Question 1:

The quality assurance checks on the previous batches of drugs found that — it is 4 times more likely that a drug is able to produce a satisfactory result than not. Given a small sample of 10 drugs, you are required to find the theoretical probability that at most, 3 drugs are not able to do a satisfactory job.

a.) ***Propose the type of probability distribution that would accurately portray the above scenario and list out the three conditions that this distribution follows.***

<u>Answer:</u>

Binomial probability distribution will accurately portray the above scenario as it satisfies all the three conditions that binomial distribution requires i.e.

1) The total number of trials is fixed at n:
2) Each trial is binary, i.e., it has only 2 possible outcomes as success and failure.
3) Probability of success is the same in all the trials denoted by p

In our case:

- Fixed trial: n = 10
- 2 outcomes are:

  Success: the drug was able to do a satisfactory job

  Failure: the drug was not able to do a satisfactory job
- Probability of success is the same in all the trials denoted by p, where p is 5/10 (0.2)

b.) ***Calculate the required probability.***

b) based on the problem statement

$n = 10$ .... no. of samples.

$r = 3$ .... no. of unsuccessful drug trials (atmost)

$p = 0.2$ ... probability of being un-successful (it is 4x more likely the drug is able to produce a satisfactory result or not).

formula

$$P(X = r) = {}^nC_r (p)^r (1-p)^{n-r}.$$

a) $P(X=0) = {}^{10}C_0 (0.2)^0 (1-0.2)^{10}$

$= 1 (0.2)^0 (0.8)^{10}$

$= 1 \times 1 \times 0.1073$

$= 10.73$

b) $P(X=1) = {}^{10}C_1 (0.2)^1 (1-0.2)^9$

$= 10 (0.2) (0.8)^9$

$= 10 (0.2) (0.1342)$

$= 0.2684$

$= 26.84$

c) $P(X=2) = {}^{10}C_2 (0.2)^2 (1-0.2)^8$

$= 45 (0.04) (0.1677)$

$= 0.30186.$

$= 30.18$

d) $P(X=3) = {}^{10}C_3 (0.2)^3 (1-0.2)^7$

$= 120 (0.008) (0.2097)$

$= 0.2013$

$= 20.13.$

∴ computing

$p(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= 10.73 + 26.84 + 30.18 + 20.13$

$= \underline{87.88.}$

∴ the theoretical probability that at most 3 drugs are not able to ~~for~~ do a satisfactory job is

$P(X \leq 3) = 87.88.$

Question 2:

For the effectiveness test, a sample of 100 drugs was taken. The mean time of effect was 207 seconds, with the standard deviation coming to 65 seconds. Using this information, you are required to estimate the range in which the population mean might lie — with a 95% confidence level.

a.) Discuss the main methodology using which you will approach this problem. State all the properties of the required method. Limit your answer to 150 words.

Answer:

My approach is using Central Limit Theorem to find the required range where the mean population would lie with a 95% confidence level.

The sampling distribution has the following properties:

- The mean is equal to the population mean i.e.,

  $\mu_{\bar{X}} = \mu$

- Sampling distributions std. deviation is called as std. error and is given by

  $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$

- For large n (usually n > 30), the sampling distribution becomes a normal Distribution

b.) Find the required range.

Q.2) Solution :-

Mean $(\bar{x})$ = 207

Std. deviation (σ) = 65

Sample Space (n) = 100

Confidence level (y) = 95%

Z associated with confidence level (95%) = $\underline{\underline{\pm 1.96}}$

Confidence Interval = $\left(\bar{x} - \frac{z * \sigma}{\sqrt{n}}, \bar{x} + \frac{z * \sigma}{\sqrt{n}}\right)$

$= \left(207 - \frac{1.96 \times 65}{\sqrt{100}}, 207 + \frac{1.96 \times 65}{\sqrt{100}}\right)$

$= \left(207 - \frac{1.96 \times 65}{10}, 207 + \frac{1.96 \times 65}{10}\right)$

$= (194.26, 219.74)$

$\therefore$ Confidence Interval = $(194.26, 219.74)$.

Estimated range the population mean lie is: **(194.26, 219.74)**

Question 3:

a) The painkiller drug needs to have a time of effect of at most 200 seconds to be considered as having done a satisfactory job. Given the same sample data (size, mean, and standard deviation) of the previous question, test the claim that the newer batch produces a satisfactory result and passes the quality assurance test. Utilize 2 hypothesis testing methods to make your decision. Take the significance level at 5 %. Clearly specify the hypotheses, the calculated test statistics, and the final decision that should be made for each method.

Answer: Approach in solving the above question

1) State the Null and Alternate Hypothesis

Null Hypothesis:

$\mathbf{H0: \mu \leq 200}$ seconds, time effect that the drug ***has done*** satisfactory job

Alternate Hypothesis:

$\mathbf{H1: \mu > 200}$ seconds, time effect that the drug ***has not done*** satisfactory job

2) Type of the test and the position of the critical region based on the '**sign' in the alternate hypothesis** it will be a Upper tailed test

> in H_1 → Upper-tailed test → Rejection region on **right side** of distribution

3) Two hypothesis method used to arrive to a conclusion are Critical-Value and P-Value Method

Using Critical Value Method:

Q.3 (a) $H_0 : \mu \leq 200 \longrightarrow$ Null hypothesis

$H_1 : \mu > 200 \longrightarrow$ alternate hypothesis.

Based on the sign of the Alternate hypothesis upper tailed test will be conducted.

$\mu = 200.$

$n = 100$

$\mu \bar{x} = 207$

$\sigma \bar{x} = 65$

$\alpha = 5\% = 0.05$

Cumulative probability $= 1 - 0.05$

$\therefore$ Acceptance Region $= 0.95$

Z score for 0.95

As 0.95 is not present in the z table.

Zscore of $0.9495 = 1.64$

Zscore of $0.9505 = 1.65$

$\therefore$ Average $= 1.645$ (Avg of two Zscores above)

$\therefore$ Z score $(0.95) = \underline{1.645}$

Formula Critical Value Method :=

$\sigma \bar{x} = \sigma / \sqrt{n} = 65 / \sqrt{100} = 6.5$

$UCV = \mu + (Z_c \times \sigma \bar{x})$

$= 200 + 1.645 \times 6.5$

$= \underline{210.69}$

Conclusion :- fail to Reject the null hypothesis since the sample mean doesn't lie in critical region. (UCV)

i.e $\mu \bar{x} < UCV$.

Conclusion:

By Using Critical value method, we fail to reject the null hypothesis since the sample mean doesn't lie in the critical region (UCV) i.e., $\mathbf{\mu \bar{x} < UCV}$

Using P-Value Method:

P-value Method.

1) Calculating the Zscore for sample mean distribution

$$Z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma/\sqrt{n}}$$

$$= \frac{207 - 200}{65/\sqrt{100}} = \frac{207 - 200}{65/10}$$

$$= 7/6.5$$

$$= \underline{\underline{1.076}}$$

2) Calculating the p-value from cumulative probability for the given Zscore using z table.

$\therefore$ Cumulative probability of sample point = $\underline{\underline{0.8577}}$

For one-tailed test (upper tailed):

$$p = 1 - \text{cumulative probability}$$
$$= 1 - 0.8577$$
$$= 0.1423$$
$$= \underline{\underline{14.23}}$$

Significan level (α) = 5% i.e., 0.05.

Conclusion :- Since the p-value is greater than significan level α i.e.,

$14.23 > 5\%$.

You fail to reject the null hypothesis ~~that~~

Conclusion:

By Using the P-Value Method we fail to reject the null hypothesis i.e., **H0: μ≤ 200**

Since the P-Value in upper tailed test and is greater than the significance level **(P-Value > α)**

Question 3:

b) You know that two types of errors can occur during hypothesis testing — namely Type-I and Type-II errors — wH0se probabilities are denoted by α and β respectively. For the current sample conditions (sample size, mean, and standard deviation), the value of α and β come out to be 0.05 and 0.45 respectively.

Now, a different sampling procedure (with different sample size, mean, and standard deviation) is proposed so that when the same hypothesis test is conducted, the values of α and β are controlled at 0.15 each. Explain under what conditions would either metH0d be more preferred than the other, i.e. give an example of a situation where conducting a hypothesis test having α and β as 0.05 and 0.45 respectively would be preferred over having them both at 0.15. Similarly, give an example for the reverse scenario - a situation where conducting the hypothesis test with both α and β values fixed at 0.15 would be preferred over having them at 0.05 and 0.45 respectively. Also, provide suitable reasons for your cH0ice (Assume that only the values of α and β as mentioned above are provided to you and no other information is available).

Question in tabular format:

	Case 1	Case 2
α	0.05	0.15
β	0.45	0.15

Type I and type II error:

<table>
<tr><td colspan="4">Reality:</td></tr>
<tr><td rowspan="3">Test</td><td></td><td>H0 - True:</td><td>H0 - False:</td></tr>
<tr><td>Reject H0</td><td>Type I error</td><td>Correct Conclusion</td></tr>
<tr><td>Fail to Reject H0</td><td>Correct Conclusion</td><td>Type II Error</td></tr>
</table>

- ***Null and Alternate Hypothesis:***
- Null: H0 = Drug produces satisfactory result
- Alternate: H1 = Drug doesn't produce the Satisfactory result

- ***Types of error:***
- **Type I:** reject the null hypothesis when it is **true** denoted by α
- **Type II:** fail to reject the null hypothesis when it is **False** denoted by β

- ***<u>Consequences of both the errors:</u>***
    - **Type I error:** we reject the null hypothesis that the drug produces satisfactory result, whereas it does produce satisfactory result
    - **Type II error:** we fail to reject the null hypothesis that the drug produces satisfactory result, whereas in reality it doesn't produce a satisfactory result

- Error that has the more dangerous consequence for company and health: **Type II**

  Type II error is more dangerous as compare to Type I error in terms on company and health as we fail to reject the null hypothesis that doesn't produce satisfactory result which may cause severe health issues (life threatening) to the consumer

  From Company perspective: the reputation/status of the company will suffer when consumers will file legal cases on it

  Whereas Type I error will be hazardous to the consumer and company and may only require less efforts to manufacture a new batch of painkiller drugs and test them for quality purpose

- ***<u>Conclusion for Case I vs to Case II:</u>***
    - **Case I**: In order to avoid the dangerous effect of Type II error β we can increase the probability of $\alpha = 0.05$ to $\alpha = 0.10$ as both the probability α & β are inversely proportional and would help in reducing the chances of committing type II error.
    - **Case II:** according to the above scenario discussed we should prefer α & $\beta = 0.15$ over $\alpha = 0.05$ & $\beta = 0.45$ as the company controls both types of errors to 15% significance level
      unlike the Case I where we would require decrease the β which will eventually increase the type I error α that may cost the company to invest in manufacturing the drugs again and testing for its effectiveness

Question 4:

Now, once the batch has passed all the quality tests and is ready to be launched in the market, the marketing team needs to plan an effective online ad campaign to attract new customers. Two taglines were proposed for the campaign, and the team is currently divided on which option to use.

Explain why and how A/B testing can be used to decide which option is more effective. Give a stepwise procedure for the test that needs to be conducted.

- ***<u>A/B Testing:</u>***

  A/B testing is one of the ways through which one can improve the user experience for a Website or App

  A/B testing provides a way for you to test two different versions of the same element and see which one performs better

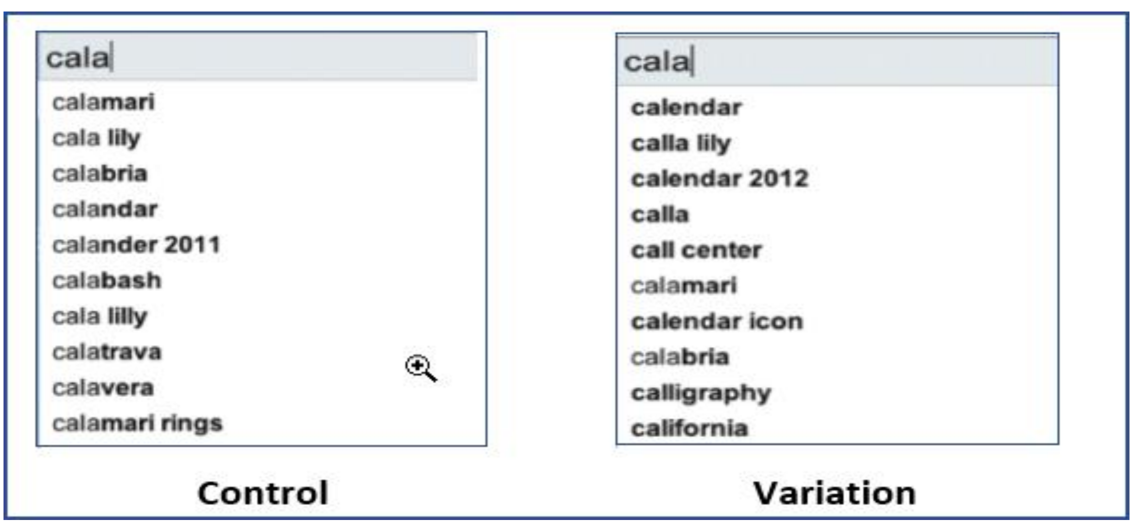
- **<u>E.g.</u>**:

  Let us consider an element **Search button** of an e-commerce Website for displaying the result that provides user to download the images from their site

  We will conduct A/B testing on the search algorithm for a website that offers downloading images

  “exact match” vs “fuzzy match”

  Refer the image below:



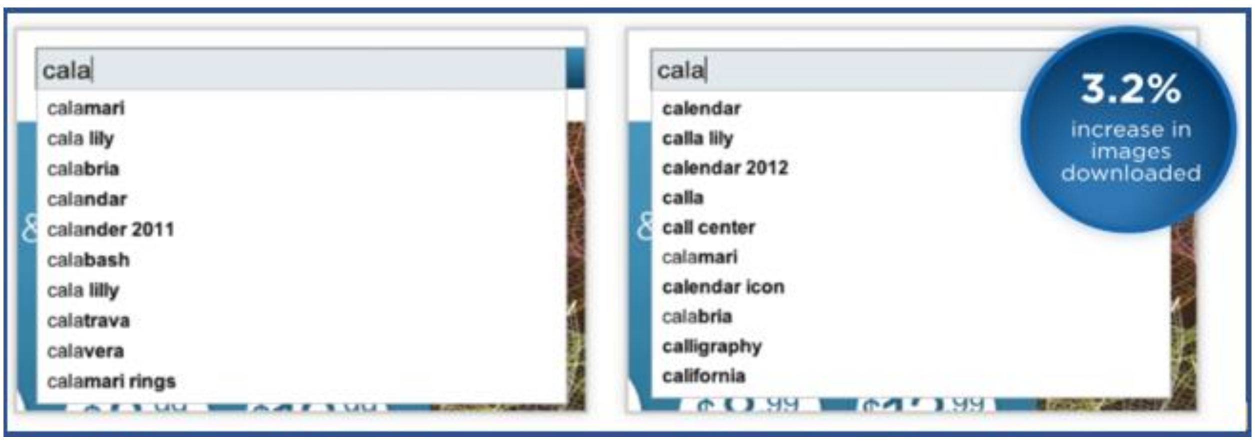
- **<u>Approach:</u>**
  - <u>Collect the data</u> to know which page or element of the website is having low traffic
    In our case: it is the search element

  - <u>Identify the goal metrics</u> that will determine whether the variation is more successful than original (control) version

  - <u>Generating hypothesis</u> by jotting down points why the variation is better than the control version
    In our case: hypothesis is fuzzy match will perform better

  - <u>Creating variation</u>: in our case it would be changing the search algorithm **from exact to fuzzy match** and successfully deploy the changes in web page

  - <u>Run the experiment</u> by randomly assigning the control and variation version to the users and their interaction would be determined, measured and counted for comparison



Control **Variation**

  - <u>Analyze</u> the experiment through s/w that will present the diff between the two version's performance
    In our eg: suppose after analyzing the two version results it was found users selected results from the fuzzy auto suggest 9.6 % more often and a 3.2% increase in downloaded images

➢ **Key Takeaway from the Current A/B Testing:**

- By conducting A/B testing we get a picture of which version of the Website or a web page will perform better.
- Therefore, before launching any product/website/App it is better to perform A/B testing to know the impact that changes have on your metrics, you can ensure that every change produces positive results because sometimes our assumptions can go wrong
- It also represents significant increase in online revenue of the company