# Notes on 'Linear Algebra, by Serge Lang'

Ghassan Shahzad

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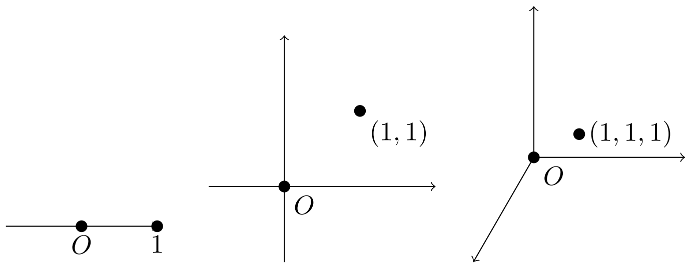
# 1 Vectors

**Definition 1.1.** A **co-ordinate** represents a point on a **line**, which we can also call 1-space. A pair of co-ordinates represent a point on a **plane**, or 2-space. A trio of co-ordinates represent a point on 3-space, and so on, such that a point with $n$ co-ordinates (what we call an $n$-tuple) is a point on $n$**-space**. An $n$-tuple can be represented as $(x_1, x_2, x_3, ..., x_n)$, and we denote an $n$-space as $\mathbb{R}^n$.

**Example 1.1.** A point on 1-space, 2-space, and 3-space respectively.




**Definition 1.2.** We can say that higher dimensional spaces are **products** of lower dimensional spaces, the result of putting them side-by-side.

**Example 1.2.** $\mathbb{R}^3 = \mathbb{R}^2 \cdot \mathbb{R}^1$

**Definition 1.3.** To add two points, we sum each corresponding co-ordinate in each, and expresss the results as a point. For instance, if $A$ and $B$ are two points in $n$-space, their sum would be: $A + B = (a_1 + b_1, a_2 + b_2, ..., a_n + b_n)$. In addition operations involving points, the following properties are observed:

1. **Associativity:** $(A + B) + C = A + (B + C)$
2. **Commutativity:** $A + B = B + A$
3. **Identity Element:** If $O$ is the origin point, $A + O = A$
4. **Inverse Element:** Let $A = (a_1, ..., a_n)$ and $-A = (-a_1, ..., -a_n)$. Then $A+(-A) = O$

**Example 1.3.** Let the points $A = (1, 3, 5)$, $B = (-3, 4, 9)$, $C = (4, 5, 9)$, and $O = (0, 0, 0)$. The previously defined properties can be demonstrated:

1. **Associativity:**

$$\begin{aligned}(A + B) + C &= (-2, 7, 14) + (4, 5, 9)\\ &= (1, 3, 5) + (1, 9, 18)\\ &= (2, 12, 23)\end{aligned}$$

2. **Commutativity:**

$$\begin{aligned}B + A &= (-3 + 1, 4 + 3, 9 + 5)\\ &= (-2, 7, 14)\\ &= A + B\end{aligned}$$

3. **Identity Element:**

$$\begin{aligned}A + O &= (1, 3, 5) + (0, 0, 0)\\ &= (1, 3, 5)\\ &= A\end{aligned}$$

**4. Inverse Element:**

$$\begin{aligned} A + (-A) &= (1 - 1, 3 - 3, 5 - 5) \\ &= (0, 0, 0) \\ &= O \end{aligned}$$

**Definition 1.4.** Representing the addition of vectors geometrically will result in a parallelogram.

**Example 1.4.**