

BEISPIEL MARKOV - CHEBYCHEV

Sei X_i die Indikatorvariable s.d.

$X_i = 1$ wenn der i -te Wurf 6 zeigt.

$$X = \sum_{i=1}^n X_i$$

wir zählen Anzahl 6
 \Rightarrow Binomial

$$X \sim \text{Bin}(n, 1/6)$$

$$\mathbb{E}[X] = n/6, \quad \text{Var}[X] = 5n/36$$

(a) $W_X \subseteq \mathbb{R}_{\geq 0}$ thus

$$\Pr[X \geq 2n/9] \leq \frac{\mathbb{E}[X]}{2n/9} = 9/12 = 3/4$$

$$(b) \{X \geq 2n/9\} \Rightarrow \{|X - n/6| \geq n/18\}$$

$\Leftarrow \downarrow$ other direction is wrong!

$$\Pr[|X - \mathbb{E}[X]| \geq n/18] \leq \frac{\text{Var}[X]}{(n/18)^2} = \frac{5n \cdot 18^2}{36n^2} = \frac{95}{n}$$

(c) Let X_i be the indicator variable s.d.

$X_i = 1$ when the i -th roll shows 6.

$$X = \sum_{i=1}^n X_i \quad \checkmark$$

$$\checkmark \quad \mathbb{E}[X] = n/6$$

$$X \geq 2n/9 \Leftrightarrow X \geq (1+\delta)\mathbb{E}[X]$$

with $\delta = 1/3$ $0 \leq \delta \leq 1$, thus

$$\begin{aligned} \Pr[X \geq 2n/9] &\leq e^{-\frac{1}{3} \delta^2 \mathbb{E}[X]} \\ &= e^{-\frac{n}{3 \cdot 9 \cdot 6}} = e^{-n/162} \end{aligned}$$