Image Processing Project

COMP 6771

Fuzzy C-Means Clustering with Spatial Information for Image Segmentation

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1. Review

FCM (fuzzy c-means clustering) is an unsupervised method used for image segmentation. It assigns a membership value to data points indicating the possibility of them belonging to each cluster. The original FCM is sensitive to noise, and a noisy pixel is classified incorrectly which is an important issue when it comes to medical applications such as MRI segmentation. To reduce the effect of noise, the authors of [1] proposed an altered version of FCM called sFCM (spatial FCM), by incorporating spatial information in the membership function. The idea behind this approach is the correlation of image pixels, meaning that pixels in the same neighborhood, most likely belong to the same clusters and the membership value of a noisy pixel can be corrected by considering the membership values of the neighboring pixels resulting in smoother and more homogenous segmentations.

In the original FCM, membership values are assigned to feature points based on their Euclidian distance with cluster centers in feature space. However, sFCM computes an additional value by summing membership values of all neighboring pixels in an $n \times n$ window. This new value is called spatial value and is combined with former membership values to compute new ones. Two parameters p and q are used to control the balance between noise reduction (contribution of neighborhood) and preserving image details (contribution of center pixel). Cluster centers are initially picked at random and are then updated iteratively based on all feature points and their newly computed membership values. The algorithm stops when changes in either cluster centers or membership values are less than a threshold. At last, the cluster with the maximum membership value is chosen for each pixel.

Evaluation of the method is done on one real MRI, one synthetic image with four gray levels, and noisy versions of them with different noise levels. However, this may not be sufficient to conclude the effectiveness of the method. For example, given that the effect of using sFCM is generating more homogenous regions and removing noise, the method should have also been tested on MRIs including small tumors to check if they are incorrectly removed. Moreover, the size of the window affects noise removal, and possibly unwanted removal of small regions while the results are only reported for 5×5 windows. Furthermore, sFCM improves V_{pc} (Partition coefficient) and V_{pe} (partition entropy) metrics but the results for V_{xb} (xie-beni metric) are not consistent. Authors claim that this was expected, since V_{xb} measures the compactness in feature space and sFCM reduces this compactness by clustering noisy pixels based on their spatial location (not their features) but they do not validate this claim with synthetic noiseless images. Another shortcoming is that no information is reported on the quality of the real MRI such as SNR or coils characteristics of the MRI machine.

When computing spatial values, sFCM gives the same weight to all pixels in a neighborhood. However, closer pixels should contribute more to spatial value construction. Moreover, with no prior knowledge of noise level, p and q are set with trial and error and are fixed for the entire image. To address these issues, [2] proposed a novel fuzzy factor G_{ki}, that uses distance from the center pixel as well as fuzziness (membership value) of pixels as their weight when computing the weighted summation of neighborhood membership values. The new method does not require choosing any parameter and the balance between noise reduction and preserving details is automatically achieved by incorporating fuzziness. G_{ki} adaptively changes for each pixel and increases the contribution of neighborhood pixels for noisy images since they are fuzzier in general. The method called FLICM (Fuzzy Local Information C-Means) is evaluated on synthetic, and real annotated images with different types and levels of noise using SA (segmentation accuracy) and index-r measures. The results indicate that FLICM outperforms sFCM, is more robust to noise regardless of its type, and preserves more details as a result of changing adaptively from one pixel to another.

2. Implementation

sFCM performs image segmentation by clustering points in n-dimensional feature space. The features used in [1], are intensity values of T1-weighted and T2-weighted images. The image matrices are flattened and used as the first and second dimensions of the feature space. The method is tested using the same images as in the paper (Figure 1).

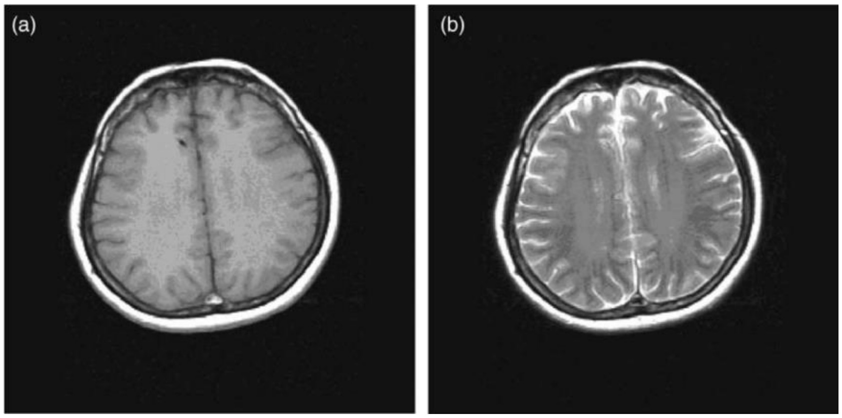

Figure 1 a) T1-weighted image and b) T2-weighted image used in [1]

The controlling parameters of sFCM are m, controlling the level of fuzziness, the window size w, a threshold for convergence, the maximum iterations, and p and q, controlling the relative importance of membership value of center pixel (u) and the total membership value of the neighborhood (h). sFCM then uses both of these values to compute the final membership value (u'). Index ij shows the relation between pixel x_j and cluster i with v_i as its center, N is the number of points, C is the number of clusters and $NB(x_j)$ is the neighborhood of pixel x_j. Membership values, cluster centers, and spatial values are computed as follows:

$$u_{ij} = \frac{1}{\sum_{k=1}^{c}\left(\frac{\left\|x_j - v_i\right\|}{\left\|x_j - v_k\right\|}\right)^{\frac{2}{m-1}}} \quad \text{Eq. 1}$$

$$v_i = \frac{\sum_{j=1}^{N} {u_{ij}}^{m} x_j}{\sum_{j=1}^{N} {u_{ij}}^{m}} \quad \text{Eq. 2}$$

$$h_{ij} = \sum_{k \in NB(x_j)} u_{ik} \quad \text{Eq. 3}$$

$$u'_{ij} = {u_{ij}^{p} h_{ij}^{q}} \Big/ {\sum_{1}^{c} u_{kj}^{p} h_{kj}^{q}} \quad \text{Eq. 4}$$

In the remaining of this section, first, qualitative results are shown, and then quantitative comparisons are presented using three different evaluation metrics.

2.1.sFCM Vs. FCM on original images

In this experiment, T1-weighted and T2-weighted images shown in Figure 1 are not added with synthetic noise but since they are real MRIs, they aren't completely noise-free either. As a result, it is expected that using sFCM gives a better result than the original FCM even when no synthetic noise is yet added to the images. To confirm this, MATLABs built-in FCM function is used and compared with the implemented sFCM method. The goal is to segment the image into 6 clusters, representing gray matter (GM), white matter (WM), cerebrospinal fluid, fat, bone, and air. The results are shown in Figure 2. m is a value greater than 1 and is set to 2, higher values of m indicate more fuzzy overlapping between clusters. The default value for MATLABs built-in FCM is 2 as well. w is set to 5 according to the paper. Higher w means more homogeneity and noise reduction, but it may decrease the accuracy of the segmentation over the edges or some small regions that are not noise. p and q are both set to 1 giving the same importance to central membership value and spatial value. As it can be seen from Figure 2, there is some noise in the output of FCM (right image) while the sFCM method with parameters mentioned above was able to remove these noises and give a more homogenous segmentation in general.

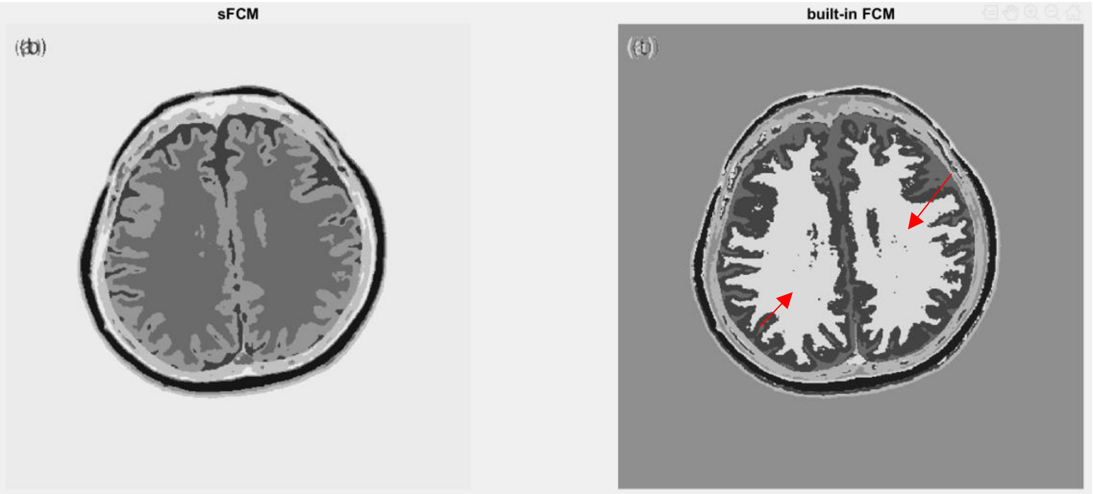

Figure 2 sFCM output on the left with parameters, w = 5, m = 2, p = q = 1 and FCM output on the right

The effect of using different window sizes and different values of p and q are shown in Figure 3-7. As it was expected, increasing window size from 3 to 9 results in better noise removal and more homogeneous segmentations. Increasing the relative importance of center pixel has the same effect as using smaller window sizes, for example Figure 6 with w = 5, p = 2 and q = 1 is highly similar to Figure 3 with w = 3 and p = q = 1. Conversely, increasing the relative importance of the neighborhood has a similar effect to using bigger window sizes as figure 7 with w = 3, p = 1 and q = 2 is slightly less noisy than figure 3 with w = 3 and p = q = 1. Further increasing p does not bring any more improvement.

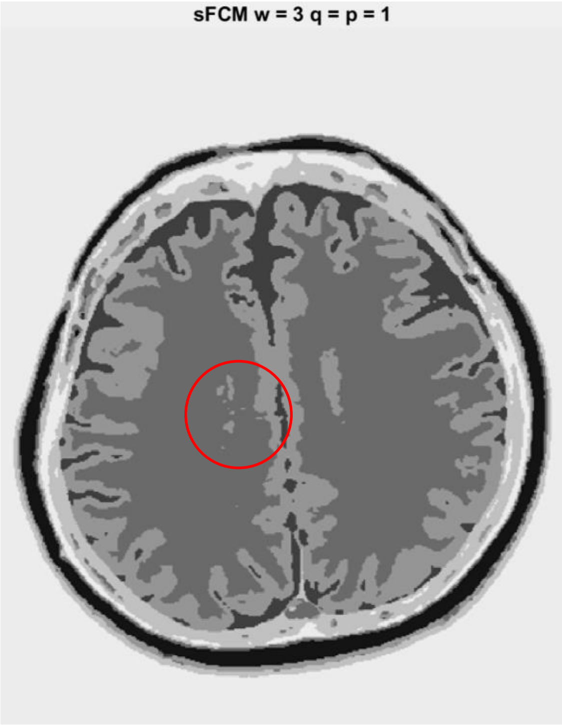

Figure 3

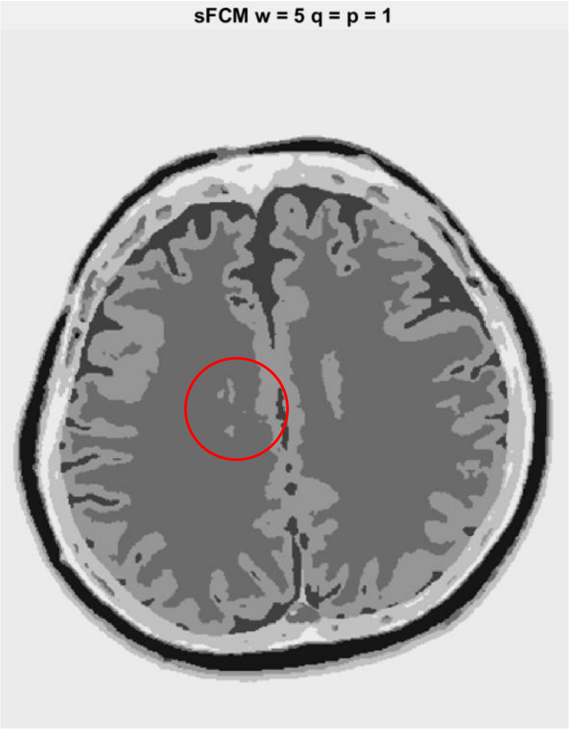

Figure 4

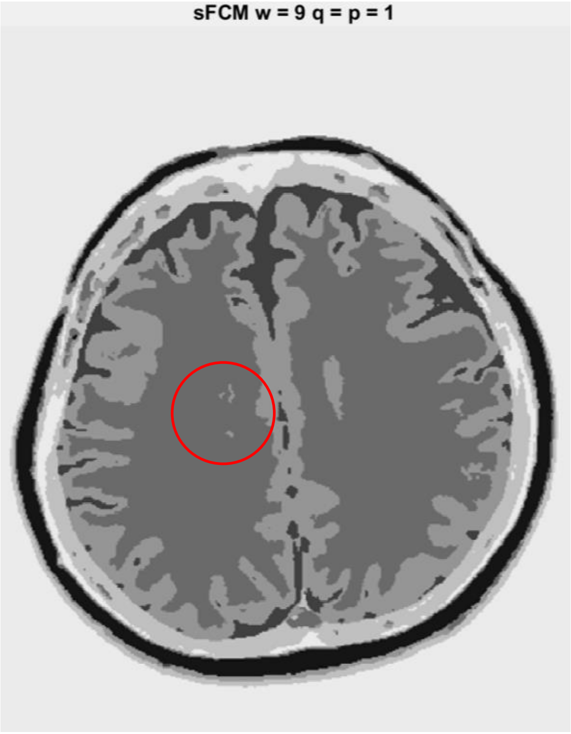

Figure 5

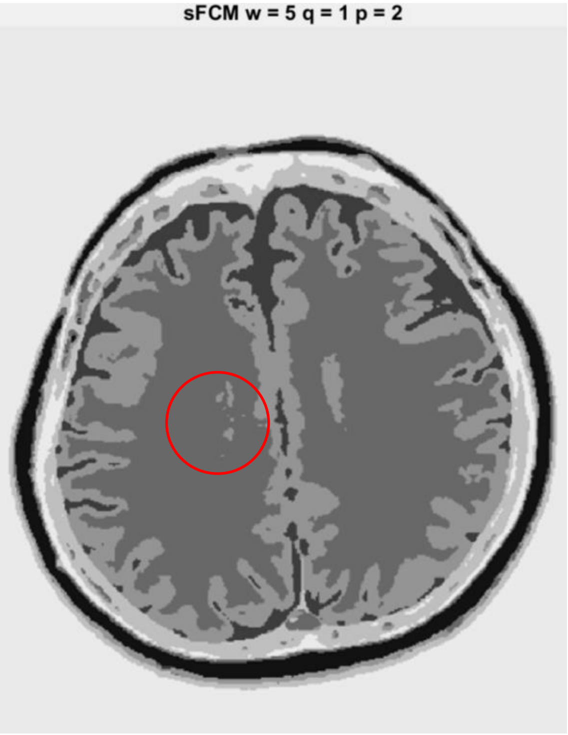

Figure 6

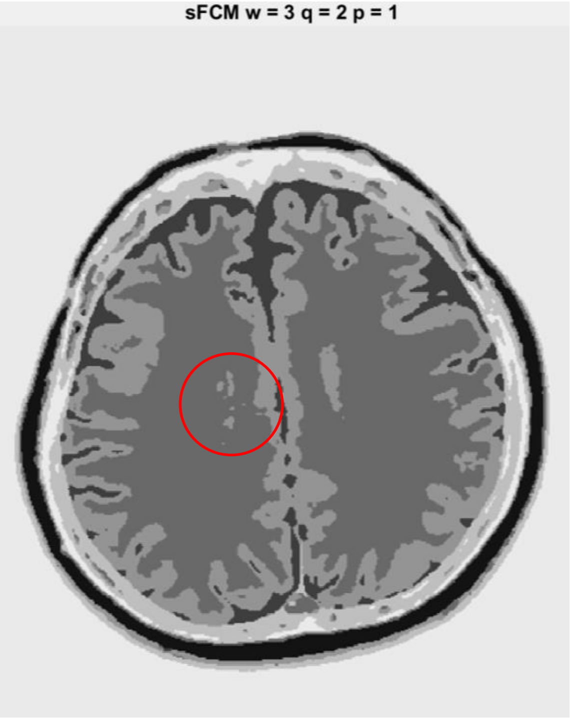

Figure 7

2.2.sFCM Vs. FCM on noisy images

Both T1 and T2 images are added with uniform random noise and the result of segmentation using $sFCM_{11}$ (p = q = 1), FCM_{02} (p = 0, q = 2), and FCM, are compared. Figures 8-11 compare the outputs of all three methods with different levels of noise. w = 5 is used in all cases. Clearly, sFCM is superior to FCM in handling noise. Using sFCM with p = 0 and q = 2, the segments are more homogenous however as the level of noise increases, they may not be accurate enough specially in edges. Although the output of $sFCM_{11}$ is noisy in figure 9, $sFCM_{11}$ and $sFCM_{02}$ were both able to distinguish the six main segments up to noise amplitude between -30 and 30, however, FCM completely loses its ability to distinguish different segments when noise amplitude is [-30 30] and higher. using random noise with amplitude between -50 and 50, the performance of all three methods is very low however, $sFCM_{02}$ still gives smooth segmentation but some of the regions are completely mixed with each other.

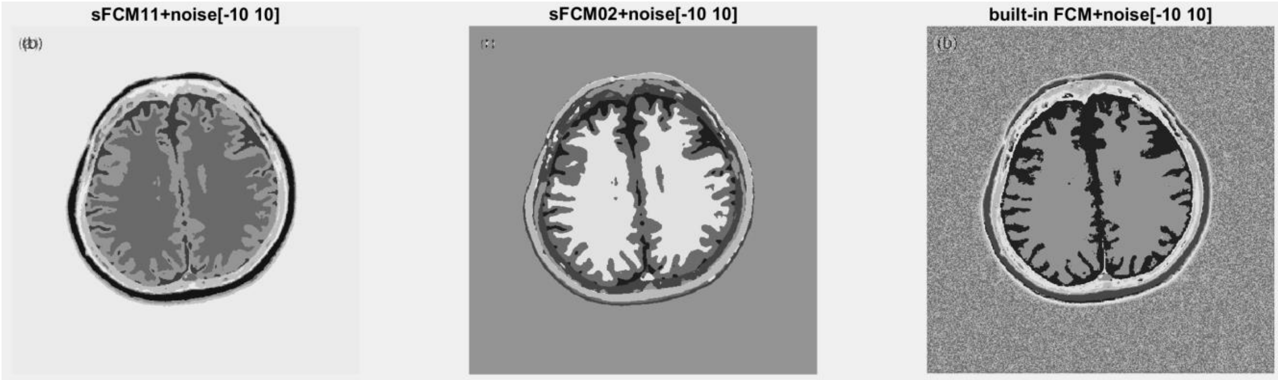

Figure 8

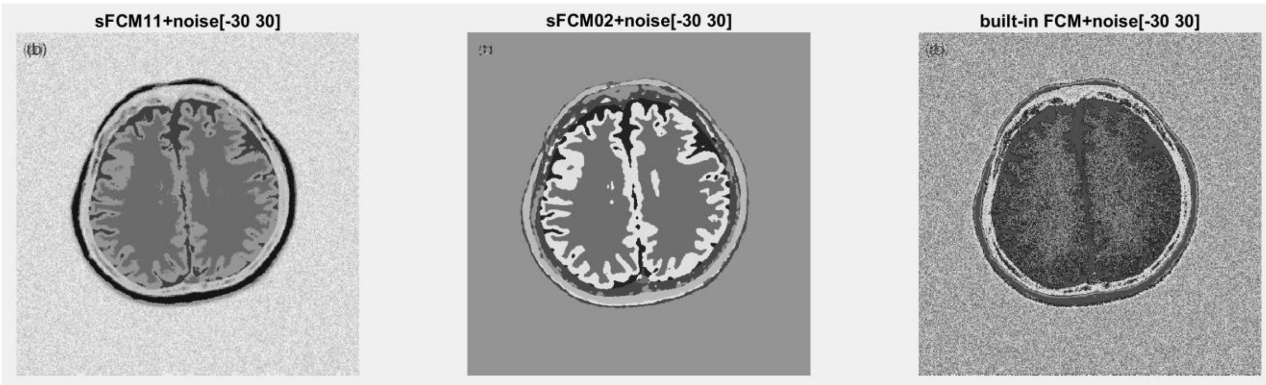

Figure 9

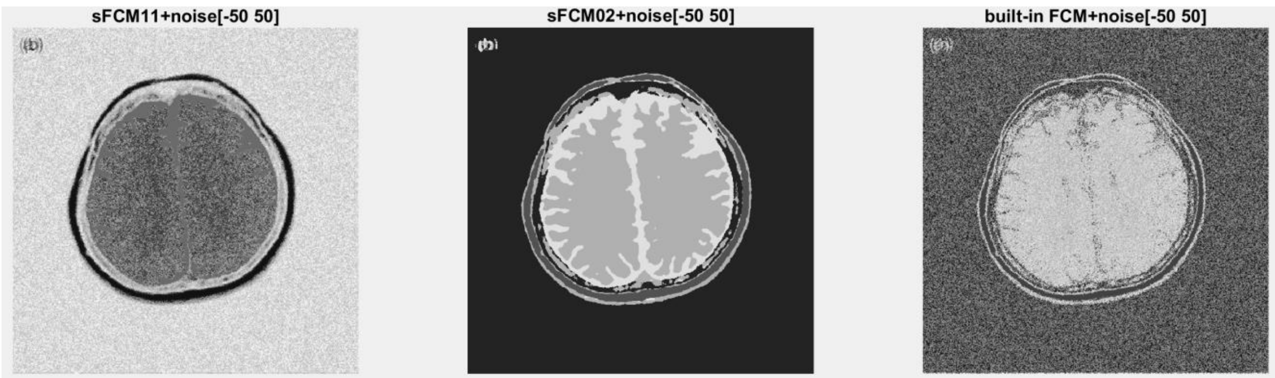

Figure 10

To evaluate the method further, a set of synthetic T1 and T2 images were generated as ground truth and were added with noise. The images are shown in Figure 11. Gray levels are 200, 150, 100, and 50. The performance of sFCM and FCM on these images with different noise magnitudes and SNR are compared in figures 12 to 14. The results on synthetic images follow the same pattern as real MRIs, sFCM can handle noise better than FCM, and $sFCM_{02}$ gives more homogeneous segmentations than the rest. Due to very low SNR in Figures 13 and 14, $sFCM_{11}$ gives very noisy output, however, the results are still better than FCM as it completely loses its ability to segment the image in Figure 14. On the other hand, $sFCM_{02}$ is still able to remove most of the noise when SNR is 10 (Figure 13) and does comparably a better job when SNR is 5 (Figure 14).

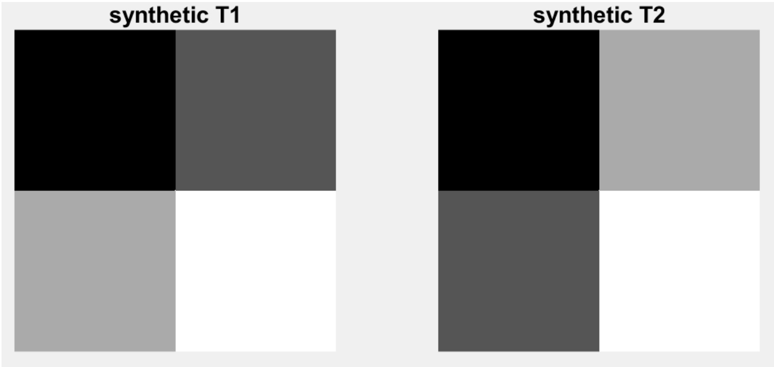

Figure 11

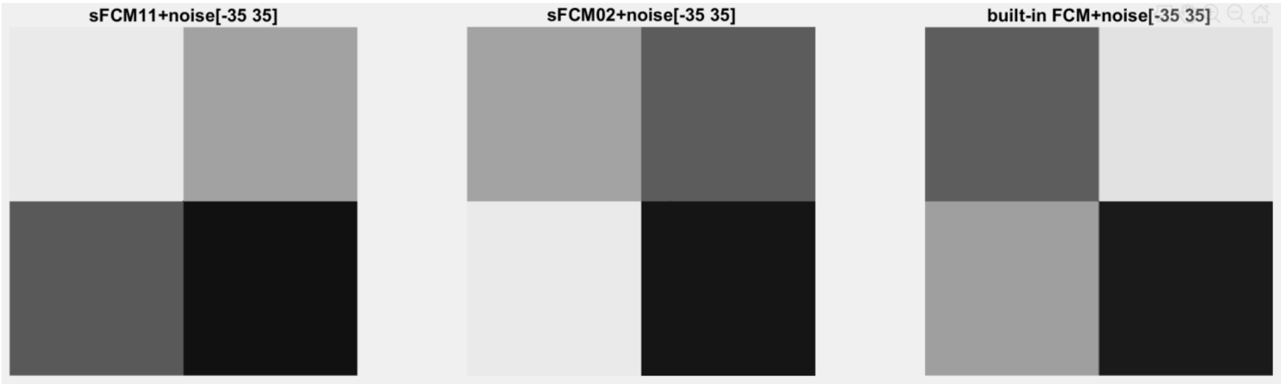

Figure 12

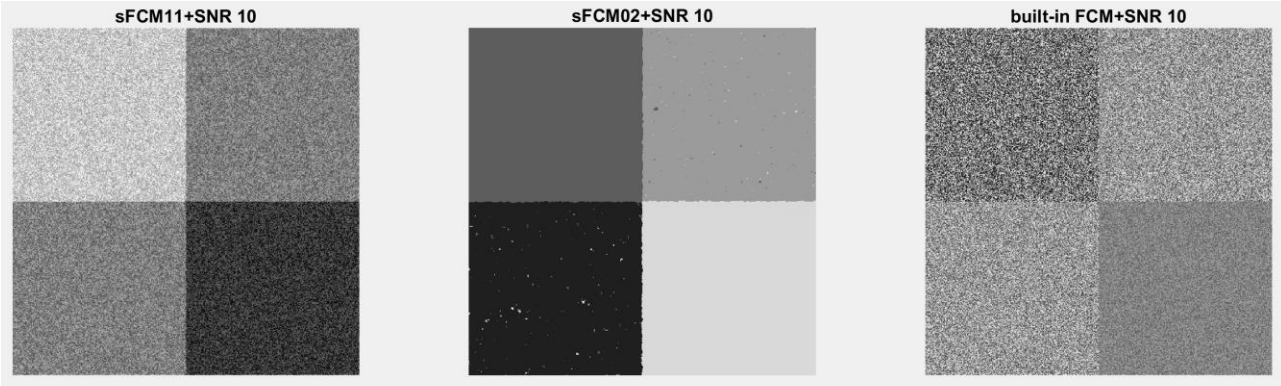

Figure 13

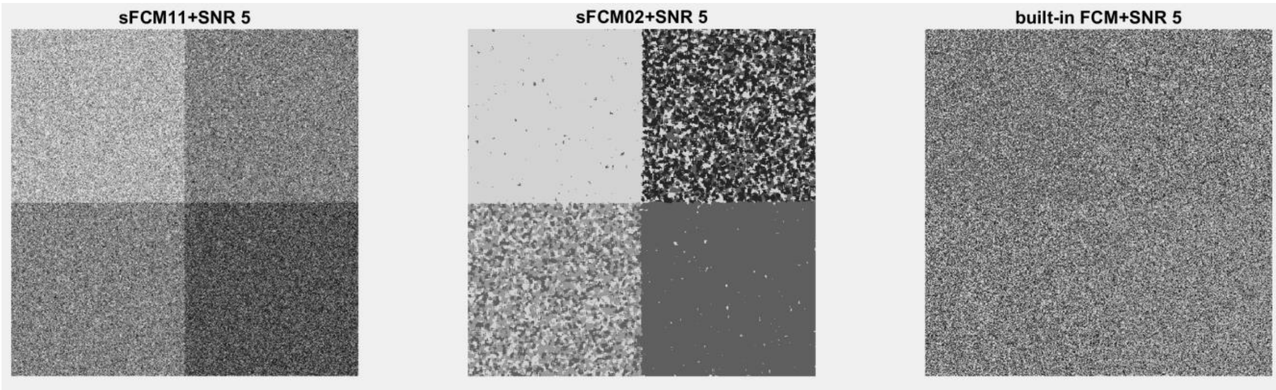

Figure 14

2.3. Validation

Table 1, shows the comparison of FCM, sFCM with p = q =1 ($sFCM_{11}$), and sFCM with p = 0 , q = 2 ($sFCM_{02}$) based on three different metrics , partition coefficient V_{pc}, partition entropy V_{pe}, and Xie-Beni index V_{xb}. V_{pc} and V_{pe} both measure the level of fuzziness assuming less fuzziness means better clustering, but with the disadvantage of not considering feature structures. V_{xb} is based on feature structures, encouraging clusters that are more compact within themselves and more separated from each other. Higher V_{pc}, lower V_{pe} and lower V_{xb} means better clustering. Higher levels of noise increase the fuzziness of the clusters, resulting in lower V_{pc}, and higher V_{pe} in most of the cases. As it is shown in the table, in real MRIs (first four rows), $sFCM_{11}$ is superior to both FCM and $sFCM_{02}$ based on V_{pc} and V_{pe}. Moreover, $sFCM_{11}$ gives lower V_{xb} than the two other methods, except for the noisy image with noise amplitude between -50 and 50. This is an expected result as V_{xb} measures the compactness in feature space and sFCM clusters noisy pixels based on their spatial location in addition to their location in feature space. Therefore, as the amplitude of noise increase, the compactness of clusters in feature space will decrease to a point where at noise amplitude of [-50 50], $sFCM_{11}$ gives higher V_{xb} compared to FCM. However, the difference is only about 0.06. $sFCM_{02}$ shows better V_{pc} and V_{pe} compared to FCM, for noiseless and noisy images with uniform random noise of amplitude between -10 and 10. However, as the amplitude of noise increases further, its performance drops drastically. This is because as the amplitude of noise increases, the entire neighborhood becomes noisier and no longer agrees on any cluster as strong as before. As a result, the summation of membership values gets lower for all the clusters and since the center pixel itself is not involved in the calculation of membership values in $sFCM_{02}$, the final membership values are low even if the center pixel isn't noisy itself. Lower membership values for all clusters mean more fuzziness in general and therefore worse V_{pc} and V_{pe}. $sFCM_{02}$ gives higher V_{xb} than both FCM and $sFCM_{02}$ in all images. As it was explained earlier, this is an expected result as $sFCM_{02}$ depends more on the spatial location than $sFCM_{11}$ when clustering noisy pixels and therefore, V_{xb} increases drastically with noise amplitude. Results on synthetic images almost follows the same pattern. $sFCM_{11}$ is superior to FCM and $sFCM_{02}$ based on V_{pc} and V_{pe}. $sFCM_{02}$ gives higher V_{pc} and lower V_{pe} compared to FCM as long as the noise level is not very high (SNR = 5) and its V_{xb} is higher than the other two methods since It depends more on the spatial location for clustering. The noise magnitude n in table 1 is equal to $3.5 \times (pixel\ value)^{1/2}$.

Image	Method	V_{pc}	V_{pe}	V_{xb}
T1T2	FCM $sFCM_{11}$ $sFCM_{02}$	0.8724 0.9396 0.9243	0.2643 0.1133 0.1402	0.1043 0.0954 0.1287
T1T2 + noise [-10 10]	FCM $sFCM_{11}$ $sFCM_{02}$	0.8411 0.9368 0.9213	0.3438 0.1195 0.1471	0.1473 0.1234 0.1595
T1T2 + noise [-30 30]	FCM $sFCM_{11}$ $sFCM_{02}$	0.7108 0.8205 0.6056	0.5706 0.3197 0.6177	0.3132 0.2041 2.6552
T1T2 + noise [-50 50]	FCM $sFCM_{11}$ $sFCM_{02}$	0.6981 0.7821 0.4732	0.6112 0.4118 0.8451	0.1747 0.2363 3.1690
Synthetic + noise [-n n]	FCM $sFCM_{11}$ $sFCM_{02}$	0.9609 0.9991 0.9950	0.1038 0.0035 0.0098	0.0144 0.0151 0.0169
Synthetic + noise [-35 35]	FCM $sFCM_{11}$ $sFCM_{02}$	0.7338 0.9628 0.9628	0.5309 0.1045 0.1045	0.1307 0.1593 0.1593
Synthetic + SNR 10	FCM $sFCM_{11}$ $sFCM_{02}$	0.6444 0.7649 0.6654	0.6665 0.4287 0.6500	0.1133 0.1090 0.5684
Synthetic + SNR 5	FCM $sFCM_{11}$ $sFCM_{02}$	0.6789 0.7160 0.4146	0.6086 0.5195 1.0846	0.1196 0.1004 1.9083

Table 1 performance comparison of $sFCM_{11}$, $sFCM_{02}$, and FCM on real and synthetic images with different noise levels

Paired sample t-test is a statistical procedure performed to check if on average a significant improvement is brought by sFCM to each of the measures, V_{pc}, V_{pe} and V_{xb}. The procedure tests two hypotheses, the null hypothesis, and the alternative hypothesis. The null hypothesis is $\mu_x = \mu_y$ ($x = sFCM_{11}, y \in \{FCM, sFCM_{02}\}$) which indicates the mean difference between V_{pc}, V_{pe} or V_{xb} of paired samples x and y, is zero and therefore no improvement is brought to these metrics by $sFCM_{11}$ compared to FCM and $sFCM_{02}$. The alternative hypothesis is $\mu_x > \mu_y$ for V_{pc} and $\mu_x < \mu_y$ for V_{pe} and V_{xb}.

	$X{:}Y$	T	$p-value$	$\mu_x =? \mu_y$
V_{pc}	$sFCM_{11}$:sFCM $sFCM_{11}$:$sFCM_{02}$	4.5124 2.5116	0.0014 0.0202	✗ ✗
V_{pe}	$sFCM_{11}$:sFCM $sFCM_{11}$:$sFCM_{02}$	-3.7152 -2.5514	0.0038 0.0190	✗ ✗
V_{xb}	$sFCM_{11}$:sFCM $sFCM_{11}$:$sFCM_{02}$	-0.5312 -2.2149	0.3059 0.0312	✓ ✗

Table 2 paired sample t-test results under 5% significance level

The $p - value$ is the probability of observing the test results under the null hypothesis. If these values are lower than the significance level (set at 5%), the null hypothesis is rejected in favor of the alternative hypothesis. The results presented in Table 2 agree with previous findings; in general, $sFCM_{11}$ improves V_{pc} and V_{pe} while V_{xb} can be improved or worsened depending on the level of noise.

3. Pros and cons of the method

In general, the sFCM method is superior to the original FCM in suppressing noise. However, there is a tradeoff between noise removal capacity and the accuracy of segmentation. As the amplitude of noise increases, q needs to be increased in order to get homogeneous segmentations. However, as it was shown in table 1, this will increase the fuzziness of segmentation and therefore reduces the confidence and accuracy, especially over the edges. In other words, increasing q has a smoothing effect that isn't always desirable in applications such as medical image processing where accurate segmentation is important.

4. Reimplementation challenges

One of the challenges of implementation was the processing time. As the images are 512×512, processing them pixel by pixel takes a very long time. The initial implementation of the method used a for loop to compute the distance with the cluster centers, and membership values for each pixel. By replacing the for loops with matrix multiplications the processing time reduced to less than a minute depending on the number of iterations needed for convergence. Getting Nan (not a number) values was another issue encountered only for the noiseless synthetic images. These images have only 4 grey levels (200,150,100,50), and the center of clusters move towards these values as well. When one of the centers is set to any of the above grey levels, the distance of some pixels with these cluster centers becomes zero and the $1/distance$ term in the membership function goes to infinity causing Nan values when dividing infinity by infinity in Eq.1. To solve this issue an epsilon (0.000001) term was added to all the distances. Another issue encountered was due to the random nature of membership initialization. As each pixel is randomly assigned to a cluster in the beginning, the final outcome isn't always the same. To solve this problem, the algorithm was repeated 5 times for the evaluation step, cluster centers are averaged, and final membership values are computed with respect to the averaged cluster centers.

References

[1] K.-S. Chuang, H.-L. Tzeng, S. Chen, J. Wu, and T.-J. Chen, “Fuzzy c-means clustering with spatial information for image segmentation,” *Computerized Medical Imaging and Graphics*, vol. 30, no. 1, pp. 9–15, Jan. 2006, doi: 10.1016/j.compmedimag.2005.10.001.

[2] S. Krinidis and V. Chatzis, “A Robust Fuzzy Local Information C-Means Clustering Algorithm,” *IEEE Trans. on Image Process.*, vol. 19, no. 5, pp. 1328–1337, May 2010, doi: 10.1109/TIP.2010.2040763.