
- ▶ Resemblance with pristine Meta-NPG
- ▶ In agreement with DFT



(a)



(e)



“A tight-binding routine has successfully been developed in Python. This makes it easier to, qualitatively, understand electron transport in nanoporous graphene devices (NPG)” ✓

# Questions

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JUNE 25, 2019

Introduction  
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Tight Binding  
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The Hamiltonian  
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Green's functions and recursion  
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Transmission  
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Exploring GNR bridges  
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