# A Review of Image Segmentation Techniques Integrating Region and Boundary Information

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## I. Introduction

One of the first and most important operations in image analysis and computer vision is segmentation (R. Haralick and R. Shapiro. 1992–1993; Rosenfeld and Kak, 1982). The aim of image segmentation is the domain-independent partition of the image into a set of regions which are visually distinct and uniform with respect to some property, such as gray level, texture, or color. Segmentation can be considered the first step and key issue in object recognition, scene understanding, and image understanding. Its application areas vary from industrial quality control to medicine, robot navigation, geophysical exploration, military applications, and so forth. In all these areas, the quality of the final results depends largely on the quality of the segmentation.

The problem of segmentation has been, and still is, an important research field, and many segmentation methods have been proposed in the literature (Fu and Mui, 1981; R. M. Haralick and L. G. Shapiro, 1985; Nevatia, 1986;



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Pal and Pal, 1993; Riseman and Arbib, 1977; Zucker, 1977). In general, segmentation methods are based on two basic properties of the pixels in relation to their local neighborhood: discontinuity and similarity. Methods that are based on some discontinuity property of the pixels are called *boundary-based methods,* whereas methods based on some similarity property are called *region-based methods*. More specifically,

- The boundary approach uses the postulate that abrupt changes occur with regard to the features of the pixels (e.g., abrupt changes in gray values) at the boundary between two regions. To find these positions, one can choose from two basic approaches: first- and second-order differentiation. In the first case, a gradient mask (Roberts, 1965, and Sobel, 1970, are well-known examples) is convolved with the image to obtain the gradient vector $\nabla f$ associated with each pixel. Edges are the places where the magnitude of the gradient vector $\|\nabla f\|$ is a local maximum along the direction of the gradient vector $\phi(\nabla f)$. For this purpose, the local value of the gradient magnitude must be compared with the values of the gradient estimated along this orientation and at unit distance on either side away from the pixel. After this process of nonmaxima suppression takes place, the values of the gradient vectors that remain are thresholded, and only pixels with a gradient magnitude exceeding the threshold are considered as edge pixels (Petrou and Bosdogianni, 1999). In the second-order derivative class, optimal edges (maxima of gradient magnitude) are found by searching for places where the second derivative is zero. The isotropic generalization of the second derivative to two dimensions is the Laplacian (Prewitt, 1970). However, when gradient operators are applied to an image, the zeros rarely fall exactly on a pixel. It is possible to isolate these zeros by finding zero crossings: places where one pixel is positive and a neighbor is negative (or vice versa). Ideally, edges of images should correspond to boundaries of homogeneous objects and object surfaces.
- The region approach tries to isolate areas of images that are homogeneous according to a given set of characteristics. Candidate areas may be grown, shrunk, merged, split, created, or destroyed during the segmentation process. There are two typical region-based segmentation algorithms: region-growing and split-and-merge algorithms. Region growing (Adams and Bischof, 1994; Zucker, 1976) is one of the most simple and popular algorithms and it starts by choosing a starting point or seed pixel. Then, the region grows by adding neighboring pixels that are similar, according to a certain homogeneity criterion, which increases the size of the region step by step. Typical split-and-merge techniques (Chen and Pavlidis, 1980; Fukada, 1980) consist of two basic steps. First, the whole image is considered as one region. If this region does not comply with a homogeneity

criterion, the region is split into four quadrants and each quadrant is tested in the same way until every square region created in this way contains homogeneous pixels. Next, in a second step, all adjacent regions with similar attributes may be merged upon compliance with other criteria.

Unfortunately, both techniques, boundary-based and region-based, often fail to produce accurate segmentation, although the locations where they fail are not necessarily identical. On the one hand, in boundary-based methods, if an image is noisy or if its region attributes differ by only a small amount between regions, characteristics very common in natural scenes, edge detection may result in spurious and broken edges. This occurs mainly because such methods rely entirely on the local information available in the image; very few pixels are used to detect the desired features. Edge-linking techniques can be employed to bridge short gaps in such a region boundary, although doing so is generally considered an extremely difficult task. On the other hand, region-based methods always provide closed-contour regions and make use of relatively large neighborhoods in order to obtain sufficient information to allow the algorithm to decide whether to aggregate a pixel into a region. Consequently, the region approach tends to sacrifice resolution and detail in the image to gain a sample large enough for the calculation of useful statistics for local properties. This sacrifice can result in segmentation errors at the boundaries of the regions and in failure to distinguish regions that would be small in comparison with the block size used. Further, in the absence of a priori information, reasonable starting seed points and stopping criteria are often difficult to choose. Finally, both approaches sometimes suffer from a lack of knowledge because they rely on the use of ill-defined hard thresholds that may lead to wrong decisions (Salotti and Garbay, 1992).

In the task of segmentation of some complex pictures, such as outdoor and natural images, it is often difficult to obtain satisfactory results by using only one approach to image segmentation. Taking into account the complementary nature of the edge-based and region-based information, it is possible to alleviate the problems related to each of them considered separately. The tendency toward the integration of several techniques seems to be the best way to produce better results. The difficulty in achieving this lies in that even though the two approaches yield complementary information, they involve conflicting and incommensurate objectives. Thus, as observed by Pavlidis and Liow (1990), although integration has long been a desirable goal, achieving it is a nontrivial task.

In the 1990s, numerous techniques for integrating region and boundary information were proposed. One of the principal characteristics that permits classification of these approaches is the time of fusion—embedded in the region detection or after both processes (Falah *et al.*, 1994):

- Embedded integration can be described as integration through the definition of new parameters or a new decision criterion for the region-based segmentation. First, the edge information is extracted, and, second, this information is then used within the segmentation algorithm which is mainly based on regions. For example, edge information can be used to define the seed points from which regions are grown.
- Postprocessing integration is performed after image processing by using the two approaches (boundary-based and region-based techniques). Edge information and region information are extracted independently in a preliminary step, and then integrated

Although many surveys on image segmentation have been published, as stated previously, none focuses specifically on the integration of region and boundary information. As a way to overcome this deficiency, this article discusses the most current and most relevant segmentation techniques that integrate region and boundary information. The remainder of this article is structured as follows: A discussion of embedded and postprocessing strategies and the related work concludes the Introduction. Section II defines and classifies the different approaches to embedded integration, whereas Section III analyzes the proposals for the postprocessing strategy. Section IV summarizes the advantages and disadvantages of the various approaches. Finally, the results of our study are summarized in Section V.

### A. *Integration Techniques: Embedded versus Postprocessing*

Many cooperative methods have been developed, all with the common objective of improving the segmentation by using integration. However, the fusion of boundary information and region information has been attempted through many different approaches. The result is a set of techniques that contains very disparate tendencies. As many authors have proposed (Falah *et al.,* 1994; Le Moigne and Tilton, 1995), one of the main characteristics that allows classification of the integration techniques is the *time of fusion.* This concept refers to the moment during the segmentation process when the integration of the dual sets of information is performed. This property allows us to distinguish two basic groups among the integration proposals: embedded and postprocessing.

The techniques based on embedded integration start with the extraction of the edge map. This information is then used in the region-detection algorithm, in which the boundary information is combined with the region information to carry out the segmentation of the image. A basic scheme of this method is indicated in Figure 1a. The additional information contributed by the edge detection can be employed in the definition of new parameters or new decision criteria.

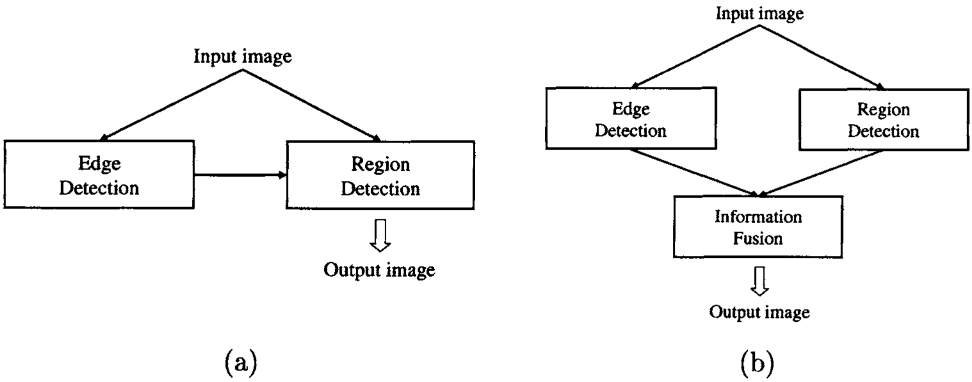


FIGURE 1. Strategy schemes for region and boundary integration according to the time of fusion: (a) embedded integration; (b) postprocessing integration.

The aim of this integration strategy is to use boundary information as the means of avoiding many of the common problems of region-based techniques.

Conversely, the techniques based on postprocessing integration extract edge information and region information independently, as depicted in the scheme of Figure 1b. This preliminary step results in two segmented images obtained by using the classical techniques of both approaches, so they probably have the typical faults that are generated by the use of a single isolated method. An a posteriori fusion process then tries to exploit the dual information in order to modify, or refine, the initial segmentation obtained by a single technique. The aim of this strategy is the improvement of the initial results and the production of a more accurate segmentation.

In the following sections (Sections II and III), we describe several key approaches that we have classified as embedded or postprocessing. Within the embedded methods we differentiate between those that use boundary information for seed-placement purposes and those that use this information to establish an appropriate decision criterion. Within the postprocessing methods, we differentiate three approaches: oversegmentation, boundary refinement, and selection evaluation. We discuss each of these approaches in depth and, in some cases, emphasize aspects related to the implementation of the methods (region-growing or split-and-merge) or to the use of fuzzy logic, which has been considered in a number of proposals.

### *B. Related Work*

Brief mention of the integration of region and boundary information for segmentation can be found in the introductory sections of several papers. As a first reference, Pavlidis and Liow (1990) introduced some earlier papers that emphasized the integration of such information. In 1994 Falah *et al.* identified two basic strategies for achieving the integration of dual information, boundaries and regions. The first strategy (postprocessing) is described as the use of the edge information to control or refine a region segmentation process. The second strategy (embedded) is to integrate edge detection and region extraction in the same process. The classification proposed by Falah *et al.* has been adopted by us and is discussed in this article.

In a different case, Le Moigne and Tilton (1995), thinking in the general case of data fusion, identified two levels of fusion: pixel and symbol. In a pixel-level integration between edges and regions, the decision for integration is made individually on each pixel, whereas the symbol-level integration is made on the basis of selected features, which simplifies the problem. In the same paper, these authors discussed embedded and postprocessing strategies and presented important arguments on the supposed superiority of the postprocessing strategy. They argued that the a posteriori fusion yields a more general approach because, for the initial task, it can employ any type of boundary and region segmentation.

A different viewpoint regarding the integration of edge and region information for segmentation proposals consists of the use of dynamic contours (snakes). In this sense, Chan *et al.* (1996) reviewed different approaches, pointing out that integration is the way to decrease the limitations of traditional deformable contours.

## II. Embedded Integration

The embedded integration strategy consists of using the edge information, previously extracted, within a region segmentation algorithm. It is well known that in most of the region-based segmentation algorithms, the manner in which initial regions are formed and the criteria for growing them are set a priori. Hence, the resulting segmentation will inevitably depend on the choice of initial region growth points (Kittler and Illingworth, 1985), whereas the region's shape will depend on the particular growth chosen (Kohler, 1981). Some proposals try to use boundary information in order to avoid these problems. According to the manner in which this information is used, it is possible to distinguish two tendencies:

1. *Guidance of seed placement:* Edge information is used as a guide to choose the most suitable position to place the seed (or seeds) of the region-growing process.
2. *Control of growing criteria:* Edge information is included in the definition of the decision criterion which controls the growth of the region.

### *A. Guidance of Seed Placement*

In 1992 Benois and Barba presented a segmentation technique that combined contour detection and a split-and-merge procedure of region growing. In this technique, the boundary information is used to choose the growing centers. More specifically, the original idea of the method is the placement of the seeds on the skeleton of nonclosed regions obtained by edge detection. The technique starts with contour detection and extraction, according to the algorithm proposed in Moulet and Barba (1988), which finds the most evident frontiers of homogeneous regions. The contours obtained as a result of this overall procedure are of high quality, but they are not always closed. Then, a region-growing procedure is used to close these regions and to obtain a more precise segmentation. Hence, as a way to obtain a uniformly spread speed of region growing constrained by original contours, the growing centers should be chosen as far as possible from these contours. To do so, the algorithm chooses them on the skeleton defined by the set of the original contours. The skeleton is computed by the Rosenfeld method of local maxima distance. Finally, the region-growing process is realized in the following steps: a splitting process that divides an initial image into homogeneous rectangular blocks, then a merging process grouping these blocks around growing centers to obtain final segments.

A similar work was proposed by Sinclair (1999), who presented an interesting integration segmentation algorithm. First, the Voronoi image generated from the edge image is used to derive the placement of the seeds. The intensity at each point in a Voronoi image is the distance to the closest edge. Second, the peaks in the Voronoi image, reflecting the farthest points from the contours, are used as seed points for region growing. In the growth, two criteria are used in order to attach unassigned pixels: the difference in color between the candidate pixel and the boundary member pixel must be less than a set threshold, and the difference in color between the candidate and the mean color of the region must be less than a second, larger threshold. In this sense, these criteria take into account local and global region information for the aggregation of a new pixel to a region. This could be especially interesting for blurred regions. From another integration aspect, edges recovered from the image act as hard barriers through which regions are not allowed to grow. Figure 2 shows the images generated during the segmentation process, including the Voronoi image, which guide the placement of the region-growing centers.

Moghaddamzadeh and Bourbakis (1997) proposed an algorithm that uses edge detection to guide initialization of an a posteriori region-growing process. Actually, this work is not specifically oriented to the placement of the seeds for the a posteriori growing process, but is focused on establishing a specific order for the processes of growing. As is well known, one disadvantage of the

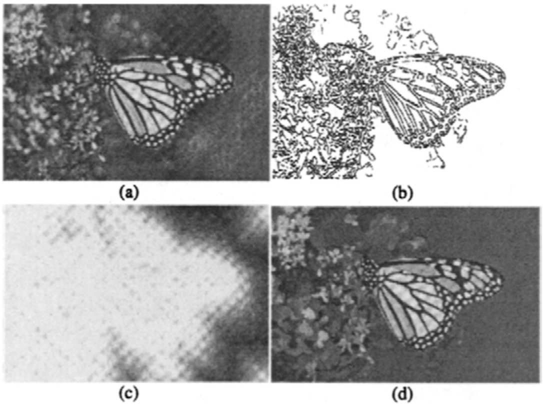

FIGURE 2. The Sinclair (1999) approach using the Voronoi image. (a) Original image. (b) Edges extracted from the original color image. (c) Voronoi image computed from the edge image. (d) Final segmentation.

region-growing and merging processes is their inherently sequential nature. Hence, the final segmentation results depend on the order in which regions are grown or merged. The objective of this proposal is to simulate the order by which we humans separate segments from each other in an image; that is, from large to small. As a way to achieve this, an edge-detection technique is applied to the image to separate large and crisp segments from the rest. The threshold of the edge-detection algorithm is fixed low enough to detect even the weakest edge pixels in order to separate regions from each other. Next, the regions obtained (considering a region as a place closed by edges) are sequentially expanded, starting from the largest segment and finishing with the smallest. *Expanding* a segment refers to merging adjacent pixels with the segment, on the basis of some conditions. Two fuzzy techniques are then proposed to expand the large segments and/or to find the smaller ones.

Another proposal, which uses the edge information to initialize the seeds of a posteriori region growing, has been presented by Cufí *et al.* (2000). Like the proposal of Moghaddamzadeh and Bourbakis, Cufí *et al.*'s proposal takes into account seed placement as well as the order by which the regions start the growing process. However, Moghaddamzadeh and Bourbakis give priority to the largest regions, whereas Cufí *et al.* prefer a concurrent growing, giving the same opportunities to the regions. The basic scheme of their technique is shown in Figure 3. The technique begins by detecting the main contours of the image following the edge extraction algorithm discussed in Cufí and Casals (1996). For each extracted contour, the algorithm places a set of growing centers at each side and along it. It is assumed that the whole set of seeds of one side of the contour belong to the same region. Then, these seeds are

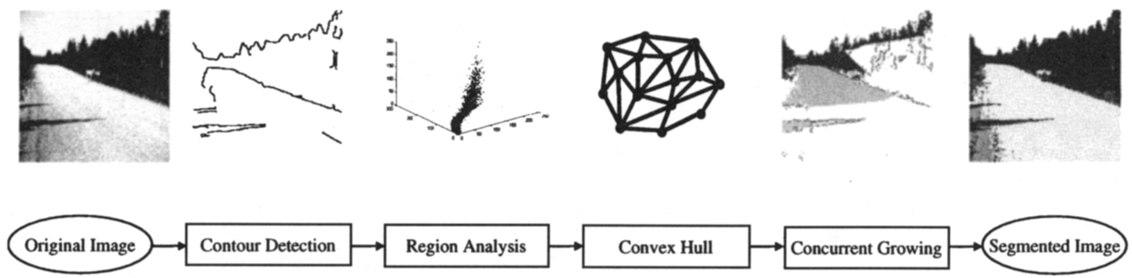


FIGURE 3. Scheme of the segmentation technique proposed by Cufí *et al.* (2000). The method is composed of four basic steps: (1) main contour detection, (2) analysis of the seeds, (3) adjustment of the homogeneity criterion, and (4) concurrent region growing.

used as samples of the corresponding regions and analyzed in the chromatic space in order to establish appropriate criteria for the posterior growing processes. The goal is to know a priori some characteristics of regions with the aim of adjusting the homogeneity criterion to the region's characteristics. Finally, the seeds simultaneously start a concurrent growth using the criterion established for each region, which is based on clustering analysis and convex hull construction.

### *B. Control of Growing Criterion*

Another way to carry out the integration from an embedded strategy is to incorporate the edge information into the growing criterion of a region-based segmentation algorithm. Thus, the edge information is included in the definition of the decision criterion that controls the growth of the region.

As discussed in the Introduction, region-growing and split-and-merge algorithms are the typical region-based segmentation algorithms. Although both share the essential concept of homogeneity, the way they carry out the segmentation process is different in the decisions taken. For this reason, and to facilitate the analysis of the surveyed algorithms, we present these two types of approaches in separate subsections.

#### 1. *Integration in Split-and-Merge Algorithms*

Bonnin and his colleagues (1989) proposed a region extraction based on a split-and-merge algorithm controlled by edge detection. The method incorporates boundary information into the homogeneity criterion of the regions to guide the region-detection procedure. The criterion to decide the split of a region takes into account edge and intensity characteristics. More specifically, if there is no edge point on the patch and if the intensity homogeneity constraints are satisfied, the region is stored; otherwise, the patch is divided into four subpatches, and the process is recursively repeated. The homogeneity intensity criterion is necessary because of possible failures of the edge detector. After the split phase, the contours are thinned and chained into edges relative to the boundaries of the initial regions. Later, a final merging process takes into account edge information in order to solve possible oversegmentation problems. In this last step, two adjacent initial regions are merged only if there are no edges on the common boundary. The general structure of the method is depicted in Figure 4, where it can be observed that edge information guides the split-and-merge procedure in both steps of the algorithm: first, to decide the split of a region, and second, in the final merging phase, to solve the possible oversegmentation.

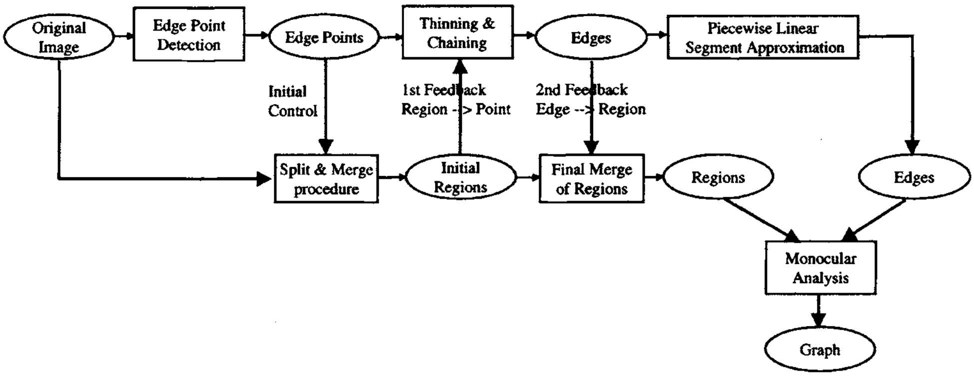


FIGURE 4. Scheme of the segmentation technique proposed by Bonnin *et al.* (1989). The edge information guides the split-and-merge procedure in both steps of the algorithm: first, to decide the split of a region, and second, in the final merging phase, to solve the possible oversegmentation.

The split-and-merge algorithm cooperating with an edge extractor was also proposed in the work of Buvry, Zagrouba *et al.* (1994). The proposed algorithm follows the basic idea introduced by Bonnin, considering the edge segmentation in the step of merging. However, a rule-based system was added to improve the initial segmentation. A scheme of the proposed algorithm is illustrated in Figure 5. These authors argued that the split-and-merge segmentation algorithm creates many horizontal or vertical boundaries without any physical meaning. To solve this problem, the authors defined a rule-based system dealing with this type of boundary. Specifically, the gradient mean of each boundary is used to decide if the boundary has a physical reality.

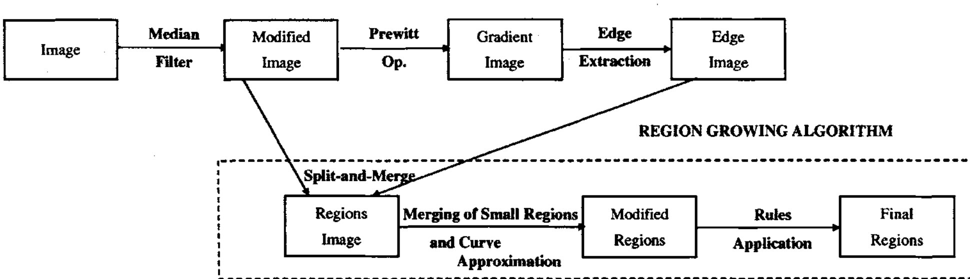


FIGURE 5. Segmentation technique proposed by Buvry, Zagrouba *et al.* (1994). Edge information is used to guide the split-and-merge region segmentation. Finally, a set of rules improve the initial segmentation by removing boundaries without corresponding edge information. Prewitt op., Prewitt operator.

In 1997, Buvry, Senard *et al.* reviewed the work presented in Buvry, Zagrouba *et al.*'s publication (1994) and proposed a new hierarchical region-detection algorithm for stereovision applications taking into account the gradient image. The method yields a hierarchical coarse-to-fine segmentation in which each region is validated by exploiting the gradient information. At each level of the segmentation process, a threshold is computed and the gradient image is binarized according to this threshold. Each closed area is labeled by applying a classical coloring process and defines a new region. Edge information is also used to determine if the split process is finished or if the next partition must be computed. As a way to do that, a gradient histogram of all pixels belonging to the region is calculated and its characteristics (mean, maximum, and entropy) are analyzed.

A proposal for enriching the segmentation by irregular pyramidal structure by using edge information can be found in the work of Bertolino and Montanvert (1996). In this proposed algorithm, a graph of adjacent regions is computed and modified according to the edge map obtained from the original image. Each graph edge* is weighted with a pair of values $(r, c)$ which represent the number of region elements and contour elements in the common boundary of both regions, respectively. Then, the algorithm goes through the graph and at each graph edge decides whether to forbid or favor the fusion between adjacent regions.

The use of edge information in a split-and-merge algorithm may not be reduced to only the decision criterion. In this sense, Gevers and Smeulders presented, in 1997, a new technique that extends the possibilities of this integration. Their proposal uses edge information to decide how the partition of the region should be made, or, in other words, where to split the region. The idea is the adjustment of this decision to boundary information and to split the region following the edges contained in it. In reference to previous works, these authors affirmed that although the quad-tree scheme is simple to implement and computationally efficient, its major drawback is that the image tessellation process is unable to adapt the tessellation grid to the underlying structure of the image. For this reason they proposed to employ the incremental Delaunay triangulation competent of forming grid edges of arbitrary orientation and position. The tessellation grid, defined by the Delaunay triangulation, is adjusted to the semantics of the image data. In the splitting phase, if a global similarity criterion is not satisfied, pixels lying on image boundaries are determined by using local difference measures and are used as new vertices to locally refine the tessellation grid.

### 2. *Integration in Region-Growing Algorithms*

One of the first integrations of edge information into a region-growing algorithm can be found in the work of Xiaohan *et al.* (1992), in which edge

*To avoid confusion, we designate *graph edge* as an edge that joins two nodes in a graph.

information was included in the decision criterion. A classic region-growing algorithm generally takes into account only the contrast between the current pixel and the region in order to decide the merging of them. Xiaohan *et al.* proposed a region-growing technique that includes the gradient region in the homogeneity criterion to make this decision. The proposed combination of region-growing and gradient information can be expressed in the following formula:

$$\begin{aligned} x(i, j) &= |X_a^N v - f(i, j)| \\ z(i, j) &= (1 - \phi)x(i, j) + \phi G(i, j) \end{aligned} \tag{1}$$

where $X_a^N v$ is the average gray value of the region which is updated pixel by pixel. The contrast of the current pixel with respect to the region is denoted by $x(i, j)$. Parameter $\phi$ controls the weight of gradient $G(i, j)$. Finally, the sum of the local and the global contrast is the final homogeneity measure, $z(i, j)$. Following this expression the proposed algorithm can be described by using only two steps:

**Step 1** If $z(i, j)$ is less than a given threshold $\beta$, then the current pixel is merged into the region.

**Step 2** Else the local maximum of the gradients on a small neighborhood of the current pixel is searched along the direction of region growing. The procedure stops at the pixel with the local gradient maximum.

The first step of the algorithm describes the growing of the region guided by the proposed homogeneity criterion. The second tries to avoid the typical error of the region-based segmentation techniques—that is, the inaccuracy of the boundaries detected—by putting the result of the segmentation in coincidence with the edge map.

A similar integration proposal was suggested by Falah *et al.* in 1994. In this work the gradient information is included in the decision criterion to restrict the growth of regions. At each iteration, only pixels having low gradient values (below a certain threshold) are allowed to be aggregated into the growing region. Another interesting aspect of this work is the choice of the seeds for the process of region growing. This selection uses the redundancy between the results obtained by several region segmentations (with different thresholds and different directions of image scanning), with the aim of placing the seeds in a proper position in which they have a high degree of certainty of belonging to a homogeneous region.

In 1992 Salotti and Garbay developed a theoretical framework of an integrated segmentation system. The core of the problem of traditional segmentation methods, as denoted by these authors, relates to the autarchy of the methods and to the schedule of conditions that are defined with a priori assumptions. As a way to solve this problem, major directives to control each decision are presented; to accumulate local information before taking difficult decisions; to use processes exploiting complementary information to cooperate successfully; to defer difficult decisions until more information is available; and, finally, to enable easy context switches to ensure an opportunistic cooperation. The main idea of these directives is that each decision must be strongly controlled. This implies that a massive collaboration must be carried out and that the segmentation task should not necessarily be achieved before the beginning of the high-level process. Finally, all these principles are used in a segmentation system with a region-growing process as main module. Pixels that seem difficult to classify because there is insufficient information for a sure decision are given to an edge-detection unit that must respond whether they correspond to an edge or not. The same directives were followed in an a posteriori work (Bellet *et al.,* 1994). that presents an edge-following techniques which uses region-based information to compute adaptive thresholds. In such situations, when it is difficult to follow the high gradient, complementary information is requested and successfully obtained through the emergence of regions on both sides of the edge. A child edge process is then created with a threshold adapted to lower gradient values. Moreover, these authors introduce the adaptability of the aggregation criterion to the region's characteristics: several types of regions are distinguished and defined. The region-growing method dynamically identifies the type of the analyzed region, and a specific adapted criterion is used.

### *C. Fuzzy Logic*

A current trend in segmentation techniques that deserves special attention is the use of *fuzzy logic* (Bezdek *et al.,* 1999). The role of fuzzy sets in segmentation techniques is becoming more important (Lambert and Carron, 1999; Pham and Prince, 1999), and the integration techniques are in the mainstream of this tendency. In this sense, we want to emphasize the growing interest of researchers to incorporate fuzzy logic methods into integrated segmentation. This interest was mainly prompted because these two integration methods are developed from complementary approaches and do not share a common measure. Hence, fuzzy logic offers the possibility to solve this problem, as it is especially suited to carry out the fusion of information of a diverse nature (Kong and Kosko, 1992; Moghaddamzadeh and Bourbakis, 1997). In the case of embedded integration of edge information into a region-growing procedure (Krishnan *et al.,* 1994; Steudel and Glesner, 1999), the fuzzy rule–based homogeneity criterion offers several advantages in contrast to ordinary feature aggregation methods. Among these advantages is its short development time as a result of the existing set of tools and methodologies for the development of fuzzy rule–based systems. An existing rule–based system can easily be modified or extended to meet the specific requirements of a certain application. Furthermore, it does not require a full knowledge of the process and it is intuitive to understand because of its human-like semantics. In addition, it is possible to include such linguistic concepts as shape, size, and color, which are difficult to handle when one is using most other mathematical methods.

A key work in using fuzzy logic was by Steudel and Glesner (1999), in which the segmentation is carried out on the basis of a region-growing algorithm that uses a fuzzy rule–based system for the evaluation of the homogeneity criterion. These authors affirmed that there are several negative aspects of using only the intensity difference for segmentation:

- Oversegmentation of the image
- Annoying false contours
- Contours that are not sufficiently smooth

Therefore, new features are introduced into the rule base of the fuzzy rule–based system which result in a better and more robust partitioning of the image while maintaining a small and compact rule base. The proposed homogeneity criterion is composed of a set of four fuzzy rules. The main criterion is the difference between the average intensity $\bar{A}$ of a region $R_j$ and the pixel $i_n$ under investigation. The corresponding fuzzy rule is as follows:

```
R1: IF DIFFERENCE IS SMALL
    THEN HOMOGENEOUS
    ELSE NOT_HOMOGENEOUS
```

Another important feature for the segmentation of regions is the gradient at the position of the pixel to be merged. A new pixel may be merged into a region $R_j$ when the gradient at that location is `low`. Conversely, when the gradient is `too high`, the pixel definitely does not belong to the region and should not be merged. In terms of a fuzzy rule,

```
R2: IF GRADIENT IS LOW
    THEN PROBABLY HOMOGENEOUS
    ELSE NOT_HOMOGENEOUS
```

With this rule, an adjacent pixel $i_n$ satisfies the premise of rule R2 with a degree of $\mu_{LOW}$ (*GRADIENT* ($i_n$)). The two remaining rules are refereed to the size and the shape of regions in order to avoid smallest regions and to benefit compact regions with smooth contours. A complete scheme of this proposal is shown in Figure 6.

Krishnan *et al.* (1994) described a boundary extraction algorithm based on the integration of fuzzy rule–based region growing and fuzzy rule–based edge detection. The properties of homogeneity and edge information of each

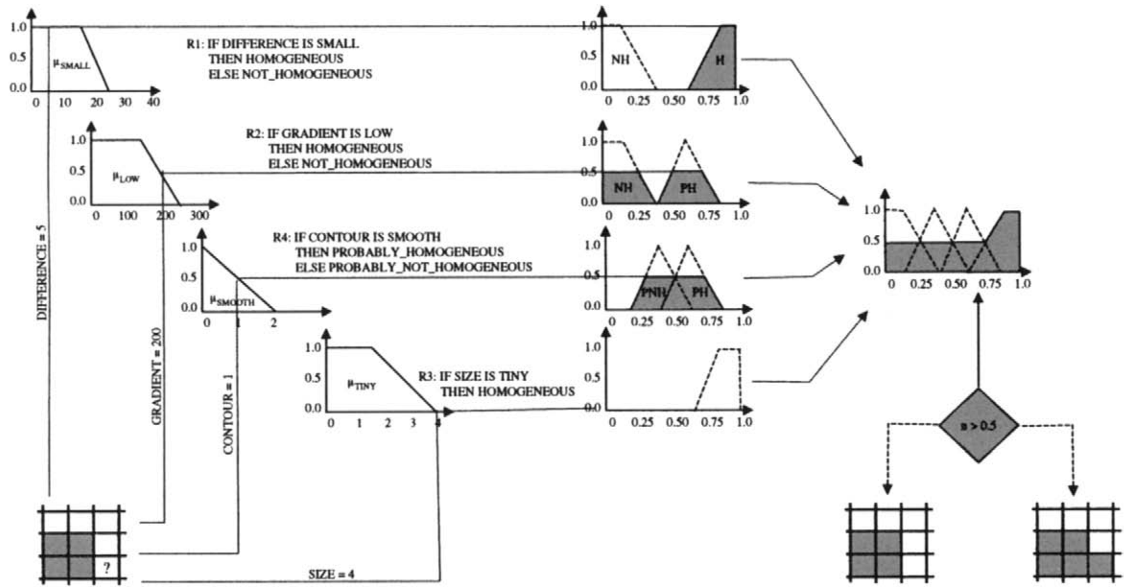


FIGURE 6. Fuzzy segmentation technique by Steudel and Glesner (1999). The method is composed of a set of fuzzy rules corresponding to the main properties of the regions: intensity, gradient, shape, and size. The united result of these rules indicates the desirability of aggregating a new pixel into the region. H, homogeneous; NH, not homogeneous; PH, probably homogeneous; PNH, probably not homogeneous. (Reprinted from Pattern Recognition, vol. 32, no. 11, A. Steudel and M. Glesner, *Fuzzy Segmented Image Coding Using Orthonormal Bases and Derivative Chain Coding,* page 1830, © 1999, with permission from Elsevier Science.)

candidate along the search directions are evaluated and compared with the properties of the seed. The fuzzy output values of edge detection and a similarity measure of the candidate pixel can be used to determine the test for the boundary pixel. This proposal was applied to colonoscopic images for the identification of closed boundaries of intestinal lumen, to facilitate diagnosis of colon abnormalities.

Another proposal for the integration of boundary information into the region-growing process was presented by Gambotto (1993), in which edge information was used to stop the growing process. The algorithm starts with the gradient image and an initial seed that must be located inside the connected region. Then, pixels that are adjacent to the region are iteratively merged if they satisfy a similarity criterion. A second criterion is used to stop this growth. The criteria assume that the gradient takes a high value over a large part of the region boundary. Thus, growth termination is based on the average gradient, $F(n)$, computed over the region boundary following the expression

$$F(n) = \sum G(k, l)/P(n) \tag{2}$$

where $P(n)$ is the perimeter of the region $R(n)$, and $G(k, l)$ is the value of the modulus of the gradient of pixels on the region boundary.

The iterative growing process is then continued until the maximum of the global contrast function, $F$, is detected. Gambotto points out that the cooperation between region growing and contour detection is desirable because the assumption of homogeneous regions is usually too restrictive. If this approach is used, the class of regions that can be characterized is wider than that characterized by using smooth gray-level variations alone.

## III. Postprocessing Integration

In contrast to the works analyzed until this point, which follow an embedded strategy, the postprocessing strategy carries out the integration a posteriori to the segmentation of the image by region-based and boundary-based algorithms. Region information and edge information are extracted in a preliminary step and then integrated. Postprocessing integration is based on fusing results from single segmentation methods attempting to combine the map of regions (generally with thick and inaccurate boundaries) and the map of edge outputs (generally with fine and sharp lines, but dislocated), with the aim of providing an accurate and meaningful segmentation. Most researchers agree on differentiating embedded methods from postprocessing methods. We have identified different approaches for performing postprocessing tasks:

1. *Oversegmentation:* This approach consists of using a segmentation method with parameters specifically fixed to obtain an oversegmented result. Then, additional information from other segmentation techniques is used to eliminate false boundaries that do not correspond with regions.

2. *Boundary refinement:* This approach considers the region segmentation result as a first approach, with regions well defined, but with inaccurate boundaries. Information from edge detection is used to refine region boundaries and to obtain a more precise result.

3. *Selection evaluation:* In this approach, edge information is used to evaluate the quality of different region-based segmentation results, with the aim of choosing the best. A third set of techniques deal with the difficulty of establishing adequate stopping criteria and thresholds in region segmentation.

### A. *Oversegmentation*

The oversegmentation approach has emerged because of the difficulty of establishing an adequate homogeneity criterion for the region growing. As Pavlidis and Liow (1990) suggested, the major reason that region growing produces false boundaries is that the definition of region uniformity is too strict, as when the definition insists on approximately constant brightness while in reality brightness may vary linearly within a region. It is very difficult to find uniformity criteria that match these requirements exactly and do not generate false boundaries. Summarizing, these authors argued that the results can be significantly improved if all region boundaries qualified as edges are checked rather than attempting to fine-tune the uniformity criteria. A basic scheme is shown in Figure 7.

A first proposal can be found in the work of Monga *et al.* (Gagalowicz and Monga, 1986; Wrobel and Monga, 1987). The algorithm starts with a region-growing or a split-and-merge procedure, in which the parameters have been set up so that an oversegmented image results. Then the region-merging process is controlled by edge information which helps to remove false contours generated by region segmentation. Every initial boundary is checked by analyzing its coherence with the edge map, where real boundaries must have high gradient values, while low values correspond to false contours. According to this assumption, two adjacent regions are merged if the average gradient on their boundary is lower than a fixed threshold.

In 1992, Kong and Kosko included fuzzy logic in the algorithm proposed by Monga *et al.* As Monga *et al.* did, Kong and Kosko computed gradient information that they called *high-frequency characteristics h,* to eliminate false contours:

$$h = \frac{|\text{high-frequency components along the boundary}|}{\text{length of the boundary}} \tag{3}$$

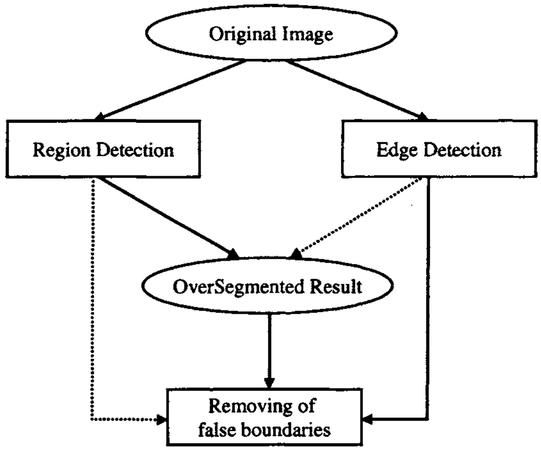


FIGURE 7. Scheme of postprocessing integration method based on oversegmentation. First, thresholds are set to obtain an initial oversegmented result. Second, complementary information allows removal of false boundaries.

For any boundary, if the high-frequency information $h$ is `small`, the algorithm concludes that the boundary is a false contour and that it can be eliminated.

Another interesting work was presented by Pavlidis and Liow (1990). The proposed algorithm shares the basic strategy of the previously described works, but these authors included a criterion in the merging decision in order to eliminate the false boundaries that resulted from the data structure used. Starting from an oversegmented image, region boundaries are eliminated or modified on the basis of criteria that integrate contrast with boundary smoothness, and variation of the image gradient along the boundary, and a final criterion that penalizes the presence of artifacts reflecting the data structure used during the segmentation. For each boundary, a merit function is computed of the form

$$f_1(\text{contrast}) + \beta f_2(\text{segmentation artifacts}) \tag{4}$$

where boundaries with low values of that sum are candidates for elimination. Finally, the proposed algorithm ends with a final step of contour refinement using snakes, which produces smoother contours.

Saber *et al.* (1997) proposed a segmentation algorithm which uses a split-and-merge process to carry out the fusion of spatial edge information and regions resulting from adaptive Bayesian color segmentation. The image is first segmented on the basis of color information only. Second, spatial edge locations are determined by using the magnitude of the gradient of the three-channel image vector field, computed as described by Lee and Cok (1991). As a way to enforce the consistency of the color segmentation map with color edge locations, a split-and-merge procedure is proposed. In the first phase, color segments that have at least one edge segment within their boundary will be split into multiple regions. The splitting is accomplished by first thresholding the gradient result and then labeling all contiguous regions therein. Next, the merging criterion favors combining two regions if there is no significant edge between the region boundaries. A flowchart of the method is depicted in Figure 8.

Using the same basic idea of starting from an oversegmented image, some authors have developed techniques that begin with edge detection to obtain oversegmentation results. The intention is the same as before, except in these cases region information allows differentiation between true and false contours. Following this strategy, Philipp and Zamperoni (1996) proposed to start with a high-resolution edge extractor, and then, according to the texture characteristics of the extracted regions, to decide whether to suppress or prolong a region. Derivative edge detectors, when employed at a high resolution, give long, rather

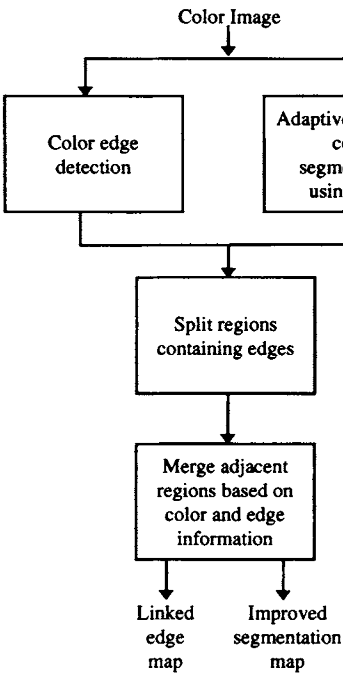


FIGURE 8. Flowchart of the method proposed by Saber *et al.* (1997). First, an initial segmentation map is computed. Second, region labels are optimized by split-and-merge procedures to enforce consistency with the edge map. GRF, Gibbs random field. (Reprinted from Image and Vision Computing, vol. 15, no. 10, E. Saber, A. M. Tekalp and G. Bozdagi, *Fusion of Color and Edge Information for Improved Segmentation and Edge Linking,* page 770, © 1997, with permission from Elsevier Science.)

isolated, and well-localized contours in nontextured areas and numerous, short, close-spaced contours in textured areas. The former correspond to true edges in the image, because they are well localized and thin, so they must be preserved, and prolonged if possible. The latter must be suppressed if they are inside a textured region but preserved and prolonged if they represent a piece of border. The feature used in this algorithm is the distance between textures on either side of the edge. As a way to obtain texture information, two seeds are put on either side of the edge and start a recursive growing until $N$ representative pixels are gathered. If the distance between textures is small, the edge is considered false and regions are merged. Otherwise, the contour is preserved and prolonged in order to maximize the distance on either side of the edge.

In 1997, Fjørtoft *et al.* presented another technique based on oversegmentation from edge detection, which was examined on synthetic aperture radar (SAR) images. These authors discussed the key role of the threshold value to extract the possible edges from an edge strength map by thresholding. The chosen threshold is related to the probability of false alarm (i.e., the probability of detecting an edge in a zone of constant reflectivity). As a way to detect all significant edges, a low threshold is set, accepting the detection of numerous false edges as well. The oversegmentation result provides, as these authors suggested, a good starting point for the merging process that eliminates false edges by merging regions. The merging step uses a likelihood ration (LR) criterion to decide the homogeneity between adjacent regions and the consequent elimination of their boundary. That is, the LR is related to the probability that the two regions have the same reflectivity.

### B. *Boundary Refinement*

As described previously, region-based segmentation yields a good detection of true regions, although, as is well known, the resultant sensitivity to noise causes the boundary of the extracted region to be highly irregular. The boundary refinement approach, which we call *result refinement,* considers region-based segmentation as a first approximation to segmentation. Typically, a region-growing procedure is used to obtain an initial estimate of a target region, which is then combined with salient edge information to achieve a more accurate representation of the target boundary. As in the oversegmentation proposals, edge information permits refinement of an initial result.

An interesting example of boundary refinement can be found in the work of Haddon and Boyce (1990), in which they proposed a segmentation algorithm consisting of two stages: after an initial region segmentation, a posteriori refinement of the generated regions is performed by means of a relaxation algorithm that uses the edge information to ensure local consistency of labeling.

Nevertheless, the main characteristic of this work is the postulate that a co-occurrence matrix may be employed as a feature space, with clusters within the matrix being identified with the regions and boundaries of an image. This postulate is proven for nearest-neighbor co-occurrence matrices derived from images whose regions satisfy Gaussian statistics; regions yield clusters on the main diagonal, and boundaries yield clusters off the main diagonal.

In 1993, Chu and Aggarwal presented an algorithm which integrates multiple region segmentation and edge maps. The proposed algorithm allows multiple input maps and applies user-selected weights on various information sources. The first step consists of transforming all inputs to edge maps. Second, a maximum likelihood estimator provides initial solutions of edge positions and strengths from multiple inputs. Third, an iterative procedure is used to smooth the resultant edge patterns. Finally, regions are merged to ensure that every region has the required properties. The strength of this proposal is that the solution is a negotiated result of all input maps rather than a selection of them. Several years later, Nair and Aggarwal (1996) made their initial proposal more sophisticated by stating the boundary refinement problem as a classification problem. Every point $s$ on the region boundary must find its new location as a selection from a set of candidate edge element locations $\bar{z} = z_j$, $j = 0 \ldots n$, where $z_0 = s$.

Using the Bayes decision rule, the algorithm chooses $z_j$ as the new location if

$$p(s|z_j) \geq p(s|z_k)P(z_k) \qquad \forall k \neq j \tag{5}$$

where $p(s|z_j)$ represents the conditional density function of $s$ given $z_j$, and $P(z_j)$ is the a priori probability of $z$. The a priori probability of each candidate location $z_j$ is estimated as the proximity of the salient edge segment to which $z_j$ belongs, to the boundary of the target region. Finally, the proposed algorithm tries to restore boundary segments by incorporating small parts of the target missed in the region segmentation; that is, for each edge pixel at the site of a break in the boundary, the algorithm tries to determine whether the pixel is part of a salient edge. If it is, the complete edge segment can be incorporated into the boundary. A scheme of this proposal is indicated in Figure 9.

A recent proposal for the boundary refinement approach was put forward by Sato *et al.* (2000). The objective of these authors is to obtain an accurate segmentation of three-dimensional medical images for clinical applications. The proposed technique takes into account the gradients of the boundary and its neighborhood and applies the gradient magnitude, based on a Sobel operator, for boundary improvement. The algorithm starts by successive steps of thresholding and ordinary region growing, which yields a first segmentation of the region of interest. The highest gradient magnitude is expected at the boundary, so a growing process starts to find this optimal boundary. For each

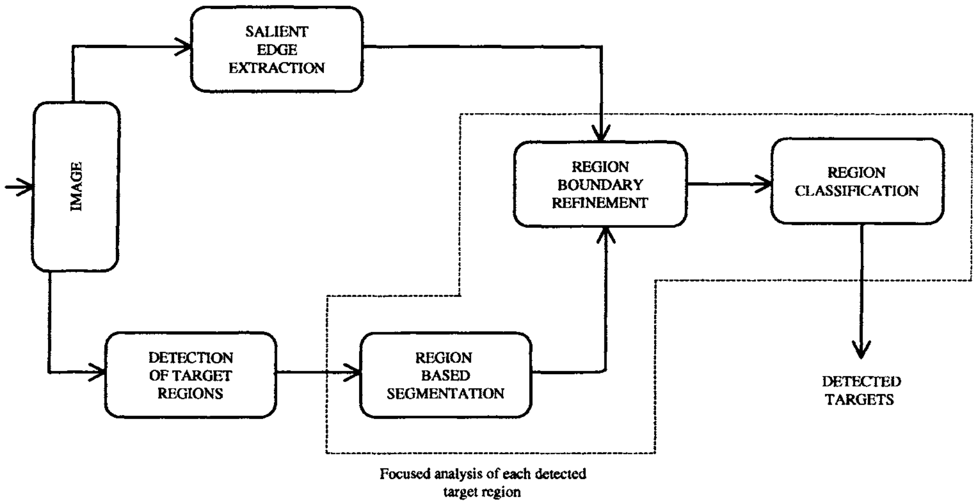


FIGURE 9. The general flow of the target segmentation paradigm proposed by Nair and Aggarwal (1996). Boundary refinement from edge information is stated as a classification problem.

voxel (three-dimensional pixel) at a boundary, neighborhoods of the voxel and outside the region are inspected by calculating their gradient magnitudes. If each voxel has a greater gradient magnitude than that of the boundary voxel, it is assigned to the next boundary region. This process is repeated recursively until no further boundary region can be created.

### 1. *Boundary Refinement by Snakes*

Although the aforementioned proposals have contributed interesting results and new ideas, the most common way to refine the boundary consists of the integration of region information with dynamic contours, also called *snakes*. The concept of the snake was introduced by Kass *et al.* (1987). A snake can be defined as an energy-minimizing spline guided by internal constraint forces and influenced by image forces. The image forces guide the snake toward salient image features such as lines, edges, and subjective contours. If we represent the position of a snake parametrically by $v(s) = (x(s), y(s))$, its energy functional can be expressed as

$$E^*_{snake} = \int_0^1 [E_{int}(v(s)) + E_{ext}(v(s))]\, ds \tag{6}$$

where $E_{int}$ represents the internal energy of the spline due to its elasticity and rigidity properties, and $E_{ext}$ gives rise to the external constraint forces.

The internal forces impose a smoothness constraint, while the external energy guides the snake to image characteristics such as edges. Unlike most other techniques for finding salient contours, the snake model is active: it is always minimizing its energy functional and therefore exhibits dynamic behavior. Because of the way the contour appears to slither while minimizing its energy, it is called a *snake*.

The snake method is known to solve boundary refinement problems by locating the object boundary from an initial plan. However, snakes do not try to solve the entire problem of finding salient image contours. The high gray-level gradient of the image may be due to object boundaries as well as noise and object textures; therefore, the optimization functions may have many local optima. Consequently, in general, active contours are sensitive to initial conditions and they are effective only when the initial position of the contour in the image is sufficiently close to the real boundary. For this reason, active contours rely on other mechanisms to place them somewhere near the desired contour. In first approximations to dynamic contours, an expert has been responsible for putting the snake close to an intended contour; its energy minimization carries it the rest of the way. The snake deforms itself into conformity with the nearest salient contour.

However, region segmentation could be the solution of the initialization problem of snakes. Proposals about integrated methods consist of using the region segmentation result as the initial contour of the snake. In this case, the segmentation process is typically divided into two steps (see Fig. 10). First, a region growing with a seed point is performed in the target region, and its

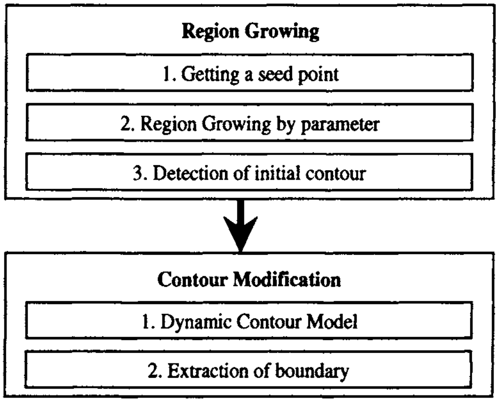


FIGURE 10. Block diagram of integration proposal using snakes. The region-based segmentation result is used to initialize the position of the dynamic contour. Next, energy minimization permits extraction of the accurate boundary of the target object.

output is used for the initial contour of the dynamic contour model. Second, the initial contour is modified on the basis of energy minimization.

Different works can be found in the literature that combine region detection and dynamic contours. In the work of Chan *et al.* (1996), the greedy algorithm proposd by Williams and Shah (1992) is used to find the minimum energy contour. This algorithm searches for the position of the minimum energy by adjusting each point on the contour during iteration to a lower energy position among its eight local neighbors. The result, although not always optimal, is comparable to that obtained by variational calculus methods and dynamic programming. The advantage is that Chan *et al.*'s method is faster. Similar proposals can be found in the works of Vérard *et al.* (1996) and Jang *et al.* (1997). Curiously, the results of all these techniques have been shown on magnetic resonance imaging (MRI) images, but this is not a simple coincidence. Accurate segmentation is critical for diagnosis in medical images. However, it is very difficult to extract the contour that matches exactly the target region in MRI images. Integrated methods seem to be a valid solution to achieve an accurate and consistent detection.

In the sense of making maximum use of information and cooperation, there exists a set of techniques that do not limit region information to initialization of the snakes. Information supplied by region segmentation is also included in the snake, more specifically in its energy functional. This functional typically has two components: an internal energy component that applies shape constraints to the model, and an external energy derived from the data to which the model is being applied. In this approach, a term derived from region information is added to the external part of the energy functional. As a result, points on the contour are allowed to expand or contract according to the fit between contour and region information.

An exemplary work about these integration methods was developed by Ivins (1996) and Ivins and Porrill (1995). In their implementation of the snake, the energy functional $E$ is specified as

$$E = \frac{\alpha}{2} \oint_A \left| \frac{\delta u}{\delta \lambda} \right|^2 d\lambda + \frac{\beta}{2} \oint_A \left| \frac{\delta^2 u}{\delta \lambda^2} \right|^2 d\lambda - \rho \int_R \int G(I(x, y))\, dx\, dy \qquad (7)$$

The first two terms in Eq. (7) correspond, respectively, to the tension energy and the stiffness energy of the contour model, and together compose the internal energy. The third term is the external energy derived from the image data. $G$ is a goodness functional that returns a measure of the likelihood that the pixel, indexed by $x$ and $y$ in the image, is part of the region of interest. $R$ is the interior of the contour, and $\alpha$, $\beta$, and $\rho$ are parameters used to weigh the three energy terms. Thus, as the energy is minimized, the contour deforms to enclose as many pixels with positive goodness as possible while excluding those with negative goodness. This seed region serves two purposes: it is used as the initial configuration of the model, and it is used to construct a statistical model of the attributes (e.g., intensity, color, texture) of the data composing the region as a whole from which the goodness functional is derived. This implementation of the method has been a posteriori revised and modified by Alexander and Buxton (1997), in order to be an effective solution to the problem of tracking the boundaries of country lanes in sequences of images from a camera mounted on an autonomous vehicle.

Another remarkable work, which is constantly evolving, was carried out by Chakraborty *et al.* (1994), who applied snakes in biomedical image analysis. The proposal uses a Fourier parameterization to define the dynamic contour. It expresses a curve in terms of an orthonormal basis, which, for most practical situations, is constrained to a limited number of harmonics. The curve is thus represented by a set of corresponding Fourier coefficients

$$p = (a_0, c_0, a_1, b_1, c_1, d_1, \cdots) \tag{8}$$

The objective function used is a function of conditional probability $P(p|I_g, I_r)$, or the probability of obtaining the $p$ contour given the region-classified image $I_r$ and the image of the scalar magnitude of the gray-level gradient $I_g$. The function is the sum of three terms:

$$M(p, I_g, I_r) = M_{prior}(p) + M_{gradient}(I_g, p) + M_{region}(I_r, p) \tag{9}$$

The first (prior) term biases the boundary toward a particular distribution of shapes generated from prior experience, while the second term in the equation (Eq. (10)), $M_{gradient}(I_g, p)$, depends on the coincidence of the parameterized boundary, with the image edges appearing as coherent features in the scalar gradient of the gray levels,

$$M_{gradient}(I_g, p) = \int_{C_p} I_g[x(p, t), y(p, t)]\, dt \tag{10}$$

such that the likelihood that $p$ represents the true boundary is proportional to the sum of the gradient values of all points in $C_p$.

Finally, term $M_{region}(I_r, p)$ (Eq. (11)) measures the goodness of match of the contour with the perimeter of the segmented interior of the object. This method rewards the boundary that contains as much of the inside region and as little of the outside as possible. This function is evaluated by integrating over the area $A_p$ bounded by the contour $p$, as expressed in

$$M_{region}(I_r, p) = \iint_{A_p} I_r(x, y)\, dA \tag{11}$$

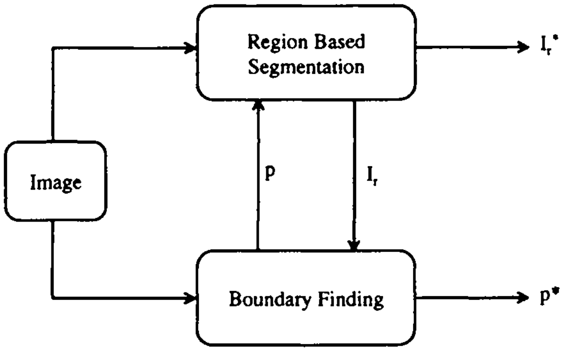


FIGURE 11. Flow diagram for game-theoretic integration of region-based segmentation and boundary finding proposed by Chakraborty and Duncan (1999). The outputs of each of the modules feed back to each other after every decision-making step. The algorithm stops when none of the modules can improve their positions unilaterally. (Reprinted from IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 21, no. 1, A. Chakraborty and J. S. Duncan, *Game-Theoretic Integration for Image Segmentation*, page 16, © 1999 IEEE.)

where pixels inside and outside $A_p$ are set equal to $+1$ and $-1$, respectively. Stating that the area integral must be evaluated many times, Chakraborty *et al.* described an alternative and faster integration method based on Green's theorem.

A recent proposal of Chakraborty and Duncan (1999) emphasized the necessity of integration. In this work, a method is proposed to integrate region segmentation and snakes by using game theory in an effort to form a unified approach. The novelty of the method is that this is a bidirectional framework, whereby both computational modules improve their results through mutual information sharing. This consists of allowing the region and boundary modules to assume the roles of individual players who are trying to optimize their individual cost functions within a game-theoretic framework. The flow of information is restricted to passing only the results of the decisions between the modules. Thus, for any one of the modules, the results of the decisions of the other moduls are used as priors, and players try to minimize their cost functions at each turn. The flow diagram for game-theoretic integration is shown in Figure 11. These authors affirm that this approach makes it unnecessary to construct a giant objective function and optimize all the parameters simultaneously.

## C. Selection

In the absence of object or scene models or ground truth data, it is critical to have a criterion that enables evaluation of the quality of a segmentation. In this sense, a set of proposals have used edge information to define an evaluation function that qualifies the quality of a region-based segmentation. The purpose is to achieve different results by changing parameters and thresholds on a region segmentation algorithm, and then to use the evaluation function to choose the best result. This strategy permits solution of the traditional problems of region segmentation, such as the definition of an adequate stopping criterion or the setting of appropriate thresholds. In 1986, Fua and Hanson developed an algorithm (published in 1987) that used edge information to evaluate region segmentation. In their proposal, high-level domain knowledge and edge-based techniques were used to select the best segmentation from a series of region-based segmented images. However, from this pioneer proposal the majority of methods based on the selection approach have been developed, as is stated in the following.

In 1995, Le Moigne and Tilton proposed choosing a stopping criterion for a region-growing procedure. This criterion is adjusted locally to select the segmentation level that provides the best local match between edge features and region segmentation contours. Figure 12 shows a basic scheme of this proposal. Desired refined segmentation is defined as the region segmentation with minimum length boundaries including all edges extracted by the Canny edge detector and for which all contours include some edge pixels. The iteration of

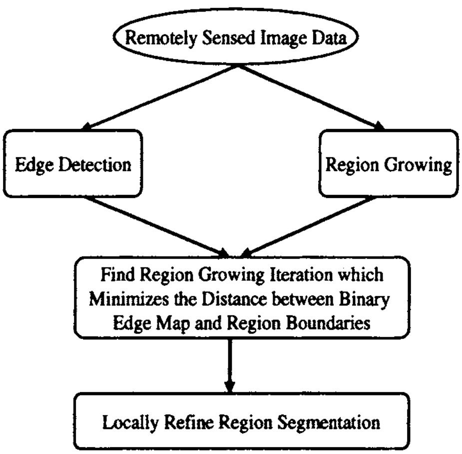


FIGURE 12. Outline of the edge/region integration algorithm proposed by Le Moigne and Tilton (1995). Edge information is used to decide the best region-growing iteration that provides the best local match of edge features and region boundaries. (Reprinted from IEEE Transactions on GeoScience and Remote Sensing, vol. 33, no. 3, J. Le Moigne and J. C. Tilton, *Refining Image Segmentation by Integration of Edge and Region Data,* page 606, © 1995 IEEE.)

the region-growing process which minimizes the *Hausdorff distance* is chosen as the best iteration. The Hausdorff distance measures the distance between two binary images—the edge pixels obtained through Canny, $A$, and the boundary of the regions obtained through the region growing, $B$—and is computed as

$$H(A, B) = \frac{1}{2}[\max_{a \in A} \min_{b \in B} d(a, b) + \max_{b \in B} \min_{a \in A} d(a, b)] \tag{12}$$

where $d(a, b)$ is a point-to-point Euclidean distance. In summary, the distance is computed by finding, for each edge pixel, the closest region boundary pixel, and respectively for each region boundary pixel the closest edge pixel, and then computing the maxima and minima expressed in Eq. (12).

Hojjatoleslami and Kittler (1998) presented a region-based segmentation which used gradient information to specify the boundary of a region. The method starts with a growing process which is stopped by using the maximum possible size $N$ of a region. Then, a reserve check on the relevant measurements is applied to detect the region boundary. Contrast and gradient are used as sequential discontinuity measurements derived by the region-growing process whose locally highest values identify the external boundary and the highest gradient boundary of each region, respectively. *Contrast* is defined as the difference between the average gray level of the region and the average of the current boundary, and it is continuously calculated. The maximum contrast corresponds to the point where the process has started to grow into the background. Finally, the last maximum gradient measure, before the maximum contrast point, specifies the best boundary for the region.

Siebert (1997) developed an interesting, simple, and faster integration technique in which edge information is used to adjust the criterion function of a region-growing segmentation. For each seed the algorithm creates a whole family of segmentation results (with different criterion functions) and then, on the basis of the local quality of the region's contour, picks the best one. As a way to measure the segmentation quality, a metric that evaluates the strength of a contour is proposed. The contour strength $cs(R)$ of a region $R$ is defined as the contrast between both sides of the boundary. More formally, the contour strength is expressed as the sum of the absolute differences between each pixel on the contour of a region and the pixels in the four-neighborhood of these contour points that are not part of the region. So that this parameter can be calculated, it is necessary to process a contour-following task, as well as several differences between integer numbers. As these authors remark, these operations are computationally inexpensive. Furthermore, these authors suggest that slightly improved results at higher computational costs can be expected if the contour strength is based on the gradient at each contour pixel rather than on the intensity difference.

A similar methodology can be found in the work of Revol-Muller *et al.* (2000), in which they proposed a region-growing algorithm for the segmentation of medical three-dimensional images. As in the work described previously, the method consists of generating a region-growing sequence by increasing the criterion function at each step. An evaluation function estimates the quality of each segmented region and permits determination of the optimal threshold. This method is illustrated schematically in Figure 13. These authors proposed different parameters based on either boundary or region criteria to be used as the evaluation function. Three choices based on boundary criteria are proposed: (1) the sum of contrasts of all transition couples (two neighboring pixels located on either side of the boundary are called a *transition couple*), normalized by the total number of transition couples; (2) the sum of all standard deviations of members of the boundary and its neighboring pixels not belonging to the segmented region, normalized by the total number of pixels belonging to the boundary; and (3) the sum of transition levels of all transition couples, normalized by the total number of transition couples. Three alternative choices based on region criteria are proposed: (1) entropy, (2) intercluster variance, and (3) inverse distance between the gray-level function of the original image and the mean of the region and its complement. Tests on three-dimensional magnetic resonance images demonstrated that the proposed algorithm achieves better results than those of manual thresholding.

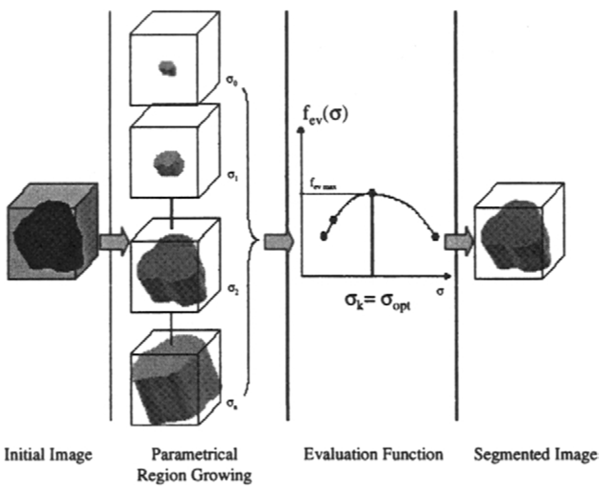


FIGURE 13. Scheme of the method proposed by Revol-Muller *et al.* (2000). A sequence of segmented regions is obtained by increasing the homogeneity threshold. Then, the evaluation function determines the optimal threshold automatically.

More ideas about the integration of different methods can be found in the work of Hibbard (1998), in which snakes are used to evaluate the quality of a segmentation result. The proposal is based on an iteratively region-growing approach, in which at each stage the region of interest grows following a deterministic criterion function based on a hierarchical classifier operating on texture features. At each stage, the optimal contour is determined by using snakes. The optimal choice is the one that best satisfies the three conditions of the objective function proposed in 1994 by Chakraborty *et al.* (see Section III.B and Eq. (9)). The function proposed by Chakraborty is used in the method as a quality measure of the current segmentation and allows choice of the best segmentation among the set of iterations of the growing process. Finally, the resulting contour corresponds to the maximum of all the iteratively computed contours.

## IV. Summary

The review of different segmentation proposals that integrate edge information and region information has permitted the identification of different strategies and methods to fuse such information. The aim of this summary is to point out the advantages and disadvantages of these approaches, as well as to remark upon new and interesting ideas that perhaps have not been properly exploited.

For the purpose of providing an overview of the presented methods, Table 1 summarizes the different ways to carry out the integration of edge and region information. The first column distinguishes the strategy according to the time of fusion: embedded or postprocessing. The second column identifies the approach used to carry out the segmentation. The next two columns describe the problem that the approach tries to solve and a description of the objective. The last column summarizes the procedure used to perform the segmentation task.

As described in Section I.A, embedded integration and postprocessing integration use different principles to perform the task of segmentation. Embedded integration is based on the design of a complex, or a superior, algorithm which uses region and edge information to avoid errors in segmentation. Conversely, the postprocessing strategy accepts faults in the elemental segmentation algorithms, but an a posteriori integration module tries to correct them. The key words that allow characterization and comparison of both strategies are as follows:

- Single algorithm and avoidance of errors by embedded integration
- Multiple algorithms and correction of errors by postprocessing integration

These two essential characteristics cause these strategies to exhibit outstanding differences. The first aspect to analyze is the complexity of both

TABLE 1
SUMMARY OF APPROACHES TO IMAGE SEGMENTATION INTEGRATING REGION AND BOUNDARY INFORMATION

| Integration | Approach | Problem to solve | Objective | Procedure |
|---|---|---|---|---|
| Embedded | Seed placement | The resulting region-based segmentation inevitably depends on the choice of initial region growth points. | Choice of reasonable starting points for region-based segmentation | Edge information is used to choose a seed (or some seeds) inside the region to start the growth. |
| | Decision criterion | The obtained region's shape depends on the particular growth decision criterion chosen. | To have in account edge information, together or not with color information, which can be used to decide about the homogeneity of a region | A region is not homogeneous when there are edges inside. For this reason a region cannot grow beyond an edge. |
| Post-processing | Over-segmentation | Uniformity criteria are too strict and generate false boundaries in segmentation. | To remove false boundaries that do not coincide with additional information | Thresholds are set to obtain a first oversegmented result. Next, boundaries that do not exist in the segmentation from a complementary approach are removed. |
| | Boundary refinement | Region-based segmentation generates errors at boundaries and this is highly irregular. | To refine the result from region-based segmentation by using edge information to arrive at a more accurate representation | A region-based segmentation is used to get an initial estimate of the region. Next, the optimal boundary that coincides with edges is searched. This process is generally carried out by using snakes. |
| | Selection | There is not a criterion to enable the evaluation of the quality of a segmentation. | To use edge information to carry out this evaluation in order to choose the best segmentation from a set of results | The quality of a region segmentation is measured as the correspondence of the boundary with the edge information. |

strategies. Embedded integration produces, in general, a more complex algorithm because, as derived from its definition, it endeavors not to commit errors or take wrong decisions. In contrast, the postprocessing strategy can be viewed as the set of many simple algorithms working in parallel and producing many wrong segmentation results. The solution of these problems is moved to an a posteriori fusion module that refines these results. Therefore, postprocessing complexity is lower because the quantity of information to process decreases, as only the results are taken into consideration.

Another aspect to analyze is the independence of these integration strategies as regards their implementation in the segmentation algorithm. In this sense, the embedded strategy is strongly dependent, because typically it implies the design of a new algorithm, which incorporates the integration internally. Hence, any change in the integration procedure will imply the modification of the algorithm. On the contrary, the postprocessing strategy produces a more general approach because it is independent of the choice of algorithms to segment the image. The fusion of the information takes into account only the results of the segmentation algorithms, so the way they are obtained is not important, and it is possible to use any established algorithms. Some researchers (Le Moigne and Tilton, 1995) indicate that postprocessing integration can also be viewed in a general data management framework, in which all incoming data are processed on-line upon acquisition, which produces basic features such as edges and regions.

However, it is necessary to remark that this independence assigned to the postprocessing strategy is not complete, and this is the weak point of this approach. It is true that it is independent concerning the chosen method, but if the results achieved by these algorithms are very poor, postprocessing fails. It is undeniable that a posteriori fusion needs to work on a relatively good set of segmentation results. Therefore, final segmentation will inevitably depend, to a larger or lesser extent, on the initial results of the segmentation. An initial fault, for example, is that inappropriate selection of seeds in a region-growing algorithm will be carried over into the totality of the segmentation process. A posteriori integration of edge information may not be able to overcome an error of this magnitude.

### A. *Disadvantages of Both Strategies*

Once a set of key proposals integrating edge information and region information have been reviewed, it can be stated that it is not feasible to determine which is the best. This problem occurs for two reasons. First, there is no generally accepted methodology in the field of computer vision which elucidates how to evaluate segmentation algorithms (Ng and Lee, 1996). Second, comparing different segmentation algorithms with each other is difficult, mainly because they differ in the properties and objectives they try to satisfy and in the image domain in which they are working. In this sense, it is well known that there is no method available for all images, since requirements related to the images to be segmented are different (e.g., the requirements for three-dimensional medical image analysis are very different from those for the outdoor color images analyzed by a road-following system).

In reference to the weak points of the approaches, a serious difficulty appears when it is required, as is usual, that the most significant edges in the image be obtained. This process is not a trivial task: for example, the gradient map has some hardships as regards the choice of an adequate threshold to achieve a reliable binarization. In this sense, the embedded proposals that use the gradient map directly as boundary information have an important advantage. Another weak point to take into account is the lack of attention that, in general, the reviewed works devote to texture. Without this property, it is not possible to discern whether a high-magnitude gradient corresponds to a boundary between regions or is the response to a textured region. Regrettably, the texture is generally forgotten in the different proposals of embedded integration. As a consequence, the algorithms are not adapted to segment heavy textured areas, which results in an oversegmentation of these regions.

Segmentation techniques based on postprocessing integration also suffer from some deficiencies. Those based on starting from an oversegmented image must solve a nontrivial problem: what should the threshold be to obtain an oversegmented result? It is well known that images have different characteristics, so this threshold cannot be a fixed value. An adequate threshold for one image may not be effective for others, and because of this the irrecoverable loss of boundaries can result. An initial mistake in such algorithms could be a serious handicap for the a posteriori fusion, which would yield an undersegmented result. As described in Section III.B, the aim of the boundary refinement approaches is to obtain reliable smooth boundaries. As a way to achieve this, the co-operation between region-based segmentation and snakes, which is the most usual technique, is a good choice. However, it should be stressed that the objective of these algorithms is, generally, to segment not a whole image, but individual objects from an image. Furthermore, these algorithms bear a deficiency that is shared with the third set of postprocessing methods: their exclusive attention to the boundary. Result refinement is reduced to the region boundary, so it is not possible to correct other mistakes inside the region. The same problem is found in the selection approach, in which the quality measure of a segmentation based on boundary information is exclusively based on the external boundary and not on any inner contour lines caused by holes. For this reason, the regions extracted may contain holes. In summary, all these weak points of postprocessing integration reaffirm the previous assertion about the necessity of having good initial segmentation results and the incapacity of the postprocessing strategy to correct some initial mistakes.

## V. Conclusions and Further Work

In this article we reviewed some key segmentation techniques integrating region information and boundary information. Special emphasis was given to the strategy performed to carry out the integration process. In this sense, a classification of cooperative segmentation techniques was proposed, and this article described several algorithms with the aim of pointing out their strengths and weaknesses.

The lack of a special treatment of textured images was noticed, and it is one of the great problems of segmentation (Deng *et al.,* 1999). If an image mainly contains homogeneous color regions, traditional methods of segmentation working in color spaces can be sufficient to achieve reasonable results. However, some real images "suffer" from texture, for example, images corresponding to natural scenes which have considerable variety of color and texture. Hence, undoubtedly, texture has a pivotal role to play in image segmentation. However, new and promising research has started in relation to the integration of color and texture (Mirmehdi and Petrou, 2000). The intention of integrating complementary information from the image may follow; it seems reasonable to think that a considerable improvement in segmentation could result from the fusion of color, texture, and boundary information.

Concerning the strategies for the integration of edge information and region information, it is obvious that there are still methods to be explored. In this sense, a hybrid strategy between embedded and postprocessing may be a solution for some of the previously mentioned typical weak points. A basic scheme of such an idea is presented in Figure 14, where an algorithm based on an embedded strategy produces an initial result, which will be a posteriori refined

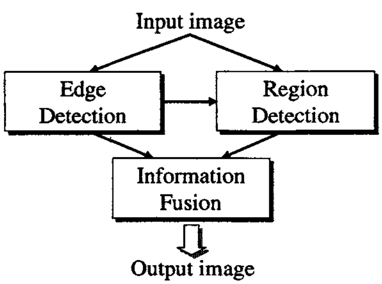


FIGURE 14. Hybrid strategy scheme for region and boundary integration.

by a postprocessing fusion with boundary information. More specifically, the first step of this new proposal could consist of the extraction of edge and region information. In this sense, an embedded algorithm permits the adequate placement of the seeds in an optimal position, and a result with regions free of holes may be obtained. Then, a posteriori fusion with boundary information could refine the segmentation, improving the resulting boundaries. This proposal combines both strategies in a hybrid scheme that uses integration of information in all steps of segmentation with the aim of obtaining a better segmentation result.

Segmentation techniques, in general, still require considerable improvement. Surveyed techniques still present some faults and there is no perfect segmentation algorithm, something which is vital for the advancement of computer vision and its applications. Nevertheless, integration of region and boundary information has allowed the improvement of previous results. Current work in this field of research has generated numerous proposals. Thus, this current interest permits us to foresee that further work and improvement of segmentation will be focused on the integration of algorithms and information.

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