$PSPACE$

Definition: $PSPACE$ is the class of languages that are decidable in polynomial space on a deterministic Turing machine. Formally,

$$PSPACE = \bigcup_k SPACE(n^k), \text{ with } k > 0.$$

We define $NPSPACE = \bigcup_k NSPACE(n^k)$, for $k > 0$. But from Savitch's theorem it follows that

$$NPSPACE = \bigcup_k NSPACE(n^k) = \bigcup_k SPACE(n^{k+2}) = \bigcup_k SPACE(n^k) = PSPACE.$$

A Hierarchy of Complexity

We can now construct a hierarchy of complexity classes.

Observe that $P \subseteq PSPACE$. To see this let a machine execute in $t(n) \geq n$ polynomial time. But this implies that the machine can use at most $t(n)$ polynomial space (one cell per computation step).

In a similar argument we have $NP \subseteq NPSPACE$ and from the argument above $NP \subseteq PSPACE$.

We already know that $P \subseteq NP$.

Finally, if we have a $f(n) \geq n$ polynomial space machine it can be shown that this machine runs in $2^{O(f(n))}$ exponential time. Thus, $PSPACE \subseteq EXPTIME$.

$$P \subseteq NP \subseteq PSPACE \subseteq EXPTIME.^{a}$$

[a] It is believed that all the set containments are proper.

A Hierarchy of Complexity