Monetary Policy and Production Networks: An Empirical Investigation*

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Abstract

This paper offers novel econometric evidence on the contribution of production networks to the effect of monetary shocks on real macroeconomic variables. In particular, we construct a highly disaggregated monthly dataset on US final sectoral consumption to estimate that at least 30% of the effect of monetary shocks on aggregate consumption comes from amplification through input-output linkages, which facilitate downstream propagation of price rigidity. At the sectoral level, we find that the network effect rises in the frequency of price non-adjustment and intermediates intensity. Moreover, the network effect is highly concentrated: sectors that jointly account for 17% of our sample aggregate consumption account for 98% of the amplification. In order to develop our econometric specification, we obtain novel analytical sector-level solutions to a forward-looking New Keynesian model with asymmetric input-output linkages.

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1. Introduction

How different economic sectors are interlinked and affect each other has long been a question of interest to economists – at least since Wassily Leontief's invention of input-output analysis in the 1950s. Nevertheless, it is only in recent years that input-output linkages, or production networks, have firmly established their presence in neoclassical macroeconomics. Indeed, production networks have recently shed light on a range of fundamental economic questions. Most notably, Acemoglu et al. (2012) build on the multi-sector framework of Long and Plosser (1983), and theoretically show that when certain sectors are disproportionally large suppliers of inputs to the rest of the economy, sectoral shocks can get amplified by input-output linkages, causing aggregate business cycle fluctuations. Such role for input-output linkages in creating aggregate fluctuations has since found support in the empirical literature (Foerster et al., 2011; Acemoglu et al., 2016).

On the side of aggregate nominal shocks, production networks have been studied as a mechanism that enables New Keynesian models to simultaneously match microeconomic evidence on frequency of price

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adjustment and macroeconomic evidence on strength of monetary transmission to real variables.[1] Basu (1995) theoretically establishes that monetary policy shocks get amplified in presence of production networks, as the latter create strategic complementarities in firms' price setting. Intuitively, the latter makes firms inherit price stickiness from their suppliers, thus creating extra short-run money non-neutrality. The above theoretical mechanism has since been studied in fully specified multi-sector quantitative models (Nakamura and Steinsson, 2010; Pasten et al., 2020; Carvalho et al., 2021). However, econometric evidence on the ability of production networks to amplify the effects of monetary policy shocks on the real economy remains almost non-existent. In what follows we make a first step towards filling this gap in the literature.

Our paper is the first study to offer econometric evidence on the role of production networks in creating short-run monetary non-neutrality of real macroeconomic variables. In particular, our empirical contribution is in establishing the following results. First, at least 30 per cent of the effect of monetary policy shocks on US real aggregate final consumption is due to production networks, which facilitate downstream propagation of price rigidity. This result is robust to varying the degree of sectoral disaggregation and the type of monetary shocks used. Second, we document substantial heterogeneity in network amplification across sectors and further examine its dimensions. Our cross-sectional analysis suggests that the network effect is, *ceteris paribus*, larger for sectors that adjust prices less frequently and those that rely more on intermediate inputs; moreover, we rationalize such results through the lens of our theoretical model. Third, we show that the aggregate network contribution is highly concentrated in the sense of being driven by a relatively small number of sectors; quantitatively, sectors that jointly account for 17 per cent of our sample aggregate consumption account for 98 per cent of the amplification by production networks.[2]

Our econometric specification is derived from a multi-sector New Keynesian model with asymmetric input-output linkages, and we make two theoretical contributions that allow us to form a tractable econometric strategy. First, we formally show that the response of sectoral final consumption to a monetary shock can be decomposed into the *direct effect*, driven by that sector's own frequency of price non-adjustment, and the *downstream effect*, driven by persistence in price setting inherited from supplier sectors and the intensity with which intermediates are used. It is the downstream effect that represents network amplification of monetary shocks. Second, we consider a finite-time version of our model, where we use backward induction in order to find novel analytical solutions for sectoral final consumptions. Those solutions directly imply a simple econometric strategy based on linear regressions that allows to consistently estimate a lower bound for the downstream effect. Our analytical solutions introduce a novel concept of the *Forward-looking Leontief Inverse*, which can be used to find analytical solutions to other problems featuring production networks and nominal rigidities in a forward-looking setting. Previous studies, such as Pasten et al. (2020), instead rely on myopic firms or other deviations from forward-looking behavior in order to obtain sector-level closed-form results.

Estimating our econometric specification requires sectoral final consumption time series whose classification matches that used in input-output accounts. Such time series are not readily available for the US, and our paper makes a further contribution by constructing a novel monthly dataset for sectoral final consumptions for up to 161 sectors of the US economy, whose classification matches that in the US Bureau of Economic Analysis (BEA) input-output accounts. In particular, we take monthly product-level consumption and consumer price time series contained in NIPA tables and use PCE Bridge Tables to match NIPA product definitions to sectoral classifications of the US BEA. In this way, our sectoral final consumption series are consistent in their classification with the sectoral classification used by BEA for their input-output-tables, and can be used together in our econometric analysis. To the best of our knowledge, we are the first ones to construct a monthly dataset for sectoral final consumption that uses classification consistent with BEA input-output accounts.

Contribution to the literature. Our results should be viewed in the context of at least two strands

[1] As discussed in Mackowiak and Smets (2008), in order to match VAR-based estimates of the effect of monetary shocks on real variables, a New Keynesian model with no additional real rigidities needs to be calibrated with a frequency of price adjustment much lower than that observed in microeconomic studies.

[2] Specifically, those broad sectoral groups are "Transportation and warehousing", "Retail trade", "Information", "Agriculture, forestry, fishing and hunting" and "Professional and business services".

of existing literature.[3] First, we contribute to the empirical literature that studies the extent to which production networks amplify the effects of economic shocks. On the side of sector-specific shocks, Acemoglu et al. (2016) identify sector-specific fiscal, imports and productivity shocks and show that the vast majority of their non-negligible impact on US sectoral value added comes from the amplification via input-output linkages. Barrot and Sauvagnat (2016) identify even more disaggregated firm-level shocks via the occurrence of natural disasters and similarly report large spillovers from suppliers of inputs to their customers.

The empirical literature on amplification of aggregate nominal shocks remains extremely thin. One exception is Ozdagli and Weber (2017) who use spatial econometric techniques to show that 50 to 85 per cent of the effect of monetary policy shocks on US stock returns comes from the amplification through production networks. Our work makes a contribution in at least two substantial ways. First, we focus on the effect of monetary shocks on real macroeconomic variables, most specifically final consumption, whereas Ozdagli and Weber (2017) study stock market responses. Second, we empirically assess a completely different theoretical mechanism: our specification estimates contribution of production networks that comes from complementarities in price setting and downstream propagation of price rigidity, whereas Ozdagli and Weber (2017) build their specification in a flexible-price framework and consider upstream propagation of demand created by monetary shocks and the effect on stock market returns.

Secondly, our results add to the quantitative theory literature on complementarities in price setting and monetary policy under production networks. Following the seminal study by Basu (1995), Carvalho (2006) shows in a DSGE setting that when the degree of price stickiness is allowed to vary across sectors, complementarities in price setting significantly increase the degree of money non-neutrality. Nakamura and Steinsson (2010) develop a multi-sector menu cost model that they calibrate for up to 14 US sectors and report large increases in short-run money non-neutrality coming from differential price stickiness across sectors coupled with intermediate inputs. Pasten et al. (2020) develop a multi-sector model with sector-specific probabilities of price adjustment that they calibrate to 341 sectors of the US economy and confirm large amplifications of monetary policy shocks coming from strategic complementarities in price setting. As for optimal monetary policy under production networks, it is studied in La'O and Tahbaz-Salehi (2019).

Several recent studies have used a mixture of aggregate and sectoral data to explicitly estimate multi-sector New Keynesian models with roundabout production and heterogeneous price stickiness. Most notably, Smets et al. (2019) use Bayesian techniques to estimate a 35-sector model on US data, and subsequently study the role of production networks for inflation dynamics; Carvalho et al. (2021) estimate sector-specific probabilities of price adjustment for 27 major sectors of the US economy, and show that the estimated model successfully replicates sectoral price responses to both idiosyncratic and aggregate shocks from Boivin et al. (2009). In addition, Bouakez et al. (2014) estimate a 30-sector menu cost model with roundabout production, and show that the implied sectoral frequencies of price adjustment match those from micro-based studies (Nakamura and Steinsson, 2008).

The rest of the paper is structured as follows. In Section 2 we build a theoretical framework and derive analytical solutions for sectoral consumptions that informs our econometric specification. Section 3 outlines the construction of our dataset. Our econometric strategy is detailed in Section 4. Sections 5 and 6 discuss the main empirical results of this paper, whose robustness is then tested in Section 7. Section 8 concludes and outlines our strategy for future work.

2. Model

In this section, we formalize our ideas in the context of a multi-sector New Keynesian model with production networks. Our theoretical contribution is twofold. First, we formally show that the response of sectoral consumption to a monetary shock can be decomposed into the *direct effect*, driven by sector's own price stickiness, and the *downstream effect*, driven by stickiness of suppliers and the intensity with which intermediates are used. Second, we find novel analytical solutions for sectoral final consumptions by considering a version of our model in finite time; all proofs are given in Appendix B.

[3] See Carvalho and Tahbaz-Salehi (2019) for a comprehensive review of the literature on production networks in macroeconomics.

2.1. Firms

On the production side, our economy consists of K sectors, indexed by $k \in \{1, 2, ..., K\}$, and a continuum of monopolistically competitive firms, indexed by j, that each belongs to one sector; let Φ_k denote the set of all firms in sector k. The production function of firm j that operates in sector k is given by:

$$Y_{kt}(j) = N_{kt}(j)^{1-\delta_k} Z_{kt}(j)^{\delta_k} - \Xi_k(j), \tag{1}$$

where $Y_{kt}(j), N_{kt}(j)$ and $Z_{kt}(j)$ are, respectively, total output, labor input and intermediate inputs used, δ_k is intermediates intensity that is allowed to be sector-specific, $\Xi_k(j)$ is a fixed cost which is set to equalize steady-state profits to zero and rule out entry and exit.

Total intermediate inputs used is an aggregate of intermediate goods purchased from all the different sectors in the economy, so that $Z_{kt}(j) \equiv \prod_{r=1}^{K} \omega_{kr}^{-\omega_{kr}} Z_{krt}^{\omega_{kr}}(j)$, where $Z_{krt}(j)$ is the intermediate inputs purchased by firm j in sector k from all firms in sector r; ω_{kr} is the relative intensity with which firms in sector k use goods produced in sector r as inputs, so that $\sum_{r=1}^{K} \omega_{kr} = 1 \quad \forall k$. In turn, $Z_{krt}(j)$ is also an aggregator for all the inputs purchased from the different firms that belong to sector r, namely $Z_{krt}(j) \equiv \left(\int_{\Phi_r} Z_{krt}(j, j')^{\frac{\theta-1}{\theta}} dj' \right)^{\frac{\theta}{\theta-1}}$, where $Z_{krt}(j, j')$ denotes total inputs purchased by firm j in sector k from firm j' in sector r.

Given the sectoral nominal wage W_{kt} and the input price index P_t^k, $\quad P_t^k \equiv \prod_{r=1}^{K} P_{rt}^{\omega_{kr}}$, firms choose $N_{kt}(j)$ and $Z_{kt}(j)$ in order to minimize their total inputs cost subject to the production function in (1). The latter delivers the the optimal labor/intermediate inputs mix condition $\frac{Z_{kt}(j)}{N_{kt}(j)} = \frac{\delta_k}{1-\delta_k} \frac{W_{kt}}{P_t^k}$ and for the nominal marginal cost, common to all firms within a given sector:

$$MC_{kt}(j) = MC_{kt} = \frac{\delta_k^{-\delta_k}}{(1-\delta_k)^{1-\delta_k}} W_{kt}^{1-\delta_k} \left[\prod_{r=1}^{K} P_{rt}^{\omega_{kr}} \right]^{\delta_k}. \tag{2}$$

One can see that the marginal cost for any firm is effectively a weighted average of its labor input cost, given by the wage, and its sector-specific inputs price index, whose relative importance increases in the share of intermediate inputs δ_k. It can also be seen how $\delta_k > 0$ makes the marginal cost of any firm sensitive to prices set by its suppliers. This is the mechanism through which production networks create complementarities in price setting, which in turn amplifies the degree of monetary non-neutrality in our economy.

Price stickiness in our economy is modeled as in Calvo (1983), although the probability of price adjustment is allowed to vary across sectors. More precisely, in any period a firm in sector k has probability $(1-\alpha_k)$ of setting its price equal to its optimal value. The optimal price at time t is chosen to maximize expected future discounted nominal profits:

$$\max_{P_{kt}(j)} \mathbb{E}_t \sum_{s=0}^{\infty} \alpha_k^s F_{t,t+s} (P_{kt}(j) Y_{k,t+s}(j) - W_{k,t+s} N_{k,t+s}(j) - P_{t+s}^k Z_{k,t+s}(j)), \tag{3}$$

subject to the production function in (1); $F_{t,t+s}$ is the stochastic discount factor between periods t and $t+s$ and is defined in the next subsection. After imposing the optimal inputs mix condition and the marginal cost equation from (2), the first order condition for the optimal reset price for any firm in sector k, P_{kt}^* is given by $P_{kt}^* = \frac{\theta}{\theta-1} \frac{\mathbb{E}_t \sum_{s=0}^{\infty} \alpha_k^s F_{t,t+s} P_{k,t+s}^{\theta} Y_{k,t+s} MC_{k,t+s}}{\mathbb{E}_t \sum_{s=0}^{\infty} \alpha_k^s F_{t,t+s} P_{k,t+s}^{\theta} Y_{k,t+s}}$. Given that the optimal reset price is identical for all firms within a sector, sectoral price index can be obtained by simple aggregation: $P_{kt} = \left[\alpha_k P_{k,t-1}^{1-\theta} + (1-\alpha_k)(P_{kt}^*)^{1-\theta} \right]^{\frac{1}{1-\theta}}$.

2.2. Households

A continuum of infinitely lived households populates our economy and owns all the firms. Markets are assumed to be complete, so a full set of Arrow-Debreu securities is available. The representative household makes choices to maximize the lifetime utility:

$$\max_{\{C_{t+s},\{N_{k,t+s}\}_{k=1}^{K},B_{t+s+1}\}_{s=0}^{\infty}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\ln(C_{t+s}) - \sum_{k=1}^{K} g_k N_{k,t+s} \right] \tag{4}$$

subject to

$$P_t^c C_t + \mathbb{E}_t \left[F_{t,t+1} B_{t+1}\right] \leq B_t + \sum_{k=1}^{K} W_{kt} N_{kt} + \sum_{k=1}^{K} \Pi_{kt}, \quad \forall t \tag{5}$$

where C_t and P_t^c are the composite consumption good and the consumption price index (defined below) respectively, N_{kt} is sectoral labor supply, B_{t+1} is the stochastic payoff of securities purchased at time t, $F_{t,t+1}$ is the stochastic discount factor to price those securities at time t and Π_{kt} denotes aggregate nominal profits of firms in sector k; β is the discount factor for future utility and g_k is a parameter capturing the relative disutility of sector-specific labor supply.

The composite consumption index C_t is an aggregator for the final consumption of goods produced in the different sectors of our economy, so that $C_t \equiv \prod_{k=1}^{K} \omega_{ck}^{-\omega_{ck}} C_{kt}^{\omega_{ck}}$, where ω_{ck} is the relative weight consumers put on goods produced in sector k, $\sum_{k=1}^{K} \omega_{ck} = 1$; C_{kt} is in turn an aggregator for the final consumption of goods produced by firms that belong to sector k: $C_{kt} \equiv \left(\int_{\Phi_r} C_{kt}(j)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}$.

The households' maximization problem delivers standard first-order conditions, namely an equation for the stochastic discount factor $F_{t,t+s} = \beta^s \left(\frac{C_{t+s}}{C_t} \right)^{-1} \frac{P_t^c}{P_{t+s}^c}$, and equations for consumption-sectoral labor supply choice $C_t^{-1} = g_k \frac{P_t^c}{W_{kt}}, \quad \forall k$, where the consumption price index is given by $P_t^c = \prod_{k=1}^{K} P_{kt}^{\omega_{ck}}$.

2.3. Monetary policy

Following Nakamura and Steinsson (2010), we assume that the monetary authority acts in a way that ensures that nominal GDP, given by $M_t \equiv P_t^c C_t$, follows an exogenous process:

$$\ln M_t = \ln M_{t-1} + r_t, \tag{6}$$

where $r_t = \rho_r r_{t-1} + \varepsilon_t^r$ defines an AR(1) process for monetary policy shocks, and innovations $\{\varepsilon_t^r\}_t$ constitute a zero-mean white noise process with variance σ_r^2. The above can be rationalized by imposing a cash-in-advance constraint on aggregate final consumption, and assuming a constant velocity of money. As discussed in Nakamura and Steinsson (2013), under such assumptions the process in (6) is equivalent to a rule for money supply.[4]

For the remainder of the main text we focus on the simplified case of non-persistent monetary shocks ($\rho_r = 0$); however, in Appendix B we provide formal analytical results for the case of persistent shocks ($|\rho_r| \in (0,1)$).

2.4. Market clearing

In addition to the optimality conditions, budget constraints and the policy rule above, equilibrium in our economy is also characterized by market-clearing conditions in the asset market: $B_t = 0$; the labor markets: $N_{kt} = \int_{\Phi_k} N_{kt}(j) dj, \quad \forall k$; and the goods markets: $Y_{kt}(j) = C_{kt}(j) + \sum_{r=1}^{K} \int_{\Phi_r} Z_{rkt}(j', j) dj', \quad \forall k, \forall j \in \Phi_k$.

2.5. Log-linearized equilibrium

We log-linearize the equilibrium conditions from the previous subsections around a symmetric zero-inflation steady-state. In Appendix A we fully characterize both the steady state and the log-linearized

[4] An alternative would be to close our model with a Taylor rule for the nominal interest rate, which comes at the cost of analytical tractability. In Section 7 we report numerical exercises with a Taylor rule.

system; here we instead focus on the resulting sectoral pricing equations (lower case letters denote log-deviations from the steady state):

$$p_{kt} = \alpha_k p_{k,t-1} + (1-\delta_k)(1-\alpha_k)m_t + \delta_k(1-\alpha_k)(1-\beta\alpha_k)\sum_{s=0}^{\infty}(\beta\alpha_k)^s \sum_{r=1}^{K}\omega_{kr}\mathbb{E}_t p_{r,t+s}, \tag{7}$$

for $k = 1, 2, ..., K$. One can clearly see that every sectoral price is now positively related to every other sectoral price in the economy, both current and those expected in future periods, weighted by the input-output entry between the respective sectors and increasing in the share of intermediate inputs δ_k. As a result, persistence in price adjustment can travel from supplier sectors to customer sectors, leading to slower adjustment of sectoral prices.

In addition, given the slower adjustment of sectoral prices, the degree of short-run money non-neutrality rises and the effect on real variables, such as consumption, becomes larger in the short run. The following proposition formalizes the above statement for the sectoral consumption:

Proposition 1. *The response of final consumption in sector k c_{kt} to a monetary shock r_t is given by:*

$$c_{kt} = \overbrace{\alpha_k c_{k,t-1} + \alpha_k r_t}^{\textbf{Direct effect}} + \underbrace{\delta_k(1-\alpha_k)(1-\beta\alpha_k)\sum_{s=0}^{\infty}(\beta\alpha_k)^s \sum_{r=1}^{K}\omega_{kr}\mathbb{E}_t c_{r,t+s}}_{\textbf{Downstream effect}}, \tag{8}$$

for $k = 1, 2, ..., K$. It consists of the the direct effect, unaffected by the intermediate inputs share δ_k, and the downstream effect, which fully vanishes under $\delta_k = 0$ and increases in δ_k, representing amplification through input-output linkages.

Proposition 1 establishes that the effect of a monetary shock r_t on sectoral final consumption can be decomposed into the *direct effect*, caused by that sector's own price stickiness and hence independent of δ_k, and the *downstream effect*, caused by price stickiness inherited from that sector's suppliers and hence increasing in δ_k and disappearing under $\delta_k = 0$. It is therefore the downstream effect that represents the amplification of the effect of monetary shocks through input-output linkages. Intuitively, the direct effect strictly increases in sector's own probability of price non-adjustment α_k. As for the downstream effect, two opposing channels of own price stickiness are at work. On the one hand, higher α_k reduces sector k's price sensitivity to its current and expected future marginal costs, making it harder to inherit stickiness from suppliers, thus dampening the downstream effect; on the other hand, higher α_k makes sector $k's$ optimal price more sensitive to expected future marginal costs (as any price set is more likely to persist in the future), thus amplifying the downstream effect.

A natural next step is to find a solution for sectoral consumption that can then be used to empirically disentangle the direct and downstream effects. Due to a number of asymmetries, however, such as heterogeneous input-output weights $\{\omega_{kr}\}_{k,r}$ and Calvo parameters $\{\alpha_k\}_k$ one cannot easily find a sector-level analytical solution to the above problem. However, in the next subsection we show how a slight modification to our model, featuring finite time, enables us to find analytical solution for sectoral consumption.[5]

2.6. Analytical solution in finite time

In this subsection we propose a slight modification to our original model that allows to find analytical solutions for sectoral price indices and final consumptions. In particular, we assume that the economy exists for a finite deterministic number of time periods $T > 1$, and firms face a Calvo lottery between periods 1 and $T-1$, whereas in the final period T all firms get to adjust their prices. The assumptions are summarized below:

[5] In Appendix C we numerically solve a simple calibrated three-sector ($K = 3$) example in the infinite-horizon case, allowing us to gain further intuition behind monetary transmission and the role of production networks.

Assumption 1. *The economy exists for a finite deterministic number of periods $T > 1$.*

Assumption 2. *In periods t=1,2,...,T-1 every firm faces a Calvo lottery with a sector-specific probability of price non-adjustment given by α_k; in period T all firms get to set their prices optimally.*

Note that the above two assumptions have no implications for the households' problem. In particular, the intratemporal consumption-labor supply choice equations remain unchanged for all $t = 1, 2, ..., T$, and the equation for the stochastic discount factor remains valid for any $1 \leq t, t + s \leq T - 1$.

Slight modifications happen on the firms' side, however. In period $t = T$, the representative firm in sector k solves a static problem of choosing price $P_{kT}(j)$ to maximize contemporaneous profits $\Pi_{kT}(j) = (P_{kT}(j)Y_{kT}(j) - W_{kT}N_{kT}(j) - P^k_T Z_{kT}(j))$, subject to the production function in (1); given no restrictions on price setting, ever firm sets the same optimal price at a constant mark-up to the marginal cost, so that $P_{kT} = P^*_{kT} = \frac{\theta}{\theta-1}MC_{kT}, \quad \forall k$, or $p_{kT} = p^*_{kT} = mc_{kT} = m_T$ in log-linear terms. Consequently, money is neutral at $t = T$, and $c_{kT} = c_T = 0, \forall k$.

In period $1 \leq t \leq T - 1$, the representative firm in sector k chooses its price to maximize expected future discounted nominal profits between periods t and $T - 1$, as in T it will get to re-set prices with certainty. The latter delivers a familiar expression for the optimal price, shared by all firms in sector k: $P^*_{kt} = \frac{\theta}{\theta-1}\frac{\mathbb{E}_t \sum_{s=0}^{T-t-1} \alpha_k^s F_{t,t+s} P^{\theta}_{k,t+s} Y_{k,t+s} MC_{k,t+s}}{\mathbb{E}_t \sum_{s=0}^{T-t-1} \alpha_k^s F_{t,t+s} P^{\theta}_{k,t+s} Y_{k,t+s}}, \quad 1 \leq t \leq T-1, \quad \forall k$, or, when written in log-linear terms: $p^*_{kt} = \frac{1-\beta\alpha_k}{1-(\beta\alpha_k)^{T-t}} \sum_{s=0}^{T-t-1} (\beta\alpha_k)^s \mathbb{E}_t mc_{k,t+s}, \quad 1 \leq t \leq T-1, \quad \forall k$. In every period $1 \leq t \leq T - 1$ every firm is still subject to the Calvo lottery with a sector-specific probability of price non-adjustment α_k. Given the optimal sectoral price p^*_{kt}, the sectoral price index is then given by $p_{kt} = \alpha_k p_{k,t-1} + (1 - \alpha_k)p^*_{kt}$, $1 \leq t \leq T - 1, \forall k$.

One can now obtain the finite-time equivalent of the sectoral price equation (7), highlighting complementarities in price setting; for compactness, we let $\mathbb{p}_t \equiv [p_{1t}, p_{2t}, ..., p_{Kt}]'$, $\mathbb{m}_t \equiv [m_t, m_t, ..., .m_t]'$, $A \equiv diag[\alpha_1, \alpha_2, ..., \alpha_K]'$, $\Delta \equiv diag[\delta_1, \delta_2, ..., \delta_K]'$, $G_{T-t} \equiv (I - \beta A)(I - (\beta A)^{T-t})^{-1}$ and write it in matrix form as:

$$\mathbb{p}_t = A\mathbb{p}_{t-1} + (I - \Delta)(I - A)\mathbb{m}_t + \Delta(I - A)G_{T-t} \sum_{s=0}^{T-t-1} (\beta A)^s \Omega \mathbb{E}_t \mathbb{p}_{t+s} \tag{9}$$

for $1 \leq t \leq T - 1$. Further, recall that $c_{kt} = p^c_t + c_t - p_{kt} = m_t - p_{kt}$, and letting $\mathbb{c}_t \equiv [c_{1t}, c_{2t}, ..., c_{Kt}]'$, $\mathbb{r}_t \equiv [r_t, r_t, ..., .r_t]'$ we can write the finite-time equation for sectoral consumption as:

$$\mathbb{c}_t = A\mathbb{c}_{t-1} + A\mathbb{r}_t + \Delta(I - A)G_{T-t} \sum_{s=0}^{T-t-1} (\beta A)^s \Omega \mathbb{E}_t \mathbb{c}_{t+s} \tag{10}$$

for $1 \leq t \leq T - 1$; which is just the finite-time version of the direct-downstream effect decomposition from Proposition 1.

It should now be clear how equations (9) and (10) allow us to use backward induction to find analytical solutions for sectoral prices and consumptions respectively. Here we focus on the case of consumptions, although the same procedure can be applied to prices. According to (10), at $t = T - 1$ sectoral consumptions vector is given by:

$$\mathbb{c}_{T-1} = A\mathbb{c}_{T-2} + A\mathbb{r}_{T-1} + \Delta(I - A)G_1\Omega\mathbb{c}_{T-1}, \tag{11}$$

and it follows that $\mathbb{c}_{T-1} = [I - \Delta(I - A)G_1\Omega]^{-1}A\mathbb{c}_{T-2} + [I - \Delta(I - A)G_1\Omega]^{-1}A\mathbb{r}_{T-1}$. Similarly, at $t = T - 2$:

$$\mathbb{c}_{T-2} = A\mathbb{c}_{T-3} + A\mathbb{r}_{T-2} + \Delta(I - A)G_2[\Omega\mathbb{c}_{T-2} + (\beta A)\Omega\mathbb{E}_{T-2}\mathbb{c}_{T-1}], \tag{12}$$

but note that we have already solved for $\mathbb{c}_{T-1}$ above, so $\mathbb{E}_{T-2}\mathbb{c}_{T-1}$ can be easily found, allowing to find a solution for $\mathbb{c}_{T-2}$. One can then repeat this procedure all the way back to $\mathbb{c}_1$, and find analytical solutions along the way. All in all, one can use the strong induction principle to show that analytical solution for generic $\mathbb{c}_t, 1 \leq t \leq T - 1$, can be written down using the notion of the *Forward-looking Leontief Inverse*, as formally introduced in the following proposition.

Proposition 2. *The response of final sectoral consumption c_{kt} to a monetary policy shock r_t is*

$$c_{kt} = \sum_{r=1}^{K} l_{kr,T-t}\alpha_r c_{r,t-1} + \tilde{\alpha}_{k,T-t} r_t, \qquad 1 \le t \le T-1 \tag{13}$$

where $\tilde{\alpha}_{k,T-t}$ is

$$\tilde{\alpha}_{k,T-t} \equiv \alpha_k + (l_{kk,T-t} - 1)\alpha_k + \sum_{r \neq k} l_{kr,T-t}\alpha_r, \tag{14}$$

and $\{l_{kj,T-t}\}_{kj}$ are entries of the Forward-looking Leontief Inverse:

$$L_{T-t} = \underbrace{\left[I - \Delta(I-A)G_{T-t}\sum_{s=0}^{T-t-1}(\beta A)^s \Omega \left\{\Pi_{j=T-t-s}^{T-t-1} L_j A\right\}\right]^{-1}}_{\text{Forward-looking Leontief Inverse}}, \tag{15}$$

where $A \equiv diag[\alpha_1, \alpha_2, ..., \alpha_K]'$, $\quad \Delta \equiv diag[\delta_1, \delta_2, ..., \delta_K]'$, $\quad G_{T-t} \equiv (I - \beta A)(I - (\beta A)^{T-t})^{-1}$ and Ω is the input-output matrix.

Note that for $\delta_k = 0, \forall k$, the Forward-looking Leontief Inverse collapses to the identity matrix, so that $L_{T-t} = I, \forall 1 \le t \le T-1$, and $c_{kt} = \alpha_k c_{k,t-1} + \alpha_k r_t$, which is exactly the *direct effect* defined earlier. However, for $\delta_k > 0$ the coefficients on lagged consumption and monetary policy shocks are instead given by a weighted sum of a sector's own Calvo parameter and the Calvo parameters of its suppliers.

2.7. Analytical Impulse Response Functions

Having obtained analytical solutions for sectoral consumptions, we can now solve for the horizon-specific impulse responses of sectoral consumptions to a monetary shock:

Corollary 1. *[Full IRF] Suppose the economy is in steady state until the beginning of period t; the impulse response of final sectoral consumption c_{kt}, $k = 1, 2, ..., K$ at horizon $H \ge 0$, following a one-time monetary policy shock r_t is given by:*

$$c_{k,t+H} = \tilde{\tilde{\alpha}}_{k,T-t}^{H} r_t, \qquad 1 \le t, t+H \le T-1 \tag{16}$$

where $\tilde{\tilde{\alpha}}_{k,T-t}^{H} \equiv \sum_{r=1}^{K}\left[\Pi_{j=T-t-H}^{T-t} L_j A\right]_{kr}$, L_j is period-j Forward-looking Leontief Inverse, $A \equiv diag[\alpha_1, \alpha_2, ..., \alpha_K]'$. Therefore, $\frac{dc_{k,t+H}}{dr_t} = \tilde{\tilde{\alpha}}_{k,T-t}^{H}$.

In other words, in a linear projection of final sectoral consumption at $t+H$ on a monetary policy shock at t, the coefficient on the latter gives the horizon-H impulse response.

Moreover, one can derive an alternative representation of the horizon-specific impulse response of sectoral final consumption, which allows to separate out an upper bound of the *direct effect*, as defined in Proposition 1. The next corollary formalizes this statement:

Corollary 2. *[Upper Bound of Direct IRF] Suppose the economy is in steady state until the beginning of period t; the impulse response of final sectoral consumption c_{kt}, $k = 1, 2, ..., K$ at horizon $H \ge 1$, following a one-time monetary policy shock r_t can be written as follows:*

$$c_{k,t+H} = \left\{\alpha_k^{H+1} + \delta_k q_{T-t-H,H}^{k}\right\} r_t + \sum_{\tau=1}^{H} \delta_k s_{T-t-H+\tau}^{k} \sum_{r=1}^{K} \omega_{kr} c_{r,t+H-\tau}, \qquad 1 \le t, t+H \le T-1 \tag{17}$$

where $q_{T-t-H,H}^{k} \equiv \sum_{j=1}^{K}[\mathbb{Q}_{T-t-H,H}]_{kj} \ge 0$, $\quad s_{T-t-H+\tau}^{k} \equiv \alpha_k^{\tau}(1-\alpha_k)g_{T-t-H+\tau}, g_{T-t-H+\tau} \equiv [G_{T-t-H+\tau}]_{kk}$, $\mathbb{Q}_{T-t,\tau} \equiv \left\{(I-A)G_{T-t}\Omega\left\{\Pi_{j=T-t}^{T-t+\tau} L_j A\right\} + \sum_{l=0}^{\tau} A^l \mathbb{R}_{T-t+l,\tau-l}\right\}$, $\mathbb{R}_{x,y} \equiv (I-A)G_x\left[\sum_{s=1}^{x-1}(\beta A)^s \Omega\left\{\Pi_{j=x-s}^{x+y} L_j A\right\}\right]$. Hence, $\frac{\partial c_{k,t+H}}{\partial r_t} = \alpha_k^{H+1} + \delta_k q_{T-t-H,H}^{k} \ge \alpha_k^{H+1}$.

Verbally, Corollary 2 establishes that in a linear projection of sectoral final consumption at $t+H$ on a monetary policy shock at t *and* a weighted average of all other sectors' final consumptions between t and $t+H-1$, the coefficient on the monetary policy shock $\left(\alpha_k^{H+1}+\delta_k q_{T-t-H,H}^k\right)$ is an upper bound for the true horizon-H direct effect $\left(\alpha_k^{H+1}\right)$.

Importantly, since in Corollary 1 $\tilde{\alpha}_{k,T-t}^H$ gives the sectoral full effect, and in Corollary 2 $\left(\alpha_k^{H+1}+\delta_k q_{T-t-H,H}^k\right)$ gives an *upper bound* for the sectoral direct effect, the difference between the two, given by $\left[\tilde{\alpha}_{k,T-t}^H-\left(\alpha_k^{H+1}+\delta_k q_{T-t-H,H}^k\right)\right]$, gives a *lower bound* for the sectoral downstream effect.[6]

In Section 4 we use the analytical expressions in Corollaries 1 and 2 to directly inform our econometric specification. However, before that, we outline the construction of our dataset.

3. Data

This section contains an overview of the data used in our empirical analysis. More details are given in the Online Appendix.

3.1. Sectoral consumption

We use the underlying detail tables of NIPA accounts published by the United States Bureau of Economic Analysis (BEA) to obtain monthly sectoral nominal Personal Consumption Expenditure (PCE) and PCE Price Index data from 1976:1 until 2015:12, where the start date was chosen to maximize the number of sectors available. In order to construct real PCE series, we combine nominal PCE and PCE Price Index data.

However, the classification used in the NIPA tables does not match that from the BEA Input-Output (IO) accounts that is used to construct our input-output weights. In order to match NIPA series to BEA IO accounts we use PCE Bridge Tables also published by the BEA. The Bridge Tables are available at two levels of aggregation: Summary and Detail that roughly correspond to 3-digit and 6-digit NAICS codes precision levels respectively. In particular, we use the PCE Bridge Tables in order construct weights $\{\zeta_{ki}\}_{ki}$, which give the share of sector k from BEA IO accounts in sector i from NIPA accounts. Then, given the time series of final sectoral consumptions based on NIPA accounts, we construct time series of final consumptions consistent with BEA IO sector definitions:

$$c_{kt}^{BEA}=\sum_{i=1}^{K^{NIPA}}\zeta_{ki}c_{it}^{NIPA},\qquad k=1,2,...,K \tag{18}$$

We construct BEA IO Accounts consistent sectoral consumption series at both levels of aggregation. The full list of sectors at each level of aggregation is available in Appendix E.

At the Detail level, which is our baseline sample, we have constructed data for 161 sectors going from 1987:1 until 2015:12 where some sectors had to be deleted due to lack of data and the starting point had to be shifted forwards to 1987:1 to minimize the number of sectors to be deleted. Apart from the advantage of a much more detailed sectoral data, the disadvantage of the Detail level accounts is that the PCE Bridge Table is only available for the year 2007, which is the table we use to match series for the entire duration of the sample under the assumption that the matching is remaining stable throughout. Overall, over the sample, the sectoral consumption series constructed account for around 83 per cent of aggregate real Personal Consumption Expenditure (PCE).

At the less disaggregated Summary level, we have only constructed data for 53 sectors, but the advantage of this level of disaggregation is the longer time series available (1976:1 until 2015:12). The consumption series constructed account for around 84 per cent of aggregate real Personal Consumption Expenditure (PCE) over the sample.[7]

[6] Appendix B formally extends Corollaries 1 and 2 for the case where $|\rho_r|\in(0,1)$.

[7] We report results using the less disaggregated Summary level accounts in Appendix K. Despite the lower degree of

3.2. Input-output weights, consumption shares and intermediates intensities

In equilibrium, the input-output weight ω_{kr} is the proportion of the total expenditure of a firm in sector k on intermediate inputs that goes to intermediate inputs produced in sector r. Similarly, the final consumption share ω_{ck} is the share of households' final consumption expenditure that goes to firms in sector k. As for the intermediates intensity δ_k, we know that in equilibrium $\delta_k/(1-\delta_k)$ is given by $P_t^k Z_{kt}(j)/W_{kt}N_{kt}(j)$. Following aggregation over all firms in a given sector, we can write ω_{kr}, ω_{ck} and δ_k as:

$$\omega_{kr} = \frac{P_{rt}Z_{krt}}{P_t^k Z_{kt}}, \quad \forall k, r \qquad \omega_{ck} = \frac{P_{kt}C_{kt}}{P_t^c C_t}, \quad \forall k \qquad \frac{\delta_k}{1-\delta_k} = \frac{P_t^k Z_{kt}}{W_{kt}N_{kt}}, \quad \forall k, \tag{19}$$

We use data from the 2007 annual "*Make*" and "*Use*" tables, published by the US Bureau of Economic Analysis (BEA) as part of its Input-Output accounts, in order to evaluate ω_{kr}, ω_{ck} and δ_k. The exact details behind the evaluation are given in Appendix F.

3.3. Sectoral Calvo paramaters

We measure sectoral Calvo parameters as one minus the sectoral frequency of price adjustment, as constructed by Pasten et al. (2020). Their measure is based on confidential firm-level pricing data from the US Bureau of Labor Statistics (BLS).[8]

3.4. Monetary policy shocks

Our baseline measure of US monetary policy shocks comes from Romer and Romer (2004), available between 1969:1 and 1996:12, and extended by Wieland and Yang (2016) until 2007:12.

We also consider monetary policy shocks identified using different strategies.[9] First, we consider shocks recursively identified from a structural vector autoregression (SVAR) in Coibion (2012).[10] Second, we consider monetary policy shocks identified using event studies and high frequency data, from Kuttner (2001).[11] In Appendix H, we provide summary statistics and time series plots for all three monetary policy shocks measures.

4. Econometric strategy

In this section we develop our econometric specification, based on the analytical solutions for sectoral consumption obtained in Section 2.

4.1. Baseline specification

Our baseline econometric specification aims to estimate both full and (an upper bound of) the direct effect of a monetary shock on final consumption, and is based on horizon-specific analytical IRFs given in Corollaries 1 and 2. In particular, we adapt the local projection methodology of Jordà (2005) in order to estimate both full and direct effect at every horizon without assuming an underlying dynamic system.

disaggregation, estimation at Summary level allows us to extend our estimation sample back to 1976:1, thus benefiting from a higher number of identified monetary shocks. When estimated sector-by-sector and aggregated, the downstream effect contribution is at least 47 per cent of the cumulative full effect, with panel fixed effects estimation yielding a contribution of around 60 per cent.

[8] We are grateful to Michael Weber for sharing sectoral data on frequency of price adjustment.

[9] We report results using alternative measures of monetary policy shocks in Appendix L.

[10] SVAR features industrial production, unemployment, consumer price index (CPI), commodity price index and the effective federal funds rate. The recursive Cholesky identification strategy is used with the effective federal funds rate ordered last. We estimate our SVAR using monthly data between 1969:1 and 2007:12 with twelve lags.

[11] We use the publicly available data from Kuttner (2001) on monthly shocks identified as changes in current month's federal funds futures within daily windows around monetary policy announcements; the series is available on Kenneth Kuttner's website: https://econ.williams.edu/faculty-pages/research/. One disadvantage of this approach is that these shocks are only available from 1989:06 (and until 2007:12 for our purposes).

First, in order to estimate the horizon-specific *full effect* of a monetary shock, we consider the following baseline specification:

$$\hat{c}_{k,t+H} = \phi^F_{k,H} r_t + \varepsilon^F_{k,t+H}, \qquad H = 1, 2, ..., \overline{H} \tag{20}$$

where $\hat{c}_{k,t+H} \equiv \left[c_{k,t+H} - c^*_{k,t+H}\right]$, $\{c_{kt}\}_t$ being series of (log) final consumption in sector k, and $\{c^*_{kt}\}_t$ is trend of (log) final consumption in sector k [12]; $\{r_t\}_t$ is the monetary policy shock process; $\varepsilon^F_{k,t+H} \equiv \left[\rho^F_{k,H} + e^F_{k,t+H}\right]$, where $\rho^F_{k,H}$ absorbs all unconditional means and unobserved time-invariant sectoral heterogeneity, whereas $\{e^F_{kt}\}_t$ is the sector-specific error process; the coefficient $\phi^F_{k,H}$ represents the horizon-H full effect of a monetary policy shock, and can be mapped back to the structural parameters of our model using Corollary 1.

Second, in order to estimate a horizon-specific (upper bound of) the *direct effect*, we use the following specification:

$$\hat{c}_{k,t+H} = \phi^D_{k,H} r_t + \sum_{\tau=1}^{H} \psi^\tau_{k,H} \sum_{r=1}^{K} \omega_{kr} \hat{c}_{r,t+H-\tau} + \varepsilon^D_{k,t+H}, \qquad H = 1, 2, ..., \overline{H} \tag{21}$$

where ω_{kr} is the share of sector k's expenditure on intermediate inputs that goes to sector r, corresponding to the $[k, r]$ entry of the input-output matrix Ω; the rest of the notation follows that in (20). The coefficient $\phi^D_{k,H}$ represents an upper bound of horizon-H direct effect of a monetary policy shock, whereas $\psi^\tau_{k,H}$ measures the sensitivity of sectoral consumption at $(t + H)$ to that of its suppliers at $(t + H - \tau)$; both coefficients can be mapped back to the structural parameters of our model using Corollary 2.

Now given that $\phi^F_{k,H}$ and $\phi^D_{k,H}$ represent, respectively, full and (upper bound of) direct effect of a monetary policy shock, it follows that $\left[\phi^F_{k,H} - \phi^D_{k,H}\right]$ gives a *lower bound* of the *downstream effect* at horizon H, representing the contribution of production networks to the effect of a monetary policy shock on final consumption.

4.2. Coefficient estimation

Our specification in (20) and (21) allows for different estimation strategies, depending on the degree of cross-sectional coefficient heterogeneity one is willing to consider.

On the one hand, we consider the case where one imposes $\phi^i_{k,H} = \phi^i_H, k = 1, 2, ..., K, \quad i \in \{F, D\}$, leaving $\rho^i_{k,H}, i \in \{F, D\}$, as a sector fixed effect. We then obtain $\hat{\phi}^F_H$ and $\hat{\phi}^D_H$ by Fixed Effects estimation, separately for every horizon, interpreting $\left[\hat{\phi}^F_H - \hat{\phi}^D_H\right]$ as a lower bound of the downstream effect at horizon H, on average.[13] We can also allow for *specific* dimensions of cross-sectional heterogeneity under fixed effects estimation by adding interaction terms of the monetary shock with sector-specific characteristics; in particular, in Section 5.2 we add interactions with sectoral Calvo parameters and intermediates intensities, and compare the results with model-based predictions regarding cross-sectional heterogeneity in responses.

On the other hand, we also consider the case with full cross-sectional coefficient heterogeneity, which amounts to estimating (20) and (21) separately for every sector by OLS. Having obtained $\hat{\phi}^F_{k,H}$ and $\hat{\phi}^D_{k,H}$, we then interpret $\left[\hat{\phi}^F_{k,H} - \hat{\phi}^D_{k,H}\right]$ as a lower bound of the downstream effect in sector k at horizon H. This allows us to see whether or not the contribution of production networks to the effect of a monetary policy shock is disproportionally larger for some sectors. We also aggregate the sectoral responses using the empirical consumption weights $\{\omega_{ck}\}_k$, thus evaluating aggregate full, (upper bound of) direct and (lower bound of) downstream effects.

Our specifications in (20) and (21) are based on theoretical expressions for IRFs, derived under the assumption of a one-time exogenous monetary shock hitting a system, which is originally in steady state.

[12] We extract trends of sectoral final consumption by HP-filtering.

[13] For inference purposes, we construct heteroskedasticity-and-autocorrelation consistent (HAC) standard errors, additionally allowing for clustering at both sector- and month-level.

(a) Responses to a 25bp tightening (without controls) (b) Responses to a 25bp tightening (with controls)

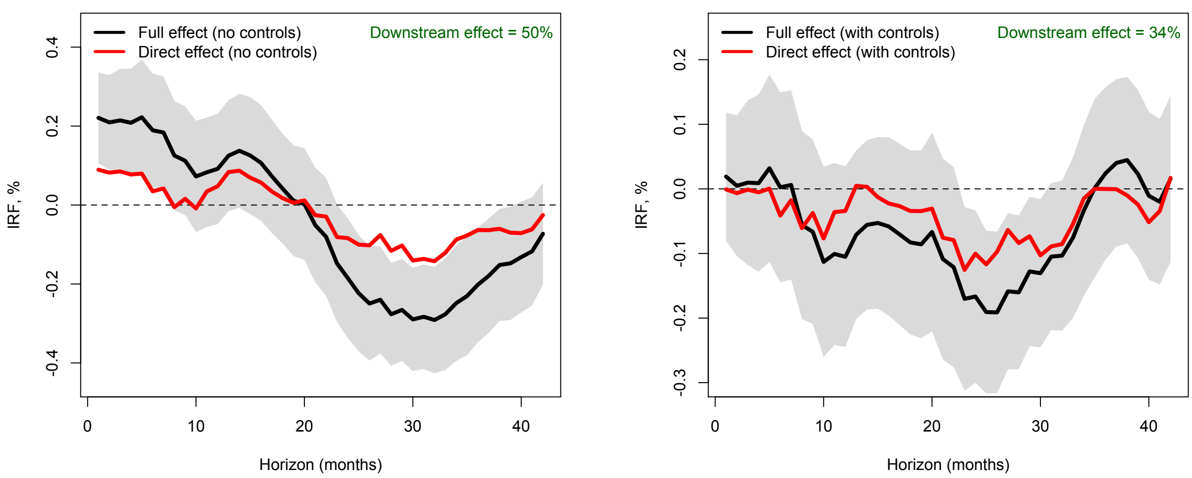


Figure 1: Final consumption response to 25bp tightening: full and direct effects (Detail level, 161 sectors)

***Note**: panel (a) shows results of panel fixed effects estimation of (20) and (21), without adding controls, reported as IRFs following a 25bp monetary tightening; panel (b) reports results of panel fixed effects estimation of (20) and (21), with a vector of controls* $\mathbb{X}_t^H = \left\{\{r_{t+\tau}\}_{\tau=1}^H, \{\hat{c}_{k,t-s}\}_{s=1}^1, \{ffr_{t-q}\}_{q=1}^4\right\}$ *added, reported as IRFs following a 25bp monetary tightening. Shaded areas denote 90% confidence bands around the full effect. The downstream effect contribution is constructed as the difference between cumulative full and direct effects as a percentage of the cumulative full effect (all cumulation is over 42 months). All reported IRFs have been smoothed with a 3-period centered moving average filter, with the original non-smoothed impulse responses reported in Appendix I (downstream effect contribution is constructed using non-smoothed IRFs.)*

In the data, however, shocks can arrive in any period and are hitting the system that is not necessarily in steady state. In order to more closely mimic the theoretical environment in our estimation, we add a vector of controls $\mathbb{X}_t^H$ to the right-hand side of both (20) and (21). The vector of controls includes a vector of leads of the monetary shock $\{r_{t+\tau}\}_{\tau=1}^H$, a vector of lags of the dependent variable $\{\hat{c}_{k,t-s}\}_{s=1}^S$, as well as a vector of lags of the federal funds rate $\{ffr_{t-q}\}_{q=1}^Q$.[14]

5. Baseline results: panel fixed effects estimation

5.1. Average full and direct effects

In Panel (a) of Figure 1 we report the horizon-specific full effect and (an upper bound of) the direct effect on sectoral consumption of a 25 basis points monetary tightening, obtained by panel fixed effects estimation of (20) and (21), without adding control variables. The black line represents the full effect impulse response function; as one can see, consumption actually rises at early horizons following the tightening, before reaching a trough of around -0.30 percentage points decline just after 30 months. As for the red line, which represents (an upper bound of) the direct effect impulse response function with no controls in estimation, it similarly exhibits a small increase immediately after the tightening, but eventually reaches a trough of approximately -0.15 percentage points. As argued earlier, the difference between the two impulse response functions gives a lower bound for the downstream effect coming from input-output linkages. We summarize the downstream effect contribution by looking at the difference between cumulative full and direct effects over 42 months following the shock, and conclude that input-output linkages contribute to at least *50 per cent* of the cumulative full effect.

[14] Adding a vector of leads of the shock additionally corrects for possible autocorrelation in monetary shock series, as shown in Alloza et al. (2019); adding lags of the dependent variable and the federal funds rate parsimoniously controls for initial conditions before the shock arrives, and we set $S = 1$ and $Q = 4$.

Panel (b) of Figure 1 reports impulse response functions estimated with a vector of controls $\mathbb{X}_t^H$ added to both (20) and (21). As can be seen from the black line, adding controls essentially removes the "consumption puzzle" observed earlier, with the full impulse response function being close to and not significantly different from zero, before turning negative; as for the trough of the consumption response, it is reached at around 26 months and is -0.20 percentage points in magnitude. Turning to the direct effect, its "consumption puzzle" is also removed by adding controls, and the peak drop achieved is at -0.12 percentage points. Summarizing the downstream effect contribution by the difference between cumulative full and direct effects over 42 months following the shock, the input-output linkages contribute to at least *34 per cent* of the cumulative full effect.

Given that adding controls helps to address the theoretically implausible "consumption puzzles" in average responses, for the rest of the paper we add the vector of controls in all exercises.

5.2. The role of heterogeneous Calvo parameters and intermediates intensity

In this subsection we report results obtained when interactions of monetary shock with sectoral Calvo parameters ($r_t \times \alpha_k$) and sectoral intermediates intensities ($r_t \times \delta_k$) are added to the right-hand side of both (20) and (21). This allows us to test theoretical predictions regarding cross-sectional heterogeneities in the downstream effect. In particular, recall that Proposition 1 establishes that the downstream effect is, *ceteris paribus*, unambiguously larger for sectors with larger intermediates intensity; on the other hand, the degree of price rigidity has an ambiguous effect on the magnitude of the downstream effect.

Panel (a) of Figure 2 shows estimated cumulative full and direct effects (over 42 months) following a 25bp tightening, for various degrees of price rigidity (while intermediates intensity is held fixed at its sample median value). As one can see, both cumulative full and direct effects become more negative as the frequency of price non-adjustment rises, although the full effect is estimated to be more sensitive to the Calvo parameter. As a result, the downstream effect, given by the difference between cumulative full and direct effects, is estimated to be larger in magnitude as the frequency of price non-adjustment rises, as is shown graphically in Panel (b) of Figure 2. The latter suggests that empirically, the effect of higher price rigidity on being more forward-looking, and hence being more sensitive to expectations of future prices set by suppliers, dominates the effect of adjusting prices less frequently and hence being less sensitive to current and expected future marginal costs.

Panel (c) of Figure 2 shows estimated cumulative full and direct effects (over 42 months) following a 25bp tightening, for various intermediates intensities (while the Calvo parameter is held fixed at its sample median value). The results suggest that both cumulative full and direct effects become more negative as the intermediates intensity rises, although the full effect is estimated to be more sensitive to intermediates intensity. As a result, the downstream effect is estimated to be larger in magnitude as intermediates intensity rises, as is shown graphically in Panel (d) of Figure 2. The latter is fully consistent with our theoretical predictions.

In this subsection we have examined *linear* relationships between consumption responses and the degrees of price rigidity and intermediates intensity. However, Proposition 2 suggests that the relationship may be non-linear. In order to account for potential non-linearities, in Appendix J we report results under additional non-linear interaction terms $r_t \times \alpha_k^2$, $r_t \times \delta_k^2$ and $r_t \times \alpha_k \delta_k$ added. The results suggest our findings regarding cross-sectional heterogeneity remain true even once non-linearities have been allowed for.

5.3. Time variation in intermediates intensity

Results in the previous subsection use values of sectoral intermediates intensities δ_k based on the 2007 BEA "*Use*" tables at the Detail level. Accounts at such high level of disaggregation are not available at annual (or higher) frequency over time, but Ghassibe (2020) constructs annual measures of intermediate intensity for less disaggregated 65 sectors of the US economy between 1987-2017, with a possibility of extending the measures back to 1963 under additional assumptions regarding the labor share of value added.[15] Further, it is shown that intermediates intensities are strongly procyclical.

[15] Prior to 1987, BEA KLEMS accounts do not report sectoral employee compensation separately from sectoral value added; hence in order to construct sectoral intermediates intensities between 1963-1986 is it assumed the sectoral labor share of value added is fixed at its 1987 level.

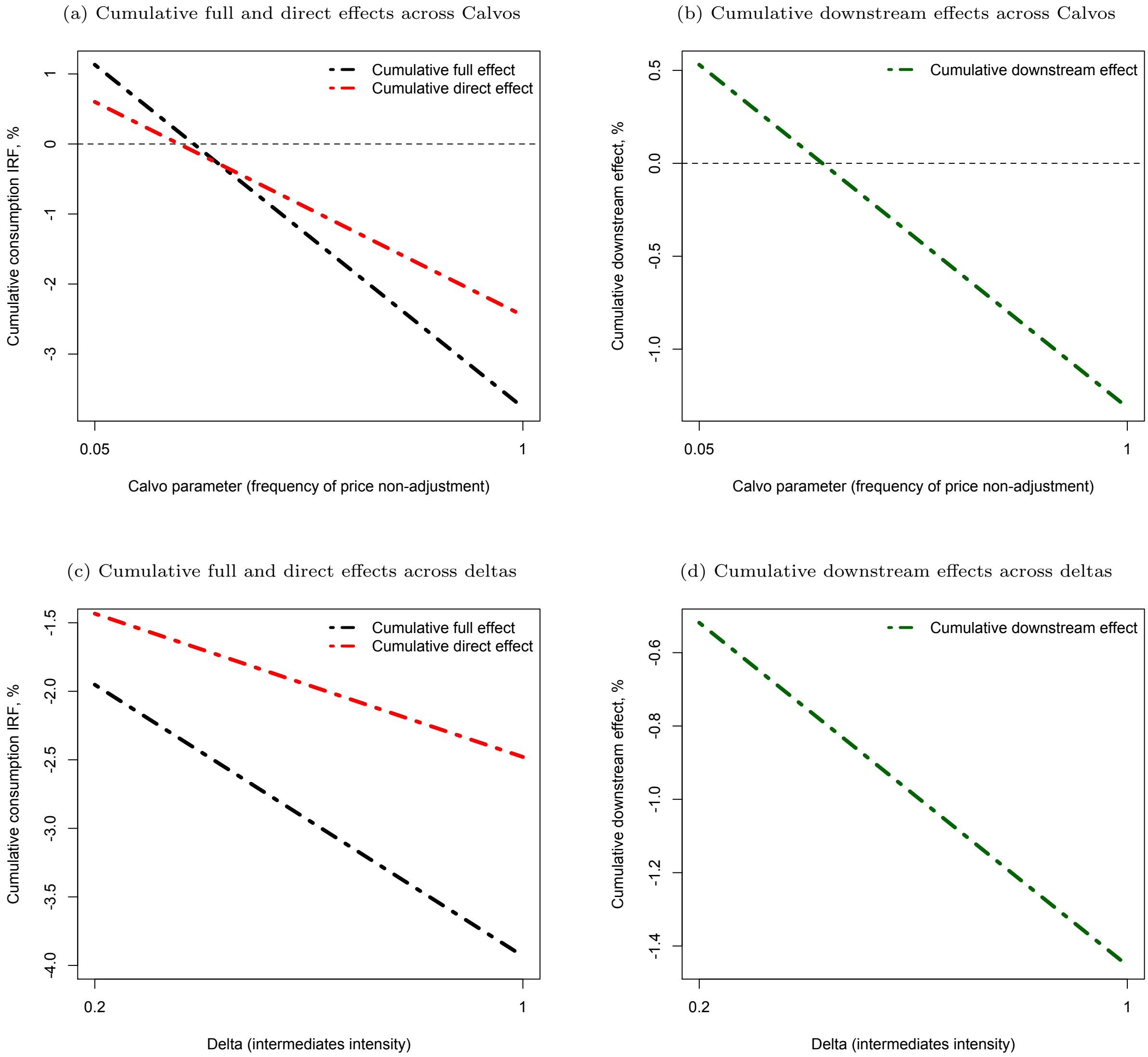


Figure 2: Response heterogeneity to 25bp tightening across frequencies of price adjustment and intermediates intensities

***Note**: Figure 2 shows results of panel fixed effects estimation of (20) and (21) with interaction terms $r_t \times \alpha_k$ and $r_t \times \delta_k$ added, as well as vector of controls $\mathbb{X}_t^H = \left\{\{r_{t+\tau}\}_{\tau=1}^H, \{\hat{c}_{k,t-s}\}_{s=1}^1, \{ffr_{t-q}\}_{q=1}^4\right\}$; panels (a) and (b) show cumulative full, direct and downstream effects for various values of the Calvo parameter, holding intermediates intensity fixed at its sample median value; panels (c) and (d) show cumulative full, direct and downstream effects for various values of intermediates intensity, holding Calvo parameter fixed at its sample median value. All cumulative effects are over 42 months.*

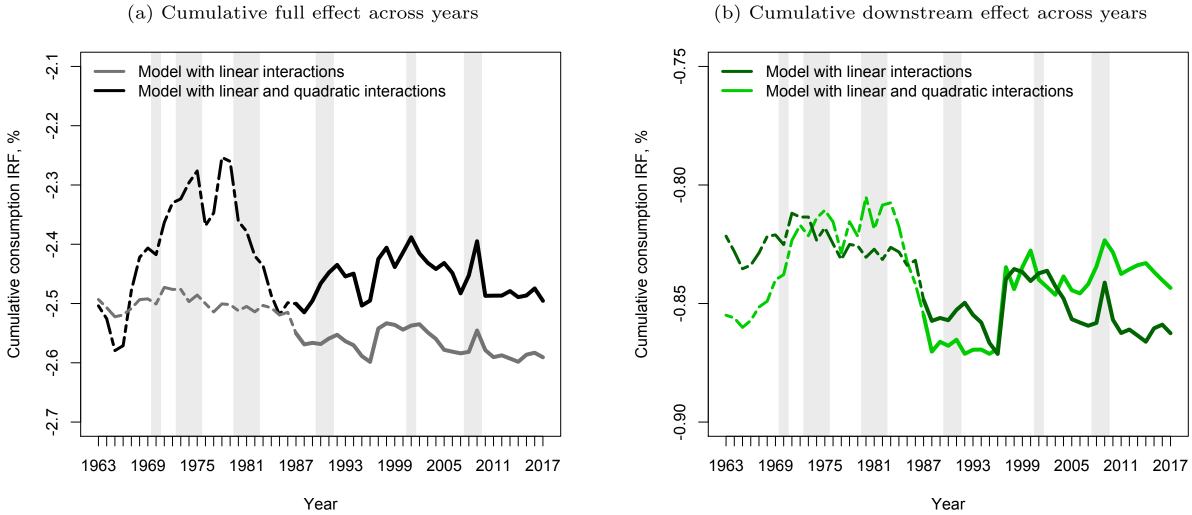


Figure 3: Cumulative response to 25bp tightening across years: the role of time variation in intermediates intensity

Note: *panels (a) and (b) show time series of cumulative (over 42 months) full and downstream effects, aggregated using sectoral consumption shares, under sectoral values of intermediates intensities across the years, as constructed by Ghassibe (2020), using estimation of (20) and (21) with controls either just linear interactions $r_t \times \alpha_k$ and $r_t \times \delta_k$ added, as well as with the addition of quadratic interaction terms $r_t \times \alpha_k^2, \quad r_t \times \delta_k^2, \quad r_t \times \alpha_k \delta_k$; solid lines use true intermediates intensities available between 1987-2017, dotted lines show results using interpolated values of intermediates intensities constructed for 1963-1986 under the additional assumption of sectoral labor shares of value added fixed at their values in 1987. Shaded bars represent NBER recession dates.*

Estimates in Panels (c) and (d) of Figure 2 give us the (cumulative) full, direct and downstream effect for various values of intermediates intensity, holding fixed the degree of price rigidity. In this subsection we combine those estimates with the annual data on intermediates intensities to perform the following exercise: for every sector, we compute the implied (cumulative) full, direct and downstream effect under its intermediates intensity in year t, $\quad t = 1963, 1964, ..., 2017$. We then aggregate the sectoral effects to gauge the aggregate (cumulative) full, direct and downstream effects following a 25bp tightening in a particular year.

Figure 3 shows cumulative (over 42 months) full and downstream effects following a 25bp tightening, between 1963-2017. We report results based both on the specification with only linear interactions with Calvo parameter and intermediates intensity, as well as the specification that additionally includes second-order interaction terms. First, note that in both specifications, the magnitude of both full and downstream effects is procyclical, which in term comes from the procyclicality of intermediates intensity. Such results provide a possible rationalization for the procyclical magnitude of GDP response to a monetary shocks documented by Tenreyro and Thwaites (2016) among others. Second, the observed procyclicality is substantially more pronounced in the non-linear specification, particularly in the Volcker disinflation episode, which witnessed a substantial drop in intermediates intensity; such empirical result suggests that the strength of monetary transmission to GDP can fall especially quickly in strong recessions which are associated with much smaller reliance on intermediate inputs.

6. Baseline results: sector-by-sector OLS estimation

6.1. Sectoral responses and aggregation

Figure 4 summarizes the results of sector-by-sector OLS estimation of both (20) and (21), yielding sectoral full and (upper bound of) direct impulse response functions. The latter are then aggregated in order to gauge economy-wide effects.

Panel (a) of Figure 4 shows the full effect impulse response functions to a 25 basis points tightening for all 161 sectors in our baseline sample. Several aspects are worth highlighting. First, there is significant heterogeneity in the responses; given that our model predicts that sectoral responses are pinned down by, among other things, sectoral Calvo parameters and intermediates intensities, such dispersion in magnitudes of responses in consistent with large cross-sectional differences in frequencies of price adjustment, as documented by Nakamura and Steinsson (2008) and Gorodnichenko and Weber (2016) among others, as well as large heterogeneity in intermediates intensities, as is documented in Appendix G. Second, the aggregated response obtained by weighing individual responses by their respective consumption weights (solid black line) follows the aggregate real consumption response (dashed purple line) very closely, and considerably more closely than the response obtained under aggregation using equal weights (dashed blue line).This suggests that the aggregate response may be driven by several large sectors, and hence that sectoral responses should indeed be aggregated with sectoral consumption weights, as opposed to equal weights.

Panel (b) reports the aggregated sectoral full effect (black line), as well as the aggregated (upper bound of) sectoral direct effects (red line), following a 25 basis points monetary tightening, where the aggregation performed with empirical consumption weights. As can be seen, the aggregated sectoral full effect is slightly positive at early horizons, but eventually reaches a trough of around -0.14 percentage points. Importantly, the aggregated full response follows very closely the impulse response of aggregate consumption, providing extra confidence in the new sectoral dataset that we have constructed. As for the aggregated (upper bound of) direct effect, it similarly starts with a slightly positive response, but then experiences a drop down to around -0.10 percentage points at the trough. When measured by the difference between cumulative aggregated full and direct effects (after 42 months), the downstream effect is estimated to contribute to (at least) 31 per cent of the cumulative full effect. The latter is very similar to the findings in the previous section under panel fixed effects estimation.

In Panels (c) and (d) we report histograms of sectoral cumulative full and (upper bounds of) cumulative direct effects (after 42 months), as well as sector-level cumulative downstream effects. One can see that the distribution of the sectoral full effects is shifted in the direction of more negative values compared to that of sectoral direct effects, which is consistent with the notion that the effect of monetary shocks on sectoral consumption is amplified by production networks; as for the sectoral downstream effects, those are estimated to be negative for the vast majority of sectors, confirming the finding that production networks amplify the effects of monetary shocks.

Note that although most of the results in this subsection are qualitatively in line with the predictions of our theory outlined in Section 2, two features cannot be reconciled using our baseline model. First, as can be seen in Panel (a) of Figure 4, quite a number of sectors appear to be experiencing an increase in their consumption at certain horizons following a monetary tightening. [16] Second, as can be seen in Panel (d) of Figure 4, though sector-level downstream effects are negative for the vast majority of sectors, it is actually positive for some sectors and thus production networks seem to actually attenuate, rather than amplify, the effect of monetary policy shocks on those sectors' consumptions.

To address this mismatch, in Appendix D we show that relaxing the assumption of unit elasticity of substitution across sectors can help jointly reconcile both of the two puzzles documented above (however, at the cost of analytical tractability at sector-level). This is because under non-unit elasticity of substitution, a sector's final consumption response is pinned down by the speed of its price response *relative* to the speed of adjustment of the aggregate consumption price index. Hence, if the sectoral goods are substitutes, following a monetary tightening the price index of the sectors with the highest speed of price adjustment drops much faster than the aggregate consumption price index, and hence those sectors can "steal" customers from other sectors and see an *increase* in their consumption; at the same time, sectors with the lowest speed of price adjustment can have their negative consumption response *dampened* by networks, as price complementarities make their price index more aligned with the aggregate consumption price index, thus making them "lose" fewer customers to the other sectors. Crucially, however, the *aggregate* consumption response is fully independent of the elasticity of substitution across sectors; hence, although sectoral responses can exhibit

[16] The latter is also found by Boivin et al. (2009) in their FAVAR study of product-level consumption responses.

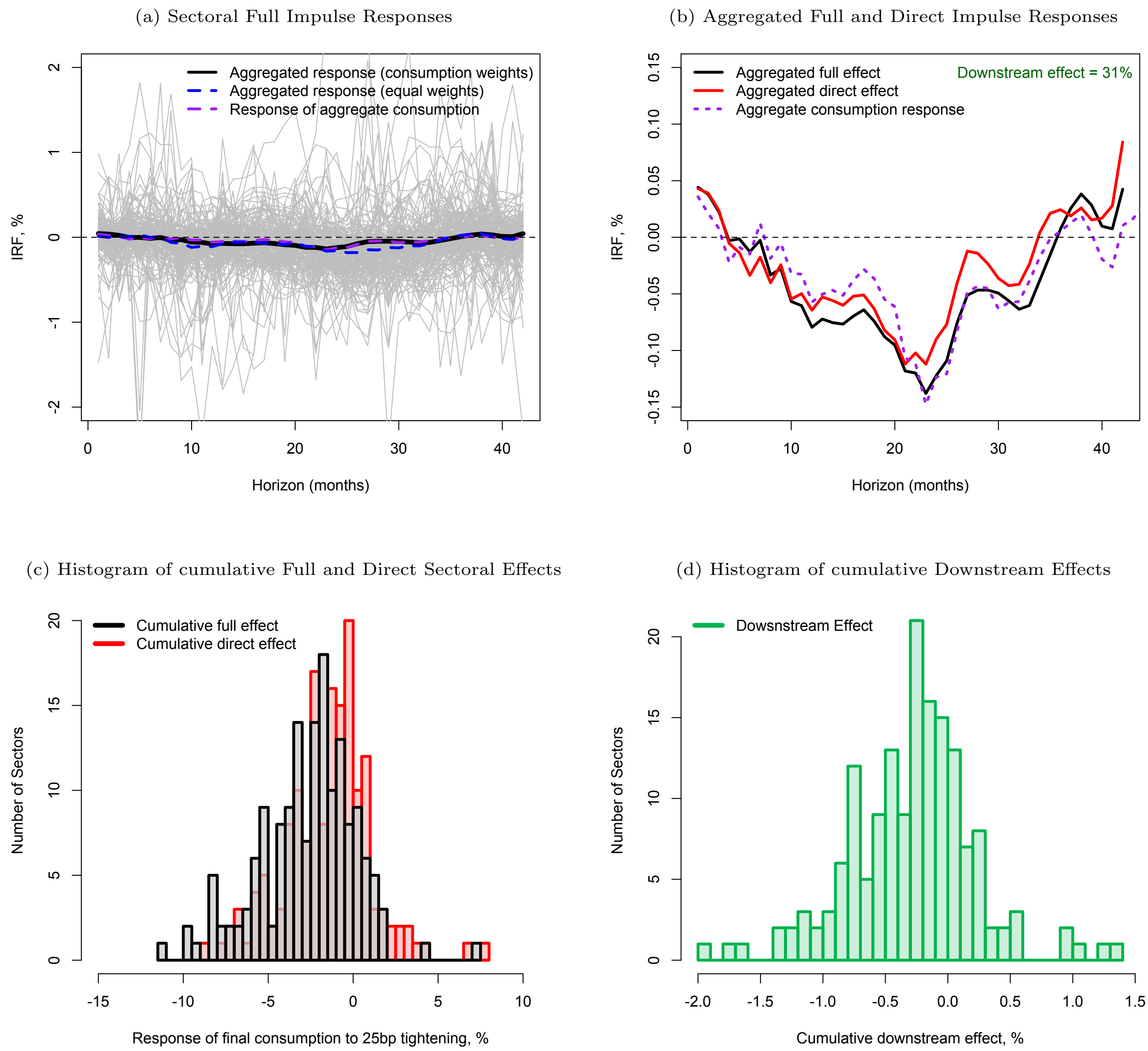


Figure 4: Sector-level consumption responses to 25bp tightening (Detail level, 161 sectors)

Note: *panel (a) shows results of sector-by-sector OLS estimation of (20) with controls, reported as sector-level IRF following a 25bp monetary tightening; panel (b) reports results of aggregation of sector-level full and direct effects using empirical final consumption shares* $\{\omega_{ck}\}_{k=1}^{161}$*; panels (c) and (d) report histograms of cumulative (over 42 months) sectoral full and direct effects, as well as the corresponding sectoral cumulative downstream effects. In panel (b) the reported impulse responses have been smoothed with a 3-period centered moving average filter, with the original non-smoothed impulse responses reported in Appendix I.*

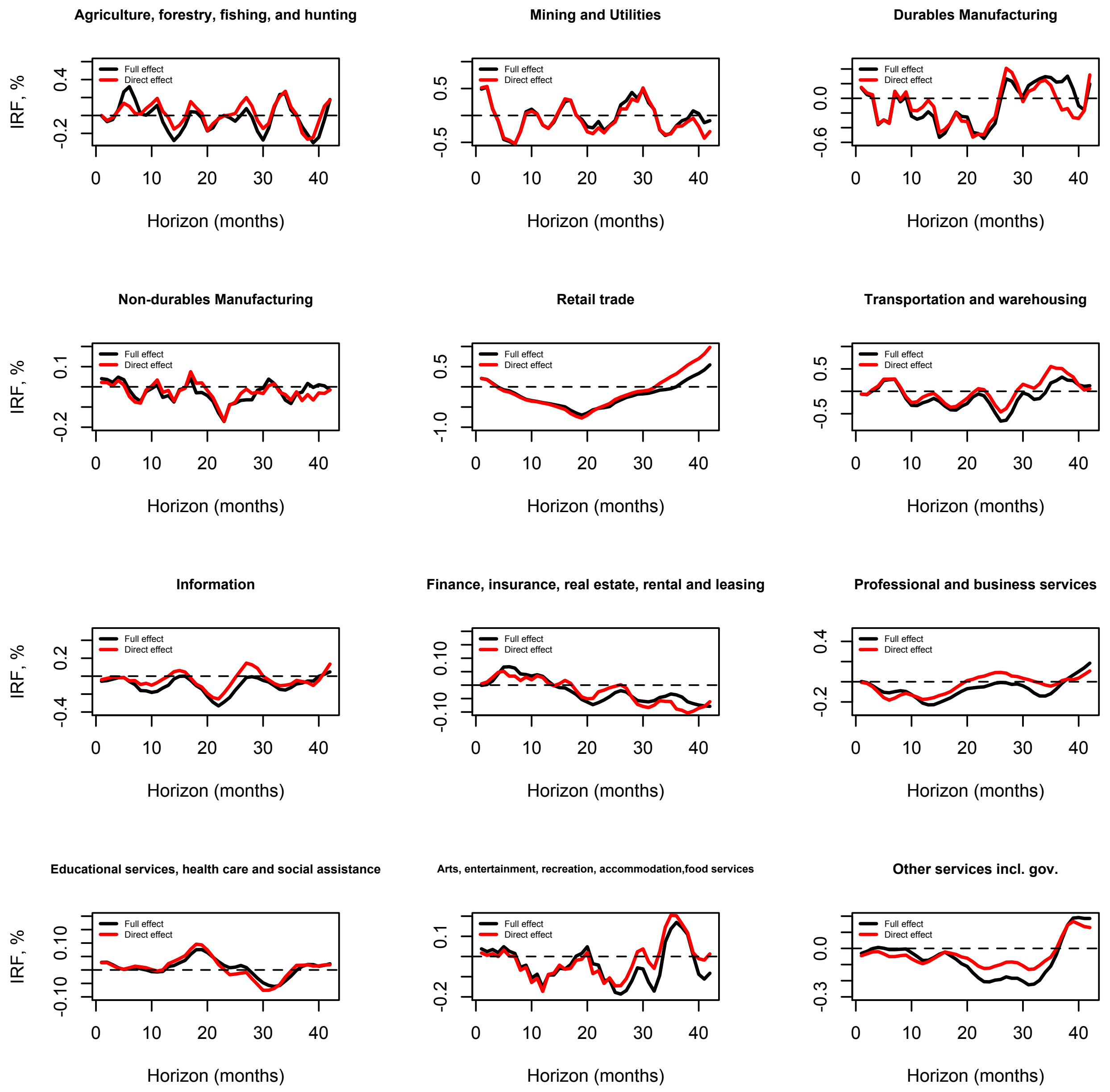


Figure 5: Consumption responses to a 25bp tightening across broad sectoral groups

***Note**: Figure 5 shows results of sector-by-sector OLS estimation of (20) and (21) with controls, reported as sector-level direct and full IRFs following a 25bp monetary tightening that have been aggregated into broad sectoral groups using empirical consumption shares $\{\omega_{ck}\}_{k=1}^{161}$. The reported impulse responses have been smoothed with a 3-period centered moving average filter, with the original non-smoothed impulse responses reported in Appendix I.*

different properties under non-unit elasticity of substitution, the aggregate consumption response is always negative following a monetary tightening and always amplified by networks regardless of the elasticity of substitution across sectors.

6.2. Full and direct effects in broad sectoral groups

In order to gain further understanding of cross-sector heterogeneities in full and direct effects, in Figure 5 we aggregate individual responses into 12 broad sectoral groups.

As before, we summarize the downstream effect by the difference between cumulative full and cumulative direct effects (over 42 months). By this metric, the group with the largest downstream effect is "Transportation and warehousing", followed by "Retail trade", "Information", "Agriculture, forestry, fishing and hunting" and "Professional and business services" as other groups with a substantial downstream effect. In fact, although those sectoral groups account for only 17 per cent of the aggregate consumption basket, their combined downstream effect equals 98 per cent of the aggregate downstream effect .

At the same time, "Other services including government" and "Arts, entertainment, recreation, accommodation, and food services" also exhibit downstream effects suggestive of amplification through production networks, though smaller in magnitude. As for "Educational services, health care, and social assistance", "Non-Durables manufacturing" and "Finance, insurance, real estate, rental and leasing" their downstream effect is essentially zero.

Notably, two sectoral groups with some of the largest full effects, namely "Mining and utilities" and "Durables manufacturing", exhibit a downstream effect of an inverted sign, implying that production networks actually slightly dampen their response to a monetary shock.

7. Further exercises and robustness checks

7.1. Monte Carlo experiments

As argued earlier, our econometric specification allows to estimate the full effect of a monetary shock, as well as an *upper bound* for the direct effect, thus giving us a *lower bound* for the downstream effect. As an additional exercise, we apply our econometric specification to synthetic data generated by our model, calibrated to the same 161 sectors used in our baseline sample, and compare our estimated (lower bound of) downstream effect, to the true known contributions of linkages in the model.[17] Panel (a) of Figure 6 shows that based on means across simulations, our estimated downstream effect accounts for 41.02 per cent of the cumulative full effect (over 42 months), while the true contribution of linkages in the model equals 41.29 per cent; hence, the lower bound that our econometric strategy recovered from simulated data is very close to the true contribution.

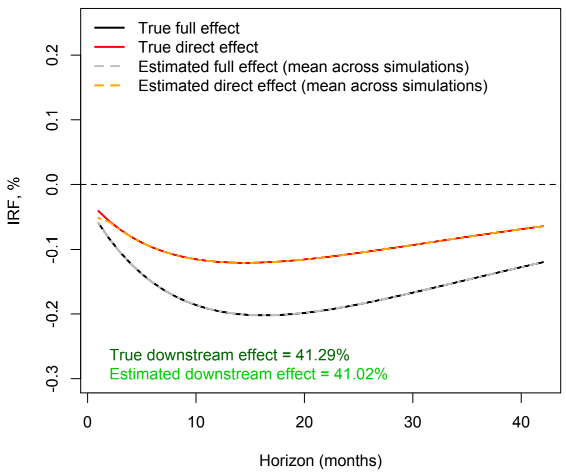
Importantly, our econometric specification is based on sector-level analytical solutions to our model, which requires simplifying assumptions such as monetary policy based on a money supply rule, unit elasticity of substitution across sectors and linear disutility of labor. Relaxing the above assumptions comes at the cost of sector-level analytical tractability. However, we can still assess the robustness of econometric strategy to model misspecification by applying it to synthetic data generated by alternative models.[18] In Panel (b) of Figure 6 we show that when data is generated by a model with a Taylor rule[19], we estimate the downstream

[17] We simulate 1,000 histories of log-linearized version of our model set out in Appendix A for 2,000 periods, discarding the first 1,748 periods as burn-in and only using the last $T = 252$ time periods (matching the time dimension of our sample), and apply specifications (20) and (21) (with controls) to estimate full and direct effects sector-by-sector with subsequent aggregation using empirical consumption weights. We calibrate the model at monthly frequency, setting $\beta = 0.9975, \rho_r = 0.95, \theta = 6$ and $\sigma_r = 0.001$, with input-output weights, sectoral Calvo parameters and intermediates intensities calibrated using data described in Section 3. In order to avoid possible singularities in the the simulated data, we include sector-specific i.i.d. productivity shocks $\{\ln \mathcal{A}_{kt}\}_t \sim^{iid} (0, \sigma_a^2)$, where we set $\sigma_a = 10^{-6} \times \sigma_r$.

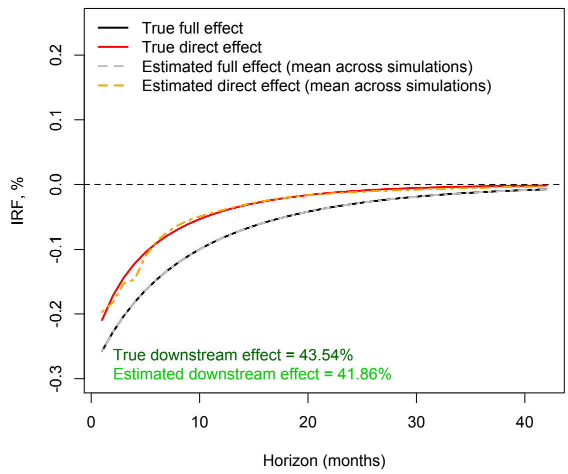
[18] Appendix A outlines a generalized version of our model that relaxes the simplifying assumptions.

[19] We consider a Taylor rule of the form $\frac{I_t}{I} = \left(\frac{I_{t-1}}{I}\right)^{\rho_i} \left[\left(\frac{P_t^c}{P_{t-1}^c}\right)^{\phi_\pi} \left(\frac{C_t}{C}\right)^{\phi_c} \left(\frac{C_t}{C_{t-1}}\right)^{\phi_{gc}}\right]^{1-\rho_i} e^{\mu_t}, \quad \mu_t = \rho_\mu \mu_{t-1} + \varepsilon_t^\mu, \quad \{\varepsilon_t^\mu\}_t \sim^{iid} (0, \sigma_\mu^2)$, where we set $\rho_i = 0.9, \phi_\pi = 1.24, \phi_c = 0.33/12, \phi_{gc} = 1.5, \rho_\mu = 0$ and $\sigma_\mu = 0.000225$.

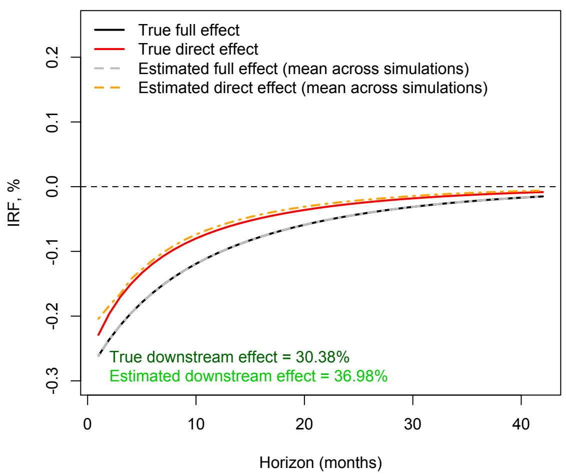
(a) Baseline model with money supply rule



(b) Model with Taylor rule



(c) Model with Taylor rule + CES aggregation



(d) Model with Taylor rule + CES agg. + finite Frisch elasticity

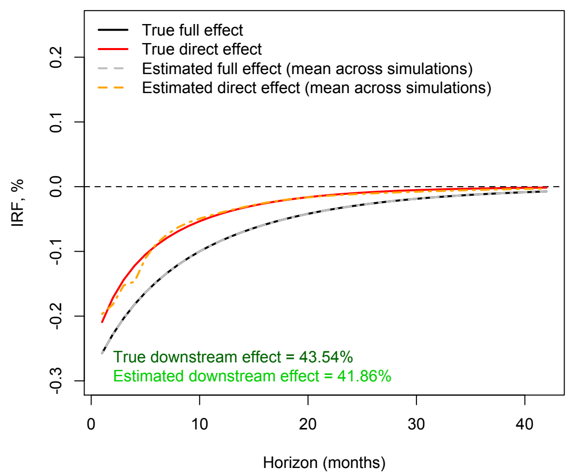


Figure 6: Theoretical full and direct effects, and those estimated from model-generated data

***Note**: Figure 6 shows results of Monte Carlo experiments using simulations of log-linearized models fully outlined in Appendix A. In each exercise, we simulate 1,000 histories of calibrated model for 2,000 periods, discarding the first 1,748 periods as burn-in and only using the last* $T = 252$ *time periods (matching the time dimension of our sample), for the same 161 sectors used in our baseline sample, and apply specifications (20) and (21) (with controls) to estimate full and direct effects sector-by-sector with subsequent aggregation using empirical consumption weights; we then use the estimates to measure the downstream contribution to cumulative aggregated full effect, and compare it to the true known contribution in the model. In panel (a) we use our baseline model with money supply rule; in panel (b) we replace the money supply rule with the Taylor rule for the nominal interest rate; in panel (c) we additionally allow for CES aggregation across sectors with elasticity above one; in panel (d) we additionally allow for finite Frisch elasticity of labor supply.*

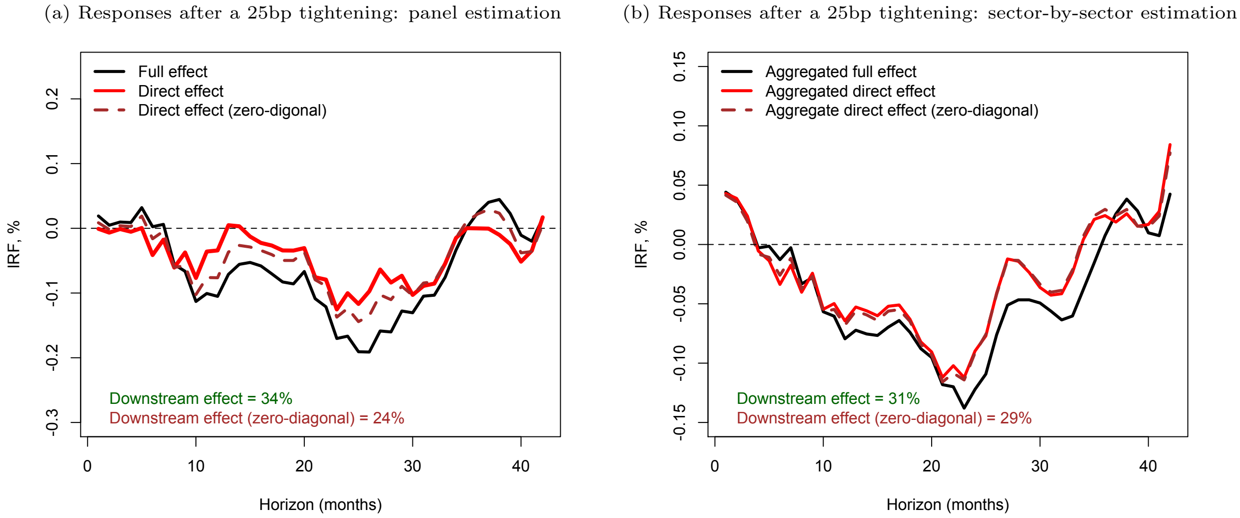


Figure 7: Final consumption response to 25bp tightening: full and direct effects under zero-diagonal (Detail level, 161 sectors)

Note*: Figure 7 shows additional estimation of the direct effect using (21) where we force the diagonal elements of the input-output matrix to be equal to zero. The new estimates are reported alongside estimation of (20) and (21) without further restrictions. Panel (a) shows results using panel fixed effects estimation, whereas panel (b) shows results using sector-by-sector OLS estimation with subsequent aggregation using empirical consumption shares $\{\omega_{ck}\}_{k=1}^{161}$. The reported impulse responses have been smoothed with a 3-period centered moving average filter, with the original non-smoothed impulse responses reported in Appendix I.*

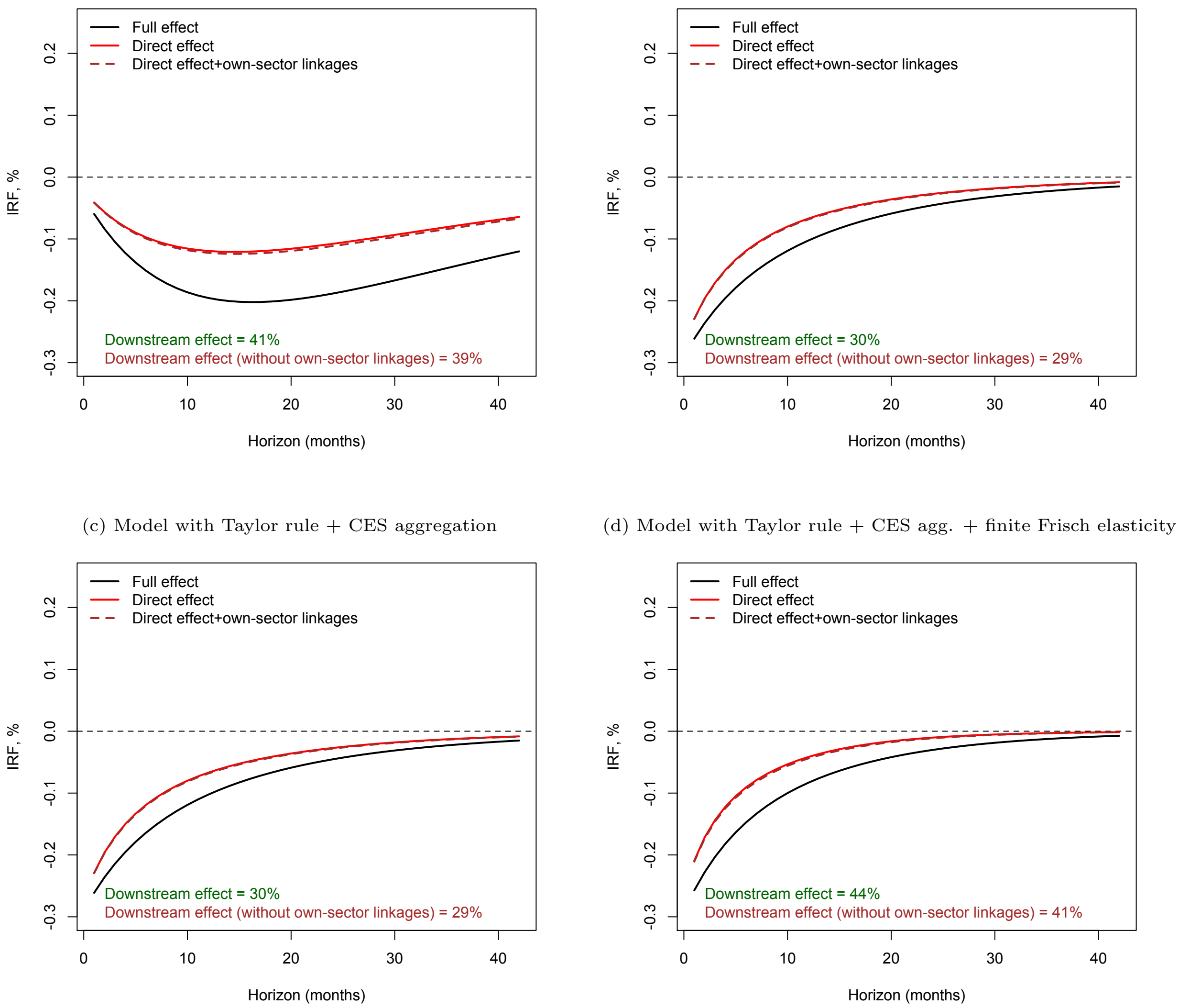
effect contribution at 35.81 per cent of the cumulative full effect, while the true contribution is 30.38 per cent. In Panel (c) of Figure 6 we additionally allow for elasticity of substitution across sectors equal to 2, the true contribution of linkages remains at 30.38 per cent, while we estimate it at 36.98 per cent. Finally, in Panel (d) of Figure 6 we generate data additionally allowing for finite Frisch elasticity of labor supply equal to 0.5, which gives a true contribution of linkages equal to 43.54 per cent, while our specification estimates it at 41.86 per cent.

7.2. Within-sector linkages vs. across-sector linkages

Our model allows for both within-sector and across-sector input-output linkages; however, our baseline econometric specification does not specifically estimate the downstream effect coming from across-sector linkages. In Figure 7 we report results of an additional exercise which attempts to separate out the effects coming from the two types of linkages. In particular, we estimate (21) forcing diagonal elements of the input-output matrix to be equal to zero ($\omega_{kk} = 0, \forall k$). If it was true that the majority of the downstream effect came from within-sector linkages, then the estimated direct effect under zero-diagonal matrix would coincide with the full effect; instead, we see in Figure 7 that the downstream effect under zero-diagonal matrix is close to that under the true matrix. Our results therefore suggest that the majority of the downstream effect comes from across-sector linkages.

Importantly, however, in the previous exercise we forced a parameter inside a reduced-form econometric model, which is why our results should be treated with caution. As an additional robustness exercise, we separate out effects of within-sector versus across-sector linkages using our calibrated structural theoretical model.[20] In Figure 8 we report aggregate consumption responses under three configurations: first, under input-output linkages completely switched off ($\delta_k = 0, \forall k$), giving us the theoretical direct effect; second, under input-output matrix set equal to the identity matrix, so that we only allow for within-sector linkages, with alternative sectoral intermediates intensities ($\bar{\delta}_k$) given by the empirical share of variable costs that

[20] We use the same calibrations as outlined in Section 7.1



(a) Baseline model with money supply rule

(b) Model with Taylor rule

(c) Model with Taylor rule + CES aggregation

(d) Model with Taylor rule + CES agg. + finite Frisch elasticity

Figure 8: Theoretical full and direct effects, and direct effects combined with own-sector linkages

Note: *Figure 8 shows aggregated full and direct effects from our calibrated theoretical model, as well effects from a calibration which only allows for within-sector input-output linkages, calibrated so that the sectoral intermediates intensities are equal to the empirical share of total variable costs that goes to intermediates bought from own sector (interpreted as direct effect+own linkages). The difference between the latter and the full effect is used to measure the (cumulative) downstream contribution without own-sector linkages. In panel (a) we use our baseline model with money supply rule; in panel (b) we replace the money supply rule with the Taylor rule for the nominal interest rate; in panel (c) we additionally allow for CES aggregation across sectors with elasticity above one; in panel (d) we additionally allow for finite Frisch elasticity of labor supply.*

are paid to firms within own sector ($\overline{\delta}_k \equiv \delta_k \omega_{kk}, \forall k$), giving us aggregate direct effect combined with the effect coming from own-sector linkages; third, under true δ_k and Ω for all sectors, giving us the theoretical full effect. As one can see, both in our baseline theoretical model, and in extensions featuring a Taylor rule, non-unit elasticity of substitution across sectors and finite Frisch elasticity, the direct effect combined with the effect of own-sector linkages is essentially indistinguishable from the direct effect. Therefore, our results using calibrated models also suggest that the majority of the downstream effect comes from across-sector linkages.

8. Conclusion and future extensions

A large theoretical literature outlining the way in which production networks create complementarities in price setting and thus amplify the effect of monetary policy shocks exists. However, there has been hardly any econometric assessment of the above mechanism. In this paper we make a first step towards filling this large gap in the literature. In particular, we offer novel empirical evidence that at least 30 per cent of the effect of monetary policy shocks on US aggregate consumption comes from the amplification through input-output linkages. We also find that the network effect is larger, *ceteris paribus*, for sectors that adjust prices less frequently and those that rely more on intermediate inputs. Our results also suggest substantial concentration: sectors that jointly account for 17 per cent of our sample aggregate consumption account for 98 per cent of the amplification by production networks.

Apart from the novel econometric evidence, we make two additional contributions. First, we obtain novel sector-level analytical solutions to a multi-sector New Keynesian models with asymmetric input-output linkages, and thus form a tractable econometric strategy. Second, we construct a new highly disaggregated monthly dataset for sectoral final consumptions in the US that can is consistent with sectoral classification used in input-output accounts of the US Bureau of Economic Analysis.

Our results create space for future research. First, our theoretical framework has only considered time-dependent pricing schedules following Calvo (1983); it would be an interesting agenda to additionally allow for state-dependent pricing and study, both theoretically an empirically, how firms with state-dependent and time-dependent pricing interact with each other in a production network, and what are the implications of such interaction for the response of aggregate variables to shocks of different magnitudes. Second, our paper has focused on production networks in a closed economy setting, and it would be an interesting extension to consider international production networks and their role in creating cross-border spillovers of monetary policy shocks.

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Online Appendix

Monetary Policy and Production Networks: An Empirical Investigation

(Mishel Ghassibe)

Appendix A. Full model details

In this section we outline a more general version of our model, where we additionally allow for sector-specific exogenous productivity process ($\mathcal{A}_{kt}$), finite Frisch elasticity of labor supply (given by $1/\varphi$), CES aggregation across sectors (with elasticity of substitution given by λ) and monetary policy implemented through a Taylor rule. Naturally, the model considered in the main text is nested as a special case where $\mathcal{A}_{kt} = 1, \varphi = 0, \lambda = 1$ and monetary policy is implemented as a money supply rule.

Appendix A.1. Full non-linear system (at firm level)

(a) Firms

$$Y_{kt}(j) = \mathcal{A}_{kt} N_{kt}(j)^{1-\delta_k} Z_{kt}(j)^{\delta_k} - \Xi_k(j), \qquad \{\ln \mathcal{A}_{kt}\}_t \sim^{iid} (0, \sigma_a^2) \tag{A.1}$$

$$Z_{kt}(j) \equiv \left[\sum_{r=1}^{K} \omega_{kr}^{\frac{1}{\lambda}} Z_{krt}(j)^{\frac{\lambda-1}{\lambda}}\right]^{\frac{\lambda}{\lambda-1}} \tag{A.2}$$

$$Z_{krt}(j) \equiv \left(\int_{j' \in \Phi_r} Z_{krt}(j,j')^{\frac{\theta-1}{\theta}} dj'\right)^{\frac{\theta}{\theta-1}} \tag{A.3}$$

$$Z_{krt}(j) = \omega_{kr} \left(\frac{P_{rt}}{P_t^k}\right)^{-\lambda} Z_{kt}(j) \tag{A.4}$$

$$Z_{krt}(j,j') = \left(\frac{P_{rt}(j')}{P_{rt}}\right)^{-\theta} Z_{krt}(j) \tag{A.5}$$

$$P_t^k = \left[\sum_{r=1}^{K} \omega_{kr} P_{rt}^{1-\lambda}\right]^{\frac{1}{1-\lambda}} \tag{A.6}$$

$$MC_{kt}(j) = MC_{kt} = \frac{1}{\mathcal{A}_{kt}} \frac{1}{1-\delta_k} \left(\frac{\delta_k}{1-\delta_k}\right)^{-\delta_k} W_{kt}^{1-\delta_k} \left(P_t^k\right)^{\delta_k} \tag{A.7}$$

$$\frac{Z_{kt}(j)}{N_{kt}(j)} = \frac{\delta_k}{1-\delta_k} \frac{W_{kt}}{P_t^k} \tag{A.8}$$

$$P_{kt}^*(j) = P_{kt}^* = \frac{\theta}{\theta-1} \frac{\mathbb{E}_t \sum_{s=0}^{\infty} \alpha_k^s F_{t,t+s} P_{k,t+s}^{\theta} Y_{k,t+s} MC_{k,t+s}}{\mathbb{E}_t \sum_{s=0}^{\infty} \alpha_k^s F_{t,t+s} P_{k,t+s}^{\theta} Y_{k,t+s}} \tag{A.9}$$

$$P_{kt} = \left[\alpha_k P_{k,t-1}^{1-\theta} + (1-\alpha_k)(P_{kt}^*)^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{A.10}$$

(b) Households

$$F_{t,t+s} = \beta^s \frac{P_t^c C_t}{P_{t+s}^c C_{t+s}} \tag{A.11}$$

$$g_k N_{kt}^{\varphi} C_t = \frac{W_{kt}}{P_t^c} \tag{A.12}$$

$$C_t \equiv \left[\sum_{r=1}^{K} \omega_{ck}^{\frac{1}{\lambda}} C_{kt}(j)^{\frac{\lambda-1}{\lambda}} \right]^{\frac{\lambda}{\lambda-1}} \tag{A.13}$$

$$C_{kt} \equiv \left(\int_{j\in\Phi_k} C_{kt}(j)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}} \tag{A.14}$$

$$C_{kt} = \omega_{ck} \left(\frac{P_{kt}}{P_t^c} \right)^{-\lambda} C_t \tag{A.15}$$

$$C_{kt}(j) = \left(\frac{P_{kt}(j)}{P_{rt}} \right)^{-\theta} C_{kt} \tag{A.16}$$

$$P_t^c = \left[\sum_{k=1}^{K} \omega_{ck} P_{kt}^{1-\lambda} \right]^{\frac{1}{1-\lambda}} \tag{A.17}$$

(c) Monetary policy

Either

$$P_t^c C_t = M_t, \quad \ln M_t = \ln M_{t-1} + r_t, \quad r_t = \rho_r r_{t-1} + \varepsilon_t^r, \quad \{\varepsilon_t^r\}_t \sim^{iid} (0, \sigma_r^2) \tag{A.18}$$

or

$$\frac{I_t}{I} = \left(\frac{I_{t-1}}{I} \right)^{\rho_i} \left[\left(\frac{P_t^c}{P_{t-1}^c} \right)^{\phi_\pi} \left(\frac{C_t}{C} \right)^{\phi_c} \left(\frac{C_t}{C_{t-1}} \right)^{\phi_{gc}} \right]^{1-\rho_i} e^{\mu_t}, \quad \mu_t = \rho_\mu \mu_{t-1} + \varepsilon_t^\mu, \quad \{\varepsilon_t^\mu\}_t \sim^{iid} (0, \sigma_\mu^2) \tag{A.19}$$

$$I_t = \frac{1}{\mathbb{E}_t F_{t,t+1}} \tag{A.20}$$

(d) Market clearing

$$Y_{kt}(j) = C_{kt}(j) + \sum_{r=1}^{K} \int_{j'\in\Phi_r} Z_{rkt}(j', j) dj' \tag{A.21}$$

$$N_{kt} = \int_{j\in\Phi_k} N_{kt}(j) dj \tag{A.22}$$

$$B_t = 0 \tag{A.23}$$

Appendix A.2. Steady state

We consider a symmetric steady state, so that $P_k(j) = P_k = P^k = P^c = P, \quad \forall k, \forall j \in \Phi_k$.

$$\frac{W_k}{P} = \left[\frac{\theta}{\theta - 1} \frac{\delta_k^{-\delta_k}}{(1-\delta_k)^{1-\delta_k}} \right]^{\frac{1}{\delta_k - 1}} \tag{A.24}$$

$$\Xi_k(j) = \frac{1}{\theta - 1} Y_k(j) \tag{A.25}$$

$$N_k(j) = (1-\delta_k) \left(\frac{W_k}{P} \right)^{-1} Y_k(j) \tag{A.26}$$

$$Z_k(j) = \delta_k Y_k(j) \tag{A.27}$$

$$[Y_1, ..., Y_K]' = [I - \Omega' diag(\delta)]^{-1} [C_1, ..., C_K]' \tag{A.28}$$

$$C_k(j) = C_k = \omega_{ck} C \tag{A.29}$$

$$Y_k(j) = Y_k \tag{A.30}$$

$$N_k(j) = N_k \tag{A.31}$$

$$C = \sum_{k=1}^{K} \frac{W_k}{P} N_k \tag{A.32}$$

Appendix A.3. Log-linearized system (at sectoral level)

(a) Firms

$$y_{kt} = \frac{\theta}{\theta - 1} [a_{kt} + (1-\delta_k) n_{kt} + \delta_k z_{kt}] \tag{A.33}$$

$$z_{kt} = \sum_{r=1}^{K} \omega_{kr} z_{krt} \tag{A.34}$$

$$z_{krt} = \lambda (p_{rt} - p_t^k) + z_{kt} \tag{A.35}$$

$$p_t^k = \sum_{r=1}^{K} \omega_{kr} p_{rt} \tag{A.36}$$

$$mc_{kt} = -a_{kt} + (1-\delta_k) w_{kt} + \delta_k p_t^k \tag{A.37}$$

$$z_{kt} = n_{kt} + w_{kt} - p_t^k \tag{A.38}$$

$$p_{kt}^* = \beta \alpha_k \mathbb{E}_t p_{k,t+1}^* + (1 - \beta \alpha_k) mc_{kt} \tag{A.39}$$

$$p_{kt} = \alpha_k p_{k,t-1} + (1-\alpha_k) p_{kt}^* \tag{A.40}$$

(b) Households

$$c_t = w_{kt} - p_t^c - \varphi n_{kt} \tag{A.41}$$

$$c_t = \sum_{k=1}^{K} \omega_{ck} c_{kt} \tag{A.42}$$

$$c_{kt} = -\lambda (p_{kt} - p_t^c) + c_t \tag{A.43}$$

(c) Monetary policy

Either

$$p_t^c + c_t = m_t, \quad m_t = m_{t-1} + r_t, \quad r_t = \rho_r r_{t-1} + \varepsilon_t^r \tag{A.44}$$

or

$$c_t = \mathbb{E}_t c_{t+1} - (i_t - \mathbb{E}_t p_{t+1}^c + p_t^c) \tag{A.45}$$

$$i_t = \rho_i i_{t-1} + (1-\rho_i)[\phi_\pi(p_t^c - p_{t-1}^c) + \phi_c c_t + \phi_{gc}(c_t - c_{t-1})] + \mu_t, \quad \mu_t = \rho_\mu \mu_{t-1} + \varepsilon_t^\mu \tag{A.46}$$

(d) Market clearing

$$y_{kt} = \frac{\omega_{ck} C}{Y_k} c_{kt} + \sum_{r=1}^{K} \omega_{rk} \frac{\delta_r Y_r}{Y_k} z_{rkt} \tag{A.47}$$

Appendix B. Proofs

In this section we present proofs of formal results in the baseline version of our model, corresponding to the model in Appendix A under $\mathcal{A}_{kt} = 1, \varphi = 0, \lambda = 1$ and monetary policy implemented as a money supply rule. Unlike in the main text, we assume persistent monetary shocks, so that $r_t = \rho_r r_{t-1} + \varepsilon_t^r, \quad |\rho_r| \in (0,1)$. Trivially, proofs of results in main text follow as a special case under $\rho_r = 0$.

Proposition 1´. *The response of final consumption in sector k c_{kt} to a monetary shock r_t, where $r_t = \rho_r r_{t-1} + \varepsilon_t^r$ and $\{\varepsilon_t^r\}_t$ is a zero-mean i.i.d. process, is given by:*

$$c_{kt} = \overbrace{\alpha_k c_{k,t-1} + \frac{\alpha_k - \beta\alpha_k\rho_r}{1-\beta\alpha_k\rho_r} r_t}^{\text{Direct effect}} + \underbrace{\delta_k(1-\alpha_k)(1-\beta\alpha_k)\sum_{s=0}^{\infty}(\beta\alpha_k)^s \sum_{r=1}^{K} \omega_{kr}\mathbb{E}_t c_{r,t+s}}_{\text{Downstream effect}},$$

for $k = 1, 2, ..., K$. It consists of the the direct effect, unaffected by the intermediate inputs share δ_k, and the downstream effect, which fully vanishes under $\delta_k = 0$ and increases in δ_k, representing amplification through input-output linkages.

Proof. From the sectoral pricing equation we know that:

$$\begin{aligned} p_{kt} &= \alpha_k p_{k,t-1} + (1-\alpha_k)(1-\beta\alpha_k)\sum_{s=0}^{\infty}(\beta\alpha_k)^s \mathbb{E}_t mc_{k,t+s} \\ &= \alpha_k p_{k,t-1} + (1-\alpha_k)(1-\beta\alpha_k)\sum_{s=0}^{\infty}(\beta\alpha_k)^s \mathbb{E}_t \left[(1-\delta_k)m_{t+s} + \delta_k \sum_{r=1}^{K}\omega_{kr} p_{r,t+s}\right] \end{aligned}$$

Given that cash-in-advance constraint implies that $p_{kt} = m_t - c_{kt}$:

$$
\begin{aligned}
p_{kt} &= \alpha_k p_{k,t-1} + (1-\alpha_k)(1-\beta\alpha_k)\sum_{s=0}^{\infty}(\beta\alpha_k)^s \mathbb{E}_t\left[(1-\delta_k)m_{t+s} + \delta_k \sum_{r=1}^{K}\omega_{kr}[m_{t+s} - c_{r,t+s}]\right] \\
&= \alpha_k p_{k,t-1} + (1-\alpha_k)(1-\beta\alpha_k)\sum_{s=0}^{\infty}(\beta\alpha_k)^s \mathbb{E}_t m_{t+s} - \delta_k(1-\alpha_k)(1-\beta\alpha_k)\sum_{s=0}^{\infty}(\beta\alpha_k)^s \sum_{r=1}^{K}\omega_{kr}\mathbb{E}_t c_{r,t+s} \\
&= \alpha_k p_{k,t-1} + (1-\alpha_k)(1-\beta\alpha_k)\sum_{s=0}^{\infty}(\beta\alpha_k)^s \left[m_t + r_t\rho_r \frac{1-\rho_r^s}{1-\rho_r}\right] \\
&\qquad - \delta_k(1-\alpha_k)(1-\beta\alpha_k)\sum_{s=0}^{\infty}(\beta\alpha_k)^s \sum_{r=1}^{K}\omega_{kr}\mathbb{E}_t c_{r,t+s} \\
&= \alpha_k p_{k,t-1} + (1-\alpha_k)m_t + (1-\alpha_k)(1-\beta\alpha_k)r_t \frac{\rho_r}{1-\rho_r}\sum_{s=0}^{\infty}(\beta\alpha_k)^s(1-\rho_r^s) \\
&\qquad - \delta_k(1-\alpha_k)(1-\beta\alpha_k)\sum_{s=0}^{\infty}(\beta\alpha_k)^s \sum_{r=1}^{K}\omega_{kr}\mathbb{E}_t c_{r,t+s}
\end{aligned}
$$

Similarly replace p_{kt} and $p_{k,t-1}$ using the cash-in-advance constraint:

$$
\begin{aligned}
m_t - c_{kt} &= \alpha_k(m_{t-1} - c_{k,t-1}) + (1-\alpha_k)m_t + (1-\alpha_k)r_t\rho_r \frac{\beta\alpha_k}{1-\beta\alpha_k} \\
&\qquad - \delta_k(1-\alpha_k)(1-\beta\alpha_k)\sum_{s=0}^{\infty}(\beta\alpha_k)^s \sum_{r=1}^{K}\mathbb{E}_t c_{r,t+s} \\
c_{kt} &= \alpha_k c_{k,t-1} + \left[\alpha_k - \frac{(1-\alpha_k)\beta\alpha_k\rho_r}{1-\beta\alpha_k\rho_r}\right] r_t + \delta_k(1-\alpha_k)(1-\beta\alpha_k)\sum_{s=0}^{\infty}(\beta\alpha_k)^s \sum_{r=1}^{K}\mathbb{E}_t c_{r,t+s} \\
c_{kt} &= \alpha_k c_{k,t-1} + \frac{\alpha_k - \beta\alpha_k\rho_r}{1-\beta\alpha_k\rho_r} r_t + \delta_k(1-\alpha_k)(1-\beta\alpha_k)\sum_{s=0}^{\infty}(\beta\alpha_k)^s \sum_{r=1}^{K}\mathbb{E}_t c_{r,t+s}.
\end{aligned}
$$

□

Proposition 2´. *The response of final sectoral consumption c_{kt} to a monetary policy shock r_t, where $r_t = \rho_r r_{t-1} + \varepsilon_t^r$ and $\{\varepsilon_t^r\}_t$ is a zero-mean i.i.d. process, is*

$$
c_{kt} = \sum_{j=1}^{K} l_{kj,T-t}\alpha_j c_{j,t-1} + \tilde{\alpha}_{k,T-t} r_t, \qquad 1 \le t \le T-1
$$

where $\tilde{\alpha}_{k,T-t}$ is

$$
\tilde{\alpha}_{k,T-t} \equiv \sum_{j=1}^{K} l_{kj,T-t}\left(\sum_{j'=1}^{K}\kappa_{jj',T-t}\right),
$$

and $\{l_{kj,T-t}\}_{kj}$ are entries of the Forward-looking Leontief Inverse:

$$
L_{T-t} = \underbrace{\left[I - \Delta(I-A)G_{T-t}\sum_{s=0}^{T-t-1}(\beta A)^s \Omega\left\{\Pi_{j=T-t-s}^{T-t-1} L_j A\right\}\right]^{-1}}_{\text{Forward-looking Leontief Inverse}},
$$

and $\{\kappa_{jj',T-t}\}_{jj'}$ *are entries of the following matrix:*

$$K_{T-t}=\left[(\underline{A}+\Gamma_{T-t})+\Delta(I-A)G_{T-t}\sum_{s=0}^{T-t-1}(\beta A)^s\Omega\sum_{\tau=0}^{s-1}\rho_r^{s-\tau}\left\{\Pi_{j=T-t-s}^{T-t-s+\tau-1}L_jA\right\}\{L_{T-t-s+\tau}K_{T-t-s+\tau}\}\right]$$

where $\underline{A}\equiv diag[\underline{\alpha}_k,\underline{\alpha}_2,...,\underline{\alpha}_K]'$, $\underline{\alpha}_k\equiv\frac{\alpha_k-\beta\alpha_k\rho_r}{1-\beta\alpha_k\rho_r}$ $\forall k$, $\Gamma_{T-t}\equiv diag[\gamma_{1,T-t},\gamma_{2,T-t},...,\gamma_{K,T-t}]'$, $\gamma_{k,T-t}\equiv\frac{\rho_r}{1-\rho_r}\frac{(1-\alpha_k)(1-\beta\alpha_k)}{1-\beta\alpha_k\rho_r}\left[\frac{1-(\beta\alpha_k\rho_r)^{T-t}}{1-(\beta\alpha_k)^{T-t}}-1\right]$ $\forall k$, $A\equiv diag[\alpha_1,\alpha_2,...,\alpha_K]'$, $\Delta\equiv diag[\delta_1,\delta_2,...,\delta_K]'$, $G_{T-t}\equiv(I-\beta A)(I-(\beta A)^{T-t})^{-1}$ *and* Ω *is the input-output matrix.*

Proof. For convenience, rewrite the equation for sectoral consumption in the proposition in matrix form:

$$\mathbb{c}_t=L_{T-t}A\mathbb{c}_{t-1}+L_{T-t}K_{T-t}\mathbb{r}_t,\qquad 1\le t\le T-1$$

where $\mathbb{c}_t\equiv[c_{1t},c_{2t},...,c_{Kt}]'$, $\mathbb{r}_t=[r_t,r_t,...,r_t]'$.

Note that from Proposition 1′ there follows a finite-time direct-downstream decomposition:

$$\mathbb{c}_t=A\mathbb{c}_{t-1}+[\underline{A}+\Gamma_{T-t}]\mathbb{r}_t+\Delta(I-A)G_{T-t}\sum_{s=0}^{T-t-1}(\beta A)^s\Omega\mathbb{E}_t\mathbb{c}_{t+s}$$

where $\underline{A}\equiv diag[\underline{\alpha}_k,\underline{\alpha}_2,...,\underline{\alpha}_K]'$, $\underline{\alpha}_k\equiv\frac{\alpha_k-\beta\alpha_k\rho_r}{1-\beta\alpha_k\rho_r}$ $\forall k$, $\Gamma_{T-t}\equiv diag[\gamma_{1,T-t},\gamma_{2,T-t},...,\gamma_{K,T-t}]'$, $\gamma_{k,T-t}\equiv\frac{\rho_r}{1-\rho_r}\frac{(1-\alpha_k)(1-\beta\alpha_k)}{1-\beta\alpha_k\rho_r}\left[\frac{1-(\beta\alpha_k\rho_r)^{T-t}}{1-(\beta\alpha_k)^{T-t}}-1\right]$ $\forall k$.

The case for $t=T-1$ is trivial and follows from the finite-time direct-downstream decomposition above:

$$\begin{aligned}\mathbb{c}_{T-1}&=A\mathbb{c}_{T-2}+[\underline{A}+\Gamma_1]\mathbb{r}_{T-1}+\Delta(I-A)G_1\Omega\mathbb{c}_{T-1},\\ \mathbb{c}_{T-1}&=\underbrace{[I-\Delta(I-A)G_1\Omega]^{-1}}_{L_1}A\mathbb{c}_{T-2}+\underbrace{[I-\Delta(I-A)G_1\Omega]^{-1}}_{L_1}\underbrace{[\underline{A}+\Gamma_1]}_{K_1}\mathbb{r}_{T-1}.\end{aligned}$$

As for $1\le t\le T-2$, we will build our proof by strong induction. First, we will show that our conjecture holds for $t=T-2$; then we will show that it holds for any $1\le t\le T-2$, if we suppose that it holds for all $t+1\le\tilde{t}\le T-2$.

For $t=T-2$, the finite-time direct-downstream decomposition is given by:

$$\mathbb{c}_{T-2}=A\mathbb{c}_{T-3}+[\underline{A}+\Gamma_2]\mathbb{r}_{T-2}+\Delta(I-A)G_2[\Omega\mathbb{c}_{T-2}+(\beta A)\Omega\mathbb{E}_{T-2}\mathbb{c}_{T-1}],$$

but we know that $\mathbb{c}_{T-1}=L_1A\mathbb{c}_{T-2}+L_1K_1\mathbb{r}_{T-1}$, and hence

$$\begin{aligned}\mathbb{c}_{T-2}&=A\mathbb{c}_{T-3}+[\underline{A}+\Gamma_2]\mathbb{r}_{T-2}+\Delta(I-A)G_2[\Omega\mathbb{c}_{T-2}+(\beta A)\Omega\mathbb{E}_{T-2}\{L_1A\mathbb{c}_{T-2}+L_1K_1\mathbb{r}_{T-1}\}]\\&=A\mathbb{c}_{T-3}+[\underline{A}+\Gamma_2]\mathbb{r}_{T-2}+\Delta(I-A)G_2\Omega\mathbb{c}_{T-2}+\Delta(I-A)G_2(\beta A)\Omega[L_1A\mathbb{c}_{T-2}+\rho_rL_1K_1\mathbb{r}_{T-2}]\\&=A\mathbb{c}_{T-3}+[(\underline{A}+\Gamma_2)+\Delta(I-A)G_2(\beta A)\Omega\rho_rL_1K_1]\mathbb{r}_{T-2}+\Delta(I-A)G_2[\Omega+(\beta A)\Omega L_1A]\mathbb{c}_{T-2}\end{aligned}$$

which allows us to solve for $\mathbb{c}_{T-2}$:

$$\begin{aligned}\mathbb{c}_{T-2}=&\underbrace{[I-\Delta(I-A)G_2(\Omega+(\beta A)\Omega L_1A)]^{-1}}_{L_2}A\mathbb{c}_{T-3}\\&+\underbrace{[I-\Delta(I-A)G_2(\Omega+(\beta A)\Omega L_1A)]^{-1}}_{L_2}\underbrace{[(\underline{A}+\Gamma_2)+\Delta(I-A)G_2(\beta A)\Omega\rho_rL_1K_1]}_{K_2}\mathbb{r}_{T-2},\\ \mathbb{c}_{T-2}=&L_2A\mathbb{c}_{T-3}+L_2K_2\mathbb{r}_{T-2}.\end{aligned}$$

as required to prove our conjecture for $t=T-2$.

Now let us write the direct-downstream decomposition for a generic period $1\le t\le T-2$, and further suppose that our conjecture holds for all $t+1 \le \tilde{t} \le T-1$:

$$
\begin{aligned}
\mathbb{c}_t &= A\mathbb{c}_{t-1} + [\underline{A} + \Gamma_{T-t}]\mathbb{r}_t + \Delta(I-A)G_{T-t}\sum_{s=0}^{T-t-1}(\beta A)^s \Omega \mathbb{E}_t \mathbb{c}_{t+s} \\
&= A\mathbb{c}_{t-1} + [\underline{A} + \Gamma_{T-t}]\mathbb{r}_t + \Delta(I-A)G_{T-t}\sum_{s=0}^{T-t-1}(\beta A)^s \Omega \mathbb{E}_t \left[L_{T-t-s}A\mathbb{c}_{t+s-1} + L_{T-t-s}K_{T-t-s}\mathbb{r}_{t+s}\right] \\
&= A\mathbb{c}_{t-1} + [\underline{A} + \Gamma_{T-t}]\mathbb{r}_t + \Delta(I-A)G_{T-t} \times \\
&\times \sum_{s=0}^{T-t-1}(\beta A)^s \Omega \mathbb{E}_t \left[L_{T-t-s}AL_{T-t-s+1}A\mathbb{c}_{t+s-2} + L_{T-t-s}AL_{T-t-s+1}K_{T-t-s+1}\mathbb{r}_{t+s-1} + L_{T-t-s}K_{T-t-s}\mathbb{r}_{t+s}\right] \\
&= \ldots \\
&= A\mathbb{c}_{t-1} + [\underline{A} + \Gamma_{T-t}]\mathbb{r}_t + \Delta(I-A)G_{T-t} \times \\
&\times \sum_{s=0}^{T-t-1}(\beta A)^s \Omega \mathbb{E}_t \left[\left\{\Pi_{j=T-t-s}^{T-t-1} L_j A\right\}\mathbb{c}_t + \sum_{\tau=0}^{s-1}\left\{\Pi_{j=T-t-s}^{T-t-s-+\tau-1} L_j A\right\} L_{T-t-s+\tau}K_{T-t-s+\tau}\mathbb{r}_{t+s-\tau}\right] \\
&= A\mathbb{c}_{t-1} + [\underline{A} + \Gamma_{T-t}]\mathbb{r}_t + \Delta(I-A)G_{T-t} \times \\
&\times \sum_{s=0}^{T-t-1}(\beta A)^s \Omega \left[\left\{\Pi_{j=T-t-s}^{T-t-1} L_j A\right\}\mathbb{c}_t + \sum_{\tau=0}^{s-1}\rho_r^{s-\tau}\left\{\Pi_{j=T-t-s}^{T-t-s-+\tau-1} L_j A\right\} L_{T-t-s+\tau}K_{T-t-s+\tau}\mathbb{r}_t\right]
\end{aligned}
$$

where we can now solve for $\mathbb{c}_t$:

$$
\begin{aligned}
\mathbb{c}_t = & \underbrace{\left[I - \Delta(I-A)G_{T-t}\sum_{s=0}^{T-t-1}(\beta A)^s \Omega \left\{\Pi_{j=T-t-s}^{T-t-1} L_j A\right\}\right]^{-1}}_{L_{T-t}} A\mathbb{c}_{t-1} + \\
& \underbrace{\left[I - \Delta(I-A)G_{T-t}\sum_{s=0}^{T-t-1}(\beta A)^s \Omega \left\{\Pi_{j=T-t-s}^{T-t-1} L_j A\right\}\right]^{-1}}_{L_{T-t}} \times \\
& \times \underbrace{\left[(\underline{A} + \Gamma_{T-t}) + \Delta(I-A)G_{T-t}\sum_{s=0}^{T-t-1}(\beta A)^s \Omega \sum_{\tau=0}^{s-1}\rho_r^{s-\tau}\left\{\Pi_{j=T-t-s}^{T-t-s-+\tau-1} L_j A\right\} L_{T-t-s+\tau}K_{T-t-s+\tau}\right]}_{K_{T-t}} \mathbb{r}_t,
\end{aligned}
$$

$$\mathbb{c}_t = L_{T-t}A\mathbb{c}_{t-1} + L_{T-t}K_{T-t}\mathbb{r}_t.$$

□

Corollary 1´. *[Full IRF] Suppose the economy is in steady state until the beginning of period t; the impulse response of final sectoral consumption c_{kt}, $k = 1, 2, ..., K$ at horizon $H \ge 0$, following a one-time monetary shock innovation ε_t^r, where $r_t = \rho_r r_{t-1} + \varepsilon_t^r \quad \forall t$, is given by:*

$$c_{k,t+H} = \tilde{\alpha}_{k,T-t}^H \varepsilon_t^r, \qquad 1 \le t, t+H \le T-1$$

where $\tilde{\alpha}_{k,T-t}^H \equiv \sum_{r=1}^K \left[\sum_{h=0}^H \rho_r^{H-h}\left\{\Pi_{j=T-t-H}^{T-t} L_j A\right\}\left\{L_{T-t-H+h}K_{T-t+H-h}\right\}\right]_{kr}$, L_j is period-j Forward-

looking Leontief Inverse, matrix K is defined in Proposition 2´ and $A \equiv diag[\alpha_1, \alpha_2, ..., \alpha_K]'$. Therefore, $\frac{dc_{k,t+H}}{d\varepsilon_t^r} = \tilde{\tilde{\alpha}}_{k,T-t}^H$.

Proof. Take the solution in Proposition 2´ written in matrix form, and iterate backwards:

$$
\begin{aligned}
\mathbb{c}_t &= L_{T-t}A\mathbb{c}_{t-1} + L_{T-t}K_{T-t}\mathbb{r}_t, \quad 1 \le t \le T-1 \\
&= L_{T-t}AL_{T-t+1}A\mathbb{c}_{t-2} + L_{T-t}AL_{T-t+1}K_{T-t+1}\mathbb{r}_{t-1} + L_{T-t}K_{T-t}\mathbb{r}_t \\
&= L_{T-t}AL_{T-t+1}AL_{T-t+2}A\mathbb{c}_{t-3} + L_{T-t}AL_{T-t+1}L_{T-t+2}K_{T-t+2}\mathbb{r}_{t-2} + L_{T-t}AL_{T-t+1}K_{T-t+1}\mathbb{r}_{t-1} + L_{T-t}K_{T-t}\mathbb{r}_t \\
&= ... \\
&= \{L_{T-t}AL_{T-t+1}A...L_{T-1}A\}\,\mathbb{c}_0 + \\
&\qquad \{L_{T-t}AL_{T-t+1}A...L_{T-1}K_{T-1}\}\,\mathbb{r}_1 + ... + \{L_{T-t}AL_{T-t+1}K_{T-t+1}\}\,\mathbb{r}_{t-1} + L_{T-t}K_{T-t}\mathbb{r}_t \\
\mathbb{c}_t &= \{L_{T-t}AL_{T-t+1}A...L_{T-1}A\}\,\mathbb{c}_0 + \sum_{h=0}^{t-1}\left\{\Pi_{j=T-t}^{T-t+h-1} L_j A\right\} L_{T-t+h}K_{T-t+h}\mathbb{r}_{t-h}.
\end{aligned}
$$

The expression above evaluated at a generic period $1 \le t+H \le T$:

$$
\mathbb{c}_{t+H} = \{L_{T-t-H}AL_{T-t+H+1}A...L_{T-1}A\}\,\mathbb{c}_0 + \sum_{h=0}^{t+H-1}\left\{\Pi_{j=T-t-H}^{T-t-H+h-1} L_j A\right\} L_{T-t-H+h}K_{T-t-H+h}\mathbb{r}_{t+H-h}.
$$

Now suppose the economy is in steady state until the beginning of period t, so that $\mathbb{c}_0 = \mathbb{r}_1 = ... = \mathbb{r}_{t-1} = \mathbb{0}_{K\times 1}$, and there is a one-time monetary innovation ε_t^r hitting the economy, which does not repeat in the future; then $r_{t+H} = \rho_r^H \varepsilon_t^r, \quad H \ge 0$, and the impulse response of sectoral consumption H periods after the innovation is given by:

$$
\mathbb{c}_{t+H} = \sum_{h=0}^{H} \rho_r^{H-h}\left\{\Pi_{j=T-t-H}^{T-t-H+h-1} L_j A\right\} L_{T-t-H+h}K_{T-t-H+h}\varepsilon_t^r,
$$

or when written in scalar form:

$$
c_{k,t+H} = \tilde{\tilde{\alpha}}_{k,T-t}^H \varepsilon_t^r,
$$

where $\tilde{\tilde{\alpha}}_{k,T-t}^H \equiv \sum_{r=1}^K \left[\sum_{h=0}^H \rho_r^{H-h}\left\{\Pi_{j=T-t-H}^{T-t} L_j A\right\}\{L_{T-t-H+h}K_{T-t+H-h}\}\right]_{kr}, \forall k = 1, 2, ..., K; 1 \le t, t+H \le T-1$. ☐

Corollary 2´. *[Upper Bound of Direct IRF] Suppose the economy is in steady state until the beginning of period t; the impulse response of final sectoral consumption c_{kt}, $k = 1, 2, ..., K$ at horizon $H \ge 1$, following a one-time monetary shock innovation ε_t^r, where $r_t = \rho_r r_{t-1} + \varepsilon_t^r \quad \forall t$, can be written as:*

$$
c_{k,t+H} = \left[\sum_{\tau=0}^{H} \rho_r^{H-\tau}\left\{\alpha_k^\tau(\underline{\alpha}_k + \gamma_{k,T-t-H+\tau}) + \delta_k q^k(T-t-H,\tau)\right\}\right]\varepsilon_t^r + \sum_{\tau=1}^{H}\delta_k s_{T-t-H+\tau}^k \sum_{r=1}^{K}\omega_{kr}c_{r,t+H-\tau},
$$

$1 \le t, t+H \le T-1$, *where* $q^k(T-t-H,\tau) \equiv \sum_{j=1}^K [\mathbb{Q}(T-t-H,\tau)]_{kj} \ge 0$,
$\underline{\alpha}_k \equiv \frac{\alpha_k - \beta\alpha_k\rho_r}{1-\beta\alpha_k\rho_r}$, $\quad \gamma_{k,T-t} \equiv \frac{\rho_r}{1-\rho_r}\frac{(1-\alpha_k)(1-\beta\alpha_k)}{1-\beta\alpha_k\rho_r}\left[\frac{1-(\beta\alpha_k\rho_r)^{T-t}}{1-(\beta\alpha_k)^{T-t}} - 1\right]$, $\quad s_{T-t-H+\tau}^k \equiv \alpha_k^\tau(1-\alpha_k)g_{T-t-H+\tau}$,
$g_{T-t-H+\tau} \equiv [G_{T-t-H+\tau}]_{kk}$. *Hence,* $\frac{\partial c_{k,t+H}}{\partial r_t} = \left[\sum_{\tau=0}^H \rho_r^{H-\tau}\left\{\alpha_k^\tau(\underline{\alpha}_k + \gamma_{k,T-t-H+\tau}) + \delta_k q^k(T-t-H,\tau)\right\}\right] \ge \left[\sum_{\tau=0}^H \rho_r^{H-\tau}\left\{\alpha_k^\tau(\underline{\alpha}_k + \gamma_{k,T-t-H+\tau})\right\}\right]$.

Proof. Substitute the expression for $\mathbb{c}_t$ obtained in Corollary 1´ into the finite-time direct-downstream decomposition (let $\mathbb{c}_0 = \mathbb{0}_{K\times 1}$):

$$\mathbb{c}_t = A\mathbb{c}_{t-1} + [\underline{A} + \Gamma_{T-t}]\mathbb{r}_t + \Delta(I-A)G_{T-t}\Omega\mathbb{c}_t +$$
$$\Delta(I-A)G_{T-t}\sum_{s=1}^{T-t-1}(\beta A)^s\Omega\left[\sum_{h=0}^{t+s-1}\left\{\Pi_{j=T-t-s}^{T-t-s+h-1}L_jA\right\}L_{T-t-s+h}K_{T-t-s+h}\mathbb{E}_t\mathbb{r}_{t+s-h}\right],$$

and separating out past and future shocks:

$$\mathbb{c}_t = A\mathbb{c}_{t-1} + [\underline{A} + \Gamma_{T-t}]\mathbb{r}_t + \Delta(I-A)G_{T-t}\Omega\mathbb{c}_t +$$
$$\Delta(I-A)G_{T-t}\sum_{s=1}^{T-t-1}(\beta A)^s\Omega\left[\sum_{h=0}^{s}\left\{\Pi_{j=T-t-s}^{T-t-s+h-1}L_jA\right\}L_{T-t-s+h}K_{T-t-s+h}\mathbb{E}_t\mathbb{r}_{t+s-h}\right] +$$
$$\Delta(I-A)G_{T-t}\sum_{s=1}^{T-t-1}(\beta A)^s\Omega\left[\sum_{h=s+1}^{t+s-1}\left\{\Pi_{j=T-t-s}^{T-t-s+h-1}L_jA\right\}L_{T-t-s+h}K_{T-t-s+h}\mathbb{r}_{t+s-h}\right],$$

$$\mathbb{c}_t = A\mathbb{c}_{t-1} + \Delta(I-A)G_{T-t}\Omega\mathbb{c}_t +$$
$$\left[\underline{A} + \Gamma_{T-t} + \Delta(I-A)G_{T-t}\sum_{s=1}^{T-t-1}(\beta A)^s\Omega\left[\sum_{h=0}^{s}\rho_r^{s-h}\left\{\Pi_{j=T-t-s}^{T-t-s+h-1}L_jA\right\}L_{T-t-s+h}K_{T-t-s+h}\right]\right]\mathbb{r}_t +$$
$$\Delta(I-A)G_{T-t}\sum_{s=1}^{T-t-1}(\beta A)^s\Omega\left[\sum_{h=s+1}^{t+s-1}\left\{\Pi_{j=T-t-s}^{T-t-s+h-1}L_jA\right\}L_{T-t-s+h}K_{T-t-s+h}\mathbb{r}_{t+s-h}\right]$$

For convenience, let $\tau \equiv (h-s)$:

$$\mathbb{c}_t = A\mathbb{c}_{t-1} + \Delta(I-A)G_{T-t}\Omega\mathbb{c}_t +$$
$$\left[\underline{A} + \Gamma_{T-t} + \Delta(I-A)G_{T-t}\sum_{s=1}^{T-t-1}(\beta A)^s\Omega\left[\sum_{h=-s}^{0}\rho_r^{-h}\left\{\Pi_{j=T-t-s}^{T-t-s+h-1}L_jA\right\}L_{T-t-s+h}K_{T-t-s+h}\right]\right]\mathbb{r}_t +$$
$$\Delta(I-A)G_{T-t}\sum_{s=1}^{T-t-1}(\beta A)^s\Omega\left[\sum_{\tau=1}^{t-1}\left\{\Pi_{j=T-t-s}^{T-t+\tau-1}L_jA\right\}L_{T-t+\tau}K_{T-t+\tau}\mathbb{r}_{t-\tau}\right],$$

$$\mathbb{c}_t = A\mathbb{c}_{t-1} + [\underline{A} + \Gamma_{T-t} + \Delta\tilde{\mathbb{R}}(T-t,\rho_r)]\mathbb{r}_t + \sum_{\tau=1}^{t-1}[\Delta\mathbb{R}(T-t,\tau)]\mathbb{r}_{t-\tau} + \Delta(I-A)G_{T-t}\Omega\mathbb{c}_t,$$

where $\tilde{\mathbb{R}}(x,\rho_r) \equiv (I-A)G_x\left[\sum_{s=1}^{x-1}(\beta A)^s\Omega\sum_{h=-s}^{0}\rho_r^{-h}\left\{\Pi_{j=x-s}^{x+h-1}L_jA\right\}L_{x+h}K_{x+h}\right]$,
$\mathbb{R}(x,y) \equiv (I-A)G_x\left[\sum_{s=1}^{x-1}(\beta A)^s\Omega\left\{\Pi_{j=x-s}^{x+y-1}L_jA\right\}L_{x+y}K_{x+y}\right]$.

Now iterate the above expression backwards:

$$\begin{aligned}
\mathbb{c}_t = {} & A^2\mathbb{c}_{t-2} + [\underline{A} + \Gamma_{T-t} + \Delta\tilde{\mathbb{R}}(T-t,\rho_r)]\mathbb{r}_t + \\
& [A(\underline{A} + \Gamma_{T-t+1}) + A\Delta\tilde{\mathbb{R}}(T-t+1,\rho_r) + \Delta\mathbb{R}(T-t,1)]\mathbb{r}_{t-1} + \\
& \sum_{\tau=2}^{t-1} [\Delta\mathbb{R}(T-t,\tau) + A\Delta\mathbb{R}(T-t+1,\tau-1)]\,\mathbb{r}_{t-\tau} + \\
& \Delta(I-A)G_{T-t}\Omega\mathbb{c}_t + A\Delta(I-A)G_{T-t+1}\Omega\mathbb{c}_{t-1} \\
\mathbb{c}_t = {} & A^3\mathbb{c}_{t-2} + [\underline{A} + \Gamma_{T-t} + \Delta\tilde{\mathbb{R}}(T-t,\rho_r)]\mathbb{r}_t + \\
& [A(\underline{A} + \Gamma_{T-t+1}) + A\Delta\tilde{\mathbb{R}}(T-t+1,\rho_r) + \Delta\mathbb{R}(T-t,1)]\mathbb{r}_{t-1} + \\
& [A^2(\underline{A} + \Gamma_{T-t+2} + \Delta\tilde{\mathbb{R}}(T-t+2,\rho_r)) + A\Delta\mathbb{R}(T-t+1,1) + \Delta\mathbb{R}(T-t,2)]\mathbb{r}_{t-2} + \\
& \sum_{\tau=3}^{t-1} \left[A^2\delta\mathbb{R}(T-t+2,\tau-2) + A\Delta\mathbb{R}(T-t+1,\tau-1) + \Delta\mathbb{R}(T-t,\tau)\right]\mathbb{r}_{t-\tau} + \\
& \Delta(I-A)G_{T-t}\Omega\mathbb{c}_t + A\Delta(I-A)G_{T-t+1}\Omega\mathbb{c}_{t-1} + A^2\Delta(I-A)G_{T-t+2}\Omega\mathbb{c}_{t-2}. \\
= {} & \ldots \\
\mathbb{c}_t = {} & A^t\mathbb{c}_0 + \sum_{\tau=0}^{t-1}\left[A^\tau(\underline{A} + \Gamma_{T-t+\tau} + \Delta\tilde{\mathbb{R}}(T-t+\tau,\rho_r)) + \sum_{l=0}^{\tau-1} A^l\Delta\mathbb{R}(T-t+l,\tau-l)\right]\mathbb{r}_{t-\tau} + \\
& \Delta(I-A)G_{T-t}\Omega\mathbb{c}_t + \sum_{\tau=1}^{t-1} A^\tau\Delta(I-A)G_{T-t+\tau}\Omega\mathbb{c}_{t-\tau} \\
\mathbb{c}_t = {} & \sum_{\tau=0}^{t-1}\left[A^\tau(\underline{A} + \Gamma_{T-t+\tau} + \Delta\tilde{\mathbb{R}}(T-t+\tau,\rho_r)) + \sum_{l=0}^{\tau-1} A^l\Delta\mathbb{R}(T-t+l,\tau-l)\right]\mathbb{r}_{t-\tau} + \\
& \Delta(I-A)G_{T-t}\Omega\left[\sum_{\tau=0}^{t-1}\left\{\Pi_{j=T-t}^{T-t+\tau-1}L_jA\right\}L_{T-t+\tau}K_{T-t+\tau}\mathbb{r}_{t-\tau}\right] + \sum_{\tau=1}^{t-1} A^\tau\Delta(I-A)G_{T-t+\tau}\Omega\mathbb{c}_{t-\tau} \\
\mathbb{c}_t = {} & \sum_{\tau=0}^{t-1}[A^\tau(\underline{A} + \Gamma_{T-t+\tau}) + \Delta\mathbb{Q}(T-t,\tau)]\,\mathbb{r}_{t-\tau} + \sum_{\tau=1}^{t-1} A^\tau\Delta(I-A)G_{T-t+\tau}\Omega\mathbb{c}_{t-\tau},
\end{aligned}$$

$$\mathbb{Q}(T-t,\tau) \equiv \left[A^\tau\tilde{\mathbb{R}}(T-t+\tau,\rho_r) + \textstyle\sum_{l=0}^{\tau-1} A^l\mathbb{R}(T-t-l,\tau-l) + (I-A)G_{T-t}\Omega\left\{\Pi_{j=T-t}^{T-t+\tau-1}L_jA\right\}L_{T-t+\tau}K_{T-t+\tau}\right].$$

The expression above evaluated at a generic period $1 \le t+H \le T$:

$$\mathbb{c}_{t+H} = \sum_{\tau=0}^{t+H-1}[A^\tau(\underline{A} + \Gamma_{T-t-H+\tau}) + \Delta\mathbb{Q}(T-t-H,\tau)]\,\mathbb{r}_{t+H-\tau} + \sum_{\tau=1}^{t+H-1} A^\tau\Delta(I-A)G_{T-t-H+\tau}\Omega\mathbb{c}_{t+H-\tau}.$$

Now suppose the economy is in steady state until the beginning of period t, so that $\mathbb{c}_0 = \mathbb{r}_1 = \ldots = \mathbb{r}_{t-1} = \mathbb{0}_{K\times 1}$, and there is a one-time monetary innovation ε_t^r hitting the economy, which does not repeat in the future; then $r_{t+H} = \rho_r^H\varepsilon_t^r, \quad H \ge 0$, and the impulse response of sectoral consumption H periods after the innovation is given by:

$$\mathbb{c}_{t+H} = \sum_{\tau=0}^{H}\rho_r^{H-\tau}[A^\tau(\underline{A} + \Gamma_{T-t-H+\tau}) + \Delta\mathbb{Q}(T-t-H,\tau)]\,\varepsilon_t^r + \sum_{\tau=1}^{H} A^\tau\Delta(I-A)G_{T-t-H+\tau}\Omega\mathbb{c}_{t+H-\tau}.$$

or when written in scalar form:

$$c_{k,t+H} = \left[\sum_{\tau=0}^{H} \rho_r^{H-\tau} \left\{ \alpha_k^\tau (\underline{\alpha}_k + \gamma_{k,T-t-H+\tau}) + \delta_k q^k(T-t-H,\tau) \right\} \right] \varepsilon_t^r + \sum_{\tau=1}^{H} \delta_k s_{T-t-H+\tau}^k \sum_{r=1}^{K} \omega_{kr} c_{r,t+H-\tau}$$

where $q^k(T-t-H,\tau) \equiv \sum_{j=1}^K [\mathbb{Q}(T-t-H,\tau)]_{kj} \geq 0, \quad s_{T-t-H+\tau}^k \equiv \alpha_k^\tau(1-\alpha_k) g_{T-t-H+\tau}, \quad g_{T-t-H+\tau} \equiv [G_{T-t-H+\tau}]_{kk}$, so that $\frac{\partial c_{k,t+H}}{\partial r_t} = \left[\sum_{\tau=0}^H \rho_r^{H-\tau} \left\{ \alpha_k^\tau(\underline{\alpha}_k + \gamma_{k,T-t-H+\tau}) + \delta_k q^k(T-t-H,\tau) \right\} \right] \geq \left[\sum_{\tau=0}^H \rho_r^{H-\tau} \left\{ \alpha_k^\tau(\underline{\alpha}_k + \gamma_{k,T-t-H+\tau}) \right\} \right], \forall k = 1,2,...,K; 1 \leq t, t+H \leq T-1.$ □

Appendix C. Three-sector economy example ($K = 3$)

Appendix C.1. Model with money supply rule

In this subsection we focus on a three-sector ($K = 3$) version of our economy, with the baseline setup considered in the main text. Its equilibrium conditions are summarized in Appendix A under $\mathcal{A}_{kt} = 1, \varphi = 0, \lambda = 1$ and monetary policy implemented through a money supply rule. We calibrate our model to monthly data by setting $\beta = 0.9975, \rho_r = 0.5, \sigma_r = 0.0025$, and the consumption shares are set to be equal ($\omega_{c1} = \omega_{c2} = \omega_{c3} = 1/3$); for simplicity, we also assume $\delta_1 = \delta_2 = \delta_3 = \delta$. We perform two exercises that consider the response of our system to a monetary contraction. In particular, we subject our system to one-time one-standard deviation negative innovation to the monetary policy shock variable r that translates into a permanent contraction of nominal GDP/money supply.

In our first exercise, Sector 1 is assumed to be almost, but not completely, flexible ($\alpha_1 = 0.1$), Sector 2 is semi-flexible ($\alpha_2 = 0.5$) and Sector 3 is almost completely rigid ($\alpha_3 = 0.9$). The input-output matrix is set to be completely symmetric, so that $\omega_{kr} = 1/3 \quad \forall k, r \in \{1,2,3\}$. Figure C.9 shows the responses of such system to the permanent contraction in money supply for three different values of the intermediate inputs share $\delta \in (0, 0.4, 0.8)$. We can see that in the economy with no intermediate inputs ($\delta = 0$), the magnitude of the reduction in consumption is proportional to own probability of price non-adjustment α_k, which is a standard result in models with no intermediate inputs and represents the *direct effect* in the notation from Proposition 1. As we increase the share of intermediate inputs, the magnitude of reduction in sectoral consumption becomes larger for all sectors, representing the addition of the *downstream effect*. Other things equal, we can see that the *degree of amplification increases in the economy's share of intermediate inputs δ*. This is because higher δ means that the sensitivity of marginal cost to suppliers' prices is higher and hence allowing for stronger "inheritance" of stickiness.

In our second exercise, Sector 1 is completely flexible ($\alpha_1 = 0$) with the other sectors remaining unchanged from the previous exercise ($\alpha_2 = 0.5, \alpha_3 = 0.9$). In addition, firms in Sector 1 are assumed to be buying inputs only from other firms in Sector 1 ($\omega_{11} = 1, \omega_{12} = \omega_{13} = 0$), whereas the input-output weights for the other two sectors are as in the previous exercise ($\omega_{kr} = 1/3 \quad k = 2,3; r = 1,2,3$). Figure C.10 shows the responses to the monetary contraction. As before, for $\delta = 0$, the magnitude of sectoral consumption fall is proportional to the degree of price stickiness, with the fully flexible Sector 1 exhibiting no change in sectoral consumption, as its price index adjusts one-for-one with the money supply. Most importantly, as we increase δ, Sector 2 and Sector 3 experience greater decrease in consumption, whereas Sector 1 still showing no response. The latter highlights the fact that the amplification of consumption response is due to *downstream propagation* of price stickiness, as it travels from suppliers firms to customer firms. Indeed, firms in Sector 1 do not purchase inputs from the sticky Sectors 2 and 3 and only from other fully flexible firms in their own sector. Hence, firms in Sector 1 do not inherit any stickiness from the other sectors and experience no amplification.

Appendix C.2. Model with Taylor rule

Consider an arguably more realistic setting, where we model the monetary authority as setting the level of nominal interest I_t in the economy, as described in Appendix A, under Taylor rule and $\mathcal{A}_{kt} = 1, \varphi = 0, \lambda = 1$. Once again, we focus on a three-sector ($K = 3$) version of our economy and perform two exercises that

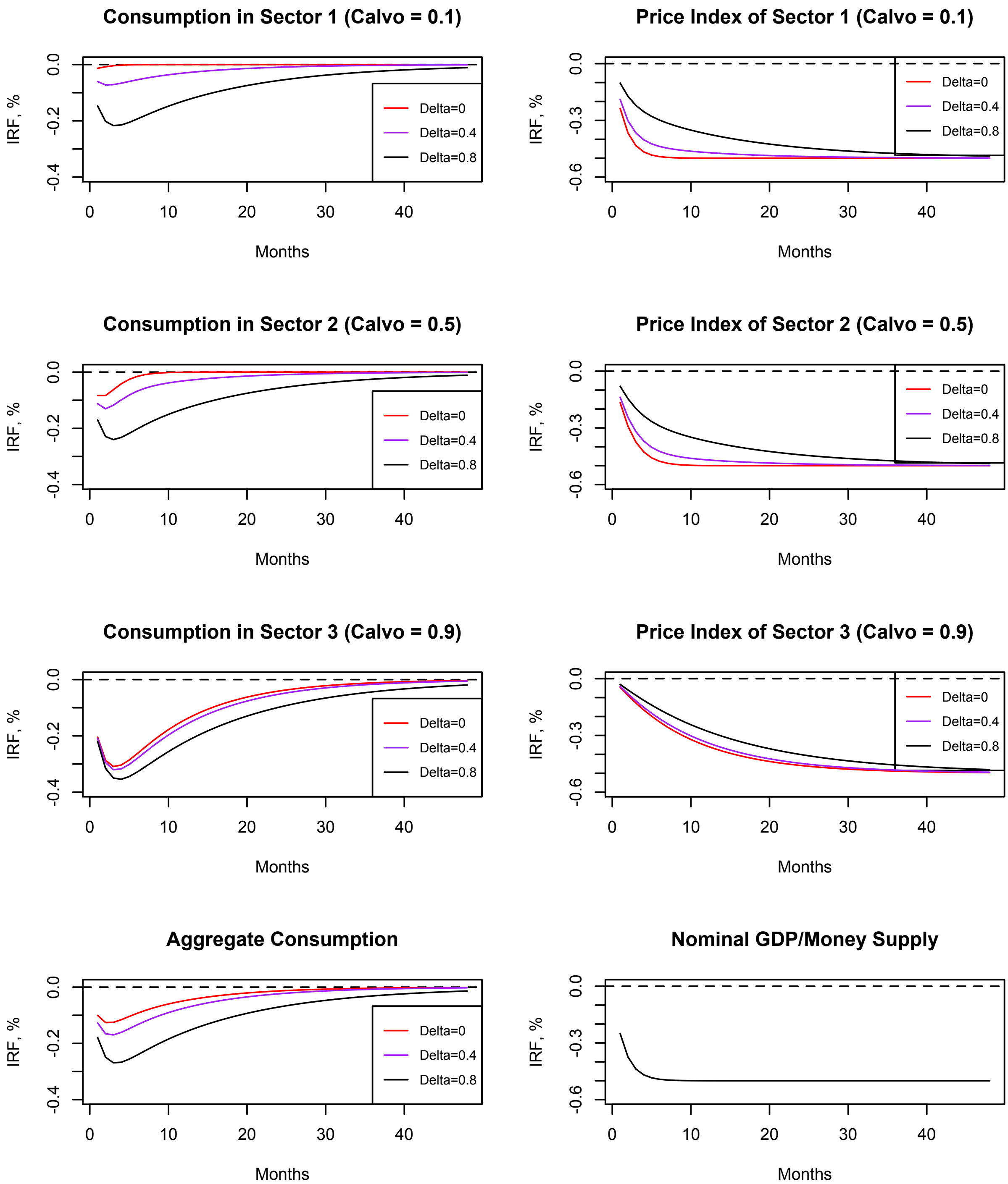


Figure C.9: Responses of sectoral consumptions and price indices to a monetary contraction

Note: *The figure is constructed by calibrating the log-linearized model in Appendix A with money supply rule and considering a one-time one standard deviation negative innovation to the monetary policy shock variable that generates a permanent contraction in money supply. The impulse responses are found numerically using Dynare for the following calibration:* $\lambda = 1, \varphi = 0, \beta = 0.9975, \rho_r = 0.5, \sigma_r = 0.0025, \alpha_1 = 0.1, \alpha_2 = 0.5, \alpha_3 = 0.9, \omega_{kr} = \omega_{cr} = 1/3 \quad \forall k, r \in \{1, 2, 3\}$.

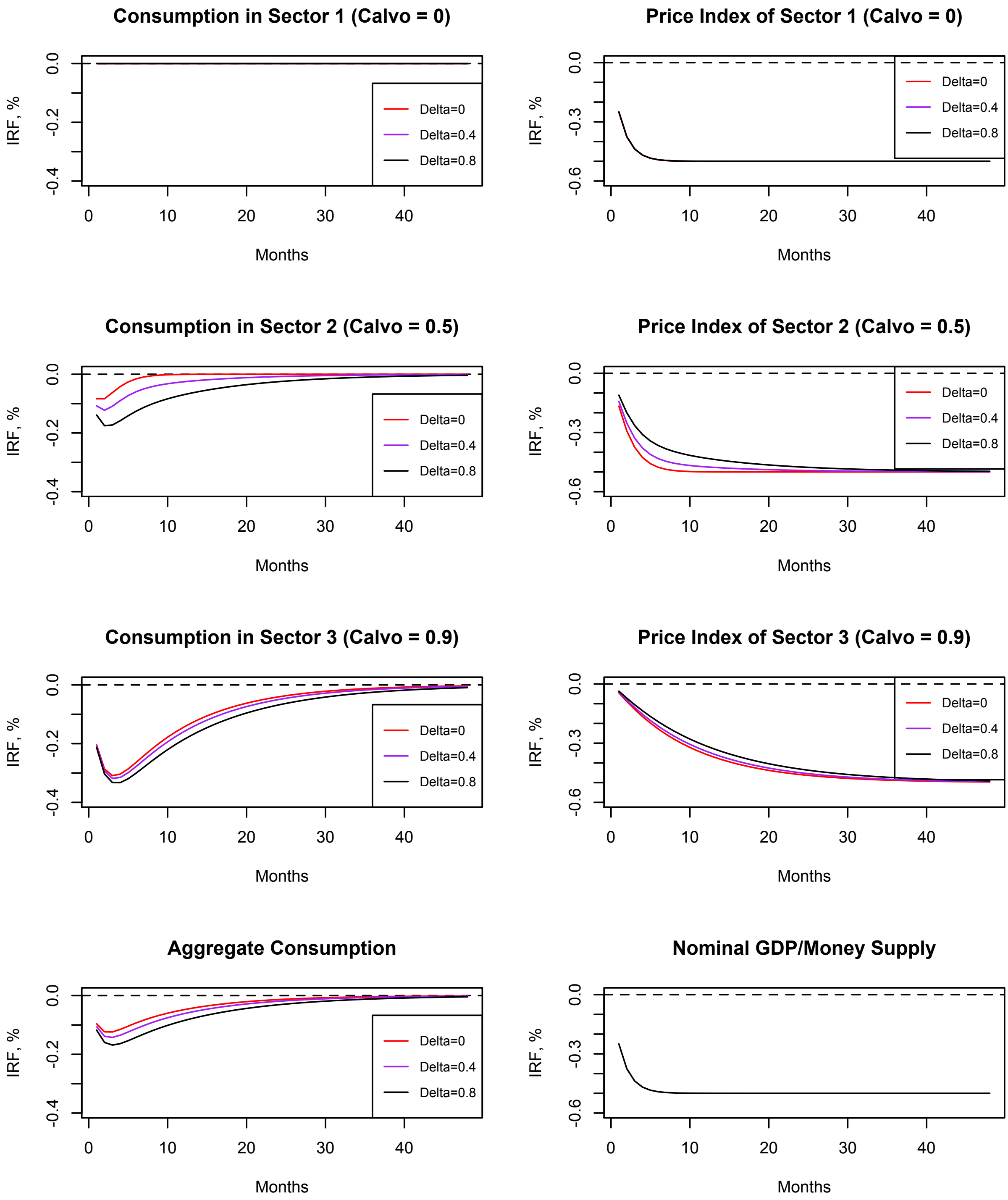


Figure C.10: Responses of sectoral consumptions and price indices to a monetary contraction

Note: *The figure is constructed by calibrating the log-linearized model in Appendix A with money supply rule for* $K = 3$ *and considering a one-time one standard deviation negative innovation to the monetary policy shock variable that generates a permanent contraction in money supply. The impulse responses are found numerically using Dynare for the following calibration:* $\lambda = 1, \varphi = 0, \beta = 0.9975, \rho_r = 0.5, \sigma_r = 0.0025, \alpha_1 = 0, \alpha_2 = 0.5, \alpha_3 = 0.9, \omega_{11} = 1, \omega_{12} = \omega_{13} = 0, \omega_{kr} = \omega_{cr} = 1/3 \quad k = 2, 3; r = 1, 2, 3.$

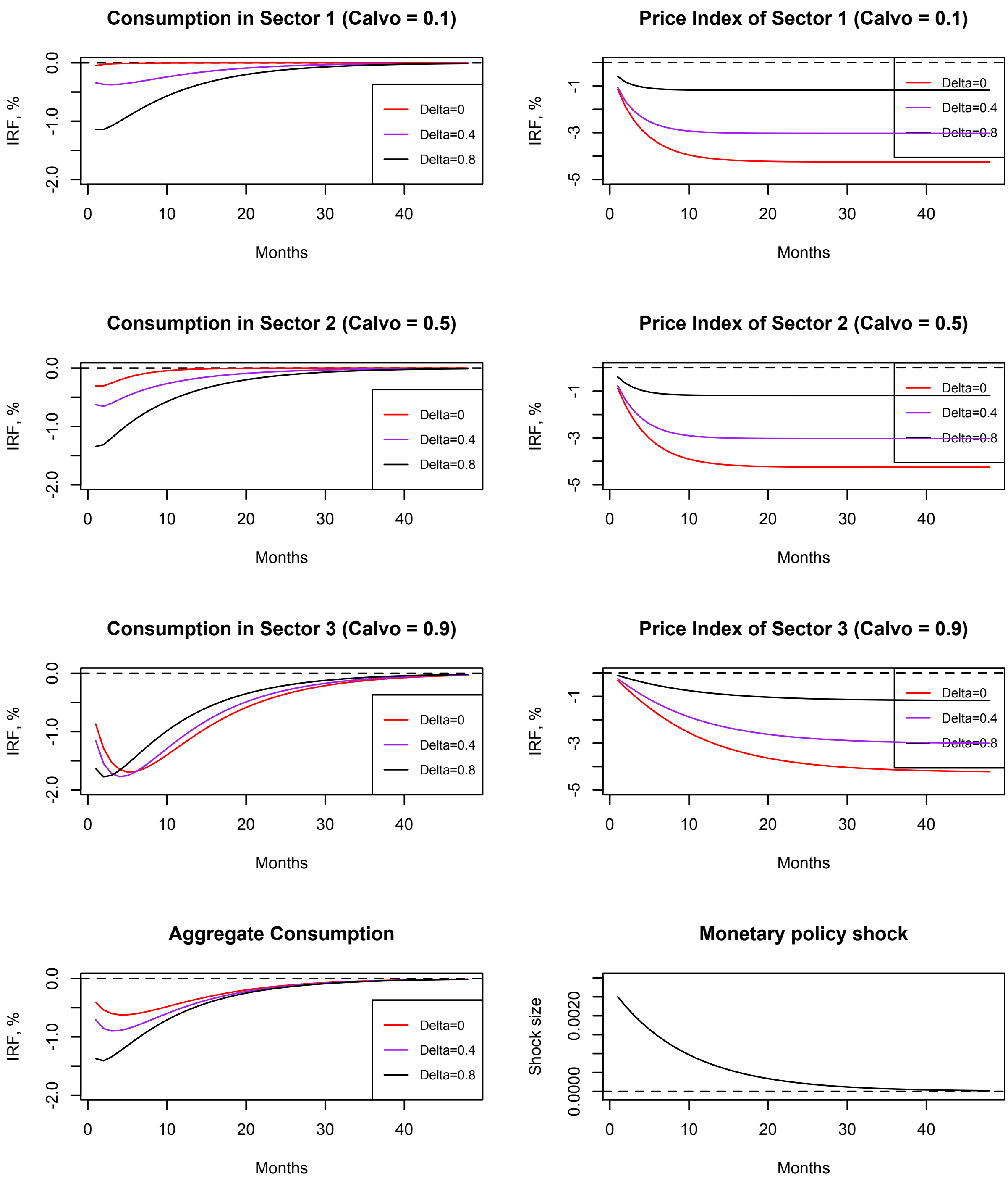


Figure C.11: Responses of sectoral consumptions and prices to a monetary policy shocks (Taylor rule)

Note: *The figure is constructed by calibrating the log-linearized model in Appendix A with Taylor rule for* $K = 3$ *and considering a one-time one standard deviation positive innovation to the monetary policy shock variable that generates a monetary contraction. The impulse responses are found numerically using Dynare for the following calibration:* $\lambda = 1, \varphi = 0, \beta = 0.9975, \rho_i = 0, \rho_\mu = 0.9, \sigma_\mu = 0.0025, \phi_\pi = 1.24, \phi_{gc} = 0, \phi_c = 0.33/12, \alpha_1 = 0.1, \alpha_2 = 0.5, \alpha_3 = 0.9, \omega_{kr} = \omega_{cr} = 1/3 \quad \forall k, r \in \{1, 2, 3\}$.

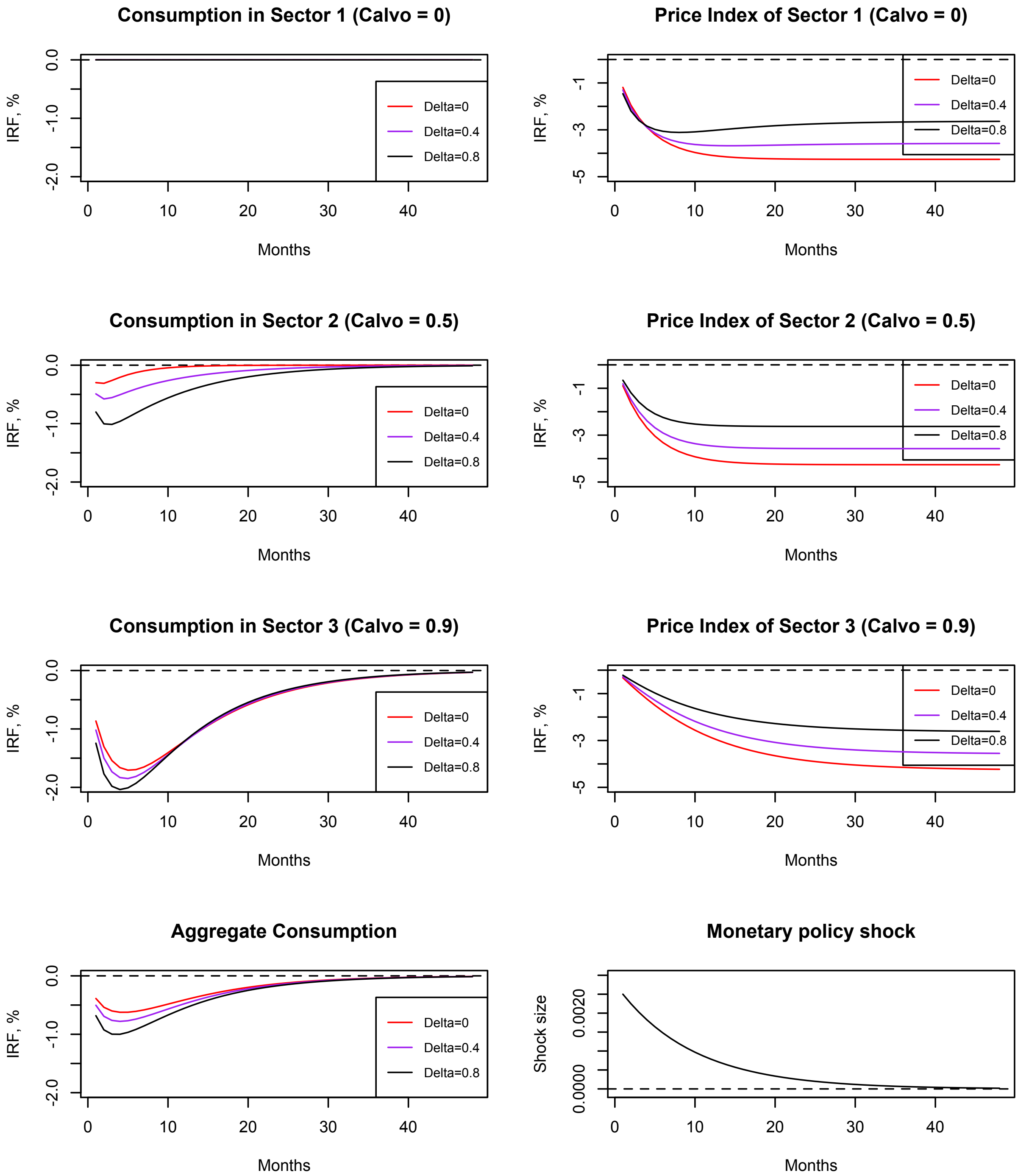


Figure C.12: Responses of sectoral consumptions and prices to a monetary policy shocks (Taylor rule)

Note: *The figure is constructed by calibrating the log-linearized model in Appendix A with Taylor rule for $K = 3$ and considering a one-time one standard deviation positive innovation to the monetary policy shock variable that generates a monetary contraction. The impulse responses are found numerically using Dynare for the following calibration:* $\lambda = 1, \varphi = 0, \beta = 0.9975, \rho_i = 0, \rho_\mu = 0.9, \sigma_\mu = 0.0025, \phi_\pi = 1.24, \phi_{gc} = 0, \phi_c = 0.33/12, \alpha_1 = 0, \alpha_2 = 0.5, \alpha_3 = 0.9, \omega_{11} = 1, \omega_{12} = \omega_{13} = 0, \omega_{kr} = \omega_{cr} = 1/3 \quad k = 2, 3; r = 1, 2, 3.$

consider the response of our system to a monetary contraction. In particular, we subject our system to a one-time one-standard deviation positive innovation to the monetary policy shock ε^{μ} that generates a monetary tightening. We calibrate our model to monthly data by setting $\beta = 0.9975, \rho_i = 0, \rho_\mu = 0.9, \sigma_\mu = 0.0025, \phi_\pi = 1.24, \phi_{gc} = 0$ and ϕ_c=0.33/12, and also additionally assume for simplicity that $\delta_1 = \delta_2 = \delta_3 = \delta$.

Just like in the previous subsection model, in our first exercise Sector 1 is assumed to be almost, but not completely flexible, ($\alpha_1 = 0.1$), Sector 2 is semi-flexible ($\alpha_2 = 0.5$ and Sector 3 is almost completely rigid ($\alpha_3 = 0.9$). The input-output matrix is set to be completely symmetric, so that $\omega_{kr} = 1/3 \quad \forall k, r \in \{1, 2, 3\}$. Figure C.11 shows the responses of our system to the monetary contraction for different values of $\delta \in \{0, 0.4, 0.8\}$. As one can see, the results are qualitatively identical to those obtained under money supply rule in our original model. Firstly, as before, under $\delta = 0$, the magnitude of contraction in sectoral consumption increases in sector's own price stickiness. Secondly, other things equal, the degree of amplification increases in δ.

In our second exercise, we assume that Sector 1 is completely flexible ($\alpha_1 = 0$), with the other sectors remaining unchanged from the previous exercise ($\alpha_2 = 0.5, \alpha_3 = 0.9$). Just as before, firms in Sector 1 buy inputs only from other firms in Sector 1 ($\omega_{11} = 1, \omega_{12} = \omega_{13} = 0$), whereas the input-output weights for the other two sectors are as in the previous exercise ($\omega_{kr} = 1/3, \quad \forall k, r \in \{2, 3\}$). Figure C.12 shows the response of such a system to a monetary contraction. We can see that, just as with the money supply rule in the original model, Sector 1 does not experience any change in consumption under $\delta = 0$, with $\delta > 0$ leaving the zero consumption response in place due to the fact that the propagation of price stickiness still goes downstream. As before, since firms in Sector 1 do not purchase any inputs from firms in sticky sectors 2 and 3, the do not inherit any stickiness.

Appendix D. Reconciling sectoral "consumption puzzles"

In this section we show how a slight modification to our baseline model in the main text can reconcile the "inverted" responses of sectoral consumption to monetary policy shocks, as well as dampening downstream effect contributions, that we documented in Section 6. Our baseline model assumed unit elasticity of substitution across sectors; however, in Appendix A we allow for a general constant elasticity of substitution (CES) across sectors λ, which nests unit elasticity of substitution under $\lambda = 1$.

The log-linearized expression for sectoral consumption under CES aggregation (while keeping the money supply rule and infinite Frisch elasticity $\varphi = 0$) becomes:

$$c_{kt} = m_t - p_{kt} + (\lambda - 1)(p^c_t - p_{kt}). \tag{D.1}$$

Under the benchmark case of $\lambda = 1$ we can see that "inverted" responses in consumption are impossible. This is because following a contraction (fall in m_t), sectoral price falls at most as quickly (happens under full flexibility) as m_t, implying a fall in sectoral consumption (and vice versa for an expansion). However, under $\lambda > 1$, for example, "inverted" sectoral consumption responses are possible. Consider a monetary contraction and let us focus on the most flexible sector in our economy. Following the contraction, the sectoral price index optimally adjusts downward faster than the overall consumption price index of the economy, so that the second term in the sectoral consumption equation above becomes positive and possibly outweighs the negative ($m_t - p_{kt}$) term, hence generating a positive sectoral consumption response. Intuitively, firms in flexible industries lower their prices faster than firms in more rigid industries and "steal" customers from the latter. Naturally, we should observe less of this behavior as the downstream effect becomes stronger as the degree of strategic complementarity in price setting becomes larger, eliminating the relative pricing effects.

To gain further intuition, consider again the three sector ($K = 3$) version of our economy, with calibration just like in the our first exercise with money supply rule in Appendix C, with the exception that we set $\lambda = 3$, so that goods produced in different sectors are now stronger substitutes. Figure D.13 shows impulse responses to a one-time one standard deviation monetary policy shock that generates a permanent contraction in nominal GDP/money supply.

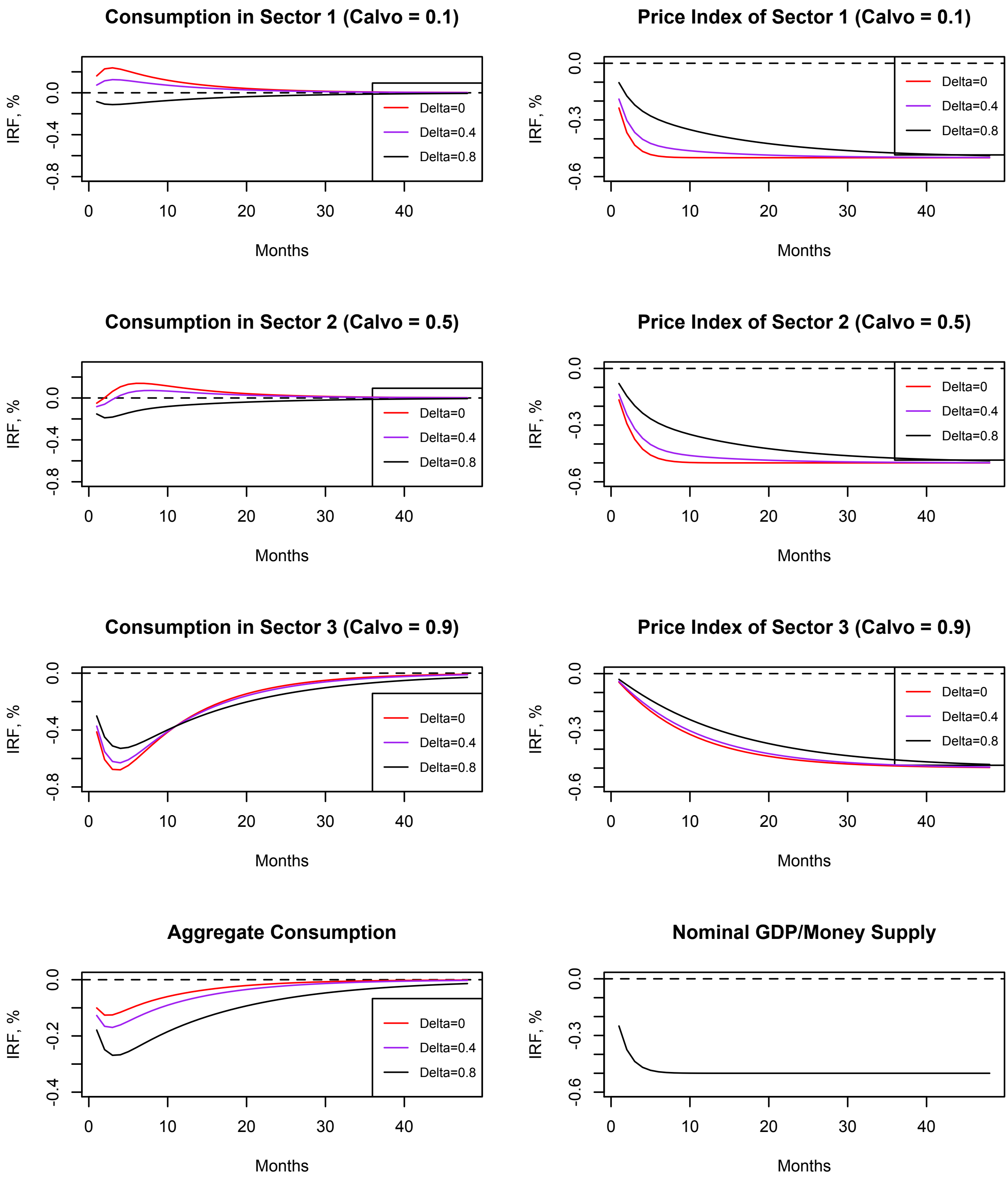


Figure D.13: Responses of sectoral consumptions and price indices to a monetary contraction

Note: *The figure is constructed by calibrating the log-linearized model in Appendix A with money supply rule for* $K = 3$ *and considering a one-time one standard deviation negative innovation to the monetary policy shock variable that generates a permanent contraction in money supply. The impulse responses are found numerically using Dynare for the following calibration:* $\lambda = 3, \varphi = 0, \beta = 0.9975, \rho_r = 0.5, \sigma_r = 0.0025, \alpha_1 = 0, \alpha_2 = 0.5, \alpha_3 = 0.9, \omega_{11} = 1, \omega_{12} = \omega_{13} = 0, \omega_{kr} = \omega_{cr} = 1/3 \quad k = 2, 3; r = 1, 2, 3.$

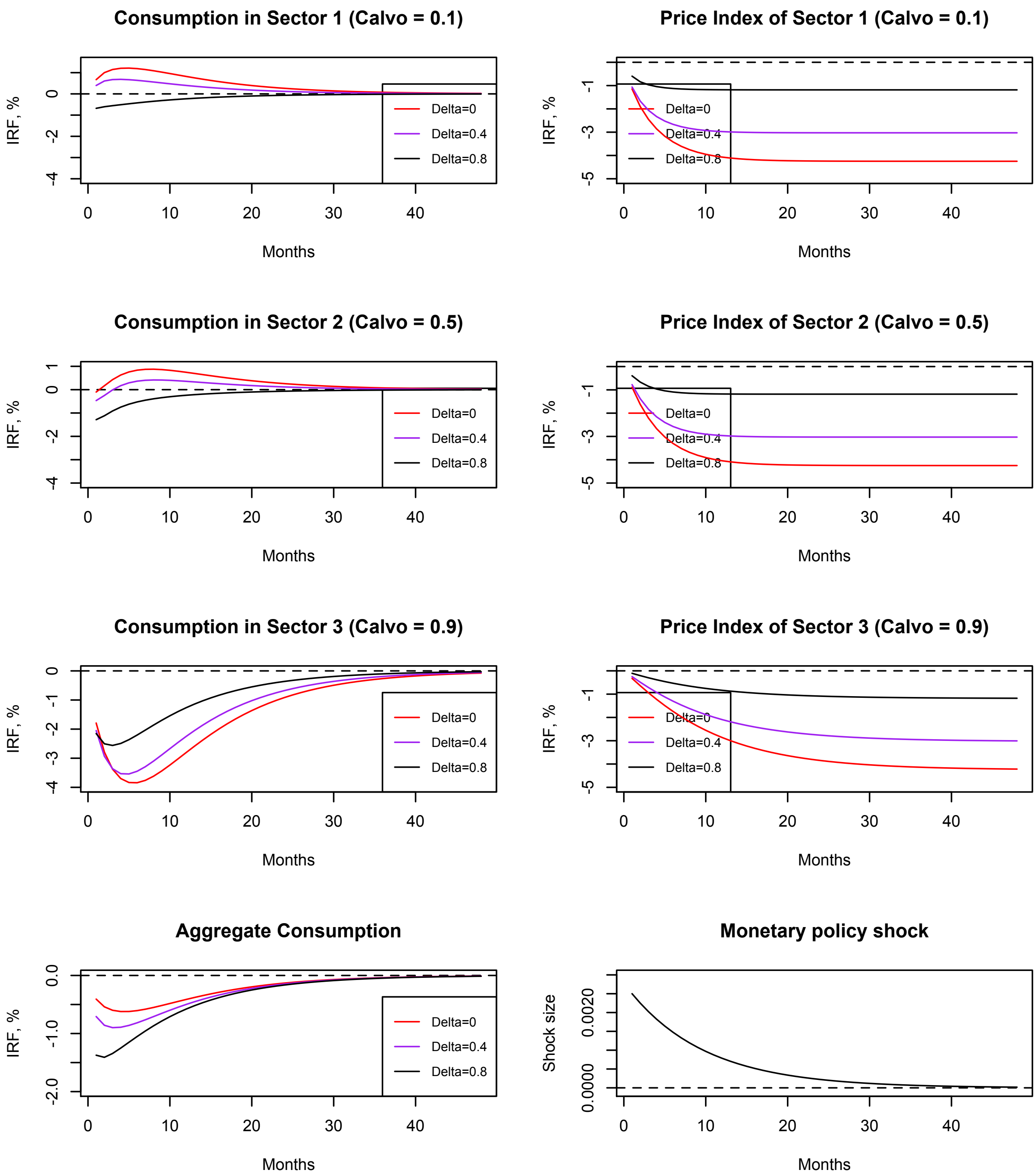


Figure D.14: Responses of sectoral consumptions and prices to a monetary policy shocks (Taylor rule)

Note: *The figure is constructed by calibrating the log-linearized model in Appendix A with Taylor rule for $K = 3$ and considering a one-time one standard deviation positive innovation to the monetary policy shock variable that generates a monetary contraction. The impulse responses are found numerically using Dynare for the following calibration:* $\lambda = 3, \varphi = 0, \beta = 0.9975, \rho_i = 0, \rho_\mu = 0.9, \sigma_\mu = 0.0025, \phi_\pi = 1.24, \phi_{gc} = 0, \phi_c = 0.33/12, \alpha_1 = 0.1, \alpha_2 = 0.5, \alpha_3 = 0.9, \omega_{kr} = \omega_{cr} = 1/3 \quad \forall k, r \in \{1, 2, 3\}$.

We can see that in the case with no intermediate inputs ($\delta = 0$) and hence no strategic complementarities in price setting, the most flexible Sector 1 shows an "inverted" response in consumption over the entire horizon, with Sector 2 also having an "inverted" response after 5 months. One can see that this is because they lower their prices much faster than firms in the rigid Sector 3, thus stealing some of the customers of firms in Sector 3. As we increase δ, the extent of "inverted" consumption responses becomes lower, with $\delta = 0.8$ generating no positive consumption responses. This is because higher share of intermediate inputs makes the degree of complementarity in price setting higher, as firms' marginal costs are more sensitive to their suppliers' prices, making the timing of price changing more uniform across sectors. Note that the move from $\delta = 0$ to $\delta = 0.4$ corresponds to a dampening contribution of downstream effect in framework we documented in Section 6.

Interestingly, although higher δ makes the consumption response more negative for the more flexible Sectors 1 and 2, the magnitude of peak drop in consumption for the rigid Sector 3 falls for higher shares for intermediate inputs. Hence, for Sector 3 the downstream effect does not lead to amplification, but rather to a contraction of the consumption response. Again, the latter corresponds to negative contribution of the downstream effect in our earlier notation.

We would also like to test whether such property would also be preserved under the Taylor rule specification for monetary policy. In Figure D.14 we consider a monetary tightening for calibration identical to the first exercise with a Taylor rule in Appendix C, with the exception that we set $\lambda = 3$. As we can see, for $\delta = 0$ and $\delta = 0.4$ the more flexible sectors 1 and 2 indeed exhibit "inverted" consumption responses, whereas for $\delta = 0.8$ all "inverted" responses are eliminated. Hence, in this respect, our model with Taylor rule behaves just like the original one with money supply rule. In addition, just as in the original model. we can observe that the magnitude of peak drop in consumption for the most rigid Sector 3 falls for higher values of δ.

Appendix E. List of sectors used

The definition of sectors here follows that used by the US Bureau of Economic Analysis (BEA) in their Input-Output accounts. The figures in parentheses are shares of corresponding real final sectoral consumption in aggregate real personal consumption expenditure (expressed in per cent). The consumption shares are constructed using the 2007 Use Tables for Detail and Summary accounts.

Appendix E.1. Detail level (161 sectors)

1. Abrasive product manufacturing (0.0019)
2. Accommodation (1.1866)
3. Accounting, tax preparation, bookkeeping, and payroll services (0.1914)
4. Air transportation (1.0285)
5. All other food and drinking places (0.8536)
6. All other food manufacturing (0.1801)
7. All other miscellaneous manufacturing (0.1879)
8. All other wood product manufacturing (0.0449)
9. Animal (except poultry) slaughtering, rendering, and processing (0.6849)
10. Animal production, except cattle and poultry and eggs (0.0438)
11. Audio and video equipment manufacturing (0.4104)
12. Automobile manufacturing (0.8281)
13. Automotive equipment rental and leasing (0.6185)
14. Automotive repair and maintenance (1.9310)
15. Beef cattle ranching and farming, including feedlots and dual-purpose ranching and farming (0.0013)
16. Book publishers (0.2493)
17. Bread and bakery product manufacturing (0.4550)
18. Breweries (0.2989)
19. Broadcast and wireless communications equipment (0.0658)

20. Business support services (0.0540)
21. Carpet and rug mills (0.0966)
22. Cheese manufacturing (0.1452)
23. Child day care services (0.5177)
24. Civic, social, professional, and similar organizations (0.4754)
25. Clay product and refractory manufacturing (0.0281)
26. Coal mining (0.0002)
27. Coffee and tea manufacturing (0.0562)
28. Commercial and industrial machinery and equipment rental and leasing (0.0046)
29. Community food, housing, and other relief services, including rehabilitation services (0.4243)
30. Computer terminals and other computer peripheral equipment manufacturing (0.1554)
31. Consumer goods and general rental centers (0.3190)
32. Couriers and messengers (0.0208)
33. Curtain and linen mills (0.1412)
34. Cut stone and stone product manufacturing (0.0446)
35. Cutlery and handtool manufacturing (0.0635)
36. Dog and cat food manufacturing (0.1561)
37. Doll, toy, and game manufacturing (0.2437)
38. Dry-cleaning and laundry services (0.1608)
39. Electric lamp bulb and part manufacturing (0.0156)
40. Electric power generation, transmission, and distribution (1.8453)
41. Electromedical and electrotherapeutic apparatus manufacturing (0.0259)
42. Electronic and precision equipment repair and maintenance (0.0502)
43. Electronic computer manufacturing (0.2082)
44. Elementary and secondary schools (0.4815)
45. Employment services (0.0188)
46. Fabric mills (0.0181)
47. Fats and oils refining and blending (0.0486)
48. Fiber, yarn, and thread mills (0.0013)
49. Fishing, hunting and trapping (0.0654)
50. Flour milling and malt manufacturing (0.0364)
51. Fluid milk and butter manufacturing (0.2456)
52. Frozen food manufacturing (0.2550)
53. Fruit and tree nut farming (0.1780)
54. Full-service restaurants (2.2680)
55. Funds, trusts, and other financial vehicles (1.5434)
56. Gambling industries (except casino hotels) (1.0675)
57. Glass and glass product manufacturing (0.0376)
58. Grain farming (0.0136)
59. Grantmaking, giving, and social advocacy organizations (0.4974)
60. Greenhouse, nursery, and floriculture production (0.1574)
61. Hardware manufacturing (0.0055)
62. Home health care services (0.8074)
63. Hospitals (8.0865)
64. Housing (18.401)
65. Independent artists, writers, and performers (0.0012)
66. Individual and family services (0.7272)
67. Industrial gas manufacturing (0.0008)
68. Insurance carriers (3.5857)
69. Investigation and security services (0.0955)
70. Iron and steel mills and ferroalloy manufacturing (0.0004)
71. Jewelry and silverware manufacturing (0.3222)

72. Junior colleges, colleges, universities, and professional schools (2.0402)
73. Lawn and garden equipment manufacturing (0.0101)
74. Leather and allied product manufacturing (0.4185)
75. Legal services (1.1241)
76. Light truck and utility vehicle manufacturing (1.4458)
77. Limited-service restaurants (2.9901)
78. Manufacturing and reproducing magnetic and optical media (0.0095)
79. Medical and diagnostic laboratories (0.3585)
80. Monetary authorities and depository credit intermediation (2.1384)
81. Motion picture and video industries (0.2758)
82. Motor home manufacturing (0.0632)
83. Motor vehicle body manufacturing (0.0006)
84. Motorcycle, bicycle, and parts manufacturing (0.1007)
85. Museums, historical sites, zoos, and parks (0.1523)
86. Natural gas distribution (0.7686)
87. News syndicates, libraries, archives and all other information services (0.0418)
88. Newspaper publishers (0.1222)
89. Nondepository credit intermediation and related activities (0.4368)
90. Nonferrous metal (except copper and aluminum) rolling, drawing, extruding and alloying (0.0003)
91. Nonferrous metal foundries (0.0024)
92. Nursing and community care facilities (1.6924)
93. Office machinery manufacturing (0.0038)
94. Office supplies (except paper) manufacturing (0.0319)
95. Offices of dentists (1.2069)
96. Offices of other health practitioners (0.7623)
97. Offices of physicians (4.3123)
98. Ophthalmic goods manufacturing (0.0916)
99. Optical instrument and lens manufacturing (0.0002)
100. Other concrete product manufacturing (0.0003)
101. Other crop farming (0.0033)
102. Other educational services (0.6037)
103. Other fabricated metal manufacturing (0.0268)
104. Other financial investment activities (1.3466)
105. Other furniture related product manufacturing (0.1166)
106. Other nonmetallic mineral mining and quarrying (0.0012)
107. Other personal services (0.6149)
108. Other petroleum and coal products manufacturing (0.0615)
109. Other retail (7.2663)
110. Other textile product mills (0.0496)
111. Outpatient care centers (0.9844)
112. Performing arts companies (0.2462)
113. Personal and household goods repair and maintenance (0.2106)
114. Personal care services (0.7316)
115. Petroleum refineries (2.6296)
116. Photographic services (0.0906)
117. Postal service (0.1233)
118. Poultry and egg production (0.0521)
119. Poultry processing (0.3536)
120. Power-driven handtool manufacturing (0.0252)
121. Printing (0.0307)
122. Promoters of performing arts and sports and agents for public figures (0.1101)
123. Radio and television broadcasting (0.0564)

124. Rail transportation (0.0813)
125. Religious organizations (0.9615)
126. Residential mental retardation, mental health, substance abuse and other facilities (0.4264)
127. Satellite, telecommunications resellers, and all other telecommunications (0.1364)
128. Scenic and sightseeing transportation and support activities for transportation (0.1168)
129. Scientific research and development services (0.0576)
130. Search, detection, and navigation instruments manufacturing (0.0004)
131. Securities and commodity contracts intermediation and brokerage (0.5098)
132. Services to buildings and dwellings (0.1895)
133. Small electrical appliance manufacturing (0.1161)
134. Software publishers (0.3259)
135. Sound recording industries (0.1035)
136. Soybean and other oilseed processing (0.0092)
137. Spectator sports (0.1946)
138. Sporting and athletic goods manufacturing (0.1699)
139. Steel product manufacturing from purchased steel (0.0057)
140. Storage battery manufacturing (0.0241)
141. Support activities for agriculture and forestry (0.0020)
142. Surgical and medical instrument manufacturing (0.0023)
143. Surgical appliance and supplies manufacturing (0.0983)
144. Telephone apparatus manufacturing (0.0078)
145. Textile and fabric finishing and fabric coating mills (0.0048)
146. Tire manufacturing (0.1232)
147. Tobacco product manufacturing (0.5646)
148. Transit and ground passenger transportation (0.3535)
149. Travel arrangement and reservation services (0.1784)
150. Truck transportation (1.0641)
151. Upholstered household furniture manufacturing (0.1503)
152. Vegetable and melon farming (0.2041)
153. Vending, commercial laundry, and other commercial and service industry machinery manufacturing (0.0002)
154. Waste management and remediation services (0.1925)
155. Watch, clock, and other measuring and controlling device manufacturing (0.0583)
156. Water transportation (0.1871)
157. Water, sewage and other systems (0.3766)
158. Wet corn milling (0.0061)
159. Wineries (0.1746)
160. Wired telecommunications carriers (1.9818)
161. Wireless telecommunications carriers (except satellite) (1.0132)

Appendix E.2. Summary level (53 sectors)

1. Accommodation (1.1568)
2. Administrative and support services (0.5408)
3. Air transportation (1.3167)
4. Ambulatory health care services (8.182)
5. Amusements, gambling, and recreation industries (1.5836)
6. Apparel and leather and allied products (2.7165)
7. Broadcasting and telecommunications (2.9244)
8. Chemical products (2.0157)
9. Computer and electronic products (0.8209)
10. Data processing, internet publishing, and other information services (0.0674)

11. Educational services (2.734)
12. Electrical equipment, appliances, and components (0.4607)
13. Fabricated metal products (0.1543)
14. Farms (0.7729)
15. Federal government enterprises (0.1824)
16. Federal Reserve banks, credit intermediation, and related activities (2.5918)
17. Food and beverage and tobacco products (6.6865)
18. Forestry, fishing, and related activities (0.1002)
19. Furniture and related products (0.5656)
20. Hospitals (7.257)
21. Housing (18.038)
22. Insurance carriers and related activities (3.6451)
23. Legal services (1.1564)
24. Machinery (0.129)
25. Mining, except oil and gas (0.008)
26. Miscellaneous manufacturing (1.1988)
27. Miscellaneous professional, scientific, and technical services (0.5387)
28. Motion picture and sound recording industries (0.4514)
29. Motor vehicles, bodies and trailers, and parts (2.8586)
30. Nonmetallic mineral products (0.0957)
31. Nursing and residential care facilities (1.9923)
32. Other retail (7.4421)
33. Other services, except government (6.3469)
34. Other transportation and support activities (0.0773)
35. Other transportation equipment (0.2029)
36. Paper products (0.2591)
37. Performing arts, spectator sports, museums, and related activities (0.5057)
38. Petroleum and coal products (1.3304)
39. Plastics and rubber products (0.327)
40. Primary metals (0.0145)
41. Printing and related support activities (0.065)
42. Publishing industries, except internet (includes software) (1.0897)
43. Rail transportation (0.075)
44. Rental and leasing services and lessors of intangible assets (1.3618)
45. Securities, commodity contracts, and investments (1.5601)
46. Social assistance (1.3046)
47. Textile mills and textile product mills (0.3438)
48. Transit and ground passenger transportation (0.3278)
49. Truck transportation (0.9186)
50. Utilities (3.167)
51. Waste management and remediation services (0.1879)
52. Water transportation (0.1175)
53. Wood products (0.0333)

Appendix F. Input-output matrix construction

The "$Make$" tables show how much of a given product is produced by a particular sector, measured in nominal (US dollar) terms. The "Use" tables show how much a given product is used as an intermediate input by a given sector (as well as its use by final users), again measured in nominal (US dollar) terms. Assume that products in the "$Make$" and "Use" tables are indexed by $q \in \{1, 2, ..., Q\}$. To calculate the $P_{rt}Z_{kt}(r)$ component, or the nominal value of inputs sold by sector r to sector k, we first work out the market share of sector r in the production of product q, given by $Share_{r,q}$, using the "$Make$" table:

$$Share_{r,q} = \frac{Make_{r,q}}{\sum_{r'=1}^{K} Make_{r',q}}, \tag{F.1}$$

where $Make_{r,q}$ denotes the nominal value of sector r's production of product q. Later, we take the nominal expenditure of sector k on every product q as an intermediate input, given by $Use_{k,q}$, and find out how much of this expenditure goes to sector r by multiplying it by r's respective market share. The latter gives us the empirical counterpart of $P_{rt}Z_{kt}(r)$, or total nominal sales of intermediate inputs from sector r to sector k:

$$Sales_{r,k} = \sum_{q=1}^{Q} Share_{r,q} Use_{k,q}. \tag{F.2}$$

Finally, the empirical counterpart of $P_t^k Z_{kt}$, or the total expenditure of sector k on intermediate inputs, is given by $\sum_{r'=1}^{K} Sales_{r,k}$ and hence ω_{kr} can be evaluated as:

$$\omega_{kr} = \frac{Sales_{r,k}}{\sum_{r'=1}^{K} Sales_{r',k}}, \qquad \forall k, r. \tag{F.3}$$

Note that the number of sectors considered in "$Make$" and "Use" tables is generally larger than that available in PCE Bridge Tables and can be matched to sectoral consumption in NIPA Tables. This is because PCE Bridge Tables exclude sectors that either don't sell anything to final consumers or those whose final consumption only accounts for a very small proportion of total output. For example, at Detail level, "$Make$" and "Use" tables consider 389 sectors, whereas PCE Bridge Tables only include data on 257 of them. After organizing our input-output weights into an input-output matrix Ω, so that $\omega_{kr} = [\Omega]_{k,r}$, we delete rows and columns for sectors that are not represented in PCE Bridge Tables and therefore don't sell a large enough proportion of their output to final consumers. In order to satisfy our theoretical requirement that the input-output weights add up to one for any sector, we normalize the rows of the reduced input-output matrix.

In addition we also use the "Use" tables to construct the consumption weights $\{\omega_{ck}\}_{k=1}^{K}$ and intermediates intensities $\{\delta_k\}_{k=1}^{K}$. Recall that in equilibrium of our baseline model those are given by (aggregating across all firms within a sector):

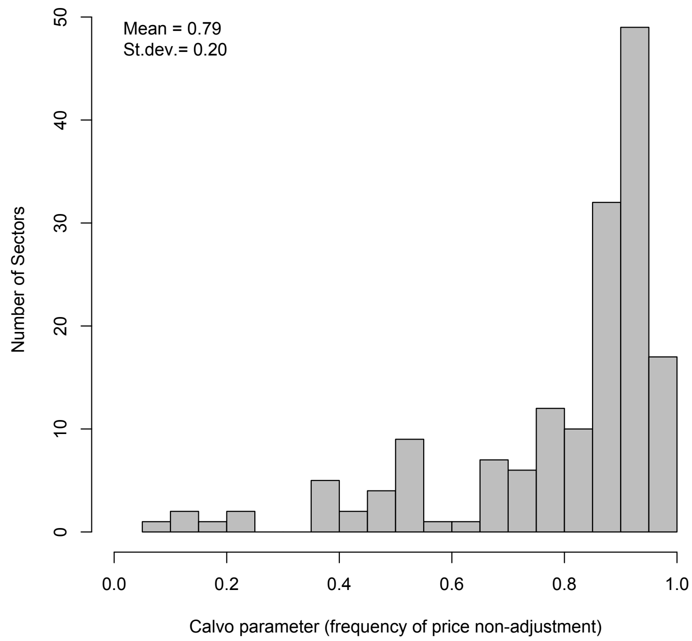
$$\omega_{ck} = \frac{P_{kt}C_{kt}}{P_t^c C_t}, \quad \forall k \qquad\qquad \frac{\delta_k}{1-\delta_k} = \frac{P_t^k Z_{kt}}{W_{kt}N_{kt}}, \quad \forall k \tag{F.4}$$

The "Use" tables contain information on Private Consumption Expenditure (PCE) for every sector that year (the empirical equivalent of $P_{kt}C_{kt}$), aggregate nominal Private Consumption Expenditure (PCE) that year (the empirical equivalent of $P_t^c C_t$), total expenditure of intermediates by every sector (the empirical equivalent of $P_t^k Z_{kt}$) as well as total compensation of employees by every sector (the empirical equivalent of $W_{kt}N_{kt}$).

At Detail level, "$Make$" and "Use" tables are only available for the year 2007, and so we construct both input-output and consumption weights based on the 2007 tables. At Summary level, those tables are available every year between 1997 and 2015; we have estimated input-output and consumption weights for every year those tables are available. However, upon inspection, the constructed weights show very little variation over time. As a result, for Summary level we have decided to use weights for the year 1997.

Appendix G. Sectoral Calvo parameters and intermediates intensities

(a) Histogram of sectoral Calvo parameters



(b) Histogram of sectoral intermediates intensities

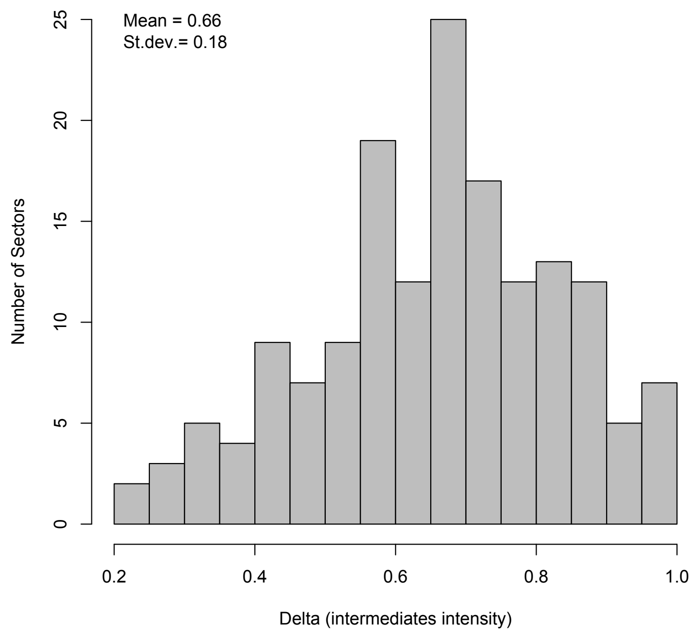


Figure G.15: Final consumption response to 25bp tightening: full and direct effects (Detail level, 161 sectors)

***Note**: panel (a) shows a histogram of sectoral Calvo parameters, given as one minus sectoral frequencies of price adjustment constructed by Pasten et al. (2020); panel (b) shows a histogram of sectoral intermediates intensities estimated using 2007 BEA "Use" table.*

Appendix H. Monetary policy shocks

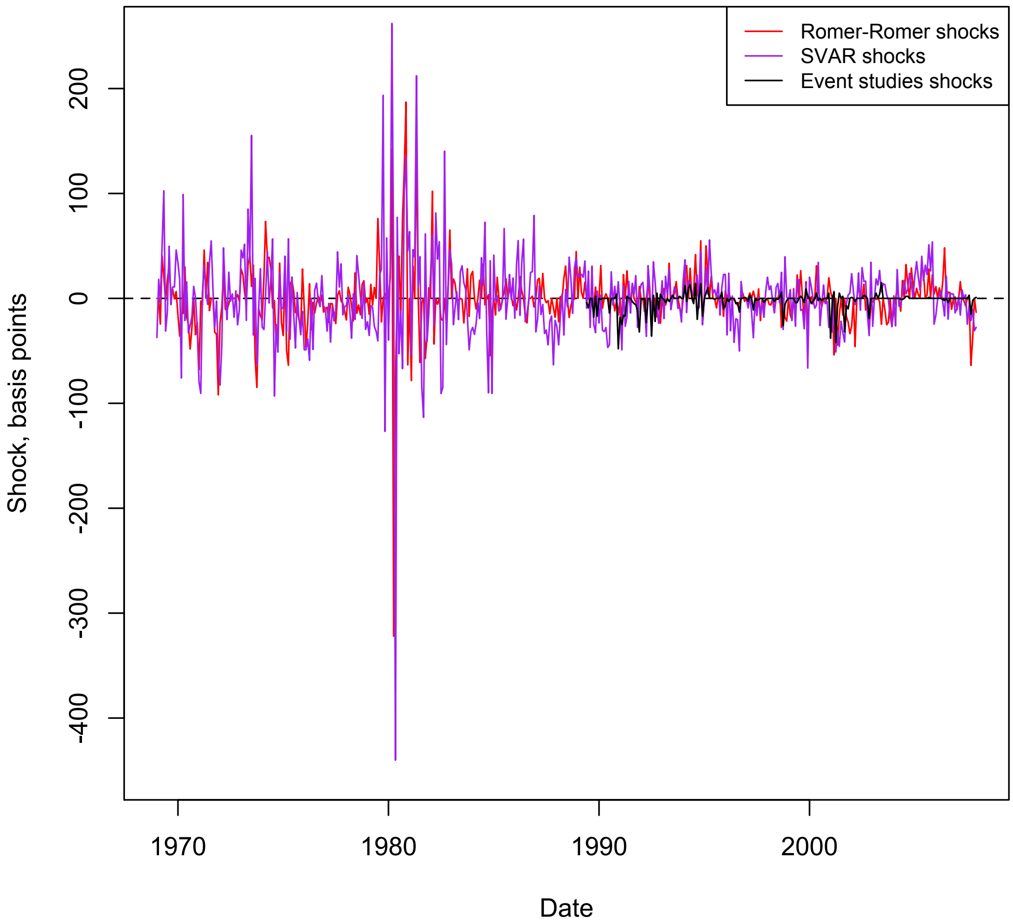


Figure H.16: Monetary policy shocks series (1969:1-2007:12)

	RR	SVAR	ES
RR	1	0.34	0.19
SVAR	0.34	1	0.29
ES	0.19	0.29	1

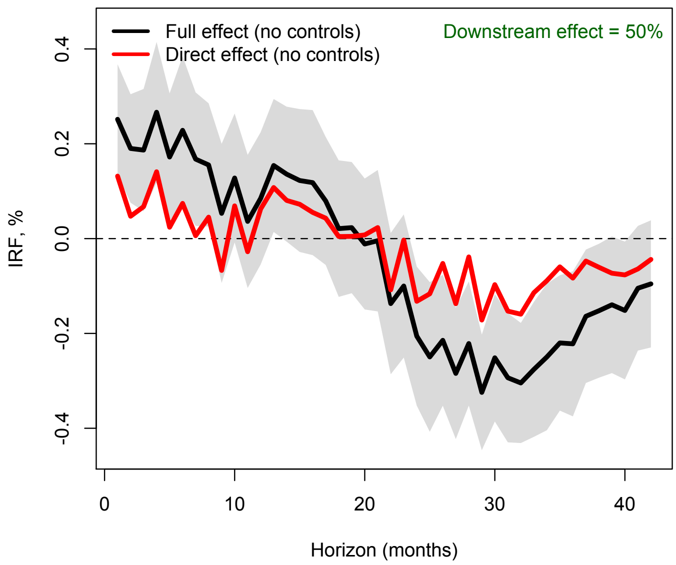
Table H.1: Correlation matrix of shocks

	RR	SVAR	ES
Mean (bp)	-0.68	0.02	-2.27
St. Dev. (bp)	29.8	43.5	8.48

Table H.2: Summary statistics of shocks

Appendix I. Non-smoothed IRFs reported in the main text

(a) Full and direct effects after a 25bp tightening (with controls)



(b) Full and direct effects after 25bp tightening (with controls)

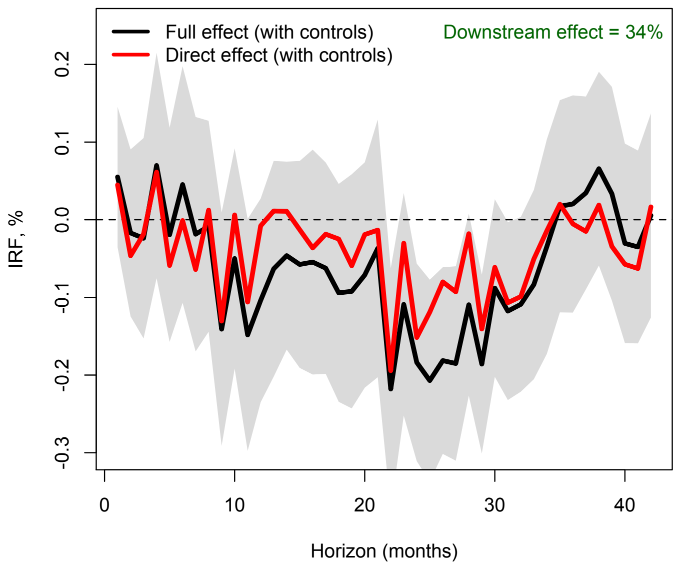


Figure I.17: Final consumption response to 25bp tightening: full and direct effects (Detail level, 161 sectors)

Note*: panel (a) shows results of panel fixed effects estimation of (20) and (21), without adding controls, reported as IRFs following a 25bp monetary tightening; panel (b) reports results of panel fixed effects estimation of (20) and (21), with a vector of controls* $\mathbb{X}_t^H = \left\{ \{r_{t+\tau}\}_{\tau=1}^H, \{\hat{c}_{k,t-s}\}_{s=1}^1, \{ffr_{t-q}\}_{q=1}^4 \right\}$ *added, reported as IRFs following a 25bp monetary tightening. Shaded areas denote 90% confidence bands around the full effect. The downstream effect contribution is constructed as the difference between cumulative full and direct effects as a percentage of the cumulative full effect (all cumulation is over 42 months).*

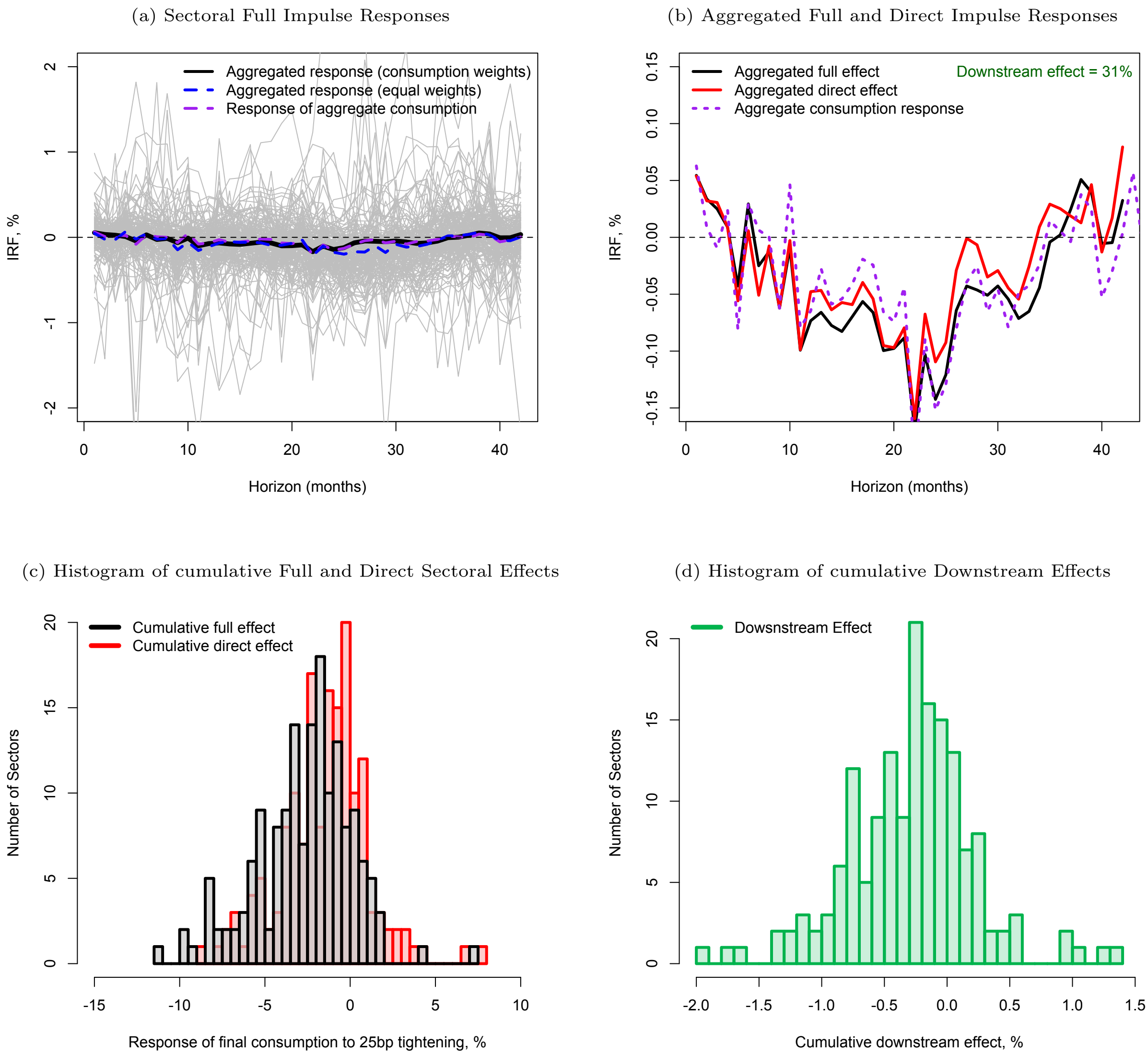


Figure I.18: Sector-level consumption responses to 25bp tightening (Detail level, 161 sectors)

***Note**: panel (a) shows results of sector-by-sector OLS estimation of (20) with controls, reported as sector-level IRF following a 25bp monetary tightening; panel (b) reports results of aggregation of sector-level full and direct effects using empirical final consumption shares $\{\omega_{ck}\}_{k=1}^{161}$; panels (c) and (d) report histograms of cumulative (over 42 months) sectoral full and direct effects, as well as the corresponding sectoral cumulative downstream effects.*

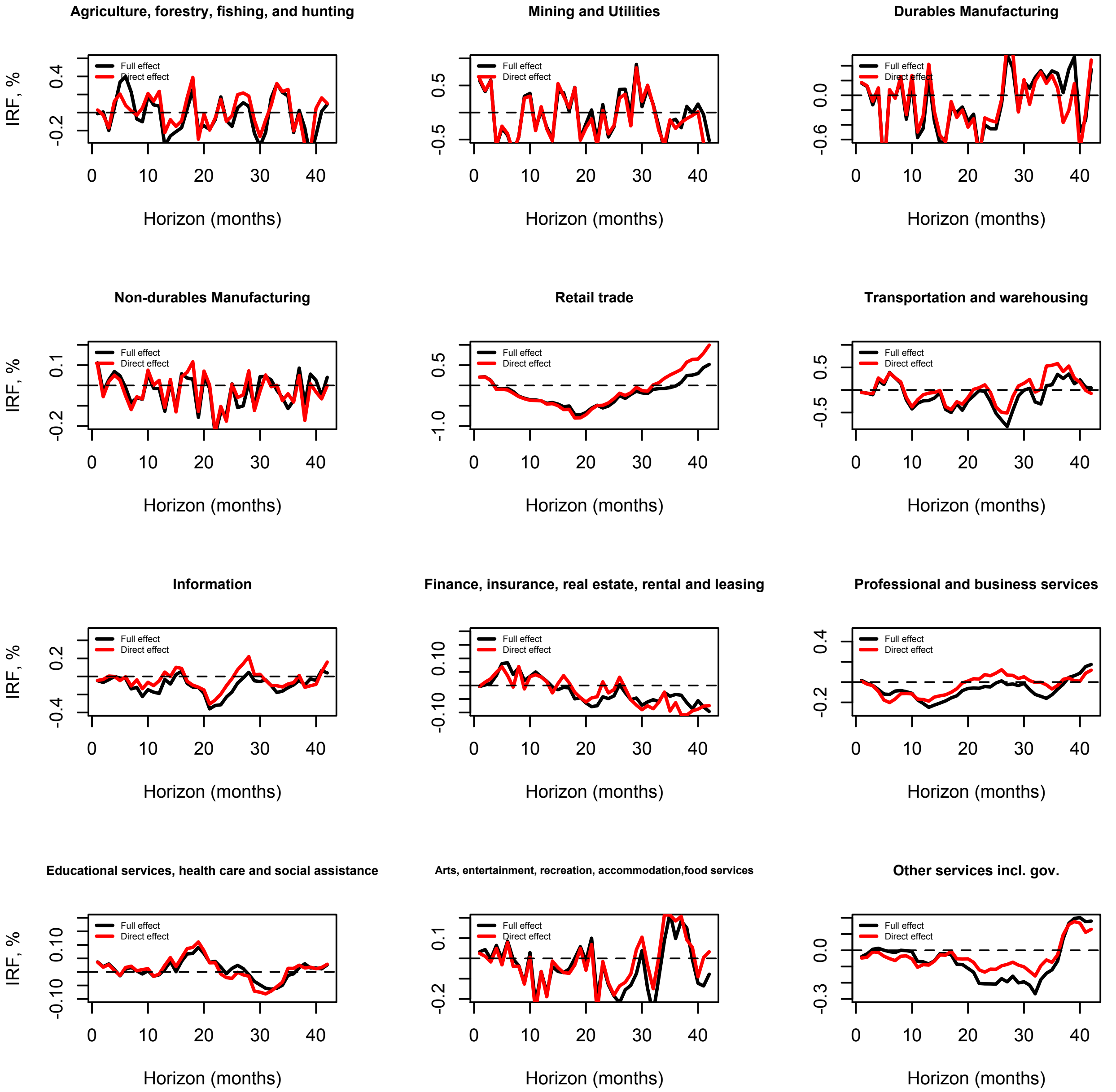
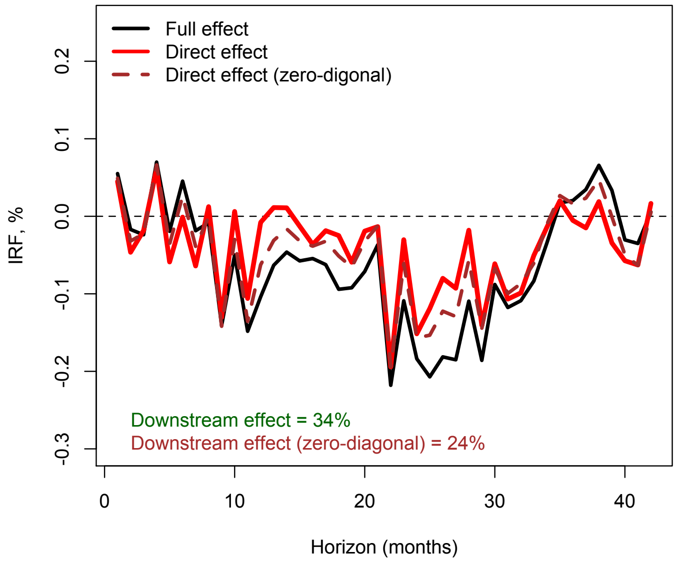


Figure I.19: Consumption responses to a 25bp tightening across broad sectoral groups

Note: *Figure I.19 shows results of sector-by-sector OLS estimation of (20) and (21) with controls, reported as sector-level direct and full IRFs following a 25bp monetary tightening that have been aggregated into broad sectoral groups using empirical consumption shares* $\{\omega_{ck}\}_{k=1}^{161}$.

(a) Aggregated full and direct effects after a 25bp tightening: panel estimation



(b) Full and direct effects after a 25bp tightening 25bp tightening: sector-by-sector estimation

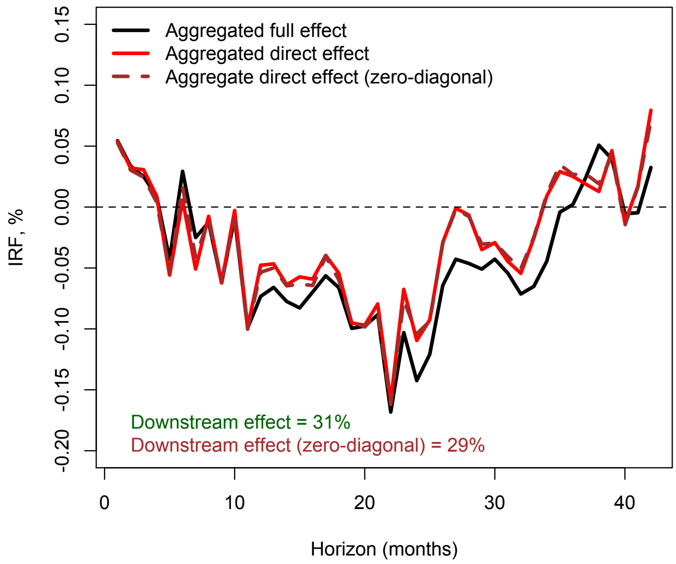
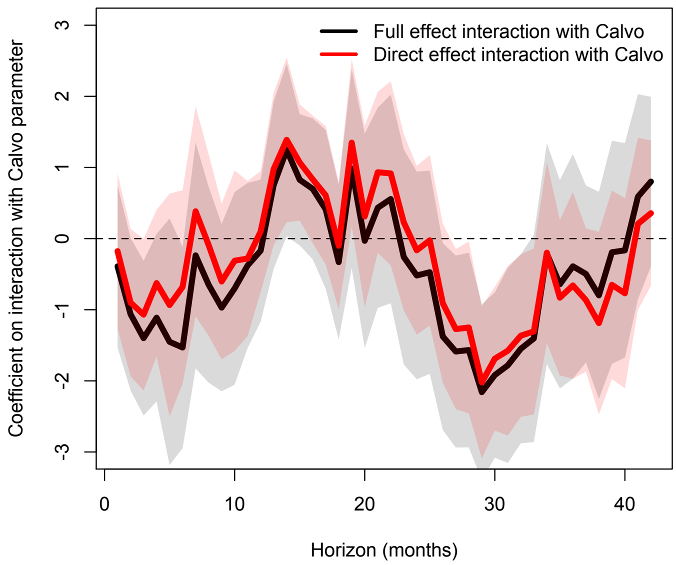


Figure I.20: Final consumption response to 25bp tightening: full and direct effects under zero-diagonal (Detail level, 161 sectors)

Note: *Figure I.20 shows additional estimation of the direct effect using (21) where we force the diagonal elements of the input-output matrix to be equal to zero. The new estimates are reported alongside estimation of (20) and (21) without further restrictions. Panel (a) shows results using panel fixed effects estimation, whereas panel (b) shows results using sector-by-sector OLS estimation with subsequent aggregation using empirical consumption shares* $\{\omega_{ck}\}_{k=1}^{161}$.

Appendix J. Additional results: the role of Calvo and delta heterogeneity

(a) Interaction with the Calvo parameter



(b) Interaction with intermediates intensity

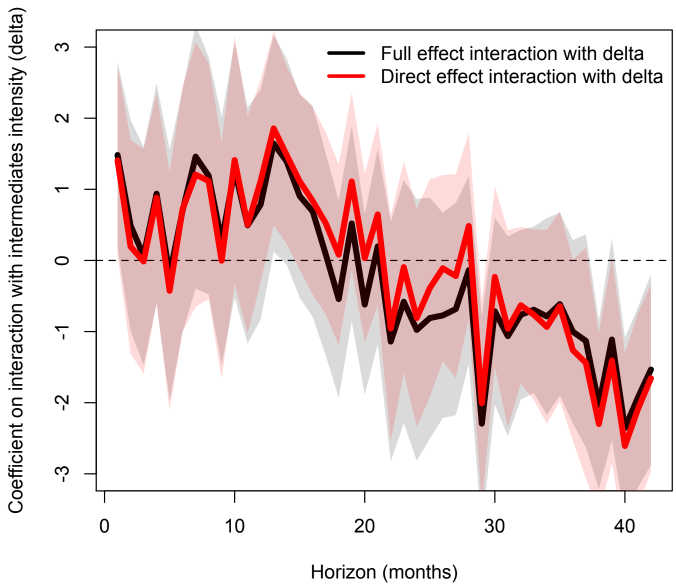
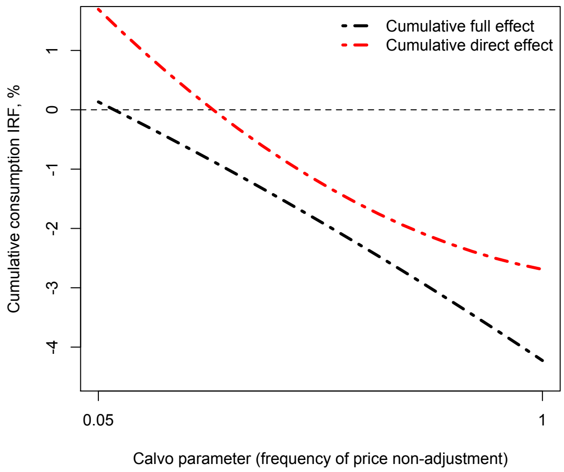


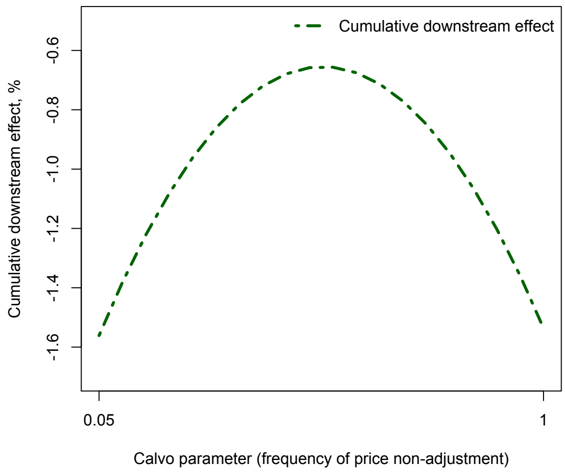
Figure J.21: Confidence bands around estimates of interaction terms

Note: *Figure J.21 shows results of panel fixed effects estimation of (20) and (21) with interaction terms $r_t \times \alpha_k$ and $r_t \times \delta_k$ added, as well as vector of controls $\mathbb{X}_t^H = \left\{ \{r_{t+\tau}\}_{\tau=1}^{H}, \{\hat{c}_{k,t-s}\}_{s=1}^{1}, \{ffr_{t-q}\}_{q=1}^{4} \right\}$; panel (a) shows point estimates and 90% confidence bands around interaction term $r_t \times \alpha_k$ for full and direct effects for every horizon; panel (a) shows point estimates and 90% confidence bands around interaction term $r_t \times \delta_k$ for full and direct effects for every horizon.*

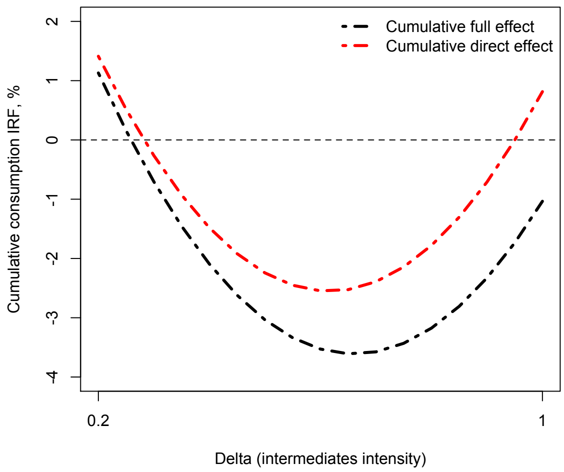
(a) Cumulative full and direct effects across Calvos



(b) Cumulative downstream effects across Calvos



(c) Cumulative full and direct effects across deltas



(d) Cumulative downstream effects across deltas

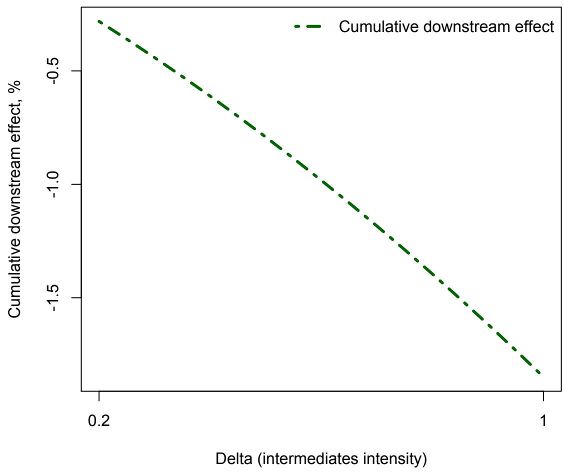


Figure J.22: Response heterogeneity across frequencies of price adjustment and intermediates intensity

Note: *Figure J.22 shows results of panel fixed effects estimation of (20) and (21) with interaction terms* $r_t \times \alpha_k, r_t \times \delta_k, r_t \times \alpha_k^2, r_t \times \delta_k^2$ *and* $r_t \times \alpha_k \delta_k$ *added, as well as vector of controls* $\mathbb{X}_t^H = \left\{ \{r_{t+\tau}\}_{\tau=1}^H, \{\hat{c}_{k,t-s}\}_{s=1}^1, \{ffr_{t-q}\}_{q=1}^4 \right\}$*; panels (a) and (b) show cumulative full, direct and downstream effects for various values of the Calvo parameter, holding intermediates intensity fixed at its sample median value; panels (c) and (d) show cumulative full, direct and downstream effects for various values of intermediates intensity, holding Calvo parameter fixed at its sample median value. All cumulative effects are over 42 months.*

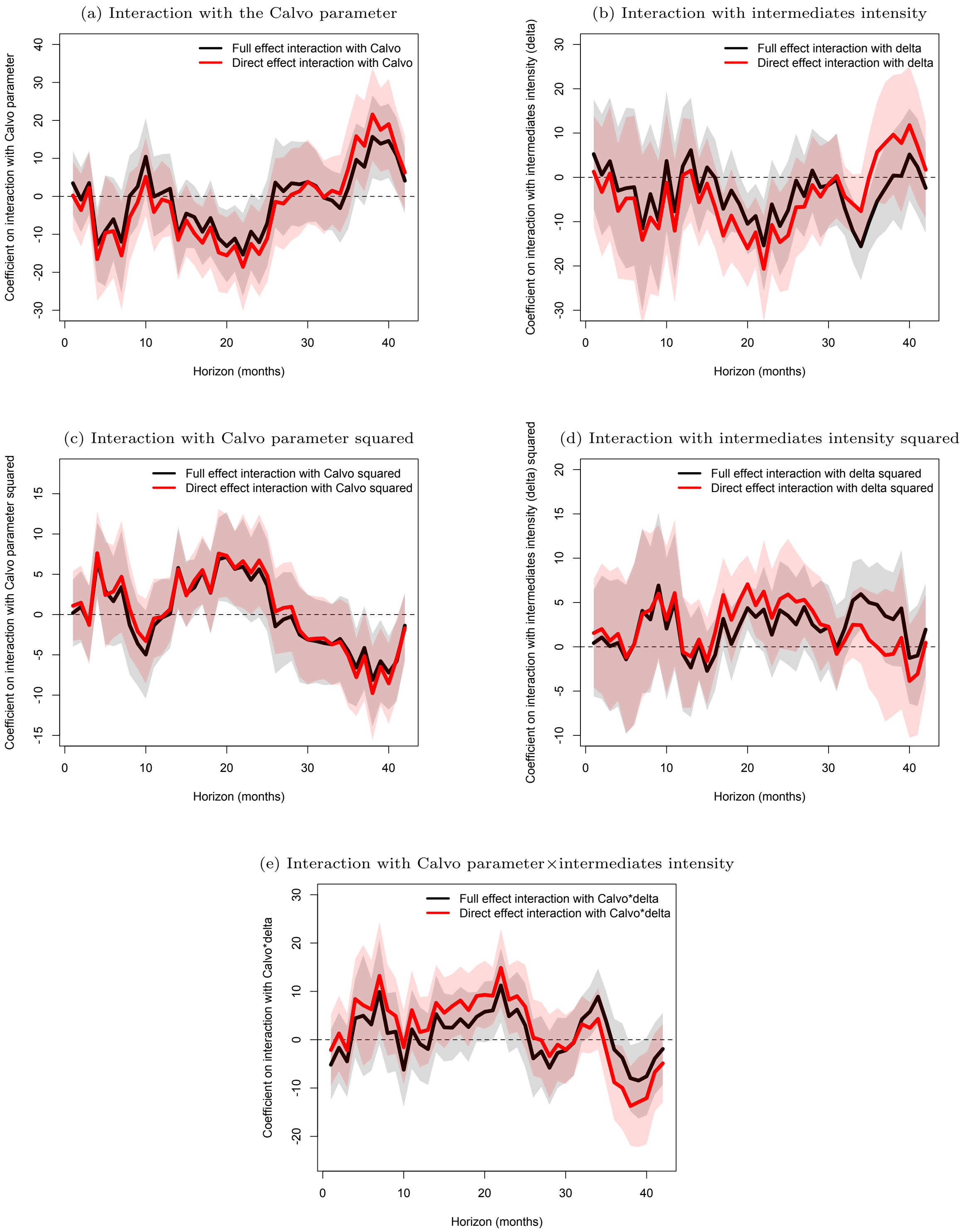
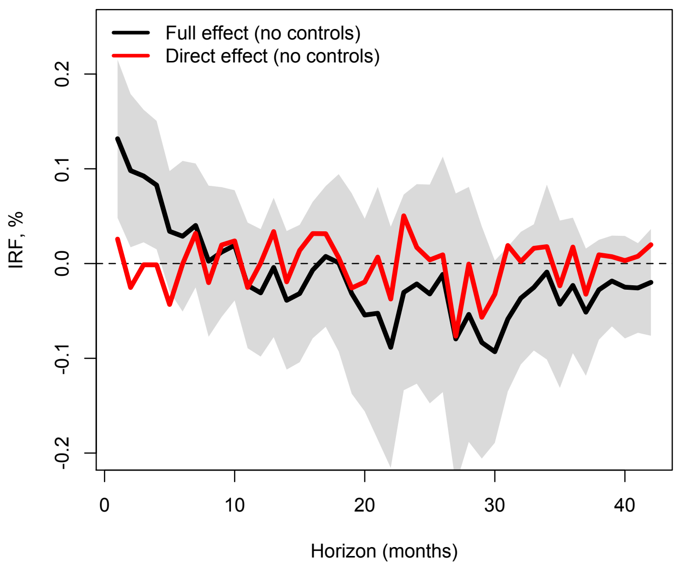


Figure J.23: Confidence bands around estimates of interaction terms

Note: *Figure J.22 shows results of panel fixed effects estimation of (20) and (21) with interaction terms* $r_t \times \alpha_k, r_t \times \delta_k, r_t \times \alpha_k^2, r_t \times \delta_k^2$ *and* $r_t \times \alpha_k \delta_k$ *added, as well as vector of controls* $\mathbb{X}_t^H = \left\{ \{r_{t+\tau}\}_{\tau=1}^{H}, \{\hat{c}_{k,t-s}\}_{s=1}^{1}, \{ffr_{t-q}\}_{q=1}^{4} \right\}$*; panels (a)-(e) show point estimates and 90% confidence bands around interaction term* $r_t \times \alpha_k, r_t \times \delta_k, r_t \times \alpha_k^2, r_t \times \delta_k^2$ *and* $r_t \times \alpha_k \delta_k$*, respectively, for full and direct effects for every horizon.*

Appendix K. Summary level of disaggregation (53 sectors)

(a) Full and direct effects after a 25bp tightening (with controls)



(b) Full and direct effects after a 25bp tightening (with controls)

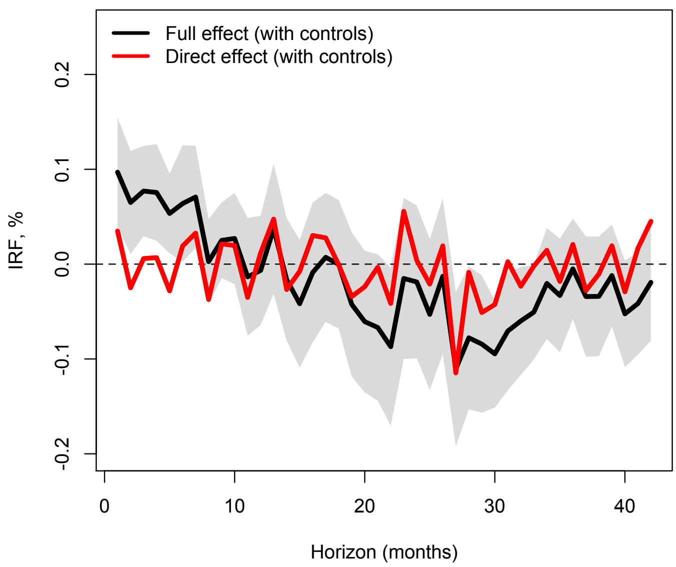


Figure K.24: Final consumption response to 25bp tightening: full and direct effects (Summary level, 53 sectors)

Note: *panel (a) shows results of panel fixed effects estimation of (20) and (21), without adding controls, reported as IRFs following a 25bp monetary tightening; panel (b) reports results of panel fixed effects estimation of (20) and (21), with a vector of controls* $\mathbb{X}_t^H = \left\{\{r_{t+\tau}\}_{\tau=1}^H, \{\hat{c}_{k,t-s}\}_{s=1}^1, \{ffr_{t-q}\}_{q=1}^4\right\}$ *added, reported as IRFs following a 25bp monetary tightening. Shaded areas denote 90% confidence bands around the full effect. The downstream effect contribution is constructed as the difference between cumulative full and direct effects as a percentage of the cumulative full effect (all cumulation is over 42 months).*

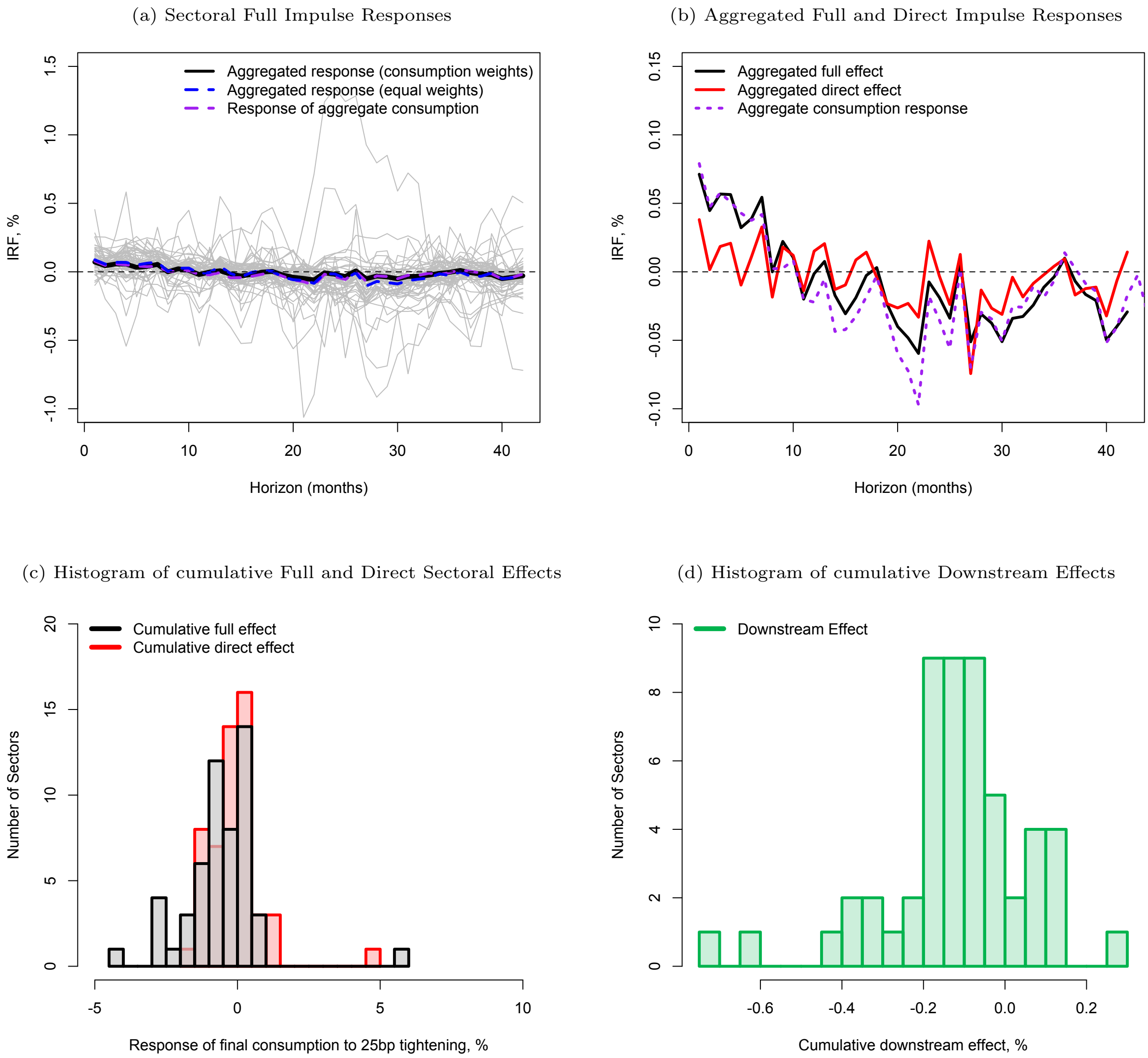
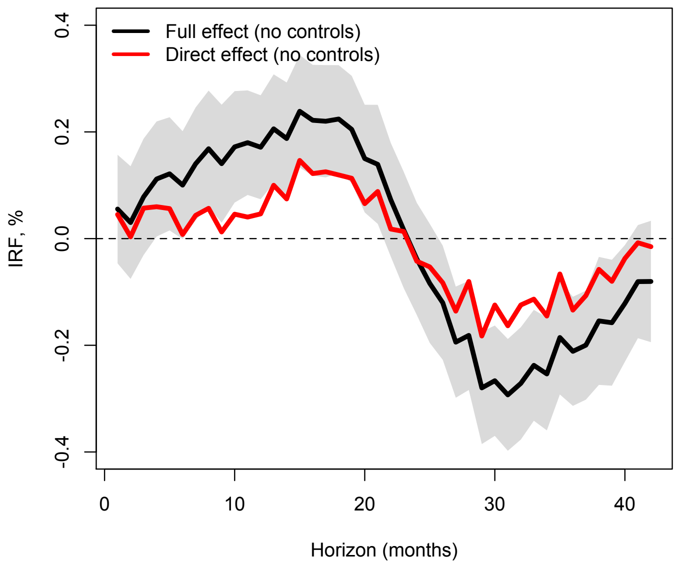


Figure K.25: Sector-level consumption responses to 25bp tightening (Summary level, 53 sectors)

***Note**: panel (a) shows results of sector-by-sector OLS estimation of (20) with controls, reported as sector-level IRF following a 25bp monetary tightening; panel (b) reports results of aggregation of sector-level full and direct effects using empirical final consumption shares $\{\omega_{ck}\}_{k=1}^{53}$; panels (c) and (d) report histograms of cumulative (over 42 months) sectoral full and direct effects, as well as the corresponding sectoral cumulative downstream effects.*

Appendix L. Alternative monetary shocks

(a) Full and direct effects after a 25bp tightening (with controls)



(b) Full and direct effects after a 25bp tightening (with controls)

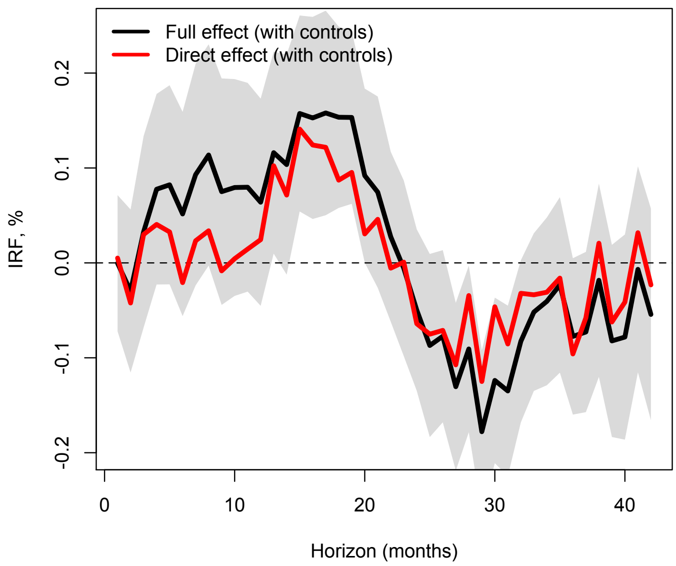


Figure L.26: Final consumption response to 25bp tightening: full and direct effects (Detail level, 161 sectors) – Coibion (2012) SVAR shocks

***Note**: panel (a) shows results of panel fixed effects estimation of (20) and (21), without adding controls, reported as IRFs following a 25bp monetary tightening; panel (b) reports results of panel fixed effects estimation of (20) and (21), with a vector of controls $\mathbb{X}_t^H = \left\{\{r_{t+\tau}\}_{\tau=1}^H, \{\hat{c}_{k,t-s}\}_{s=1}^1, \{ffr_{t-q}\}_{q=1}^4\right\}$ added, reported as IRFs following a 25bp monetary tightening. Shaded areas denote 90% confidence bands around the full effect. The downstream effect contribution is constructed as the difference between cumulative full and direct effects as a percentage of the cumulative full effect (all cumulation is over 42 months).*

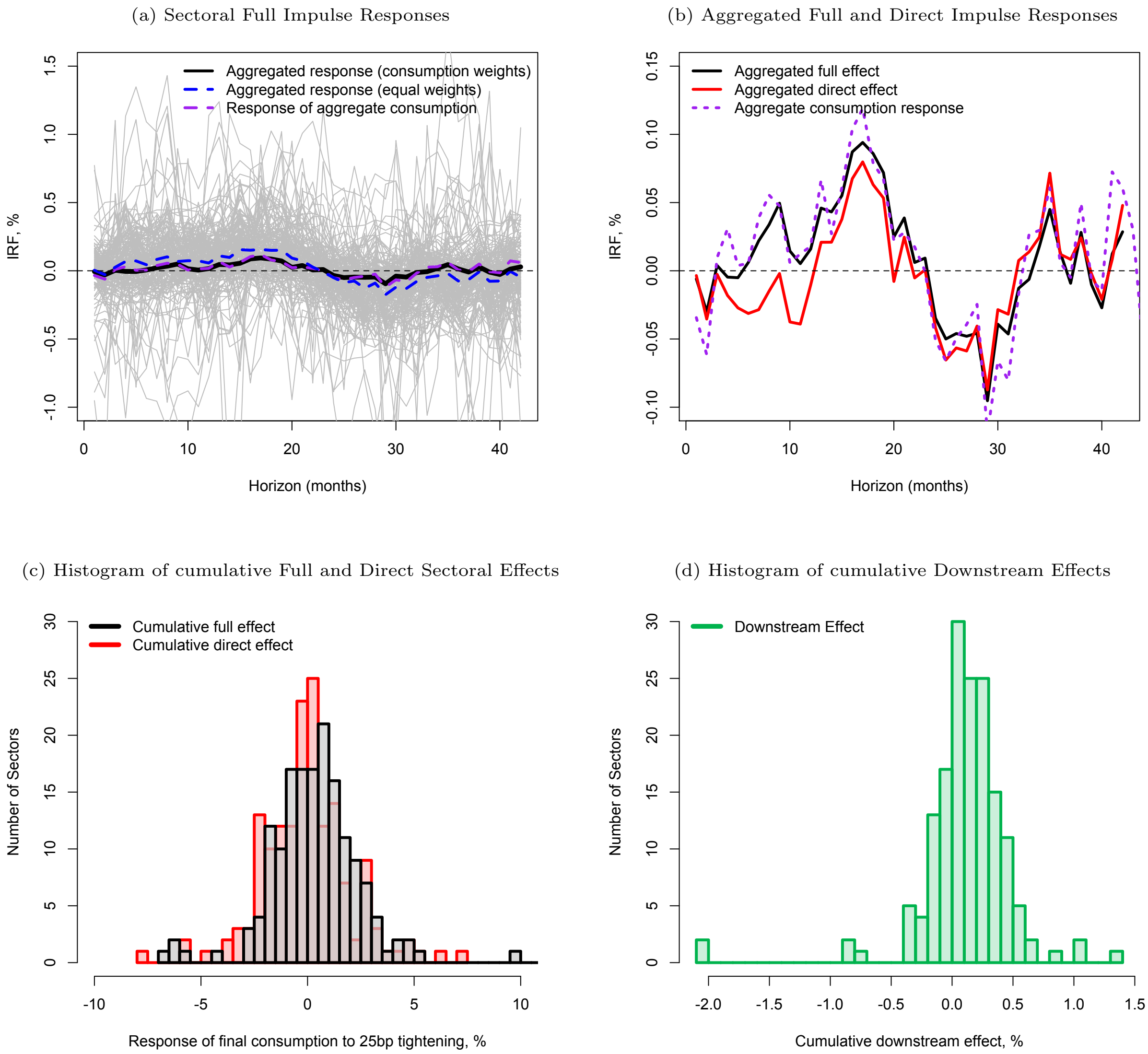
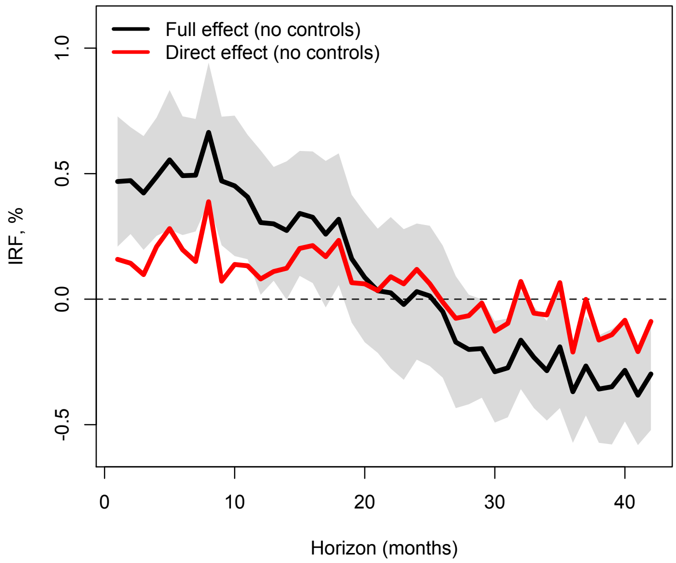


Figure L.27: Sector-level consumption responses to 25bp tightening (Detail level, 161 sectors) – Coibion (2012) SVAR shocks

***Note**: panel (a) shows results of sector-by-sector OLS estimation of (20) with controls, reported as sector-level IRF following a 25bp monetary tightening; panel (b) reports results of aggregation of sector-level full and direct effects using empirical final consumption shares $\{\omega_{ck}\}_{k=1}^{161}$; panels (c) and (d) report histograms of cumulative (over 42 months) sectoral full and direct effects, as well as the corresponding sectoral cumulative downstream effects.*

(a) Full and direct effects after a 25bp tightening (with controls)



(b) Full and direct effects after a 25bp tightening 25bp tightening (with controls)

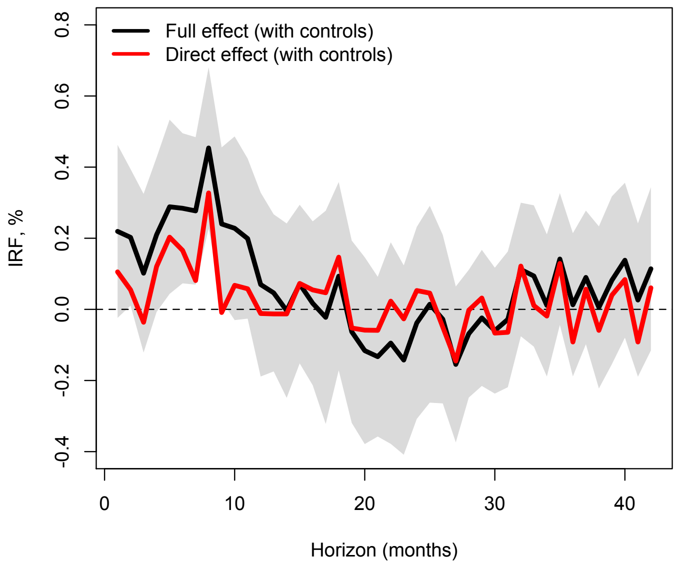


Figure L.28: Final consumption response to 25bp tightening: full and direct effects (Detail level, 161 sectors) – Kuttner (2001) event study shocks

***Note**: panel (a) shows results of panel fixed effects estimation of (20) and (21), without adding controls, reported as IRFs following a 25bp monetary tightening; panel (b) reports results of panel fixed effects estimation of (20) and (21), with a vector of controls* $\mathbb{X}_t^H = \left\{ \{r_{t+\tau}\}_{\tau=1}^{H}, \{\hat{c}_{k,t-s}\}_{s=1}^{1}, \{ffr_{t-q}\}_{q=1}^{4} \right\}$ *added, reported as IRFs following a 25bp monetary tightening. Shaded areas denote 90% confidence bands around the full effect. The downstream effect contribution is constructed as the difference between cumulative full and direct effects as a percentage of the cumulative full effect (all cumulation is over 42 months).*

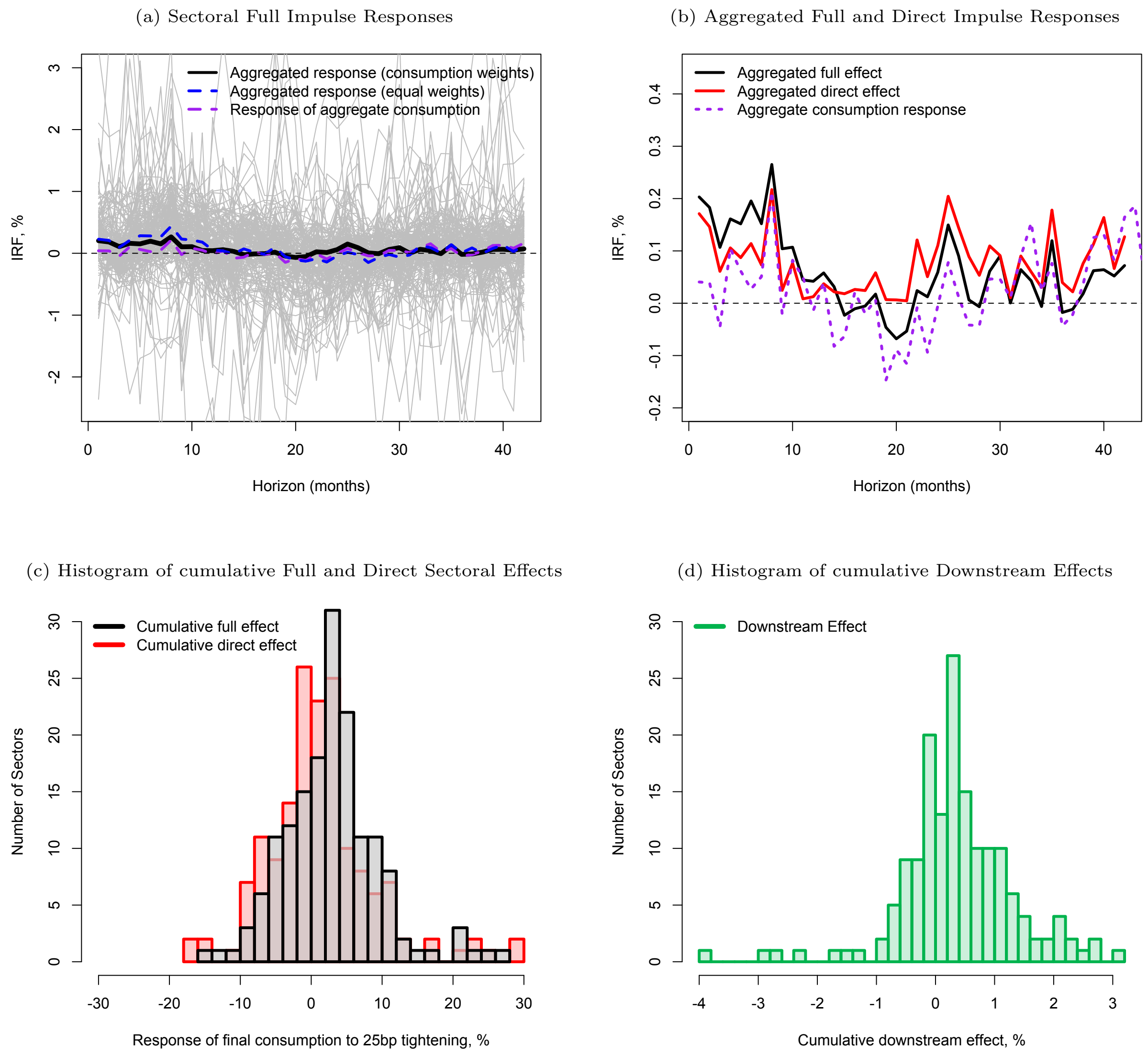


Figure L.29: Sector-level consumption responses to 25bp tightening (Detail level, 161 sectors) – Kuttner (2001) event study shocks

Note: *panel (a) shows results of sector-by-sector OLS estimation of (20) with controls, reported as sector-level IRF following a 25bp monetary tightening; panel (b) reports results of aggregation of sector-level full and direct effects using empirical final consumption shares* $\{\omega_{ck}\}_{k=1}^{161}$*; panels (c) and (d) report histograms of cumulative (over 42 months) sectoral full and direct effects, as well as the corresponding sectoral cumulative downstream effects.*