Expanding GC_ω in terms of GQ_λ. (contact: ghawkes1217@gmail.com)

The type C stable Grothendieck polynomial or k-theoretic Stanley symmetric function of type C for a given signed permutation ω is defined as:

$$GC_\omega = \sum_{f \in F_\omega} \mathbf{x}^{wt(f)}$$

where F_ω is the set of signed factorizations for ω. A signed factorization f, for ω starts with a word w, in the alphabet $\cdots - 3 < -2 < \cdots - 1 < -0 < 0 < 1 < \cdots < 2 < 3 \cdots$ such that associating i and $-i$ to the simple transposition s_i for $i \neq 0$ and associating -0 and 0 to the special generator of type C, s_0, the word w forms a Hecke expression for the signed permutation ω. This word is subdivided then into parts where each part must be strictly increasing (under the given order) to form f. The weight of f is the vector whose i^{th} entry records the number of entries in the i^{th} subdivision.

On the other hand, the Q-Grothendieck function is

$$GQ_\lambda = \sum_{q \in Q_\lambda} \mathbf{x}^{wt(q)}$$

where Q_λ is the set of semistandard shifted set valued tableaux (Q version, i.e., primes allowed on diiagonal). The weight of q is the vector whose i^{th} entry records the number of times i or i' appears in q.

We claim that we may write:

$$GC_\omega = \sum_{t \in T_\omega} GQ_{shape(t)}$$

where T_ω is the set of unimodal Hecke tableaux (defined later) for ω.

It is slightly less computation to verify the stronger statement that

$$GC_\omega^+ = \sum_{t \in T_\omega} GR_{shape(t)}$$

where GC_ω^+ differ from GC_ω by requiring there is a unique number with minimum absolute value in each factor in the definition of a factorization and that this number has a positive sign. GR_λ differs from GQ_λ by requiring that the first i or i' (reading left to right, bottom to top) in a shifted semistandard set valued tableau is an i (in particular, the first box with i or i' does not conatin both).

The latter equation is implied by the existence of a bijection, ϕ, from the set of Hecke words of length n for ω to pairs of tableaux (t, r) where t and r have the same shape, $t \in T_\omega$, and r is a standard shifted set-valued tableau with n entries. The bijection ϕ must have the following additional properties:

(1) If the word w has repeated consecutive entries, that is $w_i = wi + 1$ then the corresponding recording tableau, r must have i and $i+1$ in the same box (and conversly).
(2) The word w has a peak at i, that is $w_{i-1} < w_i > w_{i+1}$ if and only if the corresponding recording tableau has a peak at i. That is i appears in a box to the right of $i-1$ and $i+1$ appears in a box below i.

The weakest version of the conjecture check the existence of ϕ (this is what the upcoming paper of Arroyo, Hamaker, H., and Pan will prove). The next strongest checks that ϕ exists and satisfies (1). The strongest version checks that ϕ exists and satisfies (1) and (2).