

Chapter 8 Physics of the early Universe

The previous three chapters took an in-depth look at how astronomical observations enable us to measure cosmological parameters, and so to understand the ‘big picture’ of how the Universe has evolved and is continuing to expand. The cosmic microwave background (CMB) is our only direct probe of conditions in the early Universe: it provides information about both the geometry and the contents of the Universe at the time the CMB was produced (i.e. $\sim 400\,000$ years after the big bang). By this time, nucleosynthesis had occurred and ordinary matter was primarily in the form of hydrogen and helium atoms. In this chapter we will explore the physics of how particle composition evolved in the *very* early Universe to reach this state.

Objectives

Working through this chapter will enable you to:

- describe the most important particle interactions that took place in the early Universe
- explain how the changing conditions in the early Universe determined the rates of key interaction processes
- summarise the timeline of the early Universe, from the big bang to primordial nucleosynthesis
- explain the importance of two key parameters – the baryon-to-photon ratio and the neutron-to-proton ratio – for the elemental abundances of the material from which the first stars and galaxies formed
- summarise the nuclear reaction processes that took place in the early Universe.

8.1 Particle interactions in the early Universe

An understanding of particle interactions in the early Universe is necessary to fill in the first steps of a timeline for how the present-day Universe came into being. Modelling these interactions also provides a further tool for testing cosmological models: the elemental abundances in the first stars and galaxies depend critically on conditions at earlier times, and astronomers can measure these abundances by searching for and studying the oldest regions that telescopes can access.

- Why weren’t hydrogen and helium atoms present in the Universe immediately after the big bang?

- The early Universe was very hot and dominated by radiation. Nuclei and atoms cannot form in those circumstances, and even protons and neutrons break down into their constituent quarks in this environment.

Examining the earliest times in the Universe forces us to confront the limits of our knowledge. Conditions in the early Universe must have reached extremes far beyond anything that can be simulated in a lab, and cosmologists have no direct ways to test theoretical ideas of how the Universe began, or whether something – or *anything* – existed prior to the big bang. In this chapter we will focus mainly on the stages for which the physics is well understood, involving particles such as protons (p), neutrons (n), electrons (e^-) and photons (γ). But we also need to set the scene for the advent of nucleosynthesis by considering some more exotic, and less well-understood, processes.

One of the main limitations of our knowledge relates to the existence and nature of dark matter. We saw in previous chapters that, initially, radiation (and then later, matter) drove the overall rate of expansion of the Universe, and also that dark matter played a key role in the oscillations that determine the CMB structure. So dark matter is clearly an important actor in the evolution of the early Universe.

However, in this chapter we want to explore how atoms formed; to do that we need to focus on particle interactions via short-range forces. These key processes do not involve the gravitational force by which dark matter interacts, which means that we can set aside dark matter for now, and focus on particles and processes that are better understood. We will return to the evidence for, and nature of, dark matter in the next chapter.

8.1.1 Basics of early-Universe interaction physics

The particle families of the Standard Model, and the four fundamental interactions by which they interact, were introduced in Chapter 1. Before we move on to consider the main particle interaction processes of the early Universe, you should review your understanding of these particles and forces with the following exercise.

Exercise 8.1

Using your existing knowledge (supplemented by brief further research if needed), complete Table 8.1 to summarise the key properties of the four fundamental interactions. Note that there is no universal definition of ‘relative strength’, and so you may obtain somewhat different numbers from different information sources.

Table 8.1 Basic properties of the four forces. (To be completed.)

Interaction	Particles affected	Carrier particle	Relative strength
Strong			
Weak			
Electromagnetic			
Gravity			

All of the main particle families and forces are important in the early Universe but, as mentioned above, we can consider gravity somewhat separately from the other three forces because of its relative weakness in the context of interactions between individual particles. Electromagnetic, weak and strong nuclear interactions are the key processes leading to the formation of atoms in the dense early Universe.

There are a variety of different particle interactions that could, in principle, occur in most physical environments, where numerous types of particle might exist. The crucial question is which process or processes have the *most* impact on the relative proportions of different particle types at different times? We therefore want to think about **reaction rates** for different processes, and which factors control those rates.

- Which global properties were changing in the early Universe and are likely to be relevant to the rates at which particles interact?
- As the Universe expanded, its temperature and density decreased. Temperature determines the energy and velocity distributions of particles, and both particle speed and density will affect how often and how strongly particles interact.

We can write a standard expression for the rate, Γ , at which a particle will undergo interactions of a particular type without specifying (yet) which particles and type of interaction are involved:

$$\Gamma = n\sigma v \quad (8.1)$$

where n is the number density of the ‘target’ particle with which an incoming particle might interact, v is the typical relative speed of the interacting particles and σ is the **interaction cross-section**: a measure of the probability that the interaction will occur. We will consider σ in more detail shortly.

Equation 8.1 describes the rate of reaction per incoming particle, but it is often useful to consider the rate at which reactions will occur within a particular volume. This will depend on the density of both types of

particle involved, so the rate of interaction between two particle species, A and B, per unit volume is

$$R_{AB} = n_A n_B \sigma v$$

where n_A and n_B are the number densities for each species of particle, respectively.

- What units must σ have if Γ has units of s^{-1} (i.e. interactions per second) and R_{AB} has units of $s^{-1} m^{-3}$ (i.e. interactions per second per cubic metre)?
- Rationalising the other units tells us that σ must have units of m^2 .

The fact that the interaction cross-section σ has dimensions of area can be understood by analogy to the simplest type of two-body interaction in classical physics: scattering between two solid bodies. Such interactions depend on the geometric area of the target object (a larger target cross-section provides more opportunity for the scattering of incident particles to occur). For electromagnetic, weak and strong nuclear interactions, the effective area within which an interaction is likely depends on the range of the interaction and other factors, such as the energy of the particles.

Earlier in this section we noted that temperature is one of the important controlling parameters for the rate at which interactions take place in a particular environment. A key feature of the early Universe is that all of the different particle families, including radiation and neutrinos, were interacting so frequently that they were in a state of thermal equilibrium. This situation means that the particle energy distributions obey standard expressions for fermions and bosons, and the associated temperatures of matter and radiation are equal. Therefore, while thermal equilibrium still holds, we can use a single temperature value to characterise the state of the Universe at any particular time.

This temperature, T , describing both the matter and the radiation at a particular time, is a very important parameter for how matter evolved, because temperature determines a quantity known as the **interaction energy**. This quantity is the typical energy available for particle interactions and is given by:

$$E_{\text{int}} \approx k_B T \tag{8.2}$$

Particle energies are often expressed in units of eV (electronvolts), where $1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$. Use the following exercise to get a sense of the sorts of numbers that could apply to E_{int} in the very early Universe.

Exercise 8.2

Estimate the interaction energy, in units of J and GeV, when the temperature is 10^{14} K .

The available interaction energy is important for **pair production**, the process by which quarks, leptons and more massive particles were first produced in the early Universe. Pair production involves converting energy into mass (via the famous equation $E = mc^2$). When the typical interaction energy drops below the rest-mass energy of the particles that the interaction produces, then pair production of that type becomes very improbable. So, for example, because they are much less massive, electron–positron pairs can be produced at lower energies than any of the quark pairs.

Another useful way to assess the impact of changing density and temperature is to consider typical timescales for particle interactions. Recalling the definition of the interaction rate Γ for a particular process (Equation 8.1), the typical timescale for a particle to interact is $t_{\text{int}} = 1/\Gamma$.

- How would you expect Γ to change with time for typical particle interaction processes in an expanding Universe?
- The density and the velocity of particles will decrease with time, so we would expect Γ to decrease with time.

We can then ask whether, on the timescale t_{int} for some particular process, the Universe expands significantly. The simplest way to define a suitable expansion timescale for comparison is via the Hubble parameter. In Chapter 5, the Hubble time at the present epoch was defined as $t_{\text{H}} = H_0^{-1}$. Similarly, for the early Universe we can define an expansion time $t_{\text{exp}} = H(t)^{-1}$. From the relationship $H(t) = \dot{a}/a$, it can be shown that in the time $1/H(t)$ the scale factor increases by a factor of e (i.e. by a little less than three times). If the Universe nearly triples in size on this interaction timescale, then the probability of an interaction taking place is very low. Therefore, setting $t_{\text{int}} \approx t_{\text{exp}}$ provides a rough estimate of when particular interactions become improbable.

- If a particular type of interaction stops at a time $t_{\text{int}} \approx t_{\text{exp}}$, how are the interaction rate and the Hubble parameter at that time related to one another?
- Using the definitions of t_{int} and t_{exp} above, $\Gamma \approx H(t)$ at the time interactions stop.

There are, therefore, many types of interaction that are *possible* in the early Universe but become very *improbable* at later times. For example, at the particle scale, at early times there is thermal equilibrium between quarks (and later, neutrons and protons), leptons, photons and neutrinos. This equilibrium is achieved by frequent interactions that distribute energy between the different particle types. Once the interaction rate Γ for relevant processes is $\sim H(t)$ then, first, neutrinos (and later, photons) **decouple** from or **freeze out** of the cosmic ‘soup’. They are no longer in equilibrium with the other particles so travel across the Universe unimpeded by interactions. These decoupling processes also have implications for how the remaining matter (mainly neutrons, protons and electrons) evolves.

8.1.2 Key interaction processes

In the previous section we discussed particle interactions in fairly general terms. We will now review the most important types of interaction in the early Universe, in roughly chronological order of how matter evolved. Section 8.2 will then set out a full timeline for the evolution of the early Universe, in which all of these processes play crucial roles.

Two important rules that govern particle interactions are the conservation of **baryon number**, which applies to all particle interactions, and the conservation of **lepton number**, which applies to all interactions involving leptons. The sums of both of these numbers must remain the same before and after any interaction between particles. Baryons and antibaryons have baryon numbers of 1 and -1 , respectively. Similarly, leptons and antileptons have lepton numbers of 1 and -1 , respectively. Quarks and antiquarks have baryon numbers of $1/3$ and $-1/3$, respectively.

Pair production and annihilation

In the time immediately following the big bang, the first (ordinary) matter in the Universe was produced by pair production. Although this mechanism stopped being important for the overall composition of the Universe within a few seconds, it remains an astrophysically important process in extreme environments in the present-day Universe (for example, in the vicinity of black holes).

The earliest instances of the process included the production of pairs of oppositely charged quarks from gluons and/or photons, and the production of leptons (e.g. electron–positron pairs) from photons. Various other types of subatomic particle can also be generated in the same way.

Panel (a) of Figure 8.1 depicts the creation of an electron–positron pair, an interaction in which high-energy photons are the ‘reactants’. Panel (b) of the figure shows electron–positron **annihilation**, which is the opposite process to pair production; here, a particle interacts with its antiparticle and both are destroyed, with energy released as a result.

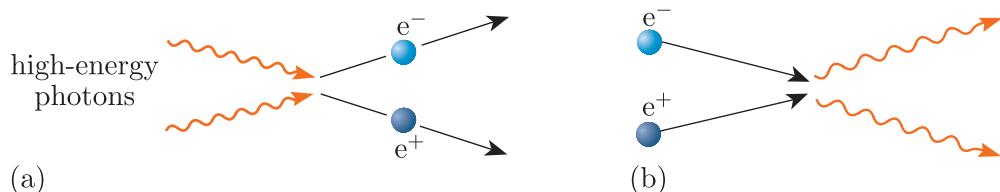


Figure 8.1 The opposing processes of (a) pair production from high-energy photons, and (b) annihilation.

Use the following exercise to consider the behaviour of different pair-production processes.

Exercise 8.3

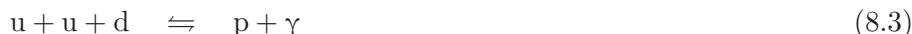
Calculate the energies needed (in units of GeV) for the creation of a proton–antiproton pair and for an electron–positron pair. Which pair is easier to create?

During a period when the interaction energy is high compared to the masses of quarks, hadrons and leptons – as in the very early Universe – pair production and annihilation will both occur, but the former will dominate. When, as the Universe expands, the available energy drops then annihilation comes to dominate instead. This leads to one of the major conundrums of early-Universe physics: if particles are created in matter and antimatter pairs, why is the present-day Universe dominated by matter? In other words, why didn't all of the matter that was created in the early Universe subsequently annihilate? The answer to this question is not yet fully understood, but we will return to it in a later section.

Hadron production and dissociation

Quarks can bind together via the strong force to form hadrons, which include baryons and mesons. Baryons are hadrons containing three quarks, and include protons and neutrons, whereas mesons (of which the π meson is perhaps the most well known) contain a quark–antiquark pair.

Hadron formation involves the release of energy, much like the more familiar process of nuclear fusion. Conversely, the presence of a high density of energetic photons will dissociate hadrons into their constituent quarks. For example, the reversible process of proton formation – from two ‘up’ (u) quarks and one ‘down’ (d) quark – and dissociation is written as



The \rightleftharpoons symbol indicates that, in the early Universe, this process operated in both directions, with energies so high that when hadrons formed they rapidly dissociated again. Similar reactions apply to the formation and dissociation of neutrons, antiprotons and antineutrons. As the available energy decreased, the bound hadrons became stable and subsequently came to make up a major component of the cosmic soup.

Weak interactions

Unlike the strong force, weak interactions don't bind particles together, but they can change particles from one type to another. Many types of weak interaction occur in the very early Universe, but the most important for understanding the timeline of how matter evolved are those involving conversions between protons and neutrons. These reactions include processes known as **beta decay** (β -decay), and electron and positron capture. There are two forms of beta decay, termed beta-minus (β^-) and beta-plus (β^+) decay. You may have encountered these processes previously as important concepts in the physics of stars.

The term ‘beta decay’ comes from nuclear physics: in the context of radioactivity, emitted electrons or positrons are referred to as beta particles.

The conversion processes between neutrons and protons are summarised below:

Neutron–proton conversion reactions

β^- -decay involves the conversion of a neutron to a proton:



while β^+ -decay involves the conversion of a proton to a neutron:



where ν_e and $\bar{\nu}_e$ refer to (electron) neutrinos and antineutrinos, respectively.

Electron capture also involves the conversion of a proton to a neutron:



while **positron capture** involves the conversion of a neutron to a proton:



Because neutrons and protons are the building blocks of atoms, the way that these reactions proceed – and the eventual balance between the number of neutrons and protons in the Universe – is important in determining the composition of matter. At the earliest times in the Universe, the weak interactions listed above could also proceed in the reverse direction but, as density decreased, the probability of neutrinos interacting with other particles became very low.

Thomson and Compton scattering

A wide range of scattering processes – in which particles exchange energy but do not bind together or change species – took place in the early Universe. These processes contribute to ensuring thermal equilibrium between different species, including photons.

The most important such process for the cosmological timeline is Thomson scattering, which was introduced in Chapter 1 and also discussed in Chapter 6. In this form of scattering, photons are deflected by the electromagnetic field of charged particles, such as electrons. Thomson scattering occurs in the limit where the photon energy is much lower than the mass energy of the particle ($h\nu \ll mc^2$), and the particle energy and photon frequency remain unchanged by the interaction.

The rate at which particles will undergo Thomson scattering is described by Equation 8.1. The cross-section for Thomson scattering depends inversely on the square of the mass of the scattering particle. So in an ionised plasma, scattering by *electrons* is the dominant process.

Figure 8.2 compares Thomson scattering, in panel (a), with a situation where the photon energy is comparable to the energy of the particle with which it is interacting ($h\nu \sim mc^2$; panel (b)). In the latter case, the particle can gain energy from the interaction and the photon frequency will decrease. This phenomenon is known as **Compton scattering**.

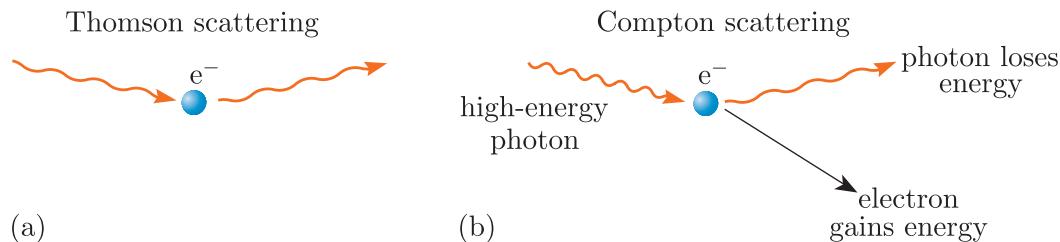


Figure 8.2 (a) Thomson scattering and (b) Compton scattering.

Finally, in situations where the particles have relativistic speeds, the reverse of the process shown in panel (b) can occur, causing photons to gain energy from the scattering process and increase their frequency. This is known as **inverse Compton scattering**, and is important in extreme environments in the present-day Universe, such as in the regions around black holes. However, it is not an important process for our discussion of the early Universe.

Nuclear fusion and photodisintegration

All of the elements in the present-day Universe formed via nuclear fusion. The majority of heavier elements were fused in stars, stellar mergers and explosions, but the fusion processes that operated in the very early Universe were the starting point for the composition of today's Universe.

The set of reactions by which fusion proceeded during the period known as **big bang nucleosynthesis** has some similarities to, but also some differences from, the fusion reaction chains that power the Sun and other stars. Both situations require high particle energies, enabled by high temperatures. But the presence of large numbers of free neutrons and photons in the early Universe, as well as the decrease in temperature as time elapsed, influenced how nucleosynthesis progressed.

The first important fusion reaction in the early Universe is the formation of deuterons, D^+ , which are nuclei containing one neutron and one proton:



Deuterons are a (heavier) form of hydrogen nuclei, and so are also sometimes written as ${}^2H^+$. You may previously have encountered deuterium, the atom consisting of a deuteron with a bound electron. This atom has an important use in nuclear reactors in the form of 'heavy water', D_2O , which is more dense than regular water and slows down the high-energy neutrons released by fission reactions, which serves to increase the likelihood of further fission reactions taking place.

Equation 8.8 shows the deuteron fusion reaction proceeding in both directions. Fusion reactions typically release energy, with the amount released corresponding to the binding energy of the reactants, as given by the difference in mass energy between the products and the reactants. However, as we saw with the dissociation of hadrons (e.g. Equation 8.3), if a *large* amount of energy is absorbed by a nucleus then it will break into its constituent parts once more. In other words, interactions with energetic photons can cause nuclei to disintegrate, a process known as **photodisintegration**. For a photon to cause a nucleus to disintegrate, its energy must be greater than the nuclear binding energy.

- How would you expect the balance between fusion reactions and photodisintegration to alter as the Universe expands?
- The density of radiation relative to matter decreases with time, so photodisintegration should become less important as the Universe expands.

Although nuclear fusion requires high energies, it could not proceed effectively at the very earliest times because of the high radiation density: nuclei that formed would rapidly break back down into their constituent hadrons. Fusion could only proceed once the photon density had dropped enough to reduce the typical photon energy. The following example quantifies this energy requirement in the context of a deuteron.

Example 8.1

Calculate the photon energy required to disintegrate a deuteron ($m_D = 3.3436 \times 10^{-27} \text{ kg}$) in units of J and MeV. Compare your findings to the interaction energy when the Universe had a temperature of $T = 10^{14} \text{ K}$: was deuteron photodisintegration feasible at this time?

Solution

In order for photodisintegration to take place, the incoming photon must have an energy greater than the binding energy of the atom. The binding energy of a nucleus is given by the **mass deficit**: the difference between the mass of the nucleus and that of the constituent particles.

The proton and neutron masses are $m_p = 1.6726 \times 10^{-27} \text{ kg}$ and $m_n = 1.6749 \times 10^{-27} \text{ kg}$. Therefore the mass deficit, Δm , is given by:

$$\Delta m = (m_p + m_n) - m_D = 3.9 \times 10^{-30} \text{ kg}$$

Using $E = \Delta m c^2$, we can calculate that this mass corresponds to a binding energy of $3.5 \times 10^{-13} \text{ J}$, which is $\sim 2.2 \text{ MeV}$.

We showed in Exercise 8.2 that at the time that the Universe's temperature was $T = 10^{14} \text{ K}$, the typical interaction energy was 8.6 GeV. Hence there was ample energy at that time for the photodisintegration of deuterons to occur.

Ionisation and recombination

The final pair of important processes to review are ionisation and recombination. Although ionisation can have multiple causes, it is photoionisation, which is caused by a particle absorbing a photon, that is of most importance in the early Universe. The ionisation of hydrogen can be written as



As earlier parts of this section have emphasised, the high radiation density and typical photon energy in the early Universe act to disrupt all types of interactions in which particles bind together. This is also the case for the binding of electrons to nuclei to form atoms. Hence when the first nuclei form, they exist as part of an ionised plasma containing oppositely charged nuclei and electrons; any atoms that form immediately dissociate again.

- How do local conditions determine the fraction of material that is ionised?
- For a predominantly hydrogen gas, the Saha equation (Equation 1.9 in Chapter 1) describes how the ratio of ionised protons and electrons to atoms depends on the temperature.

In the same way that photodisintegration decreases in importance as the Universe expands and photon energies decrease, so ionisation also becomes less favoured over time, and atoms can begin to form. In other words, the interaction described by Equation 8.9 comes to proceed mainly from right to left.

The binding of electrons to protons (and heavier nuclei) is known as recombination. This is a confusing term in the context of cosmology because the nuclei and electrons had not previously been combined. The term derives from experiments on plasmas, in which the nuclei and electrons were originally produced by ionising atoms, and it has become the accepted terminology in the study of the early Universe too, despite the potential for confusion.

8.2 A timeline towards the formation of atomic nuclei

The previous section reviewed the range of particle interactions of most importance for the evolution of matter and radiation in the early Universe. As time elapsed and the temperature of the Universe decreased, the dominant interactions changed. In this section we will set out a more detailed timeline, leading from the big bang to the point at which nuclear fusion became a significant process.

- Nuclear fusion reactions require extremely high temperatures. Why couldn't nuclear reactions take place immediately after the big bang?

- The high temperatures also meant there were very high densities of radiation. This first prevented quarks from forming protons and neutrons, and then prevented protons and neutrons from forming nuclei without immediately disintegrating.

It is therefore necessary to start at the beginning and follow the different stages by which particle composition evolved.

8.2.1 The very early Universe and inflation

The big bang itself, and conditions for the first fractions of a second afterwards, are very poorly understood. The **Planck time**, $t_P = 5.4 \times 10^{-44}$ s, is defined as the smallest time interval that can be measured, according to the theory of quantum mechanics. This means that we do not have any consistent physical theories that could be used to explain any processes occurring before t_P has elapsed.

Another reason that conventional theory fails at the very earliest times is because at the highest energies (well beyond the experimental conditions available for laboratory tests), the strong, weak and electromagnetic interactions are theorised to become a single interaction. Theories that explain this behaviour are known as **grand unified theories** (or GUTs), but a generally accepted GUT does not currently exist.

The idea of grand unification is motivated by the experimentally verified theory of **electroweak unification**, in which the weak and electromagnetic forces can be described as a single interaction at energies above ~ 250 GeV. The energy scale on which grand unification is predicted to occur is very much larger: $\sim 10^{16}$ GeV. When the age of the Universe was about 10^{-36} s, the high-energy conditions under which the strong and electroweak interactions are thought to have been unified came to an end.

Although the typical interaction energies at that time were far in excess of those in laboratory experiments, physicists do have a theoretical framework within which some predictions about this era of cosmic history can be made. It is predicted that for the brief period known as **inflation**, the scale factor of the Universe, $a(t)$, expanded exponentially (rather than more gradually, as measured observationally for much later times).

The theory of inflation was proposed by the physicist Alan Guth, and relates to the quantum field behaviour of empty space (the vacuum), which is proposed to have a non-zero associated energy. This idea of the energy density of empty space also underpins many theories of dark energy, to explain the acceleration of the Universe's expansion at the present time.

The inflationary era is thought to result from the physical state of empty space at the end of grand unification. These conditions allowed a much higher energy density of the vacuum than there is at the present day, leading to a dramatic, rapid expansion of space. Figure 8.3 provides an example of how the scale factor, a , *could* have evolved in the very early Universe.

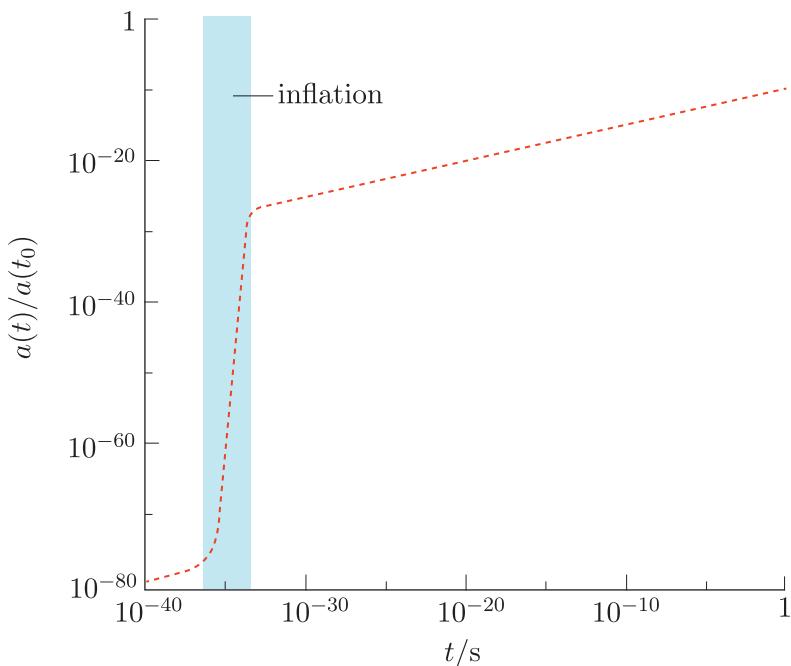


Figure 8.3 Possible evolution of the scale factor with time, including a period of inflation. Note that the numerical values shown here are not well determined.

The idea that space could expand by many tens of orders of magnitude in a fraction of a second does not immediately seem very plausible. But the reason some form of inflation is considered such a promising theory is that it resolves several hard-to-explain challenges in cosmology. These ideas will be discussed later in the module, but here we will move on to focus on the (much better-understood) physics of the next stages of the early Universe timeline.

As the period of inflation came to an end, the energy of the vacuum must have dropped to the level we see today. As the vacuum transitioned to its ‘true vacuum’ state (a more stable, lower energy state), energy was released and pair production took place. This process produced all types of quarks, antiquarks, leptons and antileptons. According to the inflationary model, the vast majority of particles in the Universe were created from the energy released as inflation came to an end.

When the temperature dropped sufficiently, the electromagnetic and weak interactions became separate; prior to this, there was no distinction in the mediating bosons. After this ‘electroweak transition’, weak and electromagnetic interactions proceeded as they do in present-day lab conditions, mediated respectively by W and Z bosons, and by photons. The constituents of the Universe immediately after this time continued to be all types of quarks and leptons and their antiparticles, as well as photons.

8.2.2 Neutrino decoupling and the baryon-to-photon ratio

When the temperature of the Universe dropped further, around 10^{-5} seconds after the big bang, quarks were able to become bound into hadrons without the process being disrupted. Although many types of hadron were formed in this way, only two were stable enough to have had a long-lasting effect on the composition of the Universe: the proton and the neutron. Both of these particles are baryons (i.e. they each contain three quarks).

The process by which the early Universe came to contain large numbers of baryons (and not antibaryons) is known as baryogenesis. How an imbalance was created – so that some fraction of the quarks that originally formed did *not* subsequently annihilate with antiquarks – is not fully understood, and must require physics beyond the Standard Model; some such theories associate baryogenesis with the GUT era, while others associate it with the separation of the weak and electromagnetic interactions.

At the start of what could be termed the hadron era, hadrons, leptons and photons were all in thermal equilibrium and undergoing frequent interactions and exchanges of energy. The interactions that were important for this thermal equilibrium were neutrino scattering processes involving electrons and positrons; for example, $\nu_e + \bar{\nu}_e \rightleftharpoons e^- + e^+$. As suggested by Equation 8.1, the rate of these scattering interactions, Γ_ν , is related to the number density of neutrinos, which decreases faster than the Hubble parameter during the radiation-dominated era.

When the interaction energy dropped to ~ 2.5 MeV, the neutrinos decoupled from the other particles. Essentially, the Universe became transparent to these primordial neutrinos, which could travel for billions of years without undergoing an interaction.*

Shortly after neutrino decoupling, when the age of the Universe was about 1 second, another important change took place: electrons and positrons have a mass energy of 0.511 MeV, and so when the interaction energy dropped to ~ 1 MeV the production of e^\pm pairs was no longer energetically favourable. As the temperature fell further, electrons and positrons began to disappear because e^\pm pairs were no longer being created, but the annihilation reaction ($e^+ + e^- \rightarrow \gamma + \gamma$) continued. As a result, the number of electrons and positrons decreased dramatically.

- How would the rapid annihilation of large numbers of electron–positron pairs affect the other constituents of the Universe at that time?

*This implies that there should be a cosmic neutrino background, analogous to the CMB. It would be another useful test for cosmological theory and early-Universe physics if we could observe such a background but, unfortunately, the very low interaction cross-section for neutrinos means that this is beyond the capability of current detectors.

- The e^\pm annihilation process produces large numbers of photons. The baryons, remaining electrons and radiation are in thermal equilibrium at this time, and so energy will be shared between them; the neutrinos will not gain any energy because they are no longer coupled to the other particles (including photons).

It is at this time – still only around 1 second after the big bang – that the value of a crucial cosmological parameter was set: the **baryon-to-photon ratio**, η . The nuclear reactions that form helium and heavier elements – and so the cosmic abundances that we can measure to test theories of early-Universe physics – all depend on this parameter.

The baryon-to-photon ratio

$$\eta = \frac{n_b}{n_\gamma} \quad (8.10)$$

where n_b and n_γ are the number densities of baryons and photons, respectively. Because the volumes over which the particles are distributed are the same, η is equivalently just the ratio of the total *numbers* of baryons to photons.

The photon density of the Universe is dominated by CMB photons – starlight and other photon-producing processes cannot compete with the number of photons produced in the early Universe. Hence n_γ can be measured from the present-day CMB. The baryon density is harder to measure, because matter in the present-day Universe is concentrated into galaxies and clusters of galaxies with large, quite empty voids between them. Nevertheless, n_b can be estimated reasonably well by observations on the scale of galaxy clusters and larger.

Example 8.2

Currently, the temperature of the CMB is 2.73 K and the baryon density is around 0.3 baryons per cubic metre. Determine the present-day number density of photons, and so estimate the value of η . (*Hint:* for a black-body distribution of photons, the mean photon energy is $E_\gamma \approx 3k_B T$.)

Solution

We will first determine the energy density of CMB radiation, ϵ .

The energy density of a relativistic gas is related to its pressure, $P = (1/3)\epsilon$, and via Equation 1.7 (in Chapter 1) to its temperature, $\epsilon = aT^4$, where a is the radiation constant. Hence at the present time:

$$\epsilon = (7.566 \times 10^{-16} \text{ J m}^{-3} \text{ K}^{-4}) \times (2.73 \text{ K})^4 = 4.20 \times 10^{-14} \text{ J m}^{-3}$$

The typical number of photons per cubic metre is given by $n_\gamma = \epsilon/E_\gamma$, where E_γ is the typical photon energy. Using the approximation

$E_\gamma \approx 3k_B T$, we therefore find that:

$$\begin{aligned} n_\gamma &= \epsilon/E_\gamma \\ &= (4.20 \times 10^{-14} \text{ J m}^{-3})/(3 \times 1.381 \times 10^{-23} \text{ JK}^{-1} \times 2.73 \text{ K}) \\ &\approx 3.7 \times 10^8 \text{ m}^{-3} \end{aligned}$$

In other words there are, on average, more than a third of a billion CMB photons per cubic metre. Given the present baryon density of $\sim 0.3 \text{ m}^{-3}$, the baryon-to-photon ratio, η , can be estimated at 8.1×10^{-10} , or $\sim 10^{-9}$.

As the example above demonstrates, it is estimated that at the present day there are approximately 1 billion photons for every baryon.

- Does η evolve with cosmic time? Briefly explain your answer.
- No. Baryon number is conserved in all particle interactions (see Section 8.1.2), and although photon production and annihilation does take place in stars and galaxies, the numbers are negligible compared with the total number of CMB photons produced in the early Universe.

The baryon-to-photon ratio must also be directly related to the baryon density parameter, Ω_b , which, as you have seen in previous chapters, can be constrained by observations of the CMB. The relationship to the present-day density parameter can be written as

$$\eta = \frac{\rho_{c,0}\Omega_{b,0}}{\langle m \rangle n_{\gamma,0}} \quad (8.11)$$

where $\rho_{c,0}$ is the present-day critical density and $\langle m \rangle$ is the mean particle mass. This relation is slightly sensitive to other cosmological parameters, because the mean baryon mass in the present-day Universe – although very close to the proton mass – is affected by the nucleosynthesis processes that occur a little later in the history of the Universe.

8.2.3 The neutron-to-proton ratio

Both the decoupling of neutrinos and electron–positron annihilation had a substantial effect on the range of weak interactions that could take place in the first seconds after the big bang. When all particles were abundant and in thermal equilibrium, neutrons could convert to protons and vice versa by interacting with neutrinos, antineutrinos, electrons and positrons, according to the following equilibrium reactions:



The ratio of neutrons to protons, n_n/n_p , turns out to be a crucial parameter for cosmic nucleosynthesis. Its value at early times is set by statistical equilibrium between the interactions of Equations 8.12 and 8.13. The probability p that particles at equilibrium will occupy a given state within these reactions depends on the energy of the state and the

gas conditions via the **Boltzmann factor**:

$$p_x \propto \exp\left(-\frac{E_x}{k_B T}\right) \quad (8.14)$$

where x represents a particular particle state. For example, if p_x is the probability that the particle is in the state of being a neutron, then E_x is the rest-mass energy of a neutron, $m_n c^2$.

The ratio of neutrons to protons in a situation of statistical equilibrium is therefore given by:

$$\frac{n_n}{n_p} = \exp\left[-\frac{(m_n - m_p)c^2}{k_B T}\right] \quad (8.15)$$

This expression may look familiar: the Saha equation describing ionisation fractions introduced in Chapter 1 is derived in a similar way. You can now test your understanding of it by completing Exercise 8.4.

Exercise 8.4

Show that the neutron-to-proton ratio is $\sim 1/5$ immediately after electron–positron annihilation (at $t \sim 1$ s), when the interaction energy $E \approx 0.8$ MeV.

When far fewer electrons and neutrinos were available for frequent interactions, the neutron-to-proton ratio no longer evolved with temperature according to Equation 8.15. Instead, the only process capable of changing baryon type was the β^- -decay of free neutrons (Equation 8.4). Therefore, the neutron-to-proton ratio slowly decreased as neutrons were converted to protons. The neutron decay lifetime, $\tau_n = 887$ s, is therefore an important cosmological parameter, and the total number N of neutrons in the Universe evolved according to:

$$N(t) = N(t_{\text{init}}) \exp\left(-\frac{t}{\tau_n}\right) \quad (8.16)$$

where t_{init} is taken as a starting time at which N is known.

Exercise 8.5

If free-neutron decay continued indefinitely, calculate the age of the Universe for which the neutron-to-proton ratio would reach a value of $n_n/n_p \approx 0.05$ (i.e. 1 neutron for every 20 protons), assuming $n_n/n_p = 1/5$ at $t = 1$.

(Hint: you may make the simplifying assumption that the number of protons remains constant, because there are already very many more neutrons than protons present when the decay starts.)

8.3 Big bang nucleosynthesis

We have now reached the point in the history of the Universe where nuclear fusion processes become important. This stage is known as big bang nucleosynthesis (or **primordial nucleosynthesis**), and it is thought to have taken place over a timescale of around 20 minutes.

The physics of big bang nucleosynthesis explains why the Universe is composed primarily of hydrogen and helium, and also provides astronomers with some direct tests of conditions of early times. The key parameters we considered in the previous section, η and n_n/n_p , strongly influence what happened next.

8.3.1 Deuterium formation

The first nuclear fusion reaction to occur in the Universe was briefly introduced in Section 8.1.2: deuterium nuclei (deuterons) formed via the fusion of a neutron and a proton, which also resulted in the release of energy (Equation 8.8). In that section we saw that the competing process of photodisintegration also plays an important role at early times: deuterons can only begin to form in large numbers when the temperature drops sufficiently to suppress photodisintegration.

Example 8.1 showed that photons of energy $\sim 2.2 \text{ MeV}$ are needed to disintegrate deuterons, which might suggest that by the time of electron–positron annihilation ($k_B T \approx 0.8 \text{ MeV}$), fusion could proceed effectively. However, we have also seen that the density of photons relative to baryons is of the order 10^9 ; this means that we need to consider the *distribution* of photon energies more carefully. Even if only a small fraction of the photons have sufficient energy to cause photodisintegration, this quantity might be enough to disrupt a large proportion of fusion interactions.

The distribution of photons according to frequency follows a black-body spectrum (see Section 1.3.2 in Chapter 1, and Figure 6.4). The number density of black-body photons with frequencies between ν and $\nu + d\nu$ is given by:

$$n(\nu, T) d\nu = \frac{8\pi}{c^3} \frac{\nu^2}{\exp(h\nu/k_B T) - 1} d\nu \quad (8.17)$$

Calculating the number of photons in the high-energy tail of the distribution requires integration of this function, which is not algebraically trivial; the following example uses results from numerical integration.

Example 8.3

Figure 8.4 plots the fraction of photons with energy greater than 2.2 MeV in a black-body distribution (i.e. $n(E > 2.2 \text{ MeV})/n_{\text{tot}}$) as a function of temperature.

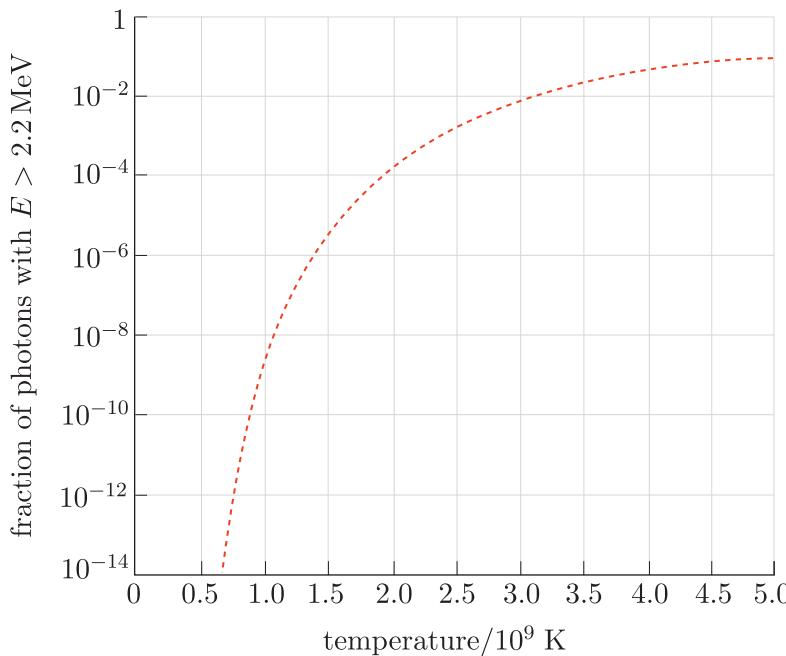


Figure 8.4 Fraction of black-body photons with $E > 2.2 \text{ MeV}$ as a function of temperature.

- (a) Based on this graph, could nuclear fusion proceed effectively immediately after electron–positron annihilation (when the interaction energy $E_{\text{int}} \approx 0.9 \text{ MeV}$)?
- (b) If not, then to what level must E_{int} drop before deuterons could survive?

Solution

- (a) In order for deuterons to survive, the number of photons energetic enough to disintegrate them must be fewer than the number of baryons. In other words,

$$n_{\gamma > 2.2} < n_b$$

where $n_{\gamma > 2.2}$ is the number density of photons with energy above 2.2 MeV .

If we divide both sides by the total number density of photons, n_{tot} , and take $f = n(E > 2.2 \text{ MeV})/n_{\text{tot}}$ (as plotted in Figure 8.4) to be the fraction of photons with energy above 2.2 MeV , then the condition for deuteron survival becomes:

$$f < \eta$$

In other words, the fraction of sufficiently energetic photons must be lower than the baryon-to-photon ratio.

An interaction energy of $E_{\text{int}} = 0.9 \text{ MeV}$ corresponds to a temperature of $T = E/k_B = 10^{10} \text{ K}$. This temperature is well beyond the right-hand edge of the graph, so it is clear that the value of f at this temperature will be higher than that for the highest plotted

temperature of 5×10^9 K, where $f \rightarrow 0.1$. In Example 8.2 you saw that $\eta \approx 10^{-9}$ so the condition for deuteron survival is not met, i.e. deuteron fusion cannot proceed effectively immediately after electron–positron annihilation.

- (b) Deuteron fusion can only proceed when $f < \eta$. The graph shows that f is lower than $\eta \approx 10^{-9}$ at temperatures $T < 1 \times 10^9$ K. This corresponds to an interaction energy $E_{\text{int}} \approx 0.09$ MeV.

The previous example has demonstrated why η is such an important parameter for how the early Universe evolved. If the value of η was substantially different then the start of nucleosynthesis would have occurred at a different temperature, and so significantly earlier or later. You may wish to explore further the distributions of photon energy at different temperatures via the following exercise.

Exercise 8.6

This is an *optional* Python exercise.

By adapting examples of numerical integration from the Python toolkit or earlier week-long practical activities, calculate the fraction of photons with $E > 2.2$ MeV for temperatures of $T = 10^9$ K and $T = 5 \times 10^9$ K, and check that your results are in agreement with Figure 8.4.

(*Hint:* to estimate n_{tot} you can assume that a reasonable upper limit for photon energy is 20 MeV.)

The overall impact of photodisintegration was to delay the onset of fusion and the formation of deuterons. The first nuclei formed at a lower temperature than would have been the case if the photon density was lower. This had an important knock-on effect for the subsequent evolution of matter. Further nuclear reactions – to form heavier elements such as helium – could only proceed once a supply of deuterons was available, so this stage of nucleosynthesis is referred to as the **deuterium bottleneck**.

- How would the subsequent evolution of matter in the Universe have been affected if deuteron formation happened at a higher or lower temperature?
- All nuclear reactions are strongly temperature-dependent, so further reactions – and the length of time over which they could operate – would differ significantly had the bottleneck occurred under different conditions. This would have the effect of altering relative abundances of elements.
- Which process in the early Universe could have resulted in a substantially different value of η , and hence a different timeline for nucleosynthesis?

- The poorly understood process of baryogenesis determined the number density of baryons in the early Universe, so strongly influenced the timeline of nucleosynthesis.

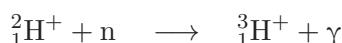
8.3.2 Helium formation

There is copious observational evidence that the Universe contains a large quantity of helium (He) – indeed the name of the element comes from its initial discovery in the outer regions of the Sun (in Greek mythology, Helios is the god of the Sun). Although helium is produced in stars, by far the bulk of the He in the present-day Universe is understood to have been synthesised in the first few hundred seconds of the early Universe.

Reaction pathways towards helium

There were multiple routes to helium production in the early Universe, with two different isotopes initially being produced: $^3_2\text{He}^{2+}$ and $^4_2\text{He}^{2+}$. The dominant processes are set out below. Note that, as indicated by the charges on each species, it is important to recall that here we are discussing the fusion of *nuclei* rather than atoms, because it is still too hot at this time for electrons to bind to the nuclei.

We can start by considering the formation from deuterium ($^2_1\text{H}^+$) of an even heavier isotope of hydrogen known as tritium ($^3_1\text{H}^+$) via one of the following reactions:



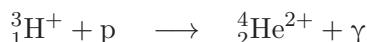
or



This newly formed tritium could then undergo further reactions to form helium-4:



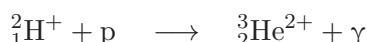
and



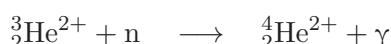
Helium-3 could also be formed from the same ingredients as formed tritium:



and



These helium-3 nuclei can then further react to form helium-4:



and



If you have previously studied nuclear fusion in stars then you may have noticed that the reactions to form helium in the early Universe differ from the proton–proton chain operating in the core region of the Sun and other stars. The reason is the high abundance of free neutrons in the early Universe (unlike in the centre of stars), which provide an additional – and in some cases easier – route to the assembly of heavier isotopes and nuclei.

- Why is it easier for a proton or a deuterium nucleus to fuse with a neutron, rather than with another identical proton or deuterium nucleus?
- Protons and deuterium nuclei are both positively charged, and so fusion reactions require the electromagnetic (Coulomb) repulsion between them to be overcome; it requires less energy for a proton or deuteron to get sufficiently close to a free neutron (if present) to fuse than for it to get close to another proton or deuteron.

Helium abundance and n_n/n_p

We have seen that the presence of free neutrons is crucial for the reaction pathways involved in fusion into helium. As a result of the reactions described in this section, almost all of the free neutrons available at early times became locked into helium. This means that the cosmic abundance of helium is closely linked to the neutron-to-proton ratio at the time of nucleosynthesis.

Let's consider the **mass fraction** of helium, Y , for a representative sample of the Universe (i.e. a region large enough to have a typical composition). Y is defined as the fraction of mass contained as helium relative to the total mass of baryons. We will see shortly that the contribution of elements heavier than helium to the total mass is very small, and so the total baryonic mass can be approximated as the sum of all of the hydrogen and helium within the region. Finally, we can make the simplifying assumption that all of the helium is helium-4.

The mass fraction of helium can then be estimated as

$$Y = \frac{N_{\text{He}}m_{\text{He}}}{N_{\text{H}}m_{\text{H}} + N_{\text{He}}m_{\text{He}}} \quad (8.18)$$

where N_{He} and N_{H} are the numbers of helium and hydrogen nuclei, and m_{He} and m_{H} are the masses of helium and hydrogen nuclei.

The next example examines the influence of the neutron-to-proton ratio on helium abundance.

Example 8.4

By expressing Equation 8.18 in terms of the numbers of protons and neutrons, show that the helium mass fraction in the Universe, Y , depends only on the neutron-to-proton ratio, n_n/n_p , and calculate the value of Y assuming that n_n/n_p is 0.16 at the start of nucleosynthesis.

Solution

We can start by making the simplifying assumption that $m_{\text{He}} = 4m_{\text{H}}$, which means Equation 8.18 can be written as

$$Y = \frac{N_{\text{He}}4m_{\text{H}}}{N_{\text{H}}m_{\text{H}} + N_{\text{He}}4m_{\text{H}}} = \frac{4N_{\text{He}}}{N_{\text{H}} + 4N_{\text{He}}} \quad (8.19)$$

We can now relate the numbers of helium and hydrogen nuclei to the numbers of protons and neutrons. Here we need to make another simplifying assumption, which is that all of the deuterium and tritium produced reacted to form helium-4. This allows us to assume that all of the neutrons are in helium, so that the number of helium nuclei will be half the number of neutrons because there are two neutrons in each helium nucleus (i.e. $N_{\text{He}} = N_{\text{n}}/2$).

Then the number of hydrogen nuclei will be the total number of protons minus the number of protons locked up in the helium nuclei, so

$$N_{\text{H}} = N_{\text{p}} - 2N_{\text{He}} = N_{\text{p}} - N_{\text{n}}.$$

Substituting in the expressions we've derived for N_{H} and N_{He} , we obtain:

$$Y = \frac{2N_{\text{n}}}{N_{\text{n}} + N_{\text{p}}}$$

To find an expression in terms of the neutron-to-proton ratio we can divide both the numerator and denominator by N_{n} to get:

$$Y = 2 \left(\frac{1}{1 + N_{\text{p}}/N_{\text{n}}} \right) = 2 \left(\frac{1}{1 + n_{\text{p}}/n_{\text{n}}} \right)$$

where the final step involved replacing the ratio of the total number of neutrons and protons with the (equivalent) ratio of their number densities.

Finally, using the given value of $n_{\text{n}}/n_{\text{p}} = 0.16$, we find that $Y = 0.28$, so our calculation predicts that helium should make up 28% of the baryonic mass of the Universe.

It is a remarkable success of the big bang model that the observed helium abundance in interstellar gas ($\sim 25\%$) is in good agreement with the predictions of early-Universe physics and big bang nucleosynthesis. In the next chapter we will compare predictions and observations of cosmic abundances more carefully.

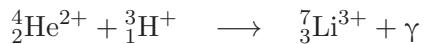
8.3.3 Formation of heavier nuclei

The major product of primordial nucleosynthesis was helium-4. The fact that nucleosynthesis did not progress further and produce large quantities of nuclides with higher mass numbers is due to two factors.

First, the rate at which two nuclei will fuse together depends very strongly on the temperature, and higher temperatures are required to fuse nuclei of higher atomic number. Because deuterons are easily photodisintegrated, the process of nucleosynthesis could only start once the deuterium bottleneck had been overcome, by which time the temperature of the

Universe was relatively low. As a consequence, the rate of fusion reactions that involved nuclides other than hydrogen and helium would have been very low.

The second factor is the lack of any stable nuclides with mass numbers of 5 or 8, which meant that helium-4 could not react either with itself or with the two most abundant species – protons and neutrons. This hurdle could, however, be overcome by reactions involving tritium or helium-3:



although because it is highly unstable, ${}^7_4\text{Be}^{4+}$ decays quickly to lithium-7:



Lithium nuclei can also undergo fusion with protons, but the result is the destruction of the lithium nucleus and the formation of two nuclei of helium-4:



The quantity of lithium produced from the above reactions (the **yield**) was small – the proportion of baryonic matter made up of lithium, or the lithium mass fraction, is $\sim 10^{-9}$, justifying the decision immediately prior to Example 8.4 to neglect heavier elements in our estimation of helium abundance! Nevertheless, predictions for the abundance of lithium are astrophysically interesting, and we will discuss in the next chapter how they have been used to test conditions in the early Universe.

8.4 Summary of Chapter 8

- The big bang model predicts that the early Universe was extremely hot and dense. At the very earliest times, conditions were sufficiently extreme that current physical theories (e.g. the Standard Model and quantum field theory) cannot fully describe them. A (poorly understood) brief period of **inflation** is thought to have taken place during which the Universe expanded rapidly.
- A ‘soup’ of many different particle types was present after inflation; the particles (e.g. quarks, antiquarks, leptons, antileptons, photons, gluons, and W and Z bosons) initially interacted very frequently with each other.
- The temperature and density of the Universe (for both matter and radiation) decreased with time, with the types of particle interaction taking place determined by the **interaction energy**, $E_{\text{int}} \approx k_B T$ (Equation 8.2), as well as the relative quantities of different types of particle.

- Important processes included:
 - **pair production** (of quarks and leptons) and **annihilation**
 - hadron production (binding of quarks via the strong force)
 - **beta decay**
 - **electron capture and positron capture**
 - Thomson scattering
 - a variety of nuclear fusion reactions
 - the **photodisintegration** of nuclei.
- The sequence of important particle interactions is summarised in Figure 8.5 (overleaf).
- The **reaction rate** for a particle of a particular type is given by $\Gamma = n\sigma v$ (Equation 8.1), where n is the number density of the target particle, σ is the **interaction cross-section**, and v is the relative speed of the interacting particles.
- Interaction rates typically decline as the Universe expands, and so the time at which a particular interaction becomes improbable can be estimated by equating Γ and the Hubble parameter, $H(t)$.
- Thermal equilibrium between different particle species was first broken when neutrinos **decoupled** from the other particles in the cosmic soup, which occurred after hadron production and was quickly followed by rapid annihilation of electrons and positrons.
- Neutrino decoupling and electron–positron annihilation set the value of the **baryon-to-photon ratio**, η , which is related to the present-day baryon density parameter via:

$$\eta = \frac{\rho_{c,0}\Omega_{b,0}}{\langle m \rangle n_{\gamma,0}} \quad (\text{Eqn 8.11})$$

- The baryon-to-photon and neutron-to-proton ratios had an important influence on the process of **big bang** or **primordial nucleosynthesis**, controlling how deuterium production evolved.
- Once a significant quantity of deuterons had been fused (the **deuterium bottleneck**), helium production rapidly consumed all of the remaining free neutrons. A small amount of lithium and beryllium were produced before the Universe became too cool to provide sufficient interaction energy for further nuclear reactions.

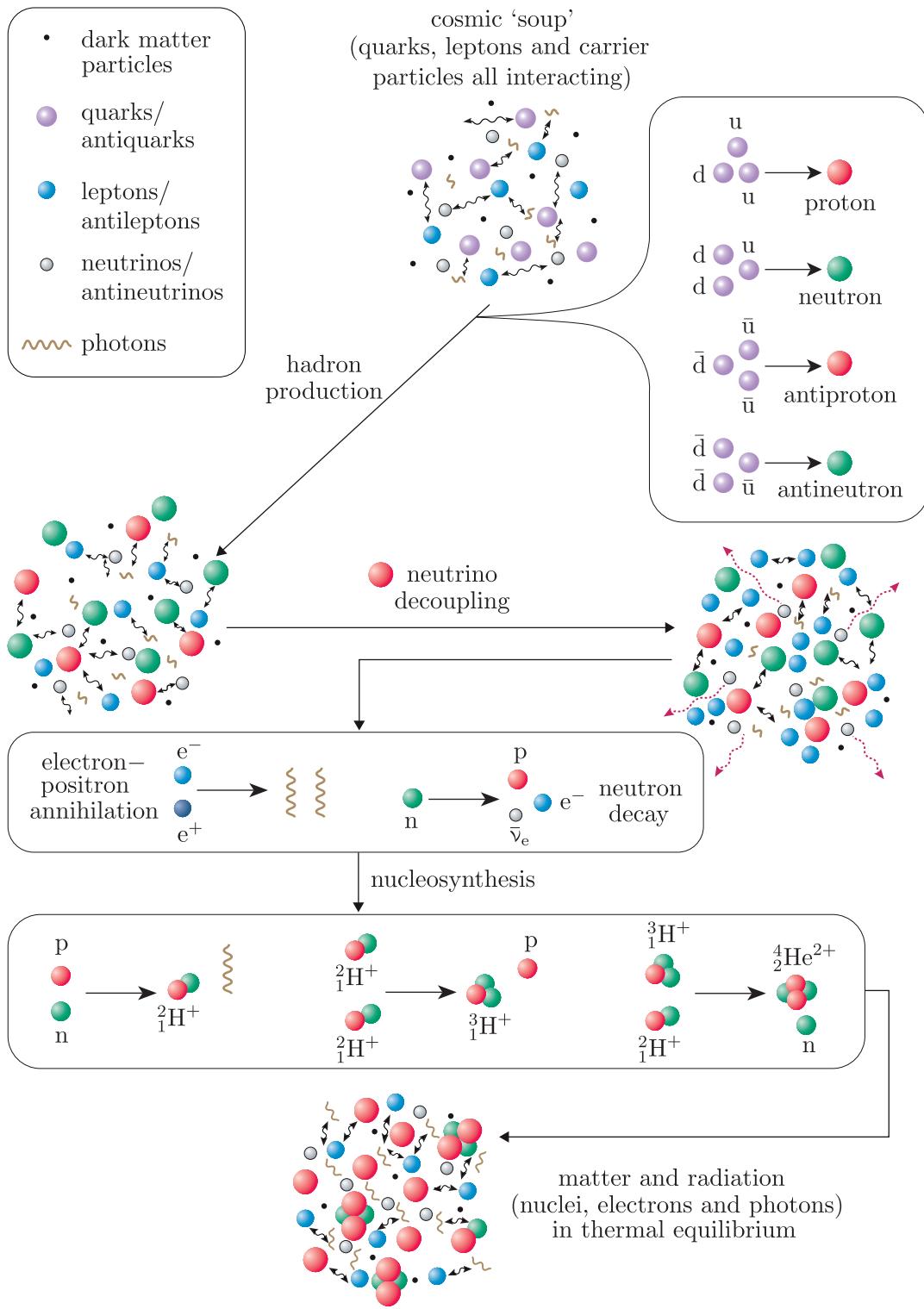


Figure 8.5 The evolution of the main particle species over the first 1000 seconds of the Universe's timeline. Note that this representation is not intended to show accurate relative quantities for different species (and does not include all the types of particles present immediately after the big bang).