A short note on skewing GPs and GQs. (contact: ghawkes1217@gmail.com)

In this short note we (conjecturally) show how to write $\mathbf{GP}_{\lambda/\mu}$ as an non-negative integral sum of $\mathbf{GP}_{\nu}$. In addition, we show how to write $\mathbf{GQ}_{\lambda/\mu}$ as an non-negative integral sum of $\mathbf{GQ}_{\nu}$. First, we must define the *backword* and *forword*. The backword of a (skew) shifted set-valued tableau (for both the P-version and the Q-version) is attained as follows. Read the entries of the tableau in the following order:

(1) Within the tableau start from the top row and work down
(2) Within each row, start with the rightmost box and move left
(3) Within a box, read primed entries first, in decreasing order, and then unprimed entries, in decreasing order

Alternatively, the forword of a (skew) shifted set-valued tableau (for both the P-version and the Q-version) is attained as follows. Read the entries of the tableau in the following order:

(1) Within the tableau start from the bottom row and work up
(2) Within each row, start with the leftmost box and move right
(3) Within a box, read unprimed entries first, in decreasing order, and then primed entries, in decreasing order

Next, we define what it means for a (skew)-shifted SVT to have the *lattice* property. To initialize set $lattice = true$. Now for each i perform the following:

- set $count_i = 0 = count_{i+1}$
- Scan through the backword, whenever you see an i set $count_i = count_i + 1$ and whenever you see an $i+1$ set $count_{i+1} = count_{i+1} + 1$. If during this process $count_{i+1}$ ever exceeds $count_i$ set $lattice = false$. Also, if while scanning an $(i+1)'$ when currently $count_i = count_{i+1}$ also set $lattice = false$.
- Scan through the forword, whenever you see an i' set $count_i = count_i + 1$ and whenever you see an $(i+1)'$ set $count_{i+1} = count_{i+1} + 1$. If during this process $count_{i+1}$ ever exceeds $count_i$ set $lattice = false$. Also, if while scanning an i when currently $count_i = count_{i+1}$ also set $lattice = false$.

Finally, we define the *primed-starting* property as follows: Read through the boxes of P by rows, top to bottom among rows and left to right within rows. If the first box you encounter that contains an i or i' (or both) contains an i' but not an i then the tableau has the primed-starting property for i. A tableau with the primed-starting property for all i is considered to have the overall primed-starting property.

Let $\mathcal{P}_{\lambda/\mu}$ denote the set of all shifted set valued P-tableau of shape λ/μ that have the lattice property and let $\mathcal{Q}_{\lambda/\mu}$ denote the set of all shifted set valued Q-tableau of shape λ/μ that have the lattice property and the primed-starting property.

Now we can expand $\mathbf{GP}$ and $\mathbf{GQ}$:

$$\mathbf{GP}_{\lambda/\mu} = \sum_{P \in \mathcal{P}_{\lambda/\mu}} GP_{wt(P)}$$

$$\mathbf{GQ}_{\lambda/\mu} = \sum_{Q \in \mathcal{Q}_{\lambda/\mu}} GQ_{wt(Q)}$$

To check that this is true for specific parameters please see:
P-version: https://github.com/ghawkes1217/Conjectures-and-Computations/blob/main/shifted-LR/skew-GP-expansion.py
Q-version: https://github.com/ghawkes1217/Conjectures-and-Computations/blob/main/shifted-LR/skew-GQ-expansion.py
The "fact" that these statements are true can be checked up to the degree 10 terms relatively quickly.