

به نام خدا

استاد: دکتر مرضیه داود آبادی
درس مبانی یادگیری عمیق

نام: فاطمه زهرا بخشنده
شماره دانشجویی: 98522157

گزارش تمرین 4:

سوال چهارم:

1- با توجه به شکل زیر عملیات را انجام می دهیم.

$$\begin{array}{|c|c|} \hline O_{11} & O_{12} \\ \hline O_{21} & O_{22} \\ \hline \end{array} = \text{Convolution} \left(\begin{array}{|c|c|c|} \hline X_{11} & X_{12} & X_{13} \\ \hline X_{21} & X_{22} & X_{23} \\ \hline X_{31} & X_{32} & X_{33} \\ \hline \end{array}, \begin{array}{|c|c|} \hline F_{11} & F_{12} \\ \hline F_{21} & F_{22} \\ \hline \end{array} \right)$$

Output \mathcal{O} Input \mathcal{X} Filter \mathcal{F}

\mathcal{X} به صورت زیر است:

2	3	4
3	1	5
4	-1	-2

\mathcal{F} نیز به صورت زیر است:

0	3
1	-2

خروجی از فرمول زیر محاسبه می شود:

$$O_{11} = X_{11}F_{11} + X_{12}F_{12} + X_{21}F_{21} + X_{22}F_{22}$$

$$O_{12} = X_{12}F_{11} + X_{13}F_{12} + X_{22}F_{21} + X_{23}F_{22}$$

$$O_{21} = X_{21}F_{11} + X_{22}F_{12} + X_{31}F_{21} + X_{32}F_{22}$$

$$O_{22} = X_{22}F_{11} + X_{23}F_{12} + X_{32}F_{21} + X_{33}F_{22}$$

خروجی لایه همگشتی:

10	3
9	18

سپس Global Average Pooling را اعمال می کنیم. خروجی به صورت زیر است:

$$Z = \frac{10 + 3 + 18 + 9}{4} = 10$$

2- در الگوریتم پس انتشار خطا داریم:

For every element of F

$$\frac{\partial L}{\partial F_i} = \sum_{k=1}^M \frac{\partial L}{\partial O_k} * \frac{\partial O_k}{\partial F_i}$$

$$\frac{\partial L}{\partial F_{11}} = \frac{\partial L}{\partial \theta_{11}} * \frac{\partial \theta_{11}}{\partial F_{11}} + \frac{\partial L}{\partial \theta_{12}} * \frac{\partial \theta_{12}}{\partial F_{11}} + \frac{\partial L}{\partial \theta_{21}} * \frac{\partial \theta_{21}}{\partial F_{11}} + \frac{\partial L}{\partial \theta_{22}} * \frac{\partial \theta_{22}}{\partial F_{11}}$$

$$\frac{\partial L}{\partial F_{12}} = \frac{\partial L}{\partial \theta_{11}} * \frac{\partial \theta_{11}}{\partial F_{12}} + \frac{\partial L}{\partial \theta_{12}} * \frac{\partial \theta_{12}}{\partial F_{12}} + \frac{\partial L}{\partial \theta_{21}} * \frac{\partial \theta_{21}}{\partial F_{12}} + \frac{\partial L}{\partial \theta_{22}} * \frac{\partial \theta_{22}}{\partial F_{12}}$$

$$\frac{\partial L}{\partial F_{21}} = \frac{\partial L}{\partial \theta_{11}} * \frac{\partial \theta_{11}}{\partial F_{21}} + \frac{\partial L}{\partial \theta_{12}} * \frac{\partial \theta_{12}}{\partial F_{21}} + \frac{\partial L}{\partial \theta_{21}} * \frac{\partial \theta_{21}}{\partial F_{21}} + \frac{\partial L}{\partial \theta_{22}} * \frac{\partial \theta_{22}}{\partial F_{21}}$$

$$\frac{\partial L}{\partial F_{22}} = \frac{\partial L}{\partial \theta_{11}} * \frac{\partial \theta_{11}}{\partial F_{22}} + \frac{\partial L}{\partial \theta_{12}} * \frac{\partial \theta_{12}}{\partial F_{22}} + \frac{\partial L}{\partial \theta_{21}} * \frac{\partial \theta_{21}}{\partial F_{22}} + \frac{\partial L}{\partial \theta_{22}} * \frac{\partial \theta_{22}}{\partial F_{22}}$$

با توجه به معادله $O_{11} = X_{11}F_{11} + X_{12}F_{12} + X_{21}F_{21} + X_{22}F_{22}$ که پیشتر داشتیم، می توان نوشت:

$$\frac{\partial L}{\partial F_{11}} = \frac{\partial L}{\partial \theta_{11}} * X_{11} + \frac{\partial L}{\partial \theta_{12}} * X_{12} + \frac{\partial L}{\partial \theta_{21}} * X_{21} + \frac{\partial L}{\partial \theta_{22}} * X_{22}$$

$$\frac{\partial L}{\partial F_{12}} = \frac{\partial L}{\partial \theta_{11}} * X_{12} + \frac{\partial L}{\partial \theta_{12}} * X_{13} + \frac{\partial L}{\partial \theta_{21}} * X_{22} + \frac{\partial L}{\partial \theta_{22}} * X_{23}$$

$$\frac{\partial L}{\partial F_{21}} = \frac{\partial L}{\partial \theta_{11}} * X_{21} + \frac{\partial L}{\partial \theta_{12}} * X_{22} + \frac{\partial L}{\partial \theta_{21}} * X_{31} + \frac{\partial L}{\partial \theta_{22}} * X_{32}$$

$$\frac{\partial L}{\partial F_{22}} = \frac{\partial L}{\partial \theta_{11}} * X_{22} + \frac{\partial L}{\partial \theta_{12}} * X_{23} + \frac{\partial L}{\partial \theta_{21}} * X_{32} + \frac{\partial L}{\partial \theta_{22}} * X_{33}$$

که می توان به صورت زیر نوشت:

$$\begin{bmatrix} \frac{\partial L}{\partial F_{11}} & \frac{\partial L}{\partial F_{12}} \\ \frac{\partial L}{\partial F_{21}} & \frac{\partial L}{\partial F_{22}} \end{bmatrix} = \text{Convolution} \left(\begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{bmatrix}, \begin{bmatrix} \frac{\partial L}{\partial O_{11}} & \frac{\partial L}{\partial O_{12}} \\ \frac{\partial L}{\partial O_{21}} & \frac{\partial L}{\partial O_{22}} \end{bmatrix} \right)$$

where

$$\begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{bmatrix} = \text{Input X} \quad \begin{bmatrix} \frac{\partial L}{\partial O_{11}} & \frac{\partial L}{\partial O_{12}} \\ \frac{\partial L}{\partial O_{21}} & \frac{\partial L}{\partial O_{22}} \end{bmatrix} = \frac{\partial L}{\partial O} \quad \text{Loss gradient from previous layer}$$

محاسبه $\frac{\partial L}{\partial O_i}$:

طبق صورت سوال $\frac{\partial L}{\partial Z} = 1$ است.

همچنین داریم $\frac{\partial L}{\partial O_i} = \frac{\partial L}{\partial Z} \times \frac{\partial Z}{\partial O_i}$.

رابطه ای که برای خروجی Z داشتیم نیز به صورت زیر است:

$$Z = \frac{O_{11} + O_{12} + O_{21} + O_{22}}{4}$$

پس $\frac{\partial Z}{\partial O_i} = \frac{1}{4}$ در نتیجه می توان گفت $\frac{\partial L}{\partial O_i} = \frac{\partial L}{\partial Z} \times \frac{\partial Z}{\partial O_i} = 1 \times \frac{1}{4} = \frac{1}{4}$.

پس حالا با داشتن $\frac{\partial L}{\partial O_i}$ طبق روابط صفحه قبل، $\frac{\partial L}{\partial F_i}$ را بدست می آوریم که به صورت زیر است:

$\frac{9}{4}$	$\frac{13}{4}$
$\frac{7}{4}$	$\frac{3}{4}$

محاسبه $\frac{\partial L}{\partial X_i}$: حالا که $\frac{\partial L}{\partial F_i}$ را داریم، از روابط زیر استفاده می کنیم.

$$\frac{\partial L}{\partial X_{11}} = \frac{\partial L}{\partial \theta_{11}} * F_{11}$$

$$\frac{\partial L}{\partial X_{12}} = \frac{\partial L}{\partial \theta_{11}} * F_{12} + \frac{\partial L}{\partial \theta_{12}} * F_{11}$$

$$\frac{\partial L}{\partial X_{13}} = \frac{\partial L}{\partial \theta_{12}} * F_{12}$$

$$\frac{\partial L}{\partial X_{21}} = \frac{\partial L}{\partial \theta_{11}} * F_{21} + \frac{\partial L}{\partial \theta_{21}} * F_{11}$$

$$\frac{\partial L}{\partial X_{22}} = \frac{\partial L}{\partial \theta_{11}} * F_{22} + \frac{\partial L}{\partial \theta_{12}} * F_{21} + \frac{\partial L}{\partial \theta_{21}} * F_{12} + \frac{\partial L}{\partial \theta_{22}} * F_{11}$$

$$\frac{\partial L}{\partial X_{23}} = \frac{\partial L}{\partial \theta_{12}} * F_{22} + \frac{\partial L}{\partial \theta_{22}} * F_{12}$$

$$\frac{\partial L}{\partial X_{31}} = \frac{\partial L}{\partial \theta_{21}} * F_{21}$$

$$\frac{\partial L}{\partial X_{32}} = \frac{\partial L}{\partial \theta_{21}} * F_{22} + \frac{\partial L}{\partial \theta_{22}} * F_{21}$$

$$\frac{\partial L}{\partial X_{33}} = \frac{\partial L}{\partial \theta_{22}} * F_{22}$$

نتیجه $\frac{\partial L}{\partial X_i}$:

0	$\frac{3}{4}$	$\frac{3}{4}$
$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$
$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$

منابع: [لینک](#)