Understanding Regression through Single Neuron Models

Objective: To explore the implementation of Simple Linear Regression (SLR), Logistic Regression, and Multiple Linear Regression (MLR) using the concept of a single neuron.

Instructions: You are free to select any random dataset for this task. The implementation should be done without relying on any external libraries; only basic Python programming is allowed.

Questions:

1. Simple Linear Regression (SLR):

- Explain the mathematical formulation of Simple Linear Regression.
- Implement a Python program that models Simple Linear Regression using a single neuron. Ensure your program includes:
 - Initialization of weights and bias.
 - A gradient descent loop for optimization.
 - Loss computation using Mean Squared Error (MSE).
 - Verify the model's predictions with test data.

2. Logistic Regression:

- Explain how a single neuron can be used to perform Logistic Regression.
- Write a Python program that implements Logistic Regression using a single neuron. Your program should:
 - Use the sigmoid function as the activation function.
 - Optimize the weights and bias using a gradient descent loop.
 - Compute the loss using Cross-Entropy Loss.
 - Test your model on a binary classification dataset and report its performance.

3. Multiple Linear Regression (MLR):

- Describe how a single neuron can be extended to handle multiple input features for MLR.
- Write a Python program to implement MLR using a single neuron. The program should:
 - Accept multiple input features.
 - Optimize the parameters (weights and bias) using gradient descent.
 - Compute the loss using Mean Squared Error (MSE).
 - Test your model on a multi-feature dataset and verify its predictions.

4. Discussion:

- Analyze the feasibility of implementing SLR, Logistic Regression, and MLR using a single neuron. Highlight the key advantages and limitations of this approach.

5. Bonus:

- Modify your Logistic Regression implementation to handle multi-class classification using a single neuron. Discuss any challenges faced and how you overcame them.

Submission Guidelines:

- Include well-documented code for each regression model.
- Submit a report (maximum 2 pages) explaining your approach, results, and observations.
- Include plots or visualizations (if applicable) to demonstrate the performance of your models.