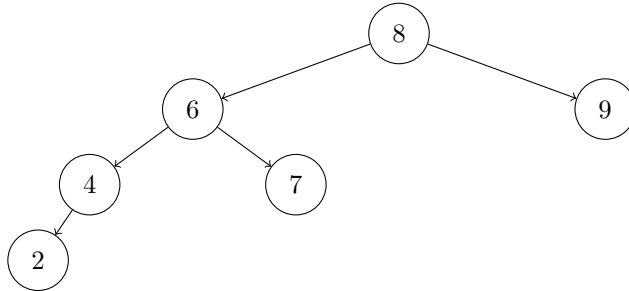
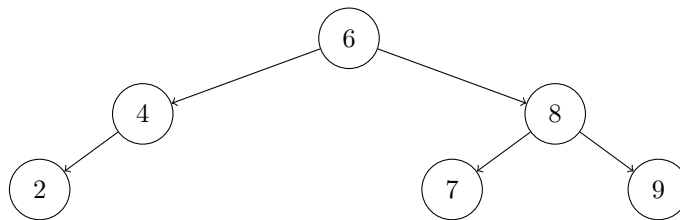


# 1 AVL Trees

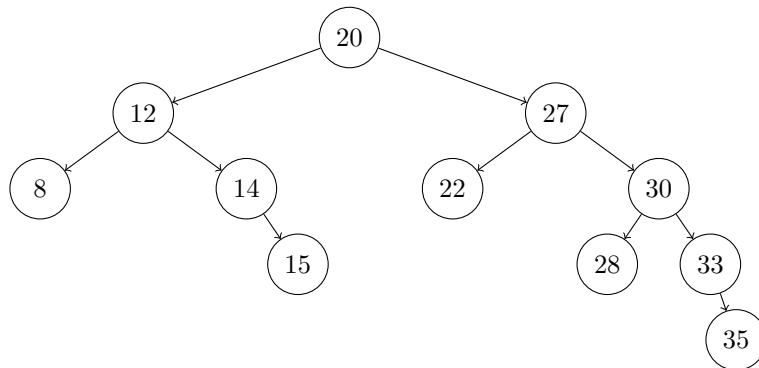
**Problem 1.** Perform a right rotation on the root of the following tree. Be sure to specify the X, Y, and Z subtrees used in the rotation.



After the left rotation, the tree will look like Subtree rooted at 4 is X Subtree rooted at 7 is y Subtree rooted as 9 is Z

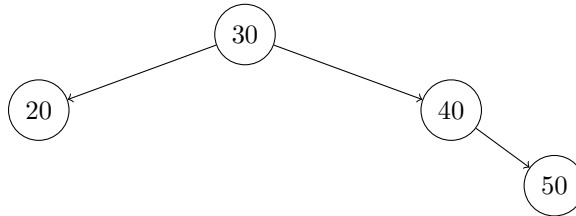


**Problem 2.** Show the right rotation of the subtree rooted at 27. Be sure to specify the X, Y, and Z subtrees used in the rotation.



**Problem 3.** Using the appropriate AVL tree algorithm, insert the value 47

into the following tree. Show the tree before and after rebalancing.



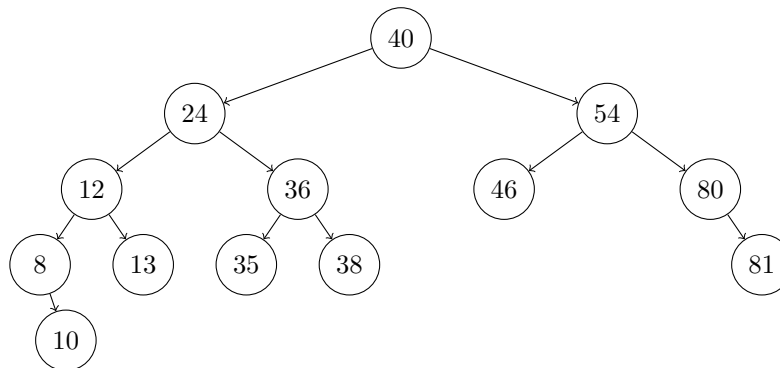
After inserting 47 (but before rebalancing), the tree will look like this:

At this point, the tree is out of balance at node ... so we rotate ...

The resulting tree looks like this:

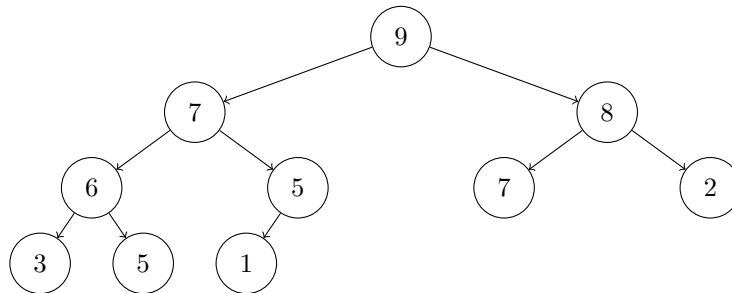
Is the tree now balanced? If not, perform the next rotation here. Repeat until the tree is balanced.

**Problem 4.** Using the appropriate AVL tree algorithm, remove the value 24 from the following tree. Show the tree before and after *each* rebalancing.



## 2 Heaps

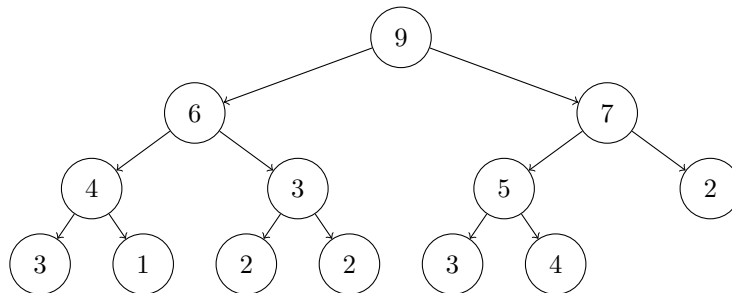
**Problem 1.** Show the addition of the element 8 to the max-heap below. First, show the addition of 8 to the tree; then, show each bubbling step.



The value 9 is initially added as follows:

After the first bubble-up step, the tree is as follows:

**Problem 2.** Show the removal of the top element of this max-heap. First, show the swap of the root node; then, show each bubbling step.



**Problem 3.** Consider the sequence of elements  $[8, 6, 1, 5, 7, 9, 4]$ . Using the representation discussed in class, show the tree to which this sequence corresponds. Then, show the *heapification* of this tree; that is, show how this tree is transformed into a heap. Demonstrate each bubbling step.